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Digital Noise Monitoring of Defect Origin

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Preface

"A considerable amount of time has passed since steam engines, steamers, and railway engines have appeared. God saw that people's life is difficult and boring. Then he decided to help them once more and gave them signals as a present. The devil saw this and created noises, which he mixed up with the signals. People could not use noises and asked scientists to spare them these troubles. People created methods and technologies of filtration and suppression of noises. However, signals were distorted and many valuable and concealed mysteries were lost together with the noise."

This book shows that characteristics of signals and noises at the output of sensors change continuously for both technical and biological objects at the origin of a defect. The known classical conditions are not satisfied. The time for a decision about the problem increases.

For these reasons, in some cases the detection of defects in information systems turns out to be overdue. Sometimes it results in catastrophic consequences.

Taking into account these and other features of the initial stage of the origin of the defect, several technologies are suggested. These technologies allow one to perform the defect monitoring at the beginning of the defect's origin by extracting information from the noise. By duplicating and combining the above technological advantages, a necessary degree of reliability to the results is reached. These technologies are proved theoretically, and the opportunity of their application for solving problems of noise monitoring at the beginning of the defect's origin in oil-gas extraction, in construction, in power engineering, in transport, in seismology, in aviation, in medicine, etc., is shown on numerous examples. In addition, they allow one to improve the results of mathematical modeling, recognition, identification, control, etc., and they can find wide application in solving numerous problems where processing and analysis of signals are required in various fields of science and technology.

The monograph is intended for teachers, post-graduate students, and university students of all professions except the humanities, and also for experts of power engineering, computer science, automation, physics, biology, geophysics, oil-gas extraction, transport, aviation, control, medicine, etc.

At present, the number of failures in power stations, sea objects of oil-gas extraction and communication, main oil-gas pipelines, petrochemical complexes, large-capacity tankers, airlines, etc., remains unjustifiably high because of mistakes in information systems in spite of the repeated increase of reliability of the element base. This often occurs by the fault of traditional information technologies, as they provide defect monitoring after the defect takes its salient character.

Reasons for a defect's origin in various objects such as living organisms and equipment are the subject of research of corresponding science directions. However, considering these problems in view of obtaining information and methods of analysis, monitoring, and diagnostics, one can notice that the problems have much in common with one another [1–14]. In many cases, the signals describing the current state and technical conditions are obtained from the sensors installed on the corresponding objects. The similar or nearly the same information technologies are used in different areas for analyzing these signals as the information carriers [1, 4, 7, 11, 14]. These technologies are realized on the same modern computers. Taking all this into account, the IT specialist does not consider the differences in solving the problem of monitoring the state of these objects despite the wide areas of specific features of each object [1, 4, 11].

However, the process of the origin and evolution of a defect before it takes its salient character has unique features for each object depending on its physical, biological, mechanical, chemical, and other properties [1, 11, 13]. These features also depend on the performed functions, the exploitation modes, etc. Due to these special features, the time period from the origin of the defect of a signal to the time it takes its salient character is unique for each object. It takes a short time for some objects. Others take considerably longer. However, despite all these differences, the information represented as the defect component of a signal has the common property to change continuously for all objects at this period. It becomes stable only after the defect takes its salient character. Thus, the reliability of results for solving the problem of monitoring the early defect's origin depends on used information technologies of analyzing the signal received as the output of the corresponding sensor of objects. As usual, these signals are accompanied by noise. That makes it difficult and sometimes impossible to solve the problem of monitoring the defect at its early origin.

In general, in solving all sorts of questions by signal processing, it is possible to get more or less acceptable results by means of known information technologies only in the case of satisfying classical conditions, i.e., analyzed signals are stationary, they follow the normal distribution law, the correlation between the noise and the legitimate signal is equal to zero, the noise is represented as "white noise," etc. Even in this case, the obtained results do not always provide enough reliability because noise from real signals differs from "white noise" and its variances and spectral components change in time.

At the same time, in many cases the above-mentioned conditions are not satisfied at all, and it is not always possible to get reliable results and form adequate solutions for the situations that arise in corresponding information systems [15–42]. For this reason, the number of failures of various objects in oil-gas extraction, petroleum chemistry, power engineering, aviation, etc., with catastrophic human, economic, and ecological consequences does not decrease, in spite of the fact that the reliability of both the element base and the equipment in information systems has increased lately. In this connection, filtration methods are often used in traditional technologies for eliminating the noise influence on the results of solved problems.

They provide good results when the filter spectrum and noise spectrum coincide. Simultaneously, for many real processes and particularly in the period of a defect's origin, the noise spectrum and variance widely vary in time. For these reasons, the "filter" spectrum range has to be widened to eliminate the noise influence on the result of signal processing. And this distorts the useful signal much more. In addition, quite often the noise arises as a result of operation of controlled objects. Thus, the noise becomes the data carrier, and this information is erased because of filtration. And in these cases the important and in some cases the only valuable information of the early defect origin is lost [42–54].

Thus, in traditional technologies, the specificity of noise influence of real signals on the desired result is not sufficiently taken into account. These technologies do not have the opportunity to extract information contained in the noise of analyzed signals.

So problems of creating technologies of noise analysis and increasing the reliability of solved problems by signal processing of results are of great importance on the contemporary stage of "signal processing" development [1, 58, 60, 61].

No doubt filtration and traditional information technology allow one to solve numerous necessary problems. However, at the same time, it is necessary and advisable to have alternative technologies possessing the property of extracting the information contained in the noise of noisy signals.

The significance of this work is also connected with the possibility of using the noise as a data carrier for creating technologies of detecting the initial stage of changes to objects. It is now that their time has come. The economy of computer resources was considered as an essential dignity of information technologies in the past for many years. But now, because of their enormous resources, one can create more effective technologies at the expense of the complication of computational process [1].

Unlike traditional technologies, where at the expense of signal noise filtration, the volume of extracted information decreases, in the suggested alternative technology due to analyzing noise as a data carrier, this disadvantage is eliminated. This significant difference opens a wide opportunity for expanding the range of solved problems on the basis of analysis of noisy signals.

I would like to express my thanks to Prof. Nozer Singpurwalla, Prof. Otto Moeschlin, Prof. Jionghua (Judy) Jin, and Prof. Yong Chen for their well-disposed and friendly attitude toward me and my works.

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1 Difficulties of Monitoring a Defect at Its Origin and Its Dataware Features

1.1 Features of Defect Origin and Difficulties of Its Monitoring

Reasons for the rise of defects on various objects such as living organisms and equipment are the subject of much research of corresponding science directions. However, considering these problems in view of obtaining information and methods of analysis, monitoring, and diagnostics, one can notice that the problems have much in common. In many cases, the signals describing the current state are obtained from the sensors installed on the corresponding objects. Similar or nearly the same information technologies are used in different areas for analyzing these signals as the information carriers. These technologies are realized on the same modern computers. Taking this into account, the IT specialist does not consider the differences in solving the problem of monitoring the state of these objects despite the wide areas of specific features of each object.

However, the rise and evolution of a defect before it takes its salient character have unique features for each object depending on its physical, biological, mechanical, chemical, and other properties. These features also depend on the performed functions, the exploitation modes, etc. Due to these special features, the time period from the rise of the defect of a signal to the time it takes to show the salient character is unique for each object. It takes a short time for some objects, while others take considerably longer. Despite these differences, the information represented as the defect component of a signal will change continuously for all objects at this period. It becomes stable only after the defect takes its salient character. Because of this, solving the problem of monitoring the rise of the defect is different from the problem of diagnosing the technical state of an object [14].

Let us consider several examples:

1. When the pin holes in an oil pipe appear, the weak whistle appears. Later the whistle grows, the oil pipe buzzes, and then it snores and gurgles. Thus, the characteristics of a signal obtained as the output of the acoustic sensor continuously change.
2. When the pin holes in the metal farms and bearings of a sea platform or pier appear, the weak peep appears. Later the peep grows and turns to a child's cry and then turns into a metal creak and so on.
3. In the initial stage of weakening the attachment of a spring of a bracket of a plane engine, the high-frequency component appears in the vibratory signals of the corresponding sensors. As the defect evolves, the frequency gradually decreases; this process goes on until the full segregation of the bracket's attachment.
4. When the deep and ultradeep boreholes under pressure are being drilled, the strap on the drilling equipment often rises. At the beginning of this process, the weak, high-frequency vibrations appear on the outputs of sensors of the mechanical speed of drilling. In some cases, when it takes its salient form, preventing the failure can be difficult. Unfortunately, sometimes the ship master detects a smooth speed disturbance of the oil-rig column when it rolls with significant vibration. Starting from this moment, he launches a technological operation to liquify the strap. This does not always end successfully.
5. It is known from literature [55] that the crack in some parts of a technological object makes intensive acoustic radiation with a frequency of 50 to 500 KHz when it appears and changes continuously during its evolution. As examples of these cracks appearing "in vulnerable places" (for example, near the rivets), it is possible to consider the cracks in the bodies of vehicles including airliners or pin holes in equipment (capacities), working under pressure (300–500 atm and more).

It is obvious from these examples that it is necessary to take into account that the spectrums and other characteristics of the signals obtained from sensors change continuously while technology to detect the influence of the rise of defects of these signals develops.

1.2 Reasons, Types, and Stages of Defect Origin Evolution in Technical Objects

The process of the defect origin during the exploitation of the equipment can be classified depending on the character of the destructive effect (chemical, thermal, mechanical) and the destructive type. In general, the elements of equipment are affected by fatigue, corrosion, pollution, overheating, overloading, burrages, wear, etc. Let us consider the most popular variants of the defect origin in detail [55–57].

The defect origin of equipment as the crack caused by metal fatigue is connected with the effects of cyclic loads. The limit fatigue is the material's property; evolution of the crack is determined by many factors and conditions of exploitation [56, 57]. As usual, the process of a crack's evolution begins when the pin holes appear, "roughening" the surface, bursting by the bounds of grains and around hard inclusions, and further penetrating deep into the material. In some cases, the pin hole turns into a macro crack and quickly spreads in metal [56, 57].

Early detection of the origin and evolution of a crack is of great importance for many modern important objects (turbines, sea platforms, planes, tankers, etc). For many objects, even after monitoring it is advisable to establish regular control under the detected cracks because it is possible to forecast how long an element can be used until a crack necessitates its replacement. The relative speed of a fatigue crack's development reasonably depends on the quality of material [12–14].

The reasons for the defect's origin vary, as the crack can be different [12–14]. During static load, the defect appears as a crack when a single load causing an effort greater than the limit fatigue of a material takes place. The defects caused by stretching lead to the local; the surface of the crack is formed by the division planes, rotated by the angle of 45° to the direction of the load [56, 57]. The defects caused by pressure take place in two basic forms, namely the timber pressure and the bend (crippling). The timber pressure takes place at the short heavy parts, which are divided by the ramps the same way as during the stretching. The difference is that there is friction between the two halves of the crack during the division [56, 57]. The defect represented as the bend takes place at long parts and causes the typical bending change of the form. The tensile and the compression potentials of the material resist the bend moment. Thus, the material destruction is similar to the formation of cracks during stretching by the external side of the bend and by pressing by the internal side of the bend [56, 57].

The bend is typical for metal plates and takes place in a way that the directions of the crests or cavities of the bend wave coincide with the diagonal of the moved plane. The two halves of the crack slide along each other; the surface is affected by friction during the defect caused by displacement (cut-off). Thus, either the crack smooths over or the burrages of the surface appear. The direction of the burrage shows the direction of applying the cut-off power.

The two halves of the broken metal sample keep some remaining bend during the defect caused by the torsion. The surface of the crack often looks the same as during stretching and is rotated by the angle of the torsion.

The defect represented as the impact tears to pieces the lower part of the product during the deformation of the middle part by the formation of the emptiness.

Wear is also one of the most common reasons of a defect's origin. The process of the origin of defects caused by wear consists of the wear faces, moving relative to each other, is the one of the basic reasons for decreasing the service life of the equipment. By analyzing the character of the change of the wear speed, it is possible to determine three distinctly distinguishable phases of the element wear during its exploitation [56, 57]. The burn-in of the elements takes place during the first phase, i.e., the micro and macro structures of the surfaces change. Wear at the second phase is named the normal period. In many cases, the linear connection between the value of the wear and the time can be assumed. The unit pressure and the relative speed of the movement of the rubbing details are the basic factors affecting the value of the wear at this phase. Abrasive wear is directly proportional to the unit pressure on the wear faces and the ways of sliding during abrasive wear. The third phase is the emergency wear, which is the result of the quantitative changes in the surface structure of the mating parts, and the process evolves with high speed.

Fatigue (pitting) wear usually takes place at the frictionless bearing and is caused by blanket fatigue. When there is the relative interfacial slip, fatigue wear can be the result of micro-roughness. Molecular wear is characterized by the evolution of the local metallic combinations and tearing away of the formed small parts from the wear faces. This type of wear takes place at high pressures and, as a rule, evolves with high speeds. Corrosive wear takes place in the presence of a hostile (oxidizing) environment. The cyclic load destroys the oxide (protective) layer and shows the latest sublayer of the metal, which is oxidized in the presence of the oxygen; the new formed oxide layer is destroyed later; and the process is repeated [56, 57].

Cavitation wear (cavitation damage) is the result of local hydraulic hits of the liquid at the cavitation zone. If the element works in a hot gas flow, the surface grows soft and is oxidized. At the same time, the broken off small parts of the metal are taken away with the gas flow (the gas erosion).

Scuffing is the reason for one of the most dangerous defects. During the contact between the surfaces of the mating parts, the pressure can tear the lubricant layer, exposing the wear faces and creating the conditions for the weld of the local parts. The scuffing is the surface damage leading to the local welds of the wear faces. This process is caused by the sudden increase of the coefficient of the friction between the wear faces.

Scuffing is typical especially for the toothings and the pistons in internal-combustion engines. It is observed as a result of the irregularity of the lubrication rate, the sudden heat, and the destruction of the blanket accompanied by the intermolecular welding of the metal of two surfaces [56, 57].

The origin of the defect represented as corrosion damage is the result of the electrochemical and chemical processes taking place on the surface of the metal, located in the corrosion-active environment [56, 57]. The nature of the corrosion of the equipment is determined by the conditions of the exploitation. Usually, stress corrosion, corrosion fatigue of metal, and cavitation damage are distinguished.

Stress corrosion results from the metal bursting by the permanent stress in the corrosion-active environment. In this case, the cracks appear by the normal to the tensile potentials, have rough bounds, and can be intercrystalline or transcrystalline depending on the material.

Corrosion fatigue is typical for details, which are in a corrosion-active environment and under the tensile cyclical loads. The cracks formed in this case go deep into the material transversely to the tensile potentials, usually have an intercrystalline character, and are filled by corrosion products.

The cavitation process of erosion usually takes place in the hydraulic gears, such as the ship screws and rudders, the liners of the engines, the friction bearings, the fuel-injection systems, the pumps, and the hydraulic systems. Destruction of the material by cavitation effects is the result of the simultaneous mechanical effect of the closing liquid vials and the electrochemical corrosion.

In some cases, corrosion in the sulfate environment is the reason of the defect's origin. Most kinds of fuel contain an admixture of the sulfur, forming sulfuric acid and the sulfate mist as the result of fuel combustion and mixing with the condensed steam of water. For example, corrosion in the sulfate environment of the staging of the marine gas turbines was described in the work. The nature of corrosion of the equipment is determined by the conditions of the exploitation. Usually, the stress corrosion, the corrosion fatigue of metal and the cavitation damage are distinguished.

Carbon formation is also one of the widespread reasons of the origin of various defects. The carburizing of the sprayers of the injectors, the loss of the mobility of the piston rings, and other damage appear in diesel engines due to carbon deposits. In the gas turbines, the carbon distorts the front and the structure of the flame and is the reason of the hogging of the combustion chambers, the wear of the turbine blades as a result of the erosion, and the corrosion, etc.

Breaking the technology during the assembly is also a reason of the defect's origin sometimes [56, 57].

Let us note that the defect's origin is not always the evidence of the critical state of objects. However, in any case, it is necessary to determine the degree of the danger of the defects and the dynamics of their change in time and to deeply understand possible results of their evolution during critical conditions. For those purposes, it is expedient to divide this process into three well-defined stages: (1) monitoring the defect's origin; (2) identifying the defect and its stage; (3) determining and controlling the dynamics of the increase of the defect.

1.3 Sensors and Features of Dataware for Monitoring a Defect at Its Origin

As usual, equipment stops working as the result of the origin of various defects [14, 55–57]. In some cases, it can lead to catastrophic consequences. To prevent it, the monitoring of defects preceding such accidents is necessary. Solving the problem of monitoring the origin of the defect, leading to the crippling and the breakdown of the capacity of work of the construction, first of all requires the creation of the corresponding dataware on the base of the analysis of the signals obtained from the output of the corresponding sensors. At the same time, the possibility of controlling the object is especially important when we consider obtaining necessary information for monitoring the origin of various defects. From this point of view, by means of the obtained statistical data related to the most dangerous defects, it is necessary to determine the types of the sensors (vulnerable places) and the places of their installation to detect the origin of the defect at early stages [12–14]. Monitoring the defect's origin means detecting the defect at early stages when negative consequences do not become apparent for reliability or the capacity for the equipment to work yet.

The reliability of monitoring depends on the quality and quantity of measured data samples, which can be obtained while the object is being exploited. At the same time, the following is necessary: (1) to perform the consecutive and systematic measuring of certain characteristics of signals obtained as the output of the corresponding sensors and (2) to find the changes of these characteristics and compare them with the initial ones.

The less information that is obtained as the result of measuring and processing of the signals and the less reliable technologies of their analysis that are used mean the less precise information it is possible to obtain. The non-precise information can affect the reliability of the results of monitoring and can lead to errors causing inevitable failures and catastrophic consequences.

In practice, an important function of a monitoring system is the increase of the reliability and the resource of the equipment by means of detecting the origin of the defect at the early stages and the optimization of the processes of the maintenance works. The monitoring system of the complicated objects must have an information base (dataware), hardware, and software.

The *information base* includes the methods of obtaining the measuring information, its storage, and the systematization of the database.

The *hardware* is the totality of the devices for obtaining and processing the information (sensors, processing properties, signaling devices, etc.).

The *software* includes the technologies of the analysis of the measuring information, solving the problem of monitoring, recognizing, decision making, etc.

The sensors allowing one to detect the origin of the most common defects are of great importance for monitoring the origin of the defect. At the same time, the parameters of various processes such as temperature, pressure, vibration, acoustic and heat radiation, etc. contain sufficient information for monitoring the origin of the corresponding defects. For example, for many technical objects such information parameters are the spectrum of the vibration of the elements of the construction, the spectrum of the acoustic vibrations, the estimates of the statistical characteristics of the signals, and other parameters characterizing the work of the system. The values of parameters in a given moment as well as their changes are of great importance.

The vibration signals are the most informative for technical objects. Connected with vibration signals, the elements of an object are moved (are affected by the vibration movements) during exploitation. Vibration movements can be caused by a cyclic process in the work of an object (the rotation of rotors, periodical loads, etc.) or by the oscillations of the elements of the construction, etc. Commonly, each point of the construction is affected by a movement represented as the geometrical sum of three components of the displacements. At each moment of time, the vibration movements can be represented as the sum of the elementary harmonic oscillations with various frequencies and amplitudes. As usual in the problems of technical diagnostics, frequencies up to 30,000 Hz (up to 10,000 Hz) and vibration accelerations up to 1,000 m/s are used.

Induction and piezometric sensors are used as vibration sensors. Modern vibration sensors have high strength and thermal resistance (through 500°C) and are installed by means of a flange or screw in a threaded opening.

Active systems can be divided into three groups [14]:

- the equipment of the periodical effect;
- the hydraulic equipment;
- the equipment of the impulse characteristics.

The internal-combustion engine, the pumps, the hydraulic valves, the elements of the ball bearing, etc. are the typical systems for which vibrations are normal.

The equipment of the periodical effect generates vibration repeating by certain time intervals. The temporal realization of this vibration can be successfully used for monitoring the defect's origin.

Vibration signals of engines contain a great number of various noises. They complicate the detection of the defect's origin by means of the traditional technologies of the analysis of the signals. Also, in some cases they are the carriers of certain information. So it is necessary to create the technologies that allow one to mark out the information component of not only the signal but also of the noise.

The choice of the place and direction of axis of the setting of the vibration sensors is very important. It is the main operation in the vibration control of the equipment. For example, the bearings are the best place for measuring the vibration of the equipment, because the basic dynamic loads and efforts take place exactly in those places. Besides, the bearings are the critical element for the engine. The vibration sensor must be installed on the body of as many machine bearings as possible. If this is impossible, measuring must be performed at the shortest distance to the bearing, with the least possible impedance between the bearing and this point.

In many cases, vibration signals containing information about the condition of the object are obtained as a random function, i.e., the values of the function cannot be predicted for the various moments of time. This is related to the fact that the vibrations appear as a result of the superposition of the great number of the various dynamic effects rising in the elements of the machine (the characteristic and the forced oscillations, impacts, effects of the work and the external environment, etc.). The noisy vibration signal having a chaotic random nature contains enough information about the technical condition of the object.

The acoustic signals are also carriers of information about the defect's origin. They have a huge information potential for detecting the process of the defect's origin both for technical and for biological objects.

In Refs. [55–57], it is often assumed that the acoustic signals are the noise of the mechanical vibration, transmitted by air, involving the combination of the various frequencies with various levels of the pressure and are the scalar values without direction.

At the technical objects, the acoustic signal is received from the output of the vibration sensors. The acoustic signal is the result of the movement and the vibrations of the parts of the machine and of the effect of the working process on the surrounding air.

The acoustic vibrations are often the stochastic process, the amplitudes and the frequencies of which have a random character. In the structure of the spectrum, its amplitude-frequency characteristic has a large informational potential about the technical condition of the machine. It is known that often the experienced mechanics can orally determine the character of imperfection of the engine, the turbine, etc.

It is natural that measuring the acoustic vibrations, their spectrum, and the correlation analysis allows one to determine the corresponding informative factors for monitoring. Microphones are used for measuring. These microphones are based on the electric or piezoelectric effects of the frequency range from 5–100 kHz (the frequency of audible sound is 20 kHz). The detachment of the useful signal from the noises is the basic difficulty in vibration-acoustic methods. In practice, the filters are often used for detecting the factors containing the diagnostic information [14, 55–57].

In some machines, for example, in aviation engines, the stream of gases exiting the jet, the acoustic radiation of the compressor blades, etc. are the strong acoustic vibrations' (noise) source.

In Ref. [55], it is shown that intensive acoustic radiation of the frequency from 50–500 kHz is formed during the appearance of the crack. The extraction of this high-frequency spectrum from the noisy acoustic signal can be used for detecting the cracks at early stages [55].

It is natural that in addition to the above–mentioned signals, the signals obtained from the output of the sensors of pressure, temperature, effort, displacement, acceleration, etc. are also analyzed in monitoring the defect. Two basic methods are used.

The first method is the study of the general structure of the noisy signal. The changes of the statistical and other characteristics of the analyzed signals are considered to be connected with the faults or other deviations of the normal condition of the machine.

The second method is the study of the separate components of the analyzed signal. In many cases, both the useful signals and the noises obtained from the output of the corresponding sensors contain the information about the change of the object's conditions. The defects can be detected by means of their system analysis and comparisons at different durations. The corresponding technologies are created for that purpose, and they are considered in Ref. [14] in detail.

So to monitoring the defect's origin, it is first necessary to choose the type and the installation position of the corresponding sensors providing the possibility of controlling the object. Besides, it is necessary to create and use the informative technologies allowing one to receive the corresponding current informative indications for the analysis of the signals obtained from the sensors. At the same time, the corresponding current and sample sets formed on their base will be the base of the dataware of the problem of monitoring the defect's origin.

1.4 Models of Signals Obtained as Output of Sensors at the Initial Stage of a Defect's Origin

As stated in Section 1.3, the dataware for solving the problem of monitoring the origin of the defects assumes the presence of the corresponding measured information. For this purpose, the sensors D are used. They transform the non-electrical values of temperatures, pressures, mechanical displacements, etc. into electrical signals. In general, the models of these signals can be represented as follows when the object is in the normal state:

$$X(t) = x(t) + b(t) + c(t) + e(t). \tag{1.1}$$

Here the value $x(t)$ is the useful signal corresponding to the measured technological parameter. The function $b(t)$ is determined by the slow change of work conditions or characteristics of the technological equipment, raw material properties, and diurnal changes of the loads. The function $c(t)$ is formed as the result of various external factors (the pressure, the temperature, the moisture, etc.). The function $e(t)$ is the measured noise appearing in the sensors, in the communication channels, in the measuring devices, and in the transformers. Thus, when the object is in the normal state, the technological parameter $X(t)$ consists of the useful signal $x(t)$ and the sum of noises $b(t)$, $c(t)$, $e(t)$, which do not have the practical informational value.

The value $b(t)$ changes slowly and so does not affect the result of the analysis of the signals. The influences of $c(t)$ on the measurement result were minimized in the modern sensors by the screening, ensuring the containment, and by the other engineering decisions. The influence on the measurement result $e(t)$ was also minimized due to the use of the modern elemental base of the microelectronics.

In spite of all this, the bandpass filters are used on the output of the sensors to remove the influence of $c(t)$ and $e(t)$ on the result of the analysis of the signals in the many informational systems. Sometimes the digital technology of fast Fourier transformation is also used for this purpose. But when using the filtration, the spectrum of the total noise

$$\varepsilon'(t) = b(t) + c(t) + e(t) \tag{1.2}$$

must coincide with the band of the filter. If not, the filtration can lead to the distortion of the spectral distribution of the useful signal. That is why in practice the band of the filter is carefully selected on the base of a priori information about the possible spectral distribution $\varepsilon'(t)$ for each signal and then is corrected by the experimental way. In this case, the operation of the filtration justifies the hopes.

The other possible way to remove the influence of noise $\varepsilon'(t)$ on the result of the analysis of the signals is the robust technology of correlation and spectral analysis [14]. According to this technology, the value of the robustness is determined in the background of the analysis of the noisy signals. Its value is used for correcting the results of processing from the influence of the noise [14].

During the exploitation of the object, when the defect is caused by fatigue, deterioration, corrosion, etc., the component $\varepsilon(t)$ is formed on the output of the corresponding sensor. The spectrum of this component continuously changes from the moment of origin of the defect until the defect takes its salient form.

These noises $\varepsilon(t)$ are divided into mechanical, acoustical, electromagnetic types, etc., depending on the source of the origin. They are also divided into the continuous and impulse types, depending on the type of signals and noises. Finally, they are divided into external and internal noises, depending on the location where the defect appears.

Besides the spectrum, the power of $\varepsilon(t)$ also continuously changes from the moment of the defect's origin until it takes its salient character. So the process of rising the defect affects the signal $g(i\Delta t)$ on the output of the sensor as $\varepsilon(i\Delta t)$. One of the possible variants of modeling the signal $g(t)$ on the output of the sensor can be represented as follows:

$$g(i\Delta t) \approx \begin{cases} X(T_0 + i\Delta t) + \varepsilon(T_0 + i\Delta t), & D_\varepsilon < 0{,}05 D_g; r_{x\varepsilon} = 0; \omega_{ng} > \omega_{ng}, \\ X(T_0 + T_1 + i\Delta t) + \varepsilon(T_0 + T_1 + i\Delta t), & D_\varepsilon < 0{,}1 D_g; r_{x\varepsilon} \neq 0; \omega_{ng} > \omega_{ng}, \\ X(T_0 + T_1 + T_2 + i\Delta t) + \varepsilon(T_0 + T_1 + T_2 + i\Delta t), & D_\varepsilon \approx 0{,}1 D_g; r_{x\varepsilon} \neq 0; \omega_{ng} \approx \omega_{ng}, \\ X(T_0 + T_1 + T_2 + T_3 + i\Delta t) & \\ + \varepsilon(T_0 + T_1 + T_2 + T_3 + i\Delta t), & D_\varepsilon < 0{,}2 D_g; r_{x\varepsilon} \neq 0{,}5; \omega_{ng} < \omega_{ng}, \\ X(T_0 + T_1 + T_2 + T_3 + T_4 + i\Delta t) & \\ + \varepsilon(T_0 + T_1 + T_2 + T_3 + T_4 + i\Delta t), & D_\varepsilon > 0{,}2 D_g; r_{x\varepsilon} \approx 0{,}5; \omega_{ng} \approx \omega_{ng}, \end{cases} \tag{1.3}$$

where $X(t)=x(t)+\varepsilon'(t)$; $\varepsilon'(t)=a(t)+b(t)+c(t)$; T_0 is the period of time before the origin of the defect; T_1, T_2, T_3 are the periods of time of the first, second, and third stages of the defect evolution, respectively; T_4 is the period of time when the defect takes its salient form; ω_{ng} and ω_{vg} are the low-frequency and high-frequency harmonics $g(i\Delta t)$ before the origin of the defect, respectively; D_ε and D_g are the variances $\varepsilon(t)$ and $g(t)$, respectively; $r_{x\varepsilon}$ is the correlation coefficient between $x(t)$ and $\varepsilon(t)$; $\omega_{n\varepsilon}$ and $\omega_{v\varepsilon}$ are the low-frequency and high-frequency harmonics of $\varepsilon(i\Delta t)$, respectively; $x(t)$ is the legitimate signal; $X(t)$ is the sum of the legitimate signal and noises caused by external factors $\varepsilon'(t)$; $g(i\Delta t)$ is the signal obtained from the output of the sensor $g(t)=X(t)+\varepsilon(t)$; and $\varepsilon(i\Delta t)$ is the noise caused by the defect.

So it is obvious that $\varepsilon(t)$ describes the influence of the defect's origin on $g(t)$ and is the carrier of the information about the origin and the evolution of the corresponding process. Besides the extraction of the information from the useful signal $x(t)$, the extraction of the information from the noise $\varepsilon(i\Delta t)$ is also needed for successfully solving the problem of monitoring the defect's origin. Thus, it is advisable to reduce solving the problem of monitoring to the separate analysis of the signal $X(t)$ and the noise $\varepsilon(t)$. These problems are considered in detail in Chapters 2–5.

1.5 Difficulties of Monitoring a Defect at Its Origin by Traditional Technologies

Many accidents with disastrous effects of such technical objects as thermoelectric power stations, nuclear power plants, large-capacity petrochemical complexes, deep-water stationary sea platforms and hydraulic structures, airplane crashes, faults of forecasting the earthquakes at the seismic stations, and the difficulties of disease diagnosis at the initial stages, etc. were supposed to be connected with the unreliability of the elemental base and the inaccuracy of the measurement equipment in the past. But now, despite the fact that both the reliability and the accuracy of the hardware have increased, the decrease of the probability of such accidents has not been significant. The performed analysis shows that in many cases the dominant reason for inadequate decisions connected with information systems is the errors of the results of processing the measured information and the absence of the opportunity to provide robustness of the unknown estimates [14]. Making adequate decisions for such situations by the corresponding systems of control and management is impossible without providing the appropriate accuracies of the estimates of the variance, correlation, and spectral and the other characteristics of the signals, obtained as the output of the corresponding sensors.

At the same time, receiving more or less appropriate estimates by using traditional technologies is possible only if the analyzed signals are stationnary and are under the normal distribution law, the correlation between the noise and the useful signal is zero, and the noise is white noise. But even in this case, the errors of the found estimates depend on the change of the variance of the noise, on the change of the correlation between the noise and the useful signal, or on the change of their distribution laws. Thus, even when all the above-mentioned conditions take place, the adequacy of the description of analyzed processes by means of traditional technologies is often unsatisfactory, and erroneous results are obtained in solving many of the most important problems. This is connected with the fact that the specificity of forming real signals was not taken into account when the corresponding algorithms were researched. However, the above-mentioned conditions do not take place at all in most cases. The performed research has shown that the signals obtained as the output of many modern sensors contain a huge informational potential. They are quite sufficient for forecasting the accidents at the thermoelectric power stations, the nuclear power plants, during boring; for the pre-flight monitoring of the technical conditions of airplanes; for forecasting the earthquakes and others various spontaneous disasters; for the diagnostics of the disease at initial stages; for increasing the effectiveness of the geophysical exploration, etc. In this vein, for the full use of the colossal informational potential of signals obtained as the output of modern sensors, the long-time necessity appears to reconsider the traditional algorithms and create new technologies that allow one to provide the adequacy of the obtained results of solving the current problems of the information systems whether or not the classical conditions take place. For that purpose, technologies extracting as much information from the signals as possible are required. One the possible solution to this problem is to create technologies that extract information from the noises of the noisy signals. The creation of such technology would promote the further development of the theory of the correlation analysis, the theory of the spectral analysis, the theory of the pattern recognition, the theory of the random process, etc. At the same time, the opportunity to expand the scopes of the solution of the most important problems of physics, biology, power engineering, geology, oil chemistry, aviation, etc. will appear by using the measured information obtained as the output of the corresponding sensors.

1.6 Factors Influencing the Adequacy of Monitoring a Defect's Origin by Methods of Correlation Analysis

Let us consider the factors complicating the solution of monitoring by the methods of correlation analysis when the measuring information $g_i(t) = X_i(t) + \varepsilon_i(t)$, $\eta(t) = Y(t) + \varphi(t)$, represented as the mixture of the useful signals $X_i(t)$, $Y(t)$ and the noises $\varepsilon_i(t)$, $\varphi(t)$, relating to the normal distribution law with the averages of distribution: $m_{\varepsilon_i} \approx 0$, $m_\varphi \approx 0$, is used for calculating the estimates of the correlation functions. It is known that the formula determining the estimates of the auto- and across-correlation functions $R_{g_i g_i}(\tau)$ and $R_{g_i \eta}(\tau)$ between the signals $g_i(t)$ and $g_j(t)$ and between $g_i(t)$ and $\eta(t)$ correspondingly can be represented as follows:

$$\begin{aligned} R_{g_i g_i}(\tau) &= \frac{1}{T}\int_0^T \dot{g}_i(t)\dot{g}_i(t+\tau)\,dt \\ &= \frac{1}{T}\int_0^T \left[\dot{X}_i(t) + \dot{\varepsilon}_i(t)\right]\left[\dot{X}_i(t+\tau) + \dot{\varepsilon}_i(t+\tau)\right]dt \\ &= R_{x_i x_i}(\tau) + \Lambda_{x_i x_i}(\tau), \end{aligned} \tag{1.4}$$

$$\begin{aligned} R_{g_i \eta}(\tau) &= \frac{1}{T}\int_0^T \dot{g}_i(t)\dot{\eta}(t+\tau)\,dt \\ &= \frac{1}{T}\int_0^T \left[\dot{X}_i(t) + \dot{\varepsilon}_i(t)\right]\left[\dot{Y}(t+\tau) + \dot{\varphi}(t+\tau)\right]dt \\ &= R_{x_i y}(\tau) + \Lambda_{x_i y}(\tau), \end{aligned} \tag{1.5}$$

where $\Lambda_{x_i x_i}(\tau)$ and $\Lambda_{x_i y}(\tau)$ are the errors of the estimates of the auto- and cross-correlation functions $R_{g_i g_i}(\tau)$ and $R_{g_i \eta}(\tau)$, respectively.

Taking into account that the following equalities take place for the real signals obtained as the outputs of the sensors:

$$\frac{1}{T}\int_0^T \dot{\varepsilon}_i(t)\dot{\varepsilon}_j(t+\tau)\,dt \approx 0\,, \quad \frac{1}{T}\int_0^T \dot{\varepsilon}_i(t)\dot{\varphi}(t+\tau)\,dt \approx 0\,, \tag{1.6}$$

we have the following expressions for the errors of cross-correlation functions $R_{g_i g_j}(\tau)$, $R_{g_i \eta}(\tau)$:

$$\Lambda_{x_i y}(\tau) = \frac{1}{T}\int_0^T X\left[\dot{x}_i(t)\dot{\varphi}(t+\tau) + \dot{\varepsilon}_i(t)\dot{Y}(t+\tau)\right]dt . \tag{1.7}$$

The value of the error $\Lambda_{x_i x_i}(\tau)$ of the estimates of the auto-correlation functions $R_{g_i g_i}(\tau)$ is determined by the following expression:

$$\Lambda_{x_i x_i}(\tau) = \frac{1}{T}\int_0^T \left[\dot{X}_i(t)\dot{\varepsilon}_i(t+\tau) + \dot{\varepsilon}_i(t)\dot{X}_i(t+\tau) + \dot{\varepsilon}_i(t)\dot{\varepsilon}_i(t+\tau)\right]dt \cdot \tag{1.8}$$

However, if we take into account that the noise $\varepsilon_i(t)$ is white noise and that the values $\varepsilon_i(t)$ and $\varepsilon_i(t+\tau)$ do not correlate with each other for $\tau \neq 0$, i.e.:

$$\frac{1}{T}\int_0^T \dot{\varepsilon}_i(t)\dot{\varepsilon}_i(t+\tau)dt = 0 \text{ for } \tau \neq 0, \tag{1.9}$$

then Eq. (1.8) is correct only for $\tau = 0$. For all other values $\tau \neq 0$, the following equality takes place:

$$\Lambda_{x_i x_i}(\tau) = \frac{1}{T}\int_0^T \left[\dot{X}_i(t)\dot{\varepsilon}_i(t+\tau) + \dot{\varepsilon}_i(t)\dot{X}_i(t+\tau)\right]dt . \tag{1.10}$$

For $\tau = 0$, the expression for the error $\Lambda_{x_i x_i}(\tau)$ can be described as follows:

$$\Lambda_{x_i x_i}(\tau) = \frac{2}{T}\int_0^T \dot{X}_i(t)\dot{\varepsilon}_i(t)dt + \frac{1}{T}\int_0^T \dot{\varepsilon}_i^2(t)dt . \tag{1.11}$$

Taking into account that the average value of the squares of the values of error is equal to the variance D_{ε_i} of the error $\varepsilon_i(t)$, i.e.,

$$\frac{1}{T}\int_0^T \dot{\varepsilon}_i^2(t)dt = D_{\varepsilon_i} \tag{1.12}$$

for $\tau = 0$, we have

$$\Lambda_{x_i x_i}(\tau) = \frac{2}{T}\int_0^T \dot{X}_i(t)\dot{\varepsilon}_i(t)dt + D_{\varepsilon_i} = \Lambda'_{x_i x_i}(\tau) + D_{\varepsilon_i}, \tag{1.13}$$

where

$$\Lambda'_{x_i x_i}(\tau) = \frac{2}{T}\int_0^T \dot{X}_i(t)\dot{\varepsilon}_i(t)\,dt.$$

It is obvious from this expression that the estimate $R_{g_i g_j}(0)$ differs from its own true values $R_{x_i x_j}(0)$ by the value of the errors $\Lambda_{x_i x_j}(0)$, and the estimates $R_{g_i g_i}(0)$, besides the values of the errors $\Lambda_{x_i x_i}(0)$, also differ from their true values $R_{x_i x_i}(0)$ by the values of the variance D_{ε_i} of the errors $\varepsilon_i(t)$. Also, the estimates $R_{g_i \eta}(0)$ differ from their own true values $R_{x_i y}(0)$ by the error $\Lambda_{x_i y}(0)$ only. So it is obvious from expressions (1.4)–(1.13) that even under the above-mentioned classic conditions of the work of an object, the reason of the inadequacy of decisions is the presence of the errors $\Lambda_{x_i x_i}(0)$, $\Lambda_{x_i x_j}(0)$ in each estimate as well as the presence of the variance D_{ε_i} of the noise $\varepsilon_i(t)$ in the estimates for $\tau = 0$.

Therefore, obtaining the assured results corresponding to these situations means it is impossible to solve the problem of monitoring the rise of the defect when the model of a signal is represented in the form of expression (1.3). If we take into consideration that the classic conditions do not take place when solving the problem of monitoring the technical state, for a great number of important objects, even for normal exploitation modes, the reasons for the inadequate results of traditional technologies of correlation analysis become obvious.

It is obvious from the above-mentioned that it is necessary to create more perfect digital technologies of correlation analysis for providing the reliability of the results of monitoring the defect at the moment of its rise.

1.7 Factors Affecting the Adequacy of Monitoring a Defect's Origin by Methods of Spectral Analysis

During spectral analysis, when the measured information consists of the useful signal and the noise, the error of the unknown estimates depends on the difference between the sum of the errors of the positive and the negative products of the samples of the total signal multiplied by the samples of the cosinusoids and the sinusoids, respectively.

If the analyzed signal $X(t)$ does not contain the noise $\varepsilon(t)$, it can be represented as the sum of the harmonic functions: the sinusoids and the cosinusoids; the sum of their ordinates is the value of the initial function for every moment of time t:

$$X(t)=\frac{a_0}{2}+\sum_{i=1}^{\infty}(a_n\cos n\omega t+b_n\sin n\omega t)\,, \tag{1.14}$$

where $a_0/2$ is the average value of the function $X(t)$ for the period T, and a_n and b_n are the amplitudes of the sinusoids and the cosinusoids with the frequency $n\omega$.

To provide the required precision of the description of the signal $X(t)$ as the sum of the sinusoids and cosinusoids, it is necessary that the following inequality take place:

$$\sum_{i=1}^{m}\lambda_i^2\le S\,, \tag{1.15}$$

where λ_i^2 are the squares of the differences between the sum of the right side of Eq. (1.14) and the samples of the signal $X(t)$ at the sampling moments $t_0, t_1,..., t_i,..., t_m$ by the step Δt; S is the permissible value of the mean-square deflection.

In formula (1.14) in the representation of the function $X(t)$, the trigonometric Fourier series ω is assumed to be equal to $2\pi/T$, and the coefficients a_n and b_n are determined in the following way:

$$a_n=\frac{2}{T}\int_0^T X(t)\cos n\omega t\,dt \text{ for } n=1,2,...; \tag{1.16}$$

$$b_n=\frac{2}{T}\int_0^T X(t)\sin n\omega t\,dt \text{ for } n=1,2,... . \tag{1.17}$$

Here the first harmonic has the frequency and the same period as the period T of the function $X(t)$. The coefficients a_1 and b_1, a_2 and b_2, and a_3 and b_3 are the amplitudes of the sinusoids and the cosinusoids obtained for $n=1$, $n=2$, $n=3$, etc.

Theoretically, condition (1.15) takes places for the given value S for the signals $X(t)$ of the bounded spectrum if they do not contain the noise $\varepsilon(t)$. But in practice, as mentioned earlier, the useful signal $X(t)$ is often accompanied by the certain noises $\varepsilon(t)$, i.e., it is the sum $g(t)=X(t)+\varepsilon(t)$. So condition (1.15) does not always take place. Nevertheless for the cases when the value of the noise changes in the acceptable bounds and is under the normal distribution law, many important problems are successfully solved by using algorithms (1.16) and (1.17). In practice, the principle of the superposition of the signals is often used during the analysis of the work of the linear elements and systems. This principle can be described in the following way. If we represent the input signal as the sum of the series of other signals, the output signal is determined as the sum of the output signals, which can be received if each input signals acts separately. This is the primary way for determining the response of the linear system or the linear part of the nonlinear system for the input signal of the arbitrary shape by means of harmonic analysis. That is why it is especially expedient to use methods and algorithms of the spectral analysis [14] while monitoring the rise of the defect. But in the cases when the corresponding classical conditions do not take place and the value of the noise is significant, this way is not able to provide inequality (1.15). So solving the problem of monitoring the rise of the defect by spectral methods has difficulties. Let us consider this problem in more detail. When the analyzed signal $g(t)$ consists of the useful random signal $X(t)$ and the noise $\varepsilon(t)$, i.e.,

$$g(t) = X(t) + \varepsilon(t) \tag{1.18}$$

formula (1.16) can be represented as follows:

$$\begin{aligned} a_n &= \frac{2}{T}\int_0^T [X(t) + \varepsilon(t)]\cos n\omega t\,dt \\ &= \frac{2}{T}\int_0^T \{[X(t)\cos n\omega t\,dt] + [\varepsilon(t)\cos n\omega t\,dt]\} \end{aligned} \tag{1.19}$$

It is obvious that the condition (1.15) may take place for the cases when

$$\sum_{i=1}^{N^+}\int_{t_i}^{t_{i+1}} \varepsilon(t)\cos n\omega t\,dt = \sum_{i=1}^{N^-}\int_{t_{i+1}}^{t_{i+2}} \varepsilon(t)\cos n\omega t\,dt, \tag{1.20}$$

where N^+, t_i, t_{i+1} is the quantity, the beginning, and the end of the positive half-periods $\cos n\omega t$, respectively, for the observation period T; N^-, t_{i+1}, t_{i+2} is the quantity, the beginning, and the end of the negative half-periods $\cos n\omega t$, respectively, for the observation period T.

For all other cases when this equality does not take place, we get the difference

$$\lambda_{an} = \sum_{i=1}^{N^+}\int_{t_i}^{t_{i+1}} \varepsilon(t)\cos n\omega t\,dt - \sum_{i=1}^{N^-}\int_{t_{i+1}}^{t_{i+2}} \varepsilon(t)\cos n\omega t\,dt, \tag{1.21}$$

leading to the error of the estimate of the coefficient a_n. The same also takes place for the determination of the estimate b_n. It is obvious from expression (1.21) that if the variance $\varepsilon(t)$ increases, the difference λ_{an} also increases. The difference of the distribution law of the analyzed signal from the normal one, and the correlation between the useful signal $X(t)$ and the noise $\varepsilon(t)$, also leads to increasing the difference λ_{an}. In this connection, for some cases the errors of the estimates λ_{an}, λ_{bn} may be commensurable to the unknown coefficients a_n, b_n.

Therefore, obtaining reliable results when solving the problem of monitoring the real objects when the above-mentioned classical conditions do not take place is not always guaranteed, even for significant defects.

The analysis of the model (1.3) makes it clear that it is necessary to create the technologies of the spectral analysis of the noisy signal obtained as the output of the sensor for periods T_1, T_2, T_3 to solve this problem successfully. At the same time, the possibility arises to provide monitoring of the defect at its rise.

For this purpose, it is necessary first to develop the algorithms and the technologies providing the inequalities $S_n >> \lambda_{an}$, $S_n >> \lambda_{bn}$ and holding the condition (1.15) by means of the removal of the reasons of the appearance of the errors λ_{an}, λ_{bn}. Also, they must satisfy the robustness conditions, i.e., they must remove the dependence between the values λ_{an}, λ_{bn} and the values of the variance of the noise $\varepsilon(t)$, must be independent on the change of the form of the distribution law of the analyzed signal, independent of the correlation coefficient between the useful signal $X(t)$ and the noise $\varepsilon(t)$, etc. Furthermore, it is necessary to provide a separate spectral analysis of the useful signal and the noise. The solution of these problems is considered in Chapter 4.

1.8 Influence of Signal Filtration on the Results of Monitoring a Defect's Origin

It is known that one of the most common methods for removing the influence of the noise on the result of the analysis of the noisy signals is its filtration. When performing the operation of the filtration of the noise for the noisy continuous signals obtained as the outputs of the sensors, it is necessary to determine the distribution of its average power by the frequencies, its spectral density of the power, the width of the spectrum, the positions and the values of the maximums of the spectral density of the power, the boundary frequencies, and other characteristics. The determination of the approximate estimates of the spectral densities of the power of the noise of the measured signals is also expedient after obtaining the above-mentioned characteristics. The determination of such characteristics of the signals during spectral analysis in most cases is based on the Fourier transformation of the signal. The method of determining the spectral density of the power by the measured correlation function in correspondence with the Wiener–Khinchin theorem is often used for noisy random signals. The spectral density of the power $G_g(f)$ gives the opportunity to make assumptions about the frequency properties of both the useful signal $X(t)$ and the noise $\varepsilon(t)$, because it defines their intensity for the various frequencies, i.e., the average power accounts for a unit of the frequency band. For this purpose, the sign functions are also used, and one method is based on the hardware application of the orthogonal functions.

The hardware determination of the spectrum of the noise is based on using the best-known method of the filtration by detecting the narrow parts of the spectrum of the analyzed signal by means of the equipment with the selective gain-frequency characteristic. It is assumed that if the spectrum of the noise is limited by the frequencies $f_1 = f - \Delta f/2$ and $f_2 = f + \Delta f/2$, the average power at the band Δf of the neighborhood of the frequency f is determined by the following expression:

$$P_x(f, \Delta f) = 2\int_{f-\Delta f/2}^{f+\Delta f/2} G_x(f)df\,. \tag{1.22}$$

If the frequency band Δf is finite but is so narrow that the spectral density of the power $G_\varepsilon(f)$ can be represented as constant for this band, it can be determined by the approximate expression

$$G_\varepsilon(f) \approx \frac{P_k(f, \Delta f)}{\Delta f}\,. \tag{1.23}$$

In correspondence with this formula, the filtration of the noise is reduced to the determination of its spectral density of the power by measuring its average power at the known narrow band Δf. In other words, it is necessary to "cut out" the narrow band of the spectrum of the total signal by the linear bandpass filter with the pass band Δf to measure the spectral density of the power $\varepsilon(t)$. For this purpose, the analyzers with the bandpass filters are used. At the same time, it is necessary to determine the spectrum of the noise, and the operation of the filtration of the noise $\varepsilon(i\Delta t)$ from the total signal $g(i\Delta t)$ can be represented as follows:

$$X(t) \approx g(t) - \varepsilon(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\Big[\big(a_{n_g}\cos n\omega t - b_{n_g}\sin n\omega t\big)$$

$$- \sum_{n=k}^{k+m}\big(a_{n_\varepsilon}\cos n_\varepsilon \omega t - b_{n_\varepsilon}\sin n_\varepsilon \omega t\big)\Big]. \tag{1.24}$$

It is clear that such an ideal division of the noise into the harmonics for the range from $n = k$ to $k + \nu$ is impossible in practice. It is obvious that in practice there are at least the low-powered spectra of the useful signal

$g(t)$ for this range. Certain low-powered spectra of the noise $\varepsilon(t)$ take place from this range. Thus, the filtration by using the bandpass filters gives good results at those uncommon ideal cases when the spectrum of the noise belongs to the specified interval. In all other cases, certain spectra, both of the noise and of the useful signal, are distorted by the filtration. At the same time, as usual, the value of the noise caused by the defect is not significant at the rise of the defect. Therefore, when solving the problem of monitoring the defect, using the operation of the filtration of the noise of the signals obtained from the sensors according to model (1.3) gives the opportunity to obtain reliable results only for the period T_4 when the defect takes its salient form. So it is necessary to create digital technologies that allow one to remove the influence of the noise on the result of the processing and to analyze the noises and the useful signals separately for the periods T_1, T_2, T_3 for detecting the rise and evolution of the defect for the periods T_1, T_2, T_3. These problems are considered in Chapter 3.

1.9 Influences of Traditional Methods of Choosing the Sampling Step on the Adequacy of Monitoring a Defect's Origin

According to the traditional methods, the sampling step Δt of the signals on the outputs of the sensors is determined on the base of its cut-off frequency f_c as follows:

$$\Delta t \leq \frac{1}{2f_c}. \tag{1.25}$$

At the same time, Δt is determined for the useful signal $X(t)$. The information of the high-frequency, low-powered spectra $\varepsilon(i\Delta t)$ of the total signal is almost lost as the result of such sampling. This leads to the loss of the information about the origin of the defect. It is necessary to take into account the information contained in $\varepsilon(i\Delta t)$ to detect the origin of the defect. For that purpose, it is necessary to determine the values of the samples of the total signal on the basis of the frequency of the spectrum of the noise $\varepsilon(i\Delta t)$, by the formula

$$\Delta t_\varepsilon \leq \frac{1}{2f_{c\varepsilon}},$$

where f_c and $f_{c\varepsilon}$ are the cut-off frequencies of $X(i\Delta t)$ and $\varepsilon(i\Delta t)$, respectively.

Let us consider the possibility of sampling the signals by this frequency in detail. It is known that the processing speed of the modern analog-digital transformers (ADT) and information systems gives the opportunity to transform the analyzed signals $g_v(i\Delta t)$ by superfluous frequency f_u:

$$f_u >> f_s, \tag{1.26}$$

where f_s is the sampling step determined according to the sampling theorem.

It is clear that for such superfluous sampling the following equality takes place:

$$P[g(i\Delta t) \approx g((i+1)\Delta t)] \approx 1, \tag{1.27}$$

where P is the sign of probability.

According to Eqs. (1.26) and (1.27), many samples $g(i\Delta t)$, following one by one, are repeated. Thus, the frequency of the change of state f_{q0} of the low order of the analog-digital transformer f_0 is sufficiently less than the superfluous frequency of sampling, i.e.,

$$f_u >> f_0. \tag{1.28}$$

It is intuitively understood that in this case the frequency of the change of the low order mainly depends on the high-frequency spectrum $\varepsilon(i\Delta t)$ of the signal $g(i\Delta t)$, and the value f_0 essentially is the frequency of the sampling noise $\varepsilon(i\Delta t)$. This problem is considered in more detail in Chapter 2.

Thus, it is obvious that in solving the problem of monitoring the origin of the defect, it is advisable to determine the samples of the total signal $g(i\Delta t)$ on the basis of the frequency of the high-frequency spectrum $\varepsilon(i\Delta t)$ containing the information about the origin of the defect. Only in that case does the opportunity to detect $\varepsilon(i\Delta t)$ in the signal $g(i\Delta t)$ for periods T_1, T_2, T_3 appear. It is obvious that technologies of analyzing the signals $g(i\Delta t)$ must be adaptive in these time intervals because the frequency characteristics $\varepsilon(i\Delta t)$ change continuously during the evolution of the defect by times T_1, T_2, T_3. The importance of providing the adaptability of sampling $g(i\Delta t)$ is connected with the fact that part of the extracted information contained in $\varepsilon(i\Delta t)$ is lost for both the insufficient and the superfluous frequency of sampling $g(i\Delta t)$. The technology of providing the adaptability of the sampling step $g(i\Delta t)$ is considered in detail in Chapter 2.

2 Position-Binary Technology of Monitoring Defect at Its Origin

2.1 Specific Properties of Periodic Effect Objects

It's known that in most cases the spectral methods are used for the experimental analysis of the cyclical (periodic) processes [12, 41]. For example, the objects of the back-and-forth motion equipment, the objects of the rotating equipment, those of the biological processes, etc. are cyclical. As a rule, the signals obtained from many cyclical objects have the complicated spasmodic leaping form and are accompanied by significant noise. At present, spectral methods and algorithms are commonly used in the experimental research of such signals [12, 37, 61]. But they are not effective enough for these objects in some cases [37]. Thus, in many cases it is necessary to use the large number of harmonic components of the corresponding amplitudes and frequencies for the appropriate description of spasmodic and leaping signals. That essentially complicates the analysis and use of the obtained results for solving the corresponding problems [37, 41]. That is why, in solving the problem of monitoring the defect origin, there is a need for methods and algorithms allowing one to (1) increase the reliability of the obtained results in comparison with the spectral method and (2) decrease the quantity of the spectrum components of the considered class of the objects [37, 41].

Let us consider the difficulties of using the spectral method for the analysis of the signals obtained from the considered objects in more detail.

It is known that when using the algorithms of this method for description of the periodic signals $X(t)$ of the bounded spectrum, the periodic signals $X(t)$ are represented as the sum of the harmonic components by means of the following expression:

$$X(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos n\omega t + b_n \sin n\omega t \right). \tag{2.1}$$

In Eq. (2.1), a_n, b_n are the amplitudes of the sinusoids and the cosinusoids with the frequency $n\omega$, which are assumed to be the informative indicators in solving the problem of monitoring the origin of the defect. It is known that for providing the accuracy of the signal restoration $X(t)$, the following inequality is required:

$$\sum_{i=1}^{n} \lambda_i^2 \leq S, \tag{2.2}$$

where λ_i^2 are the squares of deviations between the sum of the right-hand side of Eq. (2.1) and samples of signal $X(t)$ at the moments of sampling t_0, t_1, …, t_i, …, t_n with the sampling step Δt; S is the permissible value of the mean-root-square deviation.

The spasmodic leaping signals providing Eq. (2.2) lead to increasing the number of harmonic components, and that correspondingly complicates processing the experimental data. In addition, when the measured information consists of the sum of the useful signal $X(t)$ and the noise $\varepsilon(t)$, condition (2.2) takes place depending to a certain extent on the value of the noise $\varepsilon(t)$. In the existing methods, the influence of the noise is neglected in Eq. (2.1), and the error caused by the noise $\varepsilon(t)$ is assumed to be equal to zero. But for the many cyclical processes, the influence of the noise on the accuracy of the restoration of the initial signal $X(t)$ can be considerable and must be taken into account.

If we take into consideration that in the defect origin the spectrum of the signal continuously changes, the difficulties of using the technology of the spectral analysis in solving the problem of monitoring the defect origin will be clear. That it is why it is necessary to create the new spectral technologies, taking into account the specificity of the signals obtained from the periodic objects. One of the possible variants of these technologies is offered in the next section.

2.2 Position-Binary Technology of Analyzing Noisy Signals Obtained as Outputs of Sensors of Technical Objects

As stated earlier, at present the algorithms of the spectral and correlation analyses are mainly used for the analysis of the periodic processes [12]. But their application in solving the problems of monitoring the defect origin does not provide a reliable result at the origin of a defect. In this connection, the principles and algorithms allowing one to detect the defect at its origin are of both theoretical and practical interest. The availability of using the position-binary technology for this purpose is offered ahead. In that case, noisy signals are analyzed through the corresponding position-binary impulse signals (PBIS).

In practice, when measuring the signals $X(t)$, there is the minimum value of the increase, which can be provided by the used instrument depending on its resolving capacity. We will denote that minimum value of the increase as Δx. So in measuring the signal, the number of its discrete values is equal to

$$m = X / \Delta x + 1 . \tag{2.3}$$

In the process of the analog-digital conversion of the periodic signal $X(t)$, its amplitude quantization takes place for each sampling step Δt, i.e., the range of its possible changes is divided into the m sampling intervals and the value of the signal belonging to the mth sampling interval is related to the center of the sampling interval $m\Delta x$ when the following inequality takes place:

$$m\Delta x - \Delta x / 2 \le X(t) \le m\Delta x + \Delta x / 2 . \tag{2.4}$$

In this case, the values of the binary codes of the corresponding digits q_k of samples x_i of signal $X(i\Delta t)$ with sampling step Δt are determined on the basis of the following algorithm [12, 14, 37, 41]:

$$q_k(i\Delta t) = \begin{cases} 1 & \text{for} \quad x_{\text{rem}(k)}(i\Delta t) \ge \Delta x 2^k \; ; \\ 0 & \text{for} \quad x_{\text{rem}(k)}(i\Delta t) < \Delta x 2^k \; ; \end{cases} \tag{2.5}$$

$$x_{\text{rem}(k)}(i\Delta t) = x_k(i\Delta t) - \left[q_{k+1}(i\Delta t) + q_{k+2}(i\Delta t) + \ldots + q_{(n-1)}(i\Delta t)\right] ,$$

where

$$X(i\Delta t) > 2^n , \; x_{\text{rem}(n-1)}(i\Delta t) = X(i\Delta t),$$

$$n \ge \log \frac{x_{\max}}{\Delta x} , \; k = n-1, n-2, \ldots, 1, 0 .$$

First, according to this algorithm, at each step of sampling Δt, the equality $x_{\text{rem}(n-1)}(i\Delta t) = X(i\Delta t)$ is accepted. Also, according to condition (2.5), the signals $q_k(i\Delta t)$ as a code 1 or 0 are formed by iteration. In this case, in the first step $X(i\Delta t)$ is compared to the value $2^{n-1}\Delta x$. According to (2.5), at $X(i\Delta t) \geq 2^{n-1}\Delta x$, the value $q_{n-1}(i\Delta t)$ is equated to unit; according to the difference $X(i\Delta t) - 2^{n-1}\Delta x = x_{\mathrm{rem}(n-2)}$, the value of the remainder $x_{\mathrm{rem}(n-2)}$ is determined. When $X(i\Delta t) \geq 2^{n-1}\Delta x$, the value $q_{n-1}(i\Delta t)$ is set to zero and the difference remains constant. At the next iteration, the same takes place. As a result, during the cycle T_c with sampling step Δt, the signal $X(i\Delta t)$ is decomposed into the signals $q_k(i\Delta t)$ having the value 1 or 0 and whose weight depends on their positions. At the same time, the codes do not change in time when the value of the initial signal $X(i\Delta t)$ does the same at the process of sampling. Here and ahead, we will name these signals the position-binary impulse signals (PBIS). The position-binary technology is the series of the procedures of processing based on the decomposition of the continuous signal by the PBIS.

According to algorithm (2.5), the width of the PBIS is proportionate to quantity Δt when $q_k(i\Delta t)$ remains constant. Depending on the form of $X(i\Delta t)$, the same signal $q_k(i\Delta t)$ can change its value several times during one cycle after the corresponding time intervals. It is clear that if the condition of the object is constant, the combinations of the time intervals T_{k1_1}, T_{k0_1}, T_{k1_2}, T_{k0_2}, … of the PBIS at each cycle are constants, and they are repeated. Otherwise, they also change. Let us note that here T_{k1_1}, T_{k0_1}, T_{k1_2}, T_{k0_2}, … correspond to the intervals when the condition $q_k(i\Delta t) = 2^k(\Delta x = 1)$ takes place; T_{k0_1}, T_{k0_2}, … correspond to the intervals when the condition $q_k(i\Delta t) = 2^k(\Delta x = 0)$ takes place.

For example, let us suppose that the cycle time of the analyzed signal is equal to 15 microseconds and the sampling step is equal to 1 microsecond, i.e., $T_c = 15$ mcs, $\Delta t = 1$ mcs. Let us assume that PBIS $q_3(i\Delta t)$ takes the following states for one cycle: 000111100110000. In this case, the parameters of signal $q_3(i\Delta t)$ are represented as follows: 3,0; 4,1; 2,0; 2,1; 4,0. It means that during the cycle, the width of unit and zero states of signal $q_3(i\Delta t)$ corresponds to the following time intervals: 3 mcs-0; 4 mcs-1; 2 mcs-0; 2 mcs-1; 4 mcs-0. It is obvious that in each cycle the sum of all PBIS is equal to the initial signal

$$X(i\Delta t) \approx q_{n-1}(i\Delta t) + q_{n-2}(i\Delta t) + \cdots + q_1(i\Delta t) + q_0(i\Delta t) = X^*(i\Delta t). \quad (2.6)$$

Each $q_k(i\Delta t)$ can be considered as the individual signal because we can assume the sequence of time intervals when $q_k(i\Delta t)$ are in the unit and zero state to be impulse-width signals. At the same time, for the cyclic objects these PBIS $q_{kj}(i\Delta t)$ are the periodic rectangular impulses having the period T_c with unit T_1 and zero T_0 half-periods correspondingly.

We must note that the representation of the centered signals by PBIS differs only that in this case the initial signal is represented as the sum of the positive and the negative PBIS $q_k(t)$. At the same time, the signals $X(t)$ and $y(t)$ are represented as bipolar periodic PBIS, and their sum is also equal to the initial signal $X(i\Delta t)$.

At the representation of the initial signal $X(i\Delta t)$ as the sum $q_k(i\Delta t)$ at time t_i, the difference between the real value of the initial signal $X(t)$ and the sum of PBIS is

$$X(i\Delta t) - X^*(i\Delta t) = \lambda(i\Delta t). \tag{2.7}$$

Taking into account Eq. (2.4), we have

$$\lambda(i\Delta t) \le \pm\Delta x / 2 .$$

If we assume that in forming the signals $q_k(i\Delta t)$, the value of the error $\lambda(i\Delta t)$ is under the equiprobable distribution law [30], we obtain

$$P\left[\lambda_i < \frac{\Delta x}{2}\right] \approx P\left[\lambda_i > \frac{\Delta x}{2}\right], \tag{2.8}$$

where P is the sign of probability.

Thus, according to (2.7) and (2.8), the sum of the squares of deviations λ_i at t_0, t_1, ..., t_i, ... is close to zero. Inequality (2.2) can then be represented as follows:

$$\sum_{i=1}^{n} \lambda^2(i\Delta t) \le \Delta x .$$

According to this inequality, at the representation of the signal $X(t)$ as the sum of PBIS, the mean-square deviation is not greater than the value Δx, and that shows the possibility of restoration of the signal with high accuracy. For example, in solving the problems of monitoring, if the change of the object condition leads to the change of the corresponding components of the signal by a value greater than Δx, the corresponding parameters $q_k(i\Delta t)$ will be affected. Thus, the difference from the similar parameters will be detected at the initial stage of the defect origin in the process of forming the parameters as the combination of the corresponding time intervals of the signals $q_{n-1}(i\Delta t)$, $q_{n-2}(i\Delta t)$, ..., $q_0(i\Delta t)$ of the corresponding cycle. This allows one to form and provide information about changing the condition of the controlled object. So the position-binary technology opens real opportunities for detecting the defect origin, which usually precedes major failures and emergency situations.

It is obvious that the position-binary technology can also be used for the stochastic objects. In this case, the process of solving the problem of monitoring the defect origin is also greatly simplified in comparison with the spectral technologies, and its adequacy thus improves.

It is connected with the fact that the algorithms of the processing $q_k(i\Delta t)$ in practice are realized quite easily, because each position-random function has only two values. In this case, the analysis of the random process by the signals of the PBIS is similar to the analysis of the cyclic processes. The difference is that in this case the observation period of the random process T is selected according to the principles of the correlation analysis.

As follows, the average frequency $\langle f_k \rangle$ and the period $\langle T_k \rangle$ can be determined for both periodic and stochastic objects for each PBIS. It is intuitively understood that for random and periodical noisy signals $g(t)$, the average value of zero and unit half-periods of the position signals $q_k(i\Delta t)$ can be determined by the following formula for a sufficiently long observation period:

$$\langle T_{q_k} \rangle = \langle T_{1q_k} \rangle + \langle T_{0q_k} \rangle, \tag{2.9}$$

where

$$\langle T_{1q_k} \rangle = \frac{1}{\gamma} \sum_{j=1}^{\gamma} T_{1q_{kj}}, \quad \langle T_{0q} \rangle = \frac{1}{\gamma} \sum_{j=1}^{\gamma} T_{0q_{kj}}. \tag{2.10}$$

Here γ is the number of unit and zero half-periods of the PBIS for the observation period; and j is the number of the q_k th position of the PBIS.

It was shown [14] that for a sufficiently long observation period T, the estimates of the periods $\langle T_k \rangle$ of the PBIS become nonrandom values. Thus, using them can greatly simplify solving the problems of monitoring the defect, which are traditionally solved by means of the estimations of statistical and spectral characteristics of the random processes.

2.3 Opportunities of Using Position-Binary Technology for Monitoring Technical Conditions of Industrial Objects

As stated earlier, the description of the random process can be represented by means of the corresponding frequency characteristics PBIS by using the position-binary technology of the analysis [12]. The experiments connected with the frequency properties of the PBIS show that they give the opportunity to solve the problems of diagnostics and monitoring and are significantly simpler than traditional algorithms of the correlation and the spectral analysis [13]. In practice, they are realized more easily because each position-random function has only two values. At the same time, the average frequency $\bar{f}_k$ and the period $\bar{T}_k$ determined by means of the PBIS are nonrandom values. Due to the simplicity of their determination, solving the problems of monitoring, which traditionally are solved by means of the estimations of the statistical or the spectral characteristics of the random processes, are greatly simplified. For example, the signal $X(t)$ can be represented as the combination of PBIS $q_k(i\Delta t)$ in solving the problems of the diagnostics of the technical conditions of the stochastic objects. It is obvious that changing the conditions of the object leads to changing the combination of their average frequencies $\bar{f}_{q_0}, \bar{f}_{q_1}, \ldots, \bar{f}_{q_m}$. If W is the set of all possible failure states of the object, it is easy to solve the problem of diagnostics and monitoring by means of the combination or the set of combinations of the frequencies of the PBIS for each failure state of the object.

It is possible to give examples of various technical problems that can be solved be means of the PBIS. For example, voice verification can be realized by forming the combinations $q_{1э}(i\Delta t), \ldots, q_{mэ}(i\Delta t)$ for each word by means of quite simple software and hardware.

Let us consider the use of position-binary technology for the diagnostics of the cyclically worked objects on the example of the diagnostics of the depth-pump equipment of the oil well [45]. The signal obtained from the force sensor of the depth-pump equipment characterizing its technical condition is represented in Fig. 2.1(a).

At the normal condition of the equipment, the curve is a trapezoid [curve 1, Fig. 2.1(a)] for the period T_C, amplitude U_1, and constant U_0. For the sake of simplicity, let us suppose that $U = 9mV$ and the quantization step by amplitude is $\Delta x = 1mV$. In this case, $k \geq \log_2 9 = 4p$, i.e., $k = 4$ binary digits q_3, q_2, q_1, q_0 are required for sampling the initial signal by the amplitude.

The initial signal is broken down into the sequence of the PBIS in Fig. 2.1(b). As the figure shows, the frequency of the 1s and 0s in positions and the width of the unit signals and pauses for the given values Δx and Δt are correspondingly determined by the amplitude value of the initial signal. So, for example, at time t_1, $q_3 = 0$, $q_2 = 1$, $q_1 = 1$, $q_0 = 0$, i.e., the signal amplitude is determined by the four-digit binary code 0110 corresponding to 6 megavolts, etc.

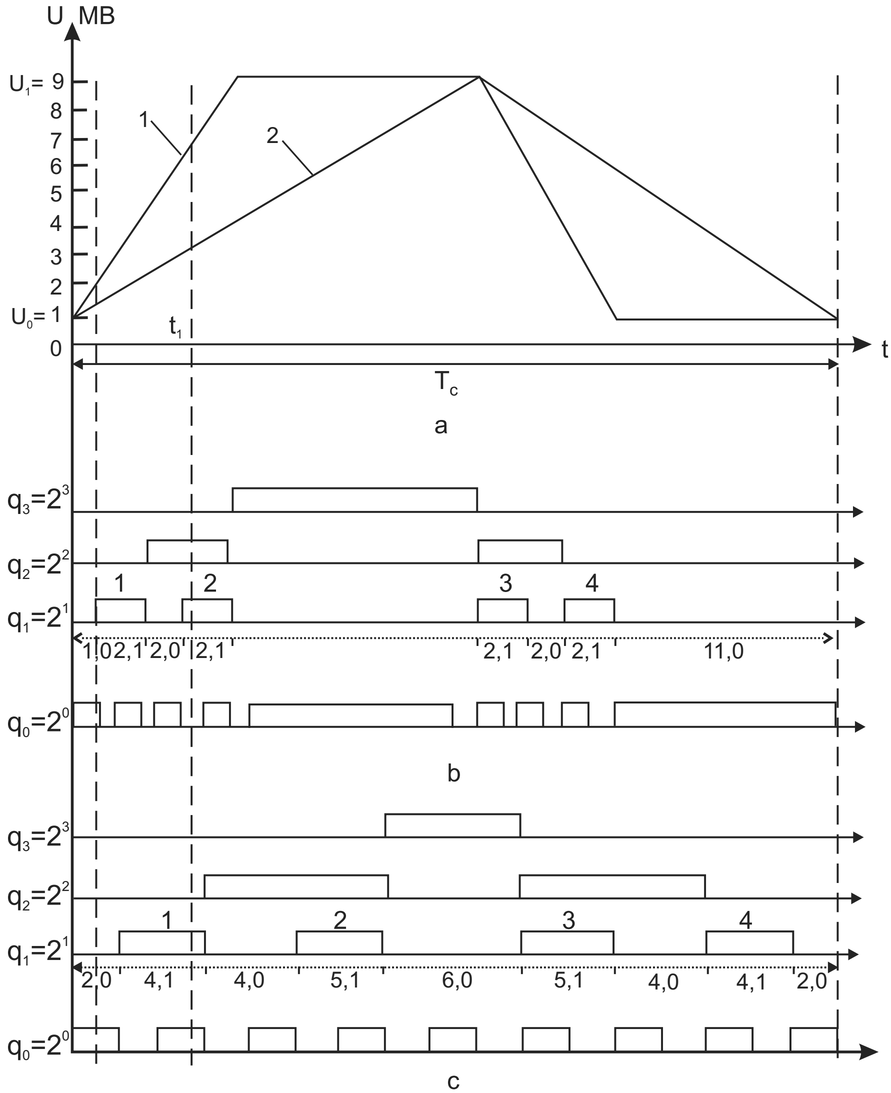

Fig. 2.1. (a) The diagram of the signal of the force sensor, (b) the PBIS for the normal state of the depth-pump equipment, (c) and that for the failure state "Plunger sticking."

In the given example, the duration of the initial signal's cycle is $T_c = 36\,\mathrm{s}$ for the step $\Delta t = 1\,\mathrm{s}$ of sampling by time. In this case, for the corresponding positions q_k, for example, for position q_1 the binary sequence 1.0, 2.1, 2.0, 2.1, 12.0, 2.1, 2.0, 2.1, 11.0, where the first number means the duration of the interval by seconds and the second shows belonging of the interval to unit or zero states, is generated for a cycle. The similar binary sequences are generated for other positions.

During the change of the technical condition of the depth-pump equipment (for example, during the appearance of the "Plunger sticking"-type failure of a pump [45]), curve 1 [Fig. 2.1(a)] becomes similar to curve 2, and, as shown in Fig. 2.1(b) and (c), the positions and the parameters of the durations and pauses of PBIS change according to that. So, for example, the new values q_k: $q_0 = 1$, $q_1 = 1$, $q_2 = 0$, $q_3 = 0$ are generated at time t_1 due to the change of the form of the initial signal, i.e., the binary code 1100 corresponds to the amplitude of the initial signal. At the same time, the new binary sequences are generated at the corresponding positions q_k. In particular, the binary sequence similar to 2.0, 4.1, 4.0, 2.1, 4.0, 4.1, 2.0 is generated for q_1.

It is obvious that other combinations of above-mentioned time intervals are received for other failures. Diagnosing the technical state of depth-pump equipment of the oil wells can be performed by these combinations.

However, despite the obvious advantage of this technology, when solving the problem of monitoring the defect, it does not allow one to detect the beginning of its origin. That explains the necessity of analyzing the noise as the data carrier appears quite often. In turn, this requires determining the sampling step Δt_ε on the basis of the high-frequency spectrum of the noise $\varepsilon(i\Delta t)$. The frequency of noise sampling can be determined by the frequency of the change of state of the lower position-binary impulse, i.e., by the value of its average period $\langle T_{q_0} \rangle$ and the average frequency $\langle f_{q_0} \rangle$ after transforming by the frequency f_u and recording the samples of the analyzed signal $g_i(i\Delta t)$ [1, 2, 58, 59].

It is obvious that if we choose the sampling step Δt_ε based on the spectrum of the noise, it will be sufficiently less than Δt.

As shown in Section 1.5, and according to the model of the signals, the spectrum and the estimates of the corresponding characteristics continuously change on the outputs of the sensors in the process of the origin and the evolution of the defect. That sufficiently affects the accuracy of the received estimates. There is the real opportunity to perform the change of the sampling step in real time, and by this way to increase the reliability of the obtained results due to the simplicity of the realization of expressions (2.9) and (2.10).

2.4 Position-Selective Adaptive Sampling of Noisy Signals

Let us consider the opportunity of determining the sampling step of the initial signal Δt by taking into account the value of the given error ε_0 by means of the frequency properties of the PBIS.

Let us assume that the analyzed signal is processed by analog-digital conversion by the current frequency f_v and by the certainly small sampling step of the quantization by time Δt. In this case, according to the inequalities $\Delta t_v << \Delta t$, many of these samples will be repeated due to the following equality:

$$P[X(i\Delta t)] \approx P[X((i+1)\Delta t)].$$

This explains why the values of the binary codes of the samples $X(i\Delta t)$ will also be repeated for each step $X((i+1)\Delta t)$ of quantization in the interval Δt_v. Due to this, the frequency f_{q_0} of the lower PBIS $q_0(t)$, which can be determined by the following formula:

$$f_{q_0} = \frac{1}{\langle T_{q_0} \rangle} \tag{2.11}$$

[where $\langle T_{q_0} \rangle$ is the average value of the period of the signal $q_0(t)$] will be sufficiently less than the current frequency of sampling f_v. At the same time, the following inequality connecting the current frequency f_v and the cutoff frequency f_C, found by the sampling theorem, takes place:

$$f_v >> f_C.$$

The value f_{q_0} can be assumed to be constant for all realizations of the same stationary random signal or cyclical signal for present ADC, i.e.,

$$f_{q_0} \approx \text{const} .$$

Thus, if we choose the value f_v satisfying this condition, the value f_{q_0} can be determined for the analyzed signal. In this case, the following condition takes place between f_{q_0} and the cutoff frequency f_C of the signal $X(i\Delta t)$ found by known methods:

$$f_C \geq f_{q_0} .$$

At the same time, taking into account that each PBIS is formed by obtaining two impulses on the lower digit of ADC, the previous condition can be represented as follows:

$$f_C \geq 2f_{q_0}.$$

On the basis of this condition, the sampling step for the useful signal $X(i\Delta t)$ can be chosen in accordance with the following inequality:

$$\Delta t \leq \frac{1}{2f_{q_0}}.$$

In this case for determining f_{q_0}, it is necessary to determine the average period of the impulses of the lower PBIS $\langle T_{q_0} \rangle$ and the average frequency of their appearances by means of the samples of the analyzed signal after its conversion and recording in memory the frequency f_v by the following expressions:

$$\langle T_{q_0} \rangle = \langle T_{1q_0} \rangle + \langle T_{0q_0} \rangle,$$

$$f_{q_0} = \frac{1}{\langle T_{q_0} \rangle}. \tag{2.12}$$

Then the values $\langle T_{1q_0} \rangle$, $\langle T_{0q_0} \rangle$ can be found in accordance with the following expressions:

$$\langle T_{1q_0} \rangle = \frac{1}{\gamma}\sum_{j=1}^{\gamma} T_{1q_{0j}} \text{ and } \langle T_{0q_0} \rangle = \frac{1}{\gamma}\sum_{j=1}^{\gamma} T_{0q_0}. \tag{2.13}$$

To ensure the necessary accuracy of the conversion is provided, it is expedient to choose Δt on the basis of the following condition:

$$\Delta t \leq \frac{1}{(2 \div 5) f_{q_0}}. \tag{2.14}$$

Experimental research has shown that in some cases measuring the time parameters of the lower digits of the PBIS is distorted by the influence of the error. Thus, in cases where the traditional technologies of the signal analysis are used, the sampling step can be determined by the frequency characteristics of the higher PBIS, i.e., by the average values of the duration of their unit T_{k1} and zero T_{k0} half-cycles. They are also determined by averaging out the time intervals in accordance with formulas (2.12), (2.13), i.e.,

$$\langle T_k \rangle = \langle T_{1q_k} \rangle + \langle T_{0q_k} \rangle,\ \langle T_{1q_k} \rangle = \frac{1}{\gamma}\sum_{j=1}^{\gamma} T_{1q_{kj}},\ \langle T_{0q_k} \rangle = \frac{1}{\gamma}\sum_{j=1}^{\gamma} T_{1q_{kj}}. \quad (2.15)$$

Taking into account that for the random stationary signals under the normal distribution law the following approximate equality takes place:

$$\langle T_{q_0} \rangle \approx \frac{1}{2}\langle T_{q_1} \rangle \approx \frac{1}{2}\langle T_{q_2} \rangle \approx \ldots \approx \frac{1}{2}\langle T_{n-1} \rangle,$$

it is advisable to determine Δt by means of the average period of the impulses of the higher digits of the PBIS. That allows one to represent the expression

$$\Delta t \geq \frac{1}{2f_{q_0}}$$

as follows:

$$\Delta t \leq \frac{1}{2 \cdot 2^k f_k}.$$

For example, for the q_1 th PBIS, the previous formula can be represented as

$$\Delta t \leq \frac{1}{2 \cdot 2f_1}. \quad (2.16)$$

For providing the given error of the formula of the determination Δt, (2.15) and (2.16) can be represented as follows:

$$\Delta t \leq \frac{1}{(2 \div 5) \cdot 2^k f_k}, \quad (2.17)$$

$$\Delta t \leq \frac{1}{(2 \div 5) \cdot 2f_1}. \quad (2.18)$$

It is obvious that the use of expressions (2.12), (2.14), and (2.16)–(2.18) allows one to sufficiently simplify the determination procedure of the sampling step Δt.

Let us consider the use of these formulas for determination Δt by the above example. Let us assume that the maximum amplitude of the trapezoid signal is 256 mV, the cycle time is 36 s, the duration of the origin-up protion is 8 s, the duration of the peak is 10 s, the fall time is 8 s, the zero value time is 10 s, and the 8-digital ADC are used for its conversion.

If we suppose that the frequency conversion of ADC is 1 kHz, the state of the first digit of ADC changes no more than 128 times because the amplitude of the signal reaches its maximum possible value of 256 mV for this time. The state of the second digit changes 64 times.

During the cycle, i.e., for 36 s, the state of the first digit changes 512 times, and the state of the second changes 256 times. It is obvious that the average frequency of the first digit is 512:36 = 14.2 Hz and the average frequency of the second digit is 256:36 = 7.1 Hz.

If we use the above-mentioned formulas, for the f_{q_0} th frequency, we get

$$\Delta t = \frac{1}{5 f_{q_0}} = \frac{1}{5 \cdot 14.2} = 0.0125\,\mathrm{s}\,,$$

and for the f_{q_1} th frequency, we get

$$\Delta t = \frac{1}{5 \cdot 2 \cdot 7.1} = 0.0125\,\mathrm{s}\,.$$

So for converting the mentioned signal by means of the 8-digital ADC, it is sufficient to realize the conversion by steps of 0.01 s, which corresponds to a sampling frequency of 100 Hz.

At the same time, the use of the sampling theorem meets various difficulties for the given signal, and the cutoff frequency appears to be more than 1000 Hz.

The given example shows that it is quite easy to determine the necessary sampling frequency taking into account the digit capacity of the ADC by software processing of the files formed as the result of the conversion of the initial signal. Thus here, in contrast to the traditional methods, the meteorological characteristics of the ADC itself are also automatically taken into account for determining the sampling step. So if the 9-digital ACD is used for the conversion of the considered signal, the found sampling frequency is equal to the average frequency of the first digit $1024:36 = 28.4$ kHz and the second $512:36 = 14.2$ Hz. At the same time, the sampling step Δt is equal to

$$\Delta t = \frac{1}{5 f_{q_0}} = \frac{1}{5 \cdot 28.4} \approx 0.0061\,\mathrm{s}\,,$$

$$\Delta t = \frac{1}{5 \cdot 2 \cdot f_{q_1}} = \frac{1}{5 \cdot 2 \cdot 14.2} \approx 0.0061\,\mathrm{s}\,.$$

That corresponds to the meteorological characteristics of the ADC, while this specific property of determining the samples of signal $g(i\Delta t)$ is not taken into account in practice during the use of the traditional methods.

So the considered algorithm of the position-selective choice of the sampling frequency is quite simple. At the same time, the meteorological properties of the measuring instruments are also taken into account. Due to this property, the sampling step chosen by this method appears to be close to the sampling step chosen by means of the other most accurate methods. The software determination of the sampling step Δt, according to the above-mentioned algorithm, can be represented as follows:

1. the initial signal $X(i\Delta t)$ is converted in digital form by the superfluous frequency f_v during the observation period T by means of ADC and the file of its samples is generated;
2. $\langle T_{q_k} \rangle$ is determined by Eq. (2.15):

$$\langle T_{q_k} \rangle = \langle T_{1q_k} \rangle + \langle T_{0q_k} \rangle;$$

3. f_{q_k} is found by Eq. (2.12):

$$f_{q_k} = \frac{1}{\langle T_{q_k} \rangle};$$

4. Δt is determined by the formula

$$\Delta t \le \frac{1}{5 \cdot 2^k f_{q_k}}\,.$$

It is necessary to perform the analysis of the noise $\varepsilon(i\Delta t)$ of the noisy signals $g(i\Delta t)$ as the data carrier when solving the problem of monitoring of the defect's origin. For this case, Δt_ε can be determined on the basis of the following condition:

$$\Delta t_\varepsilon \le \frac{1}{5 f_{q_0}}\,.$$

Here we take into consideration that the frequency of the lower PBIS represents the most high-frequency spectrum of the total signal $g(i\Delta t)$.

Taking into account that, according to the model (1.13), the high-frequency spectrum of the total signal $g(i\Delta t)$ continuously changes in the process of the evolution of the defect, it is advisable to perform the determination $\langle T_{1q_0}^m \rangle$, $\langle T_{0q_0}^m \rangle$, $\langle T_{q_0}^m \rangle$ by the expressions

$$T_{1q_0} = \frac{1}{\gamma}\left(\sum_{j=1}^{\gamma} T_{1q_{0j}} - T_{1q_{0(j-\gamma)}} + T_{1q_{0\nu}} \right), \tag{2.19}$$

$$T_{1q_0} = \frac{1}{\gamma}\left(\sum_{j=1}^{\gamma} T_{1q_{0j}} - T_{1q_{0\nu-\gamma}} + T_{1q_{0\nu}} \right), \tag{2.20}$$

$$T_{q_0} = T_{1q_0} + T_{0q_0}, \tag{2.21}$$

$$f_{q_0} = \frac{1}{5T_{q_0}}, \tag{2.22}$$

where

$$j = 1 \div \nu, \; \nu = \gamma + 1 \div \nu^*,$$

$$\nu^* = \frac{T_1 + T_2 + T_3 + T_4}{\Delta t_\varepsilon}.$$

It is easy to ensure that the opportunity of the adaptation of the sampling step in accordance with the evolution of the defect appears during the use of the expressions (2.19) and (2.20).

It is clear that Δt_ε changes gradually by the evolution of the defect. The spectra of the noise $\varepsilon(i\Delta t)$ are close to the spectra of the useful signal, and the steps Δt_ε and Δt_ε^m are the same at period T_4.

2.5 Position-Binary Detecting Defect Origin by Using Noise as a Data Carrier

Let us consider the use of the correlation between the defect origin and the value of the noise by the position-binary technology [14, 44–46]. As mentioned earlier, the values of the binary codes of the corresponding digits

q_k of the samples $g(i\Delta t)$ of the signal $g(t)$ at the beginning of each sampling step Δt are assumed to be equal to

$$g_{\mathrm{rem}(n-1)}(i\Delta t) = g(i\Delta t),$$

where

$$g(i\Delta t) > 2^n;\ g_{\mathrm{rem}(n-1)}(i\Delta t) = g(i\Delta t).$$

Then the signals $q_k(i\Delta t)$ are iteratively formed as the code 1 or 0. At the same time, the samples $X(i\Delta t)$ are compared with the value $2^{n-1}\Delta g$ at the first step. The value $q_{n-1}(i\Delta t)$ is taken to be equal to 1 for $g(i\Delta t) \geq 2^{n-1}\Delta g$. And the remainder value $g_{\mathrm{rem}(n-2)}(i\Delta t)$ is determined by the difference

$$g(i\Delta t) - 2^{n-1}\Delta g = g_{\mathrm{rem}(n-2)}(i\Delta t). \tag{2.23}$$

The sequence of these signals $q_k(i\Delta t)$ is the position-binary-impulse signals (PBIS), the sum of which is equal to the initial signal, i.e.,

$$X(i\Delta t) \approx q_{n-1}(i\Delta t) + q_{n-2}(i\Delta t) + \ldots + q_1(i\Delta t) + q_0(i\Delta t) = X^*(i\Delta t). \tag{2.24}$$

They are reflected as the noise $\varepsilon(i\Delta t)$ during the defect origin, and the signal $g(i\Delta t) = X(i\Delta t) + \varepsilon(i\Delta t)$ is formed as the output of the sensor. The short-term impulses $q_{\varepsilon k}(i\Delta t)$, the duration of which is many times less than the position signals $q_k(i\Delta t)$, are formed by influence of $\varepsilon(i\Delta t)$ in the representation of $g(i\Delta t)$ by the PBIS. In [19, 20, 22, 23] it is shown that they can be marked out by the following expressions:

$$q^*_{\varepsilon k}(i\Delta t) = \begin{cases} 1,\ \mathit{if}\ \ \overline{q_k((i-1)\Delta t)} \wedge q_k(i\Delta t) \wedge \overline{q_k((i+1)\Delta t)} \vee q_k((i-1)\Delta t) \wedge \overline{q_k(i\Delta t)} \wedge q_k((i+1)\Delta t), \\ 0,\ \mathit{if}\ \ q_k((i-1)\Delta t) \wedge q_k(i\Delta t) \wedge \overline{q_k((i+1)\Delta t)} \vee \overline{q_k((i-1)\Delta t)} \wedge \overline{q_k(i\Delta t)} \wedge q_k((i+1)\Delta t), \\ q_k((i-1)\Delta t) \wedge \overline{q_k(i\Delta t)} \wedge \overline{q_k((i+1)\Delta t)} \vee \overline{q_k((i-1)\Delta t)} \wedge q_k(i\Delta t) \wedge q_k((i+1)\Delta t), \end{cases} \tag{2.25}$$

$$q^*_{\varepsilon k}(i\Delta t) = \begin{cases} 1\ \ \mathit{if}\ \ q_k((i-1)\Delta t) = 0,\ q_k(i\Delta t) = 1,\ q_k((i+1)\Delta t) = 0, \\ 0\ \ \mathit{if}\ \ q_k((i-1)\Delta t) = q_k(i\Delta t) = q_k((i+1)\Delta t), \\ -1\ \ \mathit{if}\ \ q_k((i-1)\Delta t) = 1,\ q_k(i\Delta t) = 0,\ q_k((i+1)\Delta t) = 1. \end{cases} \tag{2.26}$$

The position noises $q_{\varepsilon k}(i\Delta t)$ of the noisy signal $X(i\Delta t)$ can be formed and marked out in the coding process of each position signal by formula (2.23) and expressions (2.25) and (2.26). It is obvious that their sum is the approximate value of the samples of the noise, i.e.,

$$\varepsilon^*(i\Delta t) \approx q_{\eta 0}(i\Delta t) + q_{\eta 1}(i\Delta t) + q_{\eta 2}(i\Delta t) + \cdots + q_{\eta k}(i\Delta t)$$

$$+ \cdots + q_{\eta(m-1)}(i\Delta t) = \sum_{k=0}^{m-1} q_{\eta k}(i\Delta t). \tag{2.27}$$

Then the approximate values of the samples of the useful signal $X^*(i\Delta t)$ can be determined by the difference

$$X^*(i\Delta t) \approx g(i\Delta t) - \varepsilon^*(i\Delta t) \approx g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t). \tag{2.28}$$

At the same time, the estimates of the variance, the estimates of the spectral characteristics of the noise, the estimates of the mutually correlation function, and the estimates of the correlation coefficient between the noise and the useful signal can be determined by the following expressions:

$$D_\varepsilon^* = \frac{1}{N}\sum_{i=1}^{N} \varepsilon^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t)\right]^2, \tag{2.29}$$

$$R_{x\varepsilon}^*(0) = \frac{1}{N}\sum_{i=1}^{N}\left[g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t)\right]\left[\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t)\right], \tag{2.30}$$

$$r_{x\varepsilon}^* = \frac{R_{x\varepsilon}(0)}{\sqrt{D_\varepsilon R_{xx}(0)}} = \frac{\sum_{i=1}^{N}\left[g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t)\right]\left[\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t)\right]}{\sqrt{\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t)\right]^2 \sum_{i=1}^{N}\left[g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t)\right]^2}}, \tag{2.31}$$

$$R_{g\varepsilon}^*(0) = \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)\varepsilon(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\left[g(i\Delta t)\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t)\right], \tag{2.32}$$

$$r_{g\varepsilon} = \frac{R_{g\varepsilon}(0)}{\sqrt{D_\varepsilon R_{gg}(0)}} \approx \frac{\sum_{i=1}^{N}\left[g(i\Delta t)\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t)\right]}{\sqrt{\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}^*(i\Delta t)\right]^2 \sum_{i=1}^{N} \dot{g}^2(i\Delta t)}}, \tag{2.33}$$

$$a_{n\varepsilon} = \frac{2}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\cos n\omega(i\Delta t) \approx \frac{2}{N}\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}^{*}(i\Delta t)\right]\cos n\omega(i\Delta t), \quad (2.34)$$

$$b_{n\varepsilon} = \frac{2}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\sin n\omega(i\Delta t) \approx \frac{2}{N}\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}^{*}(i\Delta t)\right]\sin n\omega(i\Delta t). \quad (2.35)$$

So the process of the defect origin is reflected on the lower position-binary-impulse signals $q_{\varepsilon 0}(i\Delta t)$, $q_{\varepsilon 1}(i\Delta t)$, $q_{\varepsilon 2}(i\Delta t)$, …, $q_{\varepsilon k}(i\Delta t)$, which can be marked out by the expressions (2.25) and (2.26) and can be used for determination of the estimate of the noise $\varepsilon(i\Delta t)$ by the expressions (2.29)–(2.35). Therefore, monitoring the defect at its origin becomes possible by the obtained estimates D_{ε}^{*}, $R_{x\varepsilon}^{*}(0)$, $R_{g\varepsilon}^{*}(0)$, $r_{x\varepsilon}^{*}$, $r_{g\varepsilon}^{*}$, $a_{n\varepsilon}^{*}$, and $b_{n\varepsilon}^{*}$ as a result of the use of the position-binary technology. Thus, the real opportunities of timely detection of the origin of the defects leading to the emergency condition of a diagnosed object appear.

In references [19–23] it is shown that if the condition of an object is stable, then during time T the ratio of the number $N_{\varepsilon k}$ of signals $q_{\varepsilon k}(i\Delta t)$ to the total number N_{qk} of positional-impulse signals $q_k(i\Delta t)$

$$K_{q_0} = \frac{N_{\varepsilon_0}}{N_{q_0 k}},\ K_{q_1} = \frac{N_{\varepsilon_1}}{N_{q_1 k}},\ \ldots,\ K_{q_{m-1}} = \frac{N_{\varepsilon_{(m-1)}}}{N_{q_{(m-1)}k}} \quad (2.36)$$

is the nonrandom value. At the same time, from the beginning of the process of the formation of a defect in all positional signals, the number $N_{\varepsilon k}$ is increased during time T. Hence, since this time, the magnitudes of the coefficients K_{q_0}, K_{q_1}, ..., $K_{q_{m-1}}$ will also vary. Therefore, they are the informative indicators, and they can be used to increase the reliability of monitoring results when solving the problem of detecting the defect's origin.

The performed research shows that solving the problem of monitoring with traditional methods does not give satisfactory results for a great number of the most important objects. At the same time, the use of the correlation between the defect origin and the change of the coefficients K_{q_0}, K_{q_1}, K_{q_2}, …, and other characteristics of the noise obtained by the position-binary technology give the reliable results. For example, analysis of the signals obtained in the drilling process, in the compressor station operation, etc., shows that such characteristics of the noise as D_{ε}^{*}, $R_{x\varepsilon}^{*}(0)$, $r_{x\varepsilon}^{*}$, K_{q_0}, K_{q_1}, …, K_{q_n} contain important and useful information, allowing one to detect the process of the defect's origin.

It is obvious that their use opens great possibilities for solving the corresponding problems of monitoring. As another example, it is easy to show the possibility of the use of this technology in medicine. The performed research shows that in many cases the initial stage of the various diseases has no an effect on both the corresponding signals and the estimates of their correlation and spectral characteristics. The beginning of the pathological-physiological processes is simultaneously reflected as the noise in the electrocardiograms, electroencephalograms, and other signals sufficiently early. Their detection and analysis also open great possibilities for monitoring the beginning of various diseases by means of position-binary technology.

3 Technology of Digital Analysis of Noise as a Carrier of Information About the Beginning of a Defect's Origin

3.1 Features of Analyzing Noise as a Data Carrier

As was shown in Chapter 1, in most cases the beginning of a defect's origin reflects in signals $g(i\Delta t)$ collected from sensors as a high-frequency noise. Therefore, for monitoring the defect at the beginning of its origin, it is necessary to extract the information in the noise. In this chapter, one of the possible variants for this problem is considered.

It is known that in traditional technologies for eliminating noise (noise) influence on the results of problems, methods of filtration are often used. They give good results when the spectrum of the filter coincides with the spectrum of the noise. At the same time, for many real processes the spectrum and variance of the noise change in time in a wide range and classical conditions are not fulfilled. For these reasons for eliminating noise influence on the result of signal processing, one has to enlarge the range of a "filter" spectrum. In its turn, it distorts the legitimate signal much more.

Besides, our investigations show that in some cases the noise arises as a result of certain processes occurring in controlled objects. Here, the noise became an information carrier that disappears in the process of filtration. So, the very important and, in many cases, unique information is lost. In view of this, there is the necessity of creating a technology that allows one to extract the information involved in the noise of noisy signals. The importance of this work is connected with the fact that the possibility of creating a technology of detecting the first changes in the objects on the initial stage arises. It does not mean at all that the traditional technologies are bad or unnecessary. I think that besides traditional technologies it is necessary and expedient to have alternative technologies. The time is right to have them. It is connected with the fact that for many years the main advantage of information technologies was believed to be the savings in computer resources. Nowadays, due to the great possibilities of modern computers, one can create effective technologies at the expense of sophistication of computer processes. The present monograph is written on the basis of the work performed in the above lead. In contrast to traditional technologies, where the volume of the information extracted from the total signal is reduced at the expense of the filtration, in the suggested alternative technology this defect is eliminated at the expense of separating and analyzing the noise. This qualitative difference opens wide possibilities for increasing the range of solving problems on the basis of analyzing noisy signals, because due to this there arises a number of advantages:

1. In fulfilling classical conditions, the reliability of results of signal analysis considerably increases in contrast to the traditional technologies.
2. There appears the possibility of analyzing noisy signals and of receiving the adequate results in case of lack of obedience to classical conditions. In case of applying traditional technologies, one does not receive adequate results.
3. Owing to the analysis of the noise as a carrier of legitimate information in noisy signals, the possibility of solving various problems of diagnostics, recognition, identification, forecasting, as well as the problems of monitoring a defect's origin make their appearance now. Solving these problems by means of traditional technologies is sometimes impossible.

In this case, many traditional technologies are included in suggested technologies as a special case [34, 37, 38, 40, 41, 43, 45, 57, 62]. In effect, the suggested theory allows one to use noise as legitimate information carriers because there is a relationship between the signal noise and hidden microchanges that precede the beginning of failures on real objects. Using the noise as a carrier of information gives an increase of the volume of the extracted information taken from analyzed signals. Here, long before changing object signal characteristics that reflect the beginning of abnormal microchanges in the object state, the following changes occur: noise variance estimates changes, correlations, and spectral characteristics of noise and correlation coefficient between the signal and noise change, too. Their calculation opens possibilities for solving a great number of the most important problems whose solution was difficult with traditional technologies. We give the following example of application of the suggested technologies:

1. There appears a possibility of creating technology of failure prediction by means of noise analysis on deep-seawater stationary platforms, on compressor stations and communications, on objects of drilling, on power plants, on hydrotechnical objects, and so on.

2. There appears a possibility of applying noise analysis technology for increasing the reliability of results of airplane diagnostics and forecasting changes to its technical states.
3. There appears a possibility of creating a technology of increasing a quantity of obtained information about the origination of seismic processes by means of the analysis of noise of seismic signals as information carriers.
4. There appears a possibility of increasing the reliability of results of experimental investigations in various fields of science and technology as well as in processing geophysical prospecting operations.
5. There appears a possibility of creating information technology of analysis and solving problems of control and identification of objects operating in cyclic mode.
6. There appears a possibility of improving the adequacy of mathematical models of objects of control in industry, engineering, transport, and so on.
7. There appears a possibility of creating technology of forecasting various diseases before their obvious symptoms appear.
8. There appears a possibility to considerably increase the functional possibilities and to increase the reliability of making decisions of the industrial information systems.

One can suggest that in this chapter, methods, algorithms, and information technologies on its basis would be an appropriate section of signal analysis theory, spectral analysis theory, correlation analysis theory, pattern recognition theory, random processes theory, the theory of mathematical models, and identification, and so on. They may be used in appropriate sections of many textbooks and scientific leads on cybernetics, informatics, aviation, engineering, medicine, biology, and so on, where methods, algorithms, and technologies are applied in one way or another.

3.2 Problems of Monitoring a Defect's Origin by Considering Noise as a Data Carrier

As stated earlier, in traditional methods the specific features of forming noisy signals are not sufficiently taken into consideration. In these methods the possibility of extracting and considering the information involved in noises is missing [55, 61]. In this connection on the modern stage of information technologies, the technology of noise analysis and its application is of great vital importance both for increasing reliability of solved problems of signal processing and for using the noise as a carrier of useful information [12–16, 28, 31, 32, 36, 46, 52, 56, 63].

The analysis of measuring information obtained from sensors of various technical and biological objects shows that in most cases the real signal $g(t)$ consists of a mixture of the legitimate signal $X(t)$ and the noise $\varepsilon(t)$; moreover, as a rule the noise $\varepsilon(t)$ differs from the "white noise" and the frequency of its spectrum exceeds frequencies of spectrum of legitimate signal $X(t)$ considerably, and there is often the lack of obedience to classical conditions [2, 14, 31, 52]. For this reason at the output of primary sensors after sampling with the step Δt, the real signal model can be represented as follows:

$$g(i\Delta t) = x(i\Delta t) + a(i\Delta t) + b(i\Delta t) + c(i\Delta t) + \ldots + \varepsilon_m(i\Delta t) = X(i\Delta t) + \varepsilon(i\Delta t). \quad (3.1)$$

Various factors influence the formation of the noise $a(t)$, $b(t)$, ..., $c(t)$, ..., $\varepsilon(t)$. Some of them $\varepsilon(t)$ reflect indirectly certain processes occurring in investigated and controlled objects, and the noise is the carriers of valuable information.

Presently, for eliminating the influence of the noise on the result of solving various problems, the methods of filtration are commonly used [12, 14]. If one can choose the filter eliminating only the noise spectrum of the legitimate signal, one may obtain acceptable results. If the choice of the range of the spectrum of the noise filter is unfortunate, the legitimate signal is distorted. In case the noise has useful information, the latter is irrevocably lost [12].

Besides, for real signals $g(i\Delta t)$, not only are the classical conditions not fulfilled, but also in various periods of signal realization the considerable changes of the range of the spread $\varepsilon(i\Delta t)$ as well as the changes of the spectrum occur [13, 33]. Thus, the realization of filtration gives rise to some difficulty or does not give the desired results.

By virtue of the above, without derogating the importance of the filtration, it is expedient to reduce the solution of the problem in question to the creation of the technology of extracting information from the noise. The above technologies allow one to use this information both for error correction of the result of analysis when classical algorithms are used, and for solving other vital problems for which the application of traditional technologies is impossible.

Here, the following important problems must be solved first:

1. Decomposing the noise signal $g(i\Delta t)$ into the legitimate signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$.

2. Determining the variance D_ε, the distribution law $W(\varepsilon)$, and the correlation function $R_{\varepsilon\varepsilon}(\mu)$ of noise $\varepsilon(i\Delta t)$, cross-correlation function $R_{x\varepsilon}(\mu)$, as well as the correlation coefficient $r_{x\varepsilon}$ between the legitimate signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$.
3. Realizing the spectral analysis of the noise $\varepsilon(i\Delta t)$ and the spectral analysis of the legitimate signal separately $X(i\Delta t)$.
4. Eliminating the noise influence on the result of the correlation and spectral analysis of the noisy signal $g(i\Delta t)$.
5. Creating the information technology for solving a problem of noise monitoring of the beginning of the defect's origin and the noise forecasting of the change of the condition of objects during the process of their normal operation.

Solving all these problems is considered in corresponding sections of the book in detail.

3.3 Methods of Determining the Noise Variance for the Case of Absence of Correlation Between Legitimate Signal and Noise

Various problems of recognition, identification, diagnostics, etc. are being solved by means of statistical characteristic estimates in information systems at present. The efficiency of the systems to a great degree depends on the precision of determining values of variance, estimates of auto- and cross-correlation functions of measuring information $g(t)$, $\eta(t)$ consisting of the sum of useful signals $X(t)$, $y(t)$, and noise $\varepsilon(t)$, $\phi(t)$, respectively. Application of existing algorithms for computation of estimates of correlation function $R_{gg}(\tau)$ admits that the noise $\varepsilon(t)$ is "white noise." For real technological processes, however, as a rule, the noise variance in values of the estimate $R_{gg}(\tau)$ with $\tau = 0$ is essential, and without regard for it, the results of the listed problems very often prove to be unsatisfactory [2, 33]. From the results given in papers [14, 33, 34, 37, 38, 40, 41], it is obvious that for improvement of correlation matrix, a stipulation determining the variance D_ε of the noise $\varepsilon(t)$ and estimated errors of the correlation function are of great importance. This section proposes algorithms for determining an estimate of the variance D_ε of noise $\varepsilon(t)$ for the case when measuring information $g(t)$ consists of the sum of the legitimate signal $X(t)$ and the noise $\varepsilon(t)$ [27, 38, 41].

It is known [14, 36] that for estimation of autocorrelation function $R_{xx}(\tau)$ of the legitimate signal $X(t)$ with time shift $\tau = 0$, $\tau = \Delta t$, $\tau = 2\Delta t$, the following equalities occur:

$$\left| \frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t)dt - \frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)dt \right| = \Delta R_1, \tag{3.2}$$

$$\left| \frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)dt - \frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+2\cdot\Delta t)dt \right| = \Delta R_2, \tag{3.3}$$

one can write the relations

$$\left\| \left[\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t)dt - \frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)dt \right] \right.$$

$$\left. - \left[\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)dt - \frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+2\cdot\Delta t)dt \right] \right\| << \Delta R_1 \tag{3.4}$$

or equivalent expressions

$$\left| R_{xx}(\tau = 0\cdot\Delta t) - R_{xx}(\tau = 1\cdot\Delta t) \right| = \Delta R_1, \tag{3.5}$$

$$\left| R_{xx}(\tau = 1\cdot\Delta t) - R_{xx}(\tau = 2\Delta t) \right| = \Delta R_2, \tag{3.6}$$

$$\left\| \left[R_{xx}(\tau = 0\cdot\Delta t) - R_{xx}(\tau = 1\cdot\Delta t) \right] \right.$$

$$\left. - \left[R_{xx}(\tau = 1\cdot\Delta t) - R_{xx}(\tau = 2\cdot\Delta t) \right] \right\| << \Delta R_1. \tag{3.7}$$

By virtue of the fact that the estimates of correlation function $R_{xx}(\tau)$ for most of the technological parameters at $\tau = 0$, $\tau = \Delta t$, $\tau = 2\Delta t$, and sampling step $\Delta t \le (0.1 \div 0.05) f_c$ (f_c is the cut-off frequency) become closer values, and ΔR_1, ΔR_2 prove to be commensurable with the quantization step by level ΔX, which is determined by resolution of measuring apparatus (for example, for the analog-digital converter it equals the weight of the least significant digit [33, 41]), the equalities (3.5)–(3.7) can be represented in the form

$$\left|\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t)\,dt-\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)\,dt\right|\approx \Delta X\,, \tag{3.8}$$

$$\left|\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)\,dt-\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+2\Delta t)\,dt\right|\approx \Delta X\,, \tag{3.9}$$

$$\left\|\left[\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t)\,dt-\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)\,dt\right]\right.$$

$$\left.-\left[\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)\,dt-\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+2\Delta t)\,dt\right]\right\|\le \Delta X \tag{3.10}$$

or

$$R_{xx}(\tau=0\cdot\Delta t)-R_{xx}(\tau=1\cdot\Delta t)\approx \Delta X\,,$$

$$R_{xx}(\tau=1\cdot\Delta t)-R_{xx}(\tau=2\cdot\Delta t)\approx \Delta X\,,$$

$$\left\|\left[R_{xx}(\tau=0\cdot\Delta t)-R_{xx}(\tau=1\cdot\Delta t)\right]-\left[R_{xx}(\tau=1\cdot\Delta t)-R_{xx}(\tau=2\cdot\Delta t)\right]\right\|\le \Delta X.$$

When the observation time T approaches infinity and the sampling step Δt approaches zero, the estimates $R_{xx}(\tau=0\cdot\Delta t)$, $R_{xx}(\tau=1\cdot\Delta t)$, and $R_{xx}(\tau=2\cdot\Delta t)$ prove so much closer that inequalities (3.8)–(3.10) become true in the form

$$\left|\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t)\,dt-\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)\,dt\right|\le \Delta X\,, \tag{3.11}$$

$$\left|\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)\,dt-\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+2\Delta t)\,dt\right|\le \Delta X\,, \tag{3.12}$$

$$\left\|\left[\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t)\,dt-\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)\,dt\right]\right.$$

$$-\left[\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)\,dt-\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+2\Delta t)\,dt\right]\Bigg\|\approx 0 \quad (3.13)$$

or

$$\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\left[R_{xx}(\tau=0\cdot\Delta t)-R_{xx}(\tau=1\cdot\Delta t)\right]\le \Delta X\,,$$

$$\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\left[R_{xx}(\tau=1\cdot\Delta t)-R_{xx}(\tau=2\cdot\Delta t)\right]\le \Delta X\,,$$

$$\left|\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\left[R_{xx}(\tau=0\cdot\Delta t)-R_{xx}(\tau=1\cdot\Delta t)\right]\right|$$

$$-\left|\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\left[R_{xx}(\tau=1\cdot\Delta t)-R_{xx}(\tau=2\cdot\Delta t)\right]\right|\approx 0\,,$$

which are equivalent to

$$\lim_{\substack{\Delta t\to 0\\ T\to\infty}}\left\{\left[R_{xx}(\tau=0\cdot\Delta t)-R_{xx}(\tau=1\cdot\Delta t)\right]\right.$$

$$\left.-\left[R_{xx}(\tau=1\cdot\Delta t)-R_{xx}(\tau=2\cdot\Delta t)\right]\right\}\approx 0\,.$$

Expressions (3.11)–(3.13) allow the following approximation equality to be considered true:

$$\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t)\,dt-\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)\,dt$$

$$\approx\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)\,dt-\frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+2\cdot\Delta t)\,dt \quad (3.14)$$

or

$$R_{xx}(\tau=0\cdot\Delta t)-R_{xx}(\tau=1\cdot\Delta t)\approx R_{xx}(\tau=1\cdot\Delta t)-R_{xx}(\tau=2\cdot\Delta t). \quad (3.15)$$

The validity of the equality was proved by a large number of computer experiments [14, 34, 37, 38, 40, 41]. These experiments have, first of all, shown that for continuous slow technological processes where low-frequency spectra (0.01÷10 Hz) predominate, with the relation of sampling frequency f_D and cut-off frequency f_c in the form $f_c : f_D \geq (10 \div 20) f_c$, inequality (3.14) normally holds. For instance, for technological parameters of oil refinery and oil chemical processes such as temperature, pressure, expenditure, etc., with $f_D \geq 100$ Hz and for signals of biological processes such as electroencephalogram and electrocardiogram with sampling frequency $f_D \geq (2 \div 3)$ kHz, the equality also holds. Naturally, in determining the variance D_ε of noises of higher-frequency processes, it is necessary that the sampling step be chosen by time Δt, that is, the sampling frequency f_D so that equality (3.14) can hold. Modern measuring apparatus permits the use of coding signals with frequency over 10 MHz because these relations hold for a sufficiently wide class of random processes.

$$\frac{1}{T}\int_0^T \dot{X}(t)\dot{\varepsilon}(t)\,dt \approx 0, \tag{3.16}$$

$$\frac{1}{T}\int_0^T \dot{X}(t)\dot{\varepsilon}(t+\Delta t)\,dt \approx 0, \tag{3.17}$$

$$\frac{1}{T}\int_0^T \dot{X}(t)\dot{\varepsilon}(t+2\cdot\Delta t)\,dt \approx 0, \tag{3.18}$$

then one can write

$$\frac{1}{T}\int_0^T \dot{g}(t)\dot{g}(t)\,dt \approx \frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t)\,dt + D_\varepsilon, \tag{3.19}$$

$$\frac{1}{T}\int_0^T \dot{g}(t)\dot{g}(t+\Delta t)\,dt \approx \frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+\Delta t)\,dt, \tag{3.20}$$

$$\frac{1}{T}\int_0^T \dot{g}(t)\dot{g}(t+2\Delta t)\,dt \approx \frac{1}{T}\int_0^T \dot{X}(t)\dot{X}(t+2\Delta t)\,dt \tag{3.21}$$

or

$$R_{gg}(\tau = 0 \cdot \Delta t) \approx R_{xx}(\tau = 0 \cdot \Delta t) + D_\varepsilon,$$

$$R_{gg}(\tau = \Delta t) \approx R_{xx}(\tau = \Delta t),$$

$$R_{gg}(\tau = 2 \cdot \Delta t) \approx R_{xx}(\tau = 2 \cdot \Delta t).$$

Thus, with a glance to (3.14) and (3.19)–(3.21), we shall get

$$D_\varepsilon \approx \frac{1}{T}\int_0^T \dot{g}(t)\dot{g}(t)\,dt + \frac{1}{T}\int_0^T \dot{g}(t)\dot{g}(t+2\Delta t)\,dt - 2\frac{1}{T}\int_0^T \dot{g}(t)\dot{g}(t+\Delta t)\,dt$$

$$= \frac{1}{T}\int_0^T \left[\dot{g}(t)\dot{g}(t) + \dot{g}(t)\dot{g}(t+2\Delta t) - 2\dot{g}(t)\dot{g}(t+\Delta t)\right]dt. \quad (3.22)$$

According to this expression, while conditions (3.14)–(3.21) hold, the formula for determining the estimate of the variance D_ε of noise $\dot{\varepsilon}(t)$ can be represented in the form

$$D_\varepsilon = R_{gg}(\mu = 0) + R_{gg}(\mu = 2) - 2R_{gg}(\mu = 1). \quad (3.23)$$

Expressions (3.22) and (3.23) indicate that when conditions (3.16)–(3.21) occur, noise $\dot{\varepsilon}(t)$ does not influence the result of processing until $\tau = 0$, and the estimate error $R_{gg}(\mu = 1)$ consists only of the noise variance D_ε.

3.4 Digital Technology of Analyzing Noise and Legitimate Signal in Case of Absence of Correlation

As was stated above, in practice the measuring information received from many real processes is the mixture of the legitimate signal $X(t)$ and the noise $\varepsilon(t)$. Assume that $g(i\Delta t) = X(i\Delta t) + \varepsilon(i\Delta t)$ is a centered sampled stationary random signal with a normal distribution law consisting of the legitimate signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$, with mathematical expectation close to zero: $m_\varepsilon \approx 0$. With due regard to the influence of the noise

$\varepsilon(i\Delta t)$, the autocorrelation function $R_{gg}(\mu)$ of the centered sampled random signal $g(t) = g(t) - m_g$ (m_g is the mathematical expectation of $g(t)$) can be represented as follows:

$$R_{gg}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left[\dot{X}(i\Delta t) + \dot{\varepsilon}(i\Delta t)\right]\left[\dot{X}((i+\mu)\Delta t) + \dot{\varepsilon}((i+\mu)\Delta t)\right]$$

$$= \frac{1}{N}\sum_{i=1}^{N}\Big[\dot{X}(i\Delta t)\dot{X}((i+\mu)\Delta t) + \dot{X}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t)$$

$$+\dot{\varepsilon}(i\Delta t)\dot{X}((i+\mu)\Delta t) + \dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t)\Big]$$

$$= R_{xx}(\mu) + \lambda(\mu), \tag{3.24}$$

where

$$\lambda(\mu) = \frac{1}{N}\sum_{i=1}^{N}\left[\dot{X}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t) + \dot{\varepsilon}(i\Delta t)\dot{X}((i+\mu)\Delta t) + \dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t)\right]$$

is the error of the correlation function $R_{gg}(\mu)$.

It is known that the values $\varepsilon(i\Delta t)$ and $\varepsilon((i+\mu)\Delta t)$ of the noise for $\mu \neq 0$ do not correlate with each other. Therefore,

$$\frac{1}{N}\sum_{i=1}^{N} \dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t) \approx 0. \tag{3.25}$$

The mean magnitude of the squares of the samples of the noise is equal to the estimate of the variance D_ε of the noise:

$$\frac{1}{N}\sum_{i=1}^{N} \dot{\varepsilon}^2(i\Delta t) = D_\varepsilon. \tag{3.26}$$

Taking into account expressions (3.25) and (3.26), Eq. (3.24) can be represented as follows:

$$+\dot{X}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t)+\dot{\varepsilon}(i\Delta t)\dot{X}((i+\mu)\Delta t)\Big]+D_{\varepsilon}, \quad (3.27)$$

$$R_{gg}(\mu \neq 0)=\frac{1}{N}\sum_{i=1}^{N}\Big[\dot{X}(i\Delta t)\dot{X}((i+\mu)\Delta t)$$

$$+\dot{X}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t)+\dot{\varepsilon}(i\Delta t)\dot{X}((i+\mu)\Delta t)\Big]. \quad (3.28)$$

When the sampling step is

$$\Delta t \leq (10 \div 20)\frac{1}{f_c}$$

for $\mu = 0$, Δt, and $2\Delta t$, those estimates move closer to each other and their difference becomes commensurable with the sampling step of the magnitude Δx, which is determined by means of the resolution of the measurement device, for example, for the ADC (analog-digital converter) it equals the weight of the least-significant digit [14, 38, 41]. As the observation time T approaches infinity and the sampling step Δt approaches zero, the estimates $R_{xx}(\mu = 0)$, $R_{xx}(\mu = \Delta t)$, and $R_{xx}(\mu = 2\Delta t)$ prove to be such close magnitudes that the inequality holds

$$\lim_{\substack{T\to\infty \\ \Delta t\to 0}}\left[R_{xx}(\mu=0)-R_{xx}(\mu=\Delta t)\right] << \Delta X \,, \quad (3.29)$$

$$\lim_{\substack{T\to\infty \\ \Delta t\to 0}}\left[R_{xx}(\mu=\Delta t)-R_{xx}(\mu=2\Delta t)\right] << \Delta X \,; \quad (3.30)$$

this inequality can be represented as follows:

$$R_{xx}(\mu=0)-R_{xx}(\mu=\Delta t) \approx R_{xx}(\mu=\Delta t)-R_{xx}(\mu=2\Delta t)\,. \quad (3.31)$$

In the case when conditions (3.29)–(3.31) apply and when there is no correlation between the legitimate signal $X(t)$ and the noise $\varepsilon(t)$, i.e., under the following conditions:

$$\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\dot{\varepsilon}(i\Delta t)\approx 0\,, \quad (3.32)$$

$$\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)\approx 0\,, \tag{3.33}$$

$$\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\dot{\varepsilon}((i+2)\Delta t)\approx 0\,, \tag{3.34}$$

we can write

$$R_{gg}(\mu=0)\approx R_{xx}(\mu=0)+D_{\varepsilon}\,, \tag{3.35}$$

$$R_{gg}(\mu=1)\approx R_{xx}(\mu=1)\,, \tag{3.36}$$

$$R_{gg}(\mu=2)\approx R_{xx}(\mu=2)\,. \tag{3.37}$$

Thus, taking into account expressions (3.31)–(3.37), we can write

$$D_{\varepsilon}\approx R_{gg}(\mu=0)+R_{gg}(\mu=2)-2R_{gg}(\mu=1)\,. \tag{3.38}$$

If conditions (3.32)–(3.37) hold, expression (3.38) for determining the variance D_{ε} of the noise $\varepsilon(t)$ can be represented as follows:

$$D_{\varepsilon}\approx\frac{1}{N}\sum_{i=1}^{N}\left[\dot{g}^{2}(i\Delta t)+\dot{g}(i\Delta t)\dot{g}((i+2)\Delta t)-2\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)\right]. \tag{3.39}$$

Now consider the opportunity of determining approximate values of samples of the noise $\varepsilon^{*}(i\Delta t)$.

It is clear that if we had the magnitudes of noise samples in digital form, i.e., $\varepsilon(i\Delta t)$, we could determine noise variances according to the known following expression:

$$D_{\varepsilon}=\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}^{2}\left(i\Delta t\right). \tag{3.40}$$

The direct determination of samples of noise $\varepsilon(i\Delta t)$ from samples $\varepsilon(i\Delta t)$ of noisy signal is impossible. At the same time, it is evident from (3.39) and (3.40) that approximate sample magnitudes $\varepsilon(i\Delta t)$ may be found from the following expression:

$$\dot{\varepsilon}^{*}\left(i\Delta t\right)=\sqrt{\dot{\varepsilon}^{2}\left(i\Delta t\right)}$$

$$\approx \sqrt{\left|\dot{g}^2(i\Delta t) + \dot{g}(i\Delta t)\dot{g}((i+2)\Delta t) - 2\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)\right|}. \tag{3.41}$$

The analysis of expressions (3.40) and (3.41) shows that $\varepsilon^*(i\Delta t)$ will differ from the true magnitude $\varepsilon(i\Delta t)$ by the value $\varepsilon^*(i\Delta t) - \varepsilon(i\Delta t)$.

Here, the following equality will hold:

$$P\left\{\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}^*(i\Delta t) - \frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}(i\Delta t) \approx 0\right\} = 1, \tag{3.42}$$

where P is the sign of probability.

In view of the following:

$$\dot{g}^2(i\Delta t) + \dot{g}(i\Delta t)\dot{g}((i+2)\Delta t) - 2\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t) = \varepsilon'(i\Delta t), \tag{3.43}$$

formula (3.41) for determining approximate magnitudes of noise samples may be represented as follows:

$$\dot{\varepsilon}^*(i\Delta t) = \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\dot{g}^2(i\Delta t) + \dot{g}(i\Delta t)\dot{g}((i+2)\Delta t) - 2\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)\right|}$$

$$= \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}, \tag{3.44}$$

where $\operatorname{sgn}\varepsilon'(i\Delta t)$ is the sign of the radicand.

The analysis of expressions (3.40)–(3.41) will be considered in the next paragraph. To conclude we present appropriate formulae of noise analysis and legitimate signal analysis for the case when the correlation $r_{x\varepsilon}$ between the legitimate signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ is equal to zero.

$$m_\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\varepsilon(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\varepsilon^*(i\Delta t)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}, \tag{3.45}$$

$$D_\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\varepsilon^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\varepsilon^{*2}(i\Delta t) = \frac{1}{N}\sum_{i=1}^{N}\varepsilon'(i\Delta t), \tag{3.46}$$

$$R_{x\varepsilon}(\mu) = \frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\dot{\varepsilon}(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\dot{X}^*(i\Delta t)\dot{\varepsilon}^*(i\Delta t)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left[\dot g(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}\right]\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}, \quad (3.47)$$

$$m_x = \frac{1}{N}\sum_{i=1}^{N}\dot X(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\left[\dot g(i\Delta t) - \dot\varepsilon^*(i\Delta t)\right]$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left[\dot g(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}\right], \quad (3.48)$$

$$D_x = \frac{1}{N}\sum_{i=1}^{N}\dot X^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\left[\dot g(i\Delta t) - \dot\varepsilon^*(i\Delta t)\right]^2$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left[\dot g(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}\right]^2. \quad (3.49)$$

Based on algorithms (3.39) and (3.49), one may formulate the information technology of analysis of the noise $\varepsilon(i\Delta t)$ and the legitimate signal $X(i\Delta t)$ for the case without correlation.

1. According to the technology described in Section 2.3, the step of the sampling Δt of the noisy signal $g(i\Delta t)$ and of the noise Δt_ε is determined.
2. Supposing that the cut-off frequency of the noise will be an order of magnitude greater than the cut-off frequency of the noisy signal, the condition $\Delta t_\varepsilon \le (0{,}05 \div 0{,}10)\Delta t$ is verified.
3. The estimations of the autocorrelation function $R_{gg}(\mu = 0)$, $R_{gg}(\mu = \Delta t_\varepsilon)$, $R_{gg}(\mu = 2\Delta t_\varepsilon)$, $R_{gg}(\mu = 3\Delta t_\varepsilon)$ for noisy signal $g(i\Delta t)$ sampled with the step of sampling of the noise are determined with the step Δt_ε. The validity of the following relation is verified:
   $R_{gg}(\mu = 1\Delta t_\varepsilon) - R_{gg}(\mu = 2\Delta t_\varepsilon) \approx R_{gg}(\mu = 2\Delta t_\varepsilon) - R_{gg}(\mu = 3\Delta t_\varepsilon)$.
   When fulfilling this equality, the obtained step Δt_ε is taken as satisfied. Otherwise, it is decreased again, and the procedure is repeated until the mentioned equality holds.
4. According to the expression,

   $$m_\varepsilon \approx \frac{1}{N}\sum_{i=1}^{N}\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\dot g^2(i\Delta t) + \dot g(i\Delta t)\dot g((i+2)\Delta t) - 2\dot g(i\Delta t)\dot g((i+1)\Delta t)\right|}$$

   The mathematical expectation of the noise $\varepsilon(i\Delta t)$ is determined.

5. According to the expression

$$D_{\varepsilon} \approx \frac{1}{N}\sum_{i=1}^{N}\left[\dot{g}^{2}(i\Delta t)+\dot{g}(i\Delta t)\dot{g}\big((i+2)\Delta t\big)-2\dot{g}(i\Delta t)\dot{g}\big((i+1)\Delta t\big)\right],$$

the noise variance estimate is determined.

6. According to the expression

$$R_{x\varepsilon} \approx \frac{1}{N}\sum_{i=1}^{N}\Big\{\Big[\dot{g}(i\Delta t)-\operatorname{sgn}\varepsilon'(i\Delta t)$$

$$\times\sqrt{\left|\dot{g}^{2}(i\Delta t)+\dot{g}(i\Delta t)\dot{g}\big((i+2)\Delta t\big)-2\dot{g}(i\Delta t)\dot{g}\big((i+1)\Delta t\big)\right|}\Big]$$

$$\times\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\dot{g}^{2}(i\Delta t)+\dot{g}(i\Delta t)\dot{g}\big((i+2)\Delta t\big)-2\dot{g}(i\Delta t)\dot{g}\big((i+1)\Delta t\big)\right|}\Big\},$$

we calculate the estimate of the cross-correlation function between the legitimate signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$.

7. According to the expression

$$m_{x} \approx \frac{1}{N}\sum_{i=1}^{N}\Big[\dot{g}(i\Delta t)-\operatorname{sgn}\varepsilon'(i\Delta t)$$

$$\times\sqrt{\left[\dot{g}^{2}(i\Delta t)+\dot{g}(i\Delta t)\dot{g}\big((i+2)\Delta t\big)-2\dot{g}(i\Delta t)\dot{g}\big((i+1)\Delta t\big)\right]}\Big],$$

the estimation of the mathematical expectation of the legitimate signal $X(i\Delta t)$ is determined.

8. In the following way we determine the estimation of the variance of the legitimate signal $X(i\Delta t)$:

$$D_{x} \approx \frac{1}{N}\sum_{i=1}^{N}\Big[\dot{g}(i\Delta t)-\operatorname{sgn}\varepsilon'(i\Delta t)$$

$$\times\sqrt{\left|\dot{g}^{2}(i\Delta t)+\dot{g}(i\Delta t)\dot{g}\big((i+2)\Delta t\big)-2\dot{g}(i\Delta t)\dot{g}\big((i+1)\Delta t\big)\right|}\Big]^{2}.$$

3.5 Digital Technology of Determining the Noise Variance in Case of Availability of Correlation Between Legitimate Signal and Noise

Let us consider the possibility of determining the approximate magnitudes of noise when there is a correlation between the legitimate signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$, i.e., $r_{x\varepsilon} \neq 0$, but in this case $r_{\varepsilon_i\varepsilon_j} = 0$, $i \neq j$, and

$\Delta t = \Delta t_\varepsilon$ [16]. Taking into account that $g(i\Delta t) = X(i\Delta t) + \varepsilon(i\Delta t)$, the formula (3.39) may be represented as follows:

$$D'_\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\left[\dot{g}^2(i\Delta t) + \dot{g}(i\Delta t)\dot{g}((i+2)\Delta t) - 2\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)\right]$$

$$= \frac{1}{N}\sum_{i=1}^{N}\Big\{\Big[\dot{X}(i\Delta t)\dot{X}(i\Delta t) + \dot{X}(i\Delta t)\dot{X}((i+2)\Delta t)$$

$$-2\dot{X}(i\Delta t)\dot{X}((i+1)\Delta t)\Big]$$

$$+\left[\dot{X}(i\Delta t)\dot{\varepsilon}(i\Delta t) + \dot{X}(i\Delta t)\dot{\varepsilon}((i+2)\Delta t) - 2\dot{X}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)\right]$$

$$+\left[\dot{\varepsilon}(i\Delta t)\dot{X}(i\Delta t) + \dot{\varepsilon}(i\Delta t)\dot{X}((i+2)\Delta t) - 2\dot{\varepsilon}(i\Delta t)\dot{X}((i+1)\Delta t)\right]$$

$$+\Big[\dot{\varepsilon}^2(i\Delta t) + \dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+2)\Delta t) - 2\dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)\Big]\Big\}. \quad (3.50)$$

As stated above, at $\Delta t = \Delta t_\varepsilon$, the following equality holds:

$$\dot{X}(i\Delta t) \approx \dot{X}((i+1)\Delta t) \approx \dot{X}((i+2)\Delta t). \quad (3.51)$$

Here, the results of calculating the first and third brackets in expression (3.50) will be equal to zero because of the following:

$$\lim_{N\to\infty} P\Big\{\frac{1}{N}\sum_{i=1}^{N}\Big[\dot{X}(i\Delta t)\dot{X}(i\Delta t) + \dot{X}(i\Delta t)\dot{X}((i+2)\Delta t)$$

$$-2\dot{X}(i\Delta t)\dot{X}((i+1)\Delta t)\Big] \approx 0\Big\} = 1, \quad (3.52)$$

$$\lim_{N\to\infty} P\Big\{\frac{1}{N}\sum_{i=1}^{N}\Big[\dot{\varepsilon}(i\Delta t)\dot{X}(i\Delta t) + \dot{\varepsilon}(i\Delta t)\dot{X}((i+2)\Delta t)$$

$$-2\dot{\varepsilon}(i\Delta t)\dot{X}((i+1)\Delta t)\Big] \approx 0\Big\} = 1. \quad (3.53)$$

Based on the above, expression (3.50) may be represented as follows:

$$D'_\varepsilon = \frac{1}{N}\sum_{i=1}^{N}\Big\{\left[\dot{X}(i\Delta t)\dot{\varepsilon}(i\Delta t) + \dot{X}(i\Delta t)\dot{\varepsilon}((i+2)\Delta t) - 2\dot{X}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)\right]$$

$$+\left[\dot{\varepsilon}^2(i\Delta t)+\dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+2)\Delta t)-2\dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)\right]\Big\}. \quad (3.54)$$

In this expression, the result of the first bracket will be

$$\frac{1}{N}\sum_{i=1}^{N}\left[\dot{X}(i\Delta t)\dot{\varepsilon}(i\Delta t)+\dot{X}(i\Delta t)\dot{\varepsilon}((i+2)\Delta t)-2\dot{X}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)\right]$$

$$=\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\Delta\dot{\varepsilon}(i\Delta t), \quad (3.55)$$

where

$$\Delta\dot{\varepsilon}(i\Delta t)=\dot{\varepsilon}(i\Delta t)+\dot{\varepsilon}((i+2)\Delta t)-2\dot{\varepsilon}((i+1)\Delta t). \quad (3.56)$$

For lack of the correlation between $\varepsilon(i\Delta t)$ and $\varepsilon((i+1)\Delta t)$, $\varepsilon((i+2)\Delta t)$, the sum of the products $\varepsilon(i\Delta t)\varepsilon((i+2)\Delta t)$ and $2\varepsilon(i\Delta t)\varepsilon((i+1)\Delta t)$ will be equal to zero. Based on that, the result of the fourth bracket in expression (3.50) will be

$$\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}^2(i\Delta t).$$

Therefore, expression (3.50) will be represented as follows:

$$D'_{\varepsilon}=R_{\varepsilon\varepsilon}(\mu=0)=\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\Delta\dot{\varepsilon}(i\Delta t)+\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}^2(i\Delta t). \quad (3.57)$$

Now consider the expression analogous to (3.39):

$$D''_{\varepsilon}=\frac{1}{N}\sum_{i=1}^{N}\Big[\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)+\dot{g}(i\Delta t)\dot{g}((i+3)\Delta t)$$

$$-2\dot{g}(i\Delta t)\dot{g}((i+2)\Delta t)\Big] \quad (3.58)$$

and transform it as in the case of formula (3.50). Then the results of calculation of the first, third, and fourth brackets will be zero and the results of the second bracket will be

$$D''_{\varepsilon}=\frac{1}{N}\sum_{i=1}^{N}\Big[\dot{X}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)+\dot{X}(i\Delta t)\dot{\varepsilon}((i+3)\Delta t)$$

$$-2\dot{X}(i\Delta t)\dot{\varepsilon}((i+2)\Delta t)\Big]=\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\Delta\dot{\varepsilon}((i+1)\Delta t), \tag{3.59}$$

where

$$\Delta\dot{\varepsilon}((i+1)\Delta t)=\dot{\varepsilon}((i+1)\Delta t)+\dot{\varepsilon}((i+3)\Delta t)-2\dot{\varepsilon}((i+2)\Delta t). \tag{3.60}$$

It is shown below that the results of calculations

$$\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\Delta\dot{\varepsilon}(i\Delta t) \text{ and } \frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\Delta\dot{\varepsilon}((i+1)\Delta t)$$

in expressions (3.55), (3.59) will be close magnitudes, i.e.,

$$\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\Delta\dot{\varepsilon}(i\Delta t)\approx\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\Delta\dot{\varepsilon}((i+1)\Delta t). \tag{3.61}$$

In this equality the following relations between the products $X(i\Delta t)\Delta\varepsilon(i\Delta t)$ and $X(i\Delta t)\Delta\varepsilon((i+1)\Delta t)$ may take place:

$$\dot{X}(i\Delta t)\Delta\dot{\varepsilon}((i+1)\Delta t)=\dot{X}(i\Delta t)\Delta\dot{\varepsilon}(i\Delta t), \tag{3.62}$$

$$\dot{X}(i\Delta t)\Delta\dot{\varepsilon}((i+1)\Delta t)<\dot{X}(i\Delta t)\Delta\dot{\varepsilon}(i\Delta t), \tag{3.63}$$

$$\dot{X}(i\Delta t)\Delta\dot{\varepsilon}((i+1)\Delta t)>\dot{X}(i\Delta t)\Delta\dot{\varepsilon}(i\Delta t). \tag{3.64}$$

The analysis of these inequalities shows that multipliers of these inequalities with unit time shift recur in increasing the magnitudes i, $i=1,2,3,4,...,N$, alternately.

Here, the number of inequalities (3.63) will be equal to the number of inequalities (3.64). Thus, the following equality may be considered to be true:

$$P\Big[\dot{X}(i\Delta t)\Delta\dot{\varepsilon}((i+1)\Delta t)\geq\dot{X}(i\Delta t)\Delta\dot{\varepsilon}(i\Delta t)\Big]$$

$$\approx P\Big[\dot{X}(i\Delta t)\Delta\dot{\varepsilon}((i+1)\Delta t)\leq\dot{X}(i\Delta t)\Delta\dot{\varepsilon}(i\Delta t)\Big], i=\overline{1, N}. \tag{3.65}$$

Therefore, the following equality may also be considered to hold true:

$$P\left\{\frac{1}{N}\sum_{i=1}^{N}\left[\dot{X}\left(i\Delta t\right)\Delta\dot{\varepsilon}\left(i\Delta t\right)-\dot{X}\left(i\Delta t\right)\Delta\dot{\varepsilon}\left(\left(i+1\right)\Delta t\right)\right]\approx 0\right\}=1. \quad (3.66)$$

It follows from (3.65) and (3.66) that in processing the signal $g(i\Delta t)$ according to (3.58) or (3.59) with good regard to expressions (3.62)–(3.66), the following change is allowed: $\dot{X}(i\Delta t)\Delta\dot{\varepsilon}(i\Delta t)$ to $\dot{X}(i\Delta t)\Delta\dot{\varepsilon}((i+1)\Delta t)$. Therefore, formula (3.59) may be represented as follows:

$$D''=\frac{1}{N}\sum_{i=1}^{N}\dot{X}\left(i\Delta t\right)\left[\dot{\varepsilon}\left(\left(i+1\right)\Delta t\right)+\dot{\varepsilon}\left(\left(i+3\right)\Delta t\right)-2\dot{\varepsilon}\left(\left(i+2\right)\Delta t\right)\right]$$

$$=\frac{1}{N}\sum_{i=1}^{N}\dot{X}\left(i\Delta t\right)\Delta\dot{\varepsilon}\left(i\Delta t\right). \quad (3.67)$$

It follows that in case there is a correlation between the legitimate signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$, i.e., $r_{x\varepsilon}\neq 0$, the difference $D'_{\varepsilon}-D''_{\varepsilon}$ will be a magnitude of noise variance D_{ε}, i.e. [16],

$$D_{\varepsilon}=\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}^{2}\left(i\Delta t\right)\approx D'_{\varepsilon}-D''_{\varepsilon}=\frac{1}{N}\sum_{i=1}^{N}\dot{X}\left(i\Delta t\right)\Delta\dot{\varepsilon}\left(i\Delta t\right)$$

$$+\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}^{2}\left(i\Delta t\right)-\frac{1}{N}\sum_{i=1}^{N}\dot{X}\left(i\Delta t\right)\Delta\dot{\varepsilon}\left(i\Delta t\right). \quad (3.68)$$

Therefore,

$$D_{\varepsilon}=\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}^{2}\left(i\Delta t\right)=D'_{\varepsilon}-D''_{\varepsilon}$$

$$=\frac{1}{N}\sum_{i=1}^{N}\Big\{\left[\dot{g}^{2}\left(i\Delta t\right)+\dot{g}\left(i\Delta t\right)\dot{g}\left(\left(i+2\right)\Delta t\right)-2\dot{g}\left(i\Delta t\right)\dot{g}\left(\left(i+1\right)\Delta t\right)\right]$$

$$-\left[\dot{g}\left(i\Delta t\right)\dot{g}\left(\left(i+1\right)\Delta t\right)+\dot{g}\left(i\Delta t\right)\dot{g}\left(\left(i+3\right)\Delta t\right)\right.$$

$$\left.-2\dot{g}\left(i\Delta t\right)\dot{g}\left(\left(i+2\right)\Delta t\right)\right]\Big\}. \quad (3.69)$$

By means of the expressions for determining e' and e'', one can check that the formula (3.69) holds. For this purpose, first represent the expression for determining e' (3.50) as follows:

$$D' = \left[\frac{1}{N} \sum_{i=1}^{N} \dot{X}(i\Delta t)\dot{X}(i\Delta t) + \frac{1}{N} \sum_{i=1}^{N} X(i\Delta t) X((i+2)\Delta t) - \frac{2}{N} \sum_{i=1}^{N} X(i\Delta t) X((i+1)\Delta t) \right]$$

$$+ \left[\frac{1}{N} \sum_{i=1}^{N} \varepsilon(i\Delta t) X(i\Delta t) + \frac{1}{N} \sum_{i=1}^{N} \varepsilon(i\Delta t) X((i+2)\Delta t) - \frac{2}{N} \sum_{i=1}^{N} \varepsilon(i\Delta t) X((i+1)\Delta t) \right]$$

$$+ \left[\frac{1}{N} \sum_{i=1}^{N} X(i\Delta t)\varepsilon(i\Delta t) + \frac{1}{N} \sum_{i=1}^{N} \dot{X}(i\Delta t)\varepsilon((i+2)\Delta t) \right.$$

$$\left. - \frac{2}{N} \sum_{i=1}^{N} X(i\Delta t)\varepsilon((i+1)\Delta t) \right]$$

$$+ \left[\frac{1}{N} \sum_{i=1}^{N} \dot{\varepsilon}^2(i\Delta t) + \frac{1}{N} \sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon((i+2)\Delta t) - \frac{2}{N} \sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon((i+1)\Delta t) \right]$$

$$\approx \left[R_{xx}(0) + R_{xx}(2\Delta t_\varepsilon) - 2R_{xx}(2\Delta t_\varepsilon)\right] + \left[R_{\varepsilon x}(0) + R_{\varepsilon x}(2\Delta t_\varepsilon) - 2R_{\varepsilon x}(1\Delta t_\varepsilon)\right]$$

$$+ \left[R_{x\varepsilon}(0) + R_{x\varepsilon}(2\Delta t_\varepsilon) - 2R_{x\varepsilon}(1\Delta t_\varepsilon)\right] + \left[D_\varepsilon + R_{\varepsilon\varepsilon}(2\Delta t_\varepsilon) - 2R_{\varepsilon\varepsilon}(1\Delta t_\varepsilon)\right]. \quad (3.70)$$

Taking into account that for the noise $\varepsilon(i\Delta t)$, the following equalities hold:

$$R_{\varepsilon\varepsilon}(2\Delta t) = \frac{1}{N} \sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon((i+2)\Delta t) = 0,$$

$$R_{\varepsilon\varepsilon}(1\Delta t) = \frac{1}{N} \sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon((i+1)\Delta t) = 0,$$

$$R_{\varepsilon\varepsilon}(0) = \frac{1}{N} \sum_{i=1}^{N} \varepsilon^2(i\Delta t) = D_\varepsilon,$$

and taking the notion

$$\left.\begin{array}{l} R_{xx}(0) + R_{xx}(2\Delta t_\varepsilon) - 2R_{xx}(1\Delta t_\varepsilon) = \Delta_1' \\ R_{\varepsilon x}(0) + R_{\varepsilon x}(2\Delta t_\varepsilon) - 2R_{\varepsilon x}(1\Delta t_\varepsilon) = \Delta_2' \\ R_{x\varepsilon}(0) + R_{x\varepsilon}(2\Delta t_\varepsilon) - 2R_{x\varepsilon}(1\Delta t_\varepsilon) = \Delta_3' \end{array}\right\}, \quad (3.71)$$

one can represent expression (3.50) as follows:

$$D' = \Delta_1' + \Delta_2' - \Delta_3' + D_\varepsilon. \quad (3.72)$$

Taking into account that $g(i\Delta t) = X(i\Delta t) + \varepsilon(i\Delta t)$ and using the expression for determining $e''(i\Delta t)$ (3.58), we have

$$\begin{aligned}
D'' &= \frac{1}{N}\sum_{i=1}^{N} e''(i\Delta t) = \frac{1}{N}\Bigg[\sum_{i=1}^{N} X(i\Delta t)X((i+1)\Delta t) \\
&\quad + \sum_{i=1}^{N} X(i\Delta t)X((i+3)\Delta t) - 2\sum_{i=1}^{N} X(i\Delta t)X((i+2)\Delta t) \\
&\quad + \left(\sum_{i=1}^{N} X(i\Delta t)\varepsilon((i+1)\Delta t) + \sum_{i=1}^{N} X(i\Delta t)\varepsilon((i+3)\Delta t) - 2\sum_{i=1}^{N} X(i\Delta t)\varepsilon((i+2)\Delta t)\right) \\
&\quad + \left(\sum_{i=1}^{N} \varepsilon(i\Delta t)X((i+1)\Delta t) + \sum_{i=1}^{N} \varepsilon(i\Delta t)X((i+3)\Delta t) - 2\sum_{i=1}^{N} \varepsilon(i\Delta t)X((i+2)\Delta t)\right) \\
&\quad + \left(\sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon((i+1)\Delta t) + \sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon((i+3)\Delta t) - 2\sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon((i+2)\Delta t)\right) \\
&= \left[\frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)X((i+1)\Delta t) + \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)X((i+3)\Delta t) - \frac{2}{N}\sum_{i=1}^{N} X(i\Delta t)X((i+2)\Delta t)\right] \\
&\quad + \left[\frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)X((i+1)\Delta t) + \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)X((i+3)\Delta t) - \frac{2}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)X((i+2)\Delta t)\right] \\
&\quad + \left[\frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon((i+1)\Delta t) + \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon((i+3)\Delta t) - \frac{2}{N}\sum_{i=1}^{N} X(i\Delta t)\varepsilon((i+2)\Delta t)\right] \\
&\quad + \left[\frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon((i+1)\Delta t) + \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon((i+3)\Delta t) - \frac{2}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon((i+2)\Delta t)\right] \\
&\approx \left[R_{xx}(1\Delta t) + R_{xx}(3\Delta t) - 2R_{xx}(2\Delta t)\right] + \left[R_{\varepsilon x}(1\Delta t) + R_{\varepsilon x}(2\Delta t) - 2R_{\varepsilon x}(2\Delta t)\right] \\
&\quad + \left[R_{\varepsilon\varepsilon}(1\Delta t) + R_{x\varepsilon}(3\Delta t) - 2R_{x\varepsilon}(2\Delta t)\right].
\end{aligned} \tag{3.73}$$

Taking into account that $R_{\varepsilon\varepsilon}(1\Delta t) \approx 0$, $R_{\varepsilon\varepsilon}(2\Delta t) \approx 0$, $R_{\varepsilon\varepsilon}(3\Delta t) \approx 0$, and taking a notion that is analogous to (3.71), we have

$$\left.\begin{aligned} R_{xx}(1\Delta t_\varepsilon)+R_{xx}(3\Delta t_\varepsilon)-2R_{xx}(2\Delta t)=\Delta_1'' \\ R_{\varepsilon x}(1\Delta t_\varepsilon)+R_{\varepsilon x}(2\Delta t_\varepsilon)-2R_{\varepsilon x}(2\Delta t)=\Delta_2'' \\ R_{x\varepsilon}(1\Delta t_\varepsilon)+R_{x\varepsilon}(3\Delta t_\varepsilon)-2R_{x\varepsilon}(2\Delta t)=\Delta_3'' \end{aligned}\right\}; \tag{3.74}$$

one can represent as follows the expression for determining D'':

$$D''=\Delta_1''+\Delta_2''+\Delta_3''. \tag{3.75}$$

Supposing that the following equalities hold:

$$\left.\begin{aligned} \Delta_1' \approx \Delta_1'' \\ \Delta_2' \approx \Delta_2'' \\ \Delta_3' \approx \Delta_3'' \end{aligned}\right\}, \tag{3.76}$$

and taking into account (3.72) and (3.75), we have

$$D_\varepsilon'-D_\varepsilon''=\Delta_1'+\Delta_2'+\Delta_3'+D_\varepsilon-(\Delta_1''+\Delta_2''+\Delta_3'')\approx D_\varepsilon, \tag{3.77}$$

This means that formula (3.69) holds for the case $r_{x\varepsilon}\neq 0$.

The investigations showed that in calculating the estimations of the magnitudes D', D'', one can represent the expressions for determining $e'(i\Delta t)$ and $e''(i\Delta t)$ as follows:

$$\left.\begin{aligned} e'(i\Delta t)=g(i\Delta t)g(i\Delta t)-2g(i\Delta t)g((i+2)\Delta t)+g(i\Delta t)g((i+4)\Delta t) \\ e''(i\Delta t)=g(i\Delta t)g((i+1)\Delta t)-2g(i\Delta t)g((i+3)\Delta t)+g(i\Delta t)g((i+5)\Delta t) \end{aligned}\right\}$$

$$\left.\begin{aligned} e'(i\Delta t)=g(i\Delta t)g(i\Delta t)-2g(i\Delta t)g((i+3)\Delta t)+g(i\Delta t)g((i+6)\Delta t) \\ e''(i\Delta t)=g(i\Delta t)g((i+1)\Delta t)-2g(i\Delta t)g((i+4)\Delta t)+g(i\Delta t)g((i+7)\Delta t) \end{aligned}\right\}$$

For the general case, the expressions for determining $e''(i\Delta t)$ и $e'(i\Delta t)$ have the following form:

$$e'(i\Delta t)=g(i\Delta t)g(\Delta t)-2g(i\Delta t)g((i+\mu-1)\Delta t)+g(i\Delta t)g((i+2\mu-2)\Delta t),$$

$$e''(i\Delta t)=g(i\Delta t)q((i+1)\Delta t)-2g(i\Delta t)g((i+\mu)\Delta t)+g(i\Delta t)g((i+2\mu-1)\Delta t). \tag{3.78}$$

Here μ is a time interval where the correlation between $\varepsilon(i\Delta t)$ и $X(i\Delta t)$ takes place. One can determine the maximum of this magnitude according to the following formula:

$$\mu = \frac{\Delta t}{\Delta t_{\varepsilon}}. \tag{3.79}$$

Thus, one can determine the variance of the noise D_{ε} of the noisy signal $g(i\Delta t)$ according to expression (3.69) with due regard to the time interval of the correlation $\Delta t = \mu \cdot \Delta t_{\varepsilon}$ between the noise and the legitimate signal using expressions (3.69) and (3.77).

3.6 Digital Technology of Separating Noise Samples and Legitimate Signal, and Determining Estimates of Its Statistical Characteristics

Taking into account the following:

$$\varepsilon'(i\Delta t) = \dot{g}^2(i\Delta t) + \dot{g}(i\Delta t)\dot{g}((i+2)\Delta t) - 2\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t), \tag{3.80}$$

$$\varepsilon''(i\Delta t) = \dot{g}(i\Delta t)\dot{g}((i+1)\Delta t) + \dot{g}(i\Delta t)\dot{g}((i+3)\Delta t) - 2\dot{g}(i\Delta t)\dot{g}((i+2)\Delta t), \tag{3.81}$$

the formula for determining $\varepsilon^*(i\Delta t)$ both in case there is a correlation and in case there is lack of it may be represented as follows:

$$\dot{\varepsilon}^*(i\Delta t) \approx \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}, \tag{3.82}$$

$$\dot{\varepsilon}^*(i\Delta t) \approx \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}. \tag{3.83}$$

These formulae are represented in the expanded form

$$\begin{aligned}
\dot{\varepsilon}^*(i\Delta t) \approx \operatorname{sgn}\Big\{&\left[\dot{g}^2(i\Delta t) + \dot{g}(i\Delta t)\dot{g}((i+2)\Delta t) - 2\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)\right] \\
&-\left[\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t) + \dot{g}(i\Delta t)\dot{g}((i+3)\Delta t)\right. \\
&\left.-2\dot{g}(i\Delta t)\dot{g}((i+2)\Delta t)\right]\Big\} \\
&\times\Big\{\Big|\dot{g}(i\Delta t)\left[\dot{g}(i\Delta t) + \dot{g}((i+2)\Delta t) - 2\dot{g}((i+1)\Delta t)\right]
\end{aligned}$$

$$-\dot{g}(i\Delta t)\left[\dot{g}((i+1)\Delta t)+\dot{g}((i+3)\Delta t)-2\dot{g}((i+2)\Delta t)\right]\Big]\Big\}^{\frac{1}{2}}, \quad (3.84)$$

$$\dot{\varepsilon}^*(i\Delta t)\approx \operatorname{sgn}\dot{g}(i\Delta t)\left[\dot{g}(i\Delta t)+\dot{g}((i+2)\Delta t)-2\dot{g}((i+1)\Delta t)\right]$$

$$\times\sqrt{\left|\dot{g}(i\Delta t)\left[\dot{g}(i\Delta t)+\dot{g}((i+2)\Delta t)-2\dot{g}((i+1)\Delta t)\right]\right|}. \quad (3.85)$$

It is evident that in processing signals $g(i\Delta t)$ for determining samples of the legitimate signal $X(i\Delta t)$, one can use the approximate magnitude of samples of the noise $\varepsilon^*(i\Delta t)$ [16]. Here in case $r_{x\varepsilon}=0$, the formula for determining the approximate magnitudes of samples of the legitimate signal will be represented as follows:

$$\dot{X}^*(i\Delta t)\approx \dot{g}(i\Delta t)-\dot{\varepsilon}^*(i\Delta t)=\dot{g}(i\Delta t)-\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}. \quad (3.86)$$

When $r_{x\varepsilon}\neq 0$, the samples $X^*(i\Delta t)$ will be found according to the formula

$$\dot{X}^*(i\Delta t)\approx \dot{g}(i\Delta t)-\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)|}. \quad (3.87)$$

It is clear that the possibility of determining approximate magnitudes of samples of noise and the legitimate signal open wide prospects for solving various application problems for which the application of filtration technology was not successful [16].

Summing up the above, the formulae of the analysis of the noise and the legitimate signal in case the correlation between them is not zero, i.e., $r_{x\varepsilon}\neq 0$, are represented as follows:

$$m_\varepsilon=\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}(i\Delta t)\approx\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}^*(i\Delta t)$$

$$=\frac{1}{N}\sum_{i=1}^{N}\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)|}, \quad (3.88)$$

$$D_\varepsilon=\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}^2(i\Delta t)\approx\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}^{*2}(i\Delta t)=\frac{1}{N}\sum_{i=1}^{N}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right], \quad (3.89)$$

$$R_{x\varepsilon}(\mu)=\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\dot{\varepsilon}(i\Delta t)=\frac{1}{N}\sum_{i=1}^{N}\dot{X}^{*}(i\Delta t)\dot{\varepsilon}^{*}(i\Delta t)$$

$$=\frac{1}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t)-\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right|}\right]$$

$$\times\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right|}, \tag{3.90}$$

$$m_x=\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\approx\frac{1}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t)-\dot{\varepsilon}^{*}(i\Delta t)\right]$$

$$=\frac{1}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t)-\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right|}\right], \tag{3.91}$$

$$D_x=\frac{1}{N}\sum_{i=1}^{N}\dot{X}^{2}(i\Delta t)\approx\frac{1}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t)-\dot{\varepsilon}^{*}(i\Delta t)\right]^{2}$$

$$=\frac{1}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t)-\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right|}\right]^{2}. \tag{3.92}$$

3.7 Algorithms for Determining the Distribution Law of Noise and the Correlation Coefficient

Having the approximate magnitudes $\varepsilon^{*}(i\Delta t)$, one can determine the correlation coefficient when the correlation between the legitimate signal and the noise exists according to the following expression [16]:

$$r_{x\varepsilon}=\frac{R_{x\varepsilon}(0)}{\sqrt{R_{xx}(0)R_{\varepsilon\varepsilon}(0)}}=\frac{R_{x\varepsilon}(0)}{\sqrt{D_x D_\varepsilon}}$$

$$\approx\frac{1}{N}\sum_{i=1}^{N}\left[\left[\dot{g}(i\Delta t)-\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right|}\right]\right.$$

$$\times \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\,\Bigg]$$

$$\times\left\{\left[\frac{1}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t) - \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\right]^2\right]\right.$$

$$\left.\times\left[\frac{1}{N}\sum_{i=1}^{N}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\right]\right\}^{-\frac{1}{2}}. \tag{3.93}$$

It is known that for determining $R_{x\varepsilon}(0)$, $R_{xx}(0)$, and $R_{\varepsilon\varepsilon}(0)$, one can also use the formulae of signed correlation functions [14]:

$$R_{x\varepsilon}^{zn}(0) = \frac{1}{N}\sum_{i=1}^{N}\operatorname{sgn}\dot{X}(i\Delta t)\operatorname{sgn}\dot{\varepsilon}(i\Delta t), \tag{3.94}$$

$$R_{xx}^{zn}(0) = \frac{1}{N}\sum_{i=1}^{N}\operatorname{sgn}\dot{X}(i\Delta t)\operatorname{sgn}\dot{X}(i\Delta t), \tag{3.95}$$

$$R_{\varepsilon\varepsilon}^{zn}(0) = \frac{1}{N}\sum_{i=1}^{N}\operatorname{sgn}\dot{\varepsilon}(i\Delta t)\operatorname{sgn}\dot{\varepsilon}(i\Delta t). \tag{3.96}$$

Considering that

$$\operatorname{sgn}\dot{X}(i\Delta t)\operatorname{sgn}\dot{X}(i\Delta t) = 1,$$

$$\operatorname{sgn}\dot{\varepsilon}(i\Delta t)\operatorname{sgn}\dot{\varepsilon}(i\Delta t) = 1,$$

$$\sum_{i=1}^{N}\operatorname{sgn}\dot{X}(i\Delta t)\operatorname{sgn}\dot{X}(i\Delta t) = N,$$

and

$$\sum_{i=1}^{N}\operatorname{sgn}\dot{\varepsilon}(i\Delta t)\operatorname{sgn}\dot{\varepsilon}(i\Delta t) = N,$$

we have

$$R_{xx}^{zn}(0) = \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \dot{X}\left(i\Delta t\right) \operatorname{sgn} \dot{X}\left(i\Delta t\right) = 1, \tag{3.97}$$

$$R_{\varepsilon\varepsilon}^{zn}(0) = \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \dot{\varepsilon}\left(i\Delta t\right) \operatorname{sgn} \dot{\varepsilon}\left(i\Delta t\right) = 1. \tag{3.98}$$

Then the formula for calculating the coefficient of correlation $r_{x\varepsilon}$ between the legitimate signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ can be represented as follows:

$$r_{x\varepsilon} = \frac{1}{N}\sum_{i=1}^{N} \ \operatorname{sgn}\ \dot{X}(i\Delta t) \operatorname{sgn}\ \dot{\varepsilon}(i\Delta t) \tag{3.99}$$

or

$$\begin{aligned} r_{x\varepsilon} \approx r_{x\varepsilon}^{*} &= \frac{1}{N}\sum_{i=1}^{N} \ \operatorname{sgn}\ \dot{X}^{*}(i\Delta t) \operatorname{sgn}\ \dot{\varepsilon}^{*}(i\Delta t) \\ &= \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn}\Big[\dot{g}\left(i\Delta t\right) - \operatorname{sgn}\left[\varepsilon'\left(i\Delta t\right) \right. \\ &\quad \left. - \varepsilon''\left(i\Delta t\right)\right] \sqrt{\left|\varepsilon'\left(i\Delta t\right) - \varepsilon''\left(i\Delta t\right)\right|}\Big] \\ &\quad \times \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right] \sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}. \end{aligned} \tag{3.100}$$

Clearly, the formula for calculating the coefficient of correlation $r_{g\varepsilon}$ between the noisy signal $g(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ has the following form:

$$r_{g\varepsilon} = \frac{1}{N}\sum_{i=1}^{N} \ \operatorname{sgn}\ \dot{g}(i\Delta t) \operatorname{sgn}\ \dot{\varepsilon}(i\Delta t) \tag{3.101}$$

or

$$\begin{aligned} r_{g\varepsilon} \approx r_{g\varepsilon}^{*} &= \frac{1}{N}\sum_{i=1}^{N} \ \operatorname{sgn}\ \dot{g}(i\Delta t) \operatorname{sgn}\ \dot{\varepsilon}^{*}(i\Delta t) \\ &= \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \dot{g}\left(i\Delta t\right) \operatorname{sgn}\left[\varepsilon'\left(i\Delta t\right)\right. \\ &\quad \left. - \varepsilon''\left(i\Delta t\right)\right] \sqrt{\left|\varepsilon'\left(i\Delta t\right) - \varepsilon''\left(i\Delta t\right)\right|}. \end{aligned} \tag{3.102}$$

In solving some applied problems, one may also use the estimates of the relay correlation functions [14].

$$R_{x\varepsilon}^{rl}(0) = \frac{1}{N}\sum_{i=1}^{N} \dot{X}(i\Delta t)\,\mathrm{sgn}\,\dot{\varepsilon}(i\Delta t), \tag{3.103}$$

$$R_{g\varepsilon}^{rl}(0) = \frac{1}{N}\sum_{i=1}^{N} \dot{g}(i\Delta t)\,\mathrm{sgn}\,\dot{\varepsilon}(i\Delta t), \tag{3.104}$$

or

$$\begin{aligned} R_{x\varepsilon}^{rl}(0) \approx R_{x\varepsilon}^{**}(0) &= \frac{1}{N}\sum_{i=1}^{N} \dot{X}^{*}(i\Delta t)\,\mathrm{sgn}\,\dot{\varepsilon}^{*}(i\Delta t) \\ &= \frac{1}{N}\sum_{i=1}^{N}\Big[\dot{g}(i\Delta t) - \mathrm{sgn}\big[\varepsilon'(i\Delta t) \\ &\quad - \varepsilon''(i\Delta t)\big]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\Big] \\ &\quad \times \mathrm{sgn}\big[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\big]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}, \end{aligned} \tag{3.105}$$

$$\begin{aligned} R_{g\varepsilon}^{rl}(0) \approx R_{g\varepsilon}^{**}(0) &= \frac{1}{N}\sum_{i=1}^{N} \dot{g}(i\Delta t)\,\mathrm{sgn}\,\dot{\varepsilon}^{*}(i\Delta t) \\ &= \frac{1}{N}\sum_{i=1}^{N} \dot{g}(i\Delta t)\,\mathrm{sgn}\big[\varepsilon'(i\Delta t) \\ &\quad - \varepsilon''(i\Delta t)\big]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}. \end{aligned} \tag{3.106}$$

It has been generally established that for realizing a noise distribution law, it is necessary to determine the number N of appropriate values of its curve $W(\varepsilon)$ according to the number N of the samples of the noise $\varepsilon(i\Delta t)$ during the time $T = N\Delta t$. For this purpose one may use an approximate magnitude of samples of an analogue of the noise $\varepsilon^{*}(i\Delta t)$. Here, in case there is no correlation between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$ and using the sample number N_1, N_2, …, N_m of the magnitude

$$\varepsilon^{*}(i\Delta t) = \mathrm{sgn}\,\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}$$

in the range from 0 to $\varepsilon_{\max}$ at regular intervals $\Delta\varepsilon$ with appropriate values $\varepsilon^{*}(i\Delta t)$, one may easily realize the distribution law $W(\varepsilon^{*})$. In case there is a correlation between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$, using the number of samples N_1, N_2, …, N_m of the magnitude

$$\dot{\varepsilon}^* = \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|},$$

one realizes the noise distribution law. According to expressions (3.82) and (3.83), in increasing $N \to \infty$ the calculated estimates N_1, N_2, …, N_m will tend to the required values of the distribution law of the noise $W(\varepsilon)$.

3.8 Algorithm for Determining the Arithmetic Mean Relative Error of Samples of Noisy Signals Caused by Noise

It is known that for calculating of the relative error in samples of noisy signal $g(i\Delta t)$, one can use the following expression [14, 26]:

$$\lambda_{\text{rel}}(i\Delta t) = \frac{\dot{\varepsilon}(i\Delta t)}{\dot{X}(i\Delta t)}. \tag{3.107}$$

It is obvious that for realization of this expression, one needs to know the samples $X(i\Delta t)$ and $\varepsilon(i\Delta t)$ preliminarily. Clearly, having the approximate values of samples of the noise $\varepsilon^*(i\Delta t)$ and the legitimate signal $X^*(i\Delta t)$, one can determine the approximate values of relative errors of samples $g(i\Delta t)$ according to the formula

$$\frac{\dot{\varepsilon}^*(i\Delta t)}{\dot{X}^*(i\Delta t)} = \lambda^*_{\text{rel}}(i\Delta t) \tag{3.108}$$

or

$$\lambda^*_{\text{rel}} = \begin{cases} \dfrac{\operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}}{\dot{g}(i\Delta t) - \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}} & \text{for} \quad r_{x\varepsilon} \neq 0, \\ \dfrac{\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}}{\dot{g}(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}} & \text{for} \quad r_{x\varepsilon} = 0. \end{cases} \tag{3.109}$$

At the same time, in many cases having the mean value of relative errors of samples $g(i\Delta t)$, one can realize the correction of errors of signal processing.

Clearly, using the relative errors $\lambda_{\text{rel}}(i\Delta t)$ of samples $g(i\Delta t)$, one can determine the estimate of average relative errors according to the formula

$$\overline{\lambda}_{\mathrm{rel}} \approx \frac{1}{N}\sum_{i=1}^{N}\lambda_{\mathrm{rel}}(i\Delta t) = \frac{1}{N}\sum_{i=1}^{N}\frac{\dot{\varepsilon}\left(i\Delta t\right)}{\dot{X}\left(i\Delta t\right)}. \tag{3.110}$$

Having the approximate values of samples of the noise $\varepsilon^*(i\Delta t)$ and the legitimate signal $X^*(i\Delta t)$, the formula for calculating the average relative error can be represented as follows:

$$\overline{\lambda}^*_{\mathrm{rel}} \approx \frac{1}{N}\sum_{i=1}^{N}\frac{\dot{\varepsilon}^*\left(i\Delta t\right)}{\dot{X}^*\left(i\Delta t\right)}. \tag{3.111}$$

This formula reflects the degree of the influence of the noise on the error of samples $g(i\Delta t)$ of the measurement information. Clearly, when the correlation between the legitimate signal and the noise differs from zero, i.e., $r_{x\varepsilon} \neq 0$, the formula $\overline{\lambda}^*_{\mathrm{rel}}$ has the followings form:

$$\overline{\lambda}^*_{\mathrm{rel}} \approx \frac{1}{N}\sum_{i=1}^{N}\frac{\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right|}}{\dot{g}(i\Delta t)-\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right|}}. \tag{3.112}$$

In case when there is no correlation between the legitimate signal and the noise, i.e., $r_{x\varepsilon} = 0$, this expression takes the form

$$\overline{\lambda}^*_{\mathrm{rel}} \approx \frac{1}{N}\sum_{i=1}^{N}\frac{\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}}{\dot{g}(i\Delta t)-\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}}. \tag{3.113}$$

In Refs. [5, 14, 26], it is shown that the determination $\overline{\lambda}^*_{\mathrm{rel}}$ according to this formula is of great practical interest for both spectral and correlation analysis.

However, it should be noted that in case we do not have the approximate estimates of the noise $\varepsilon^*(i\Delta t)$ and the legitimate signal $X^*(i\Delta t)$, one can determine the more approximate estimate of the average relative error $\overline{\lambda}_{\mathrm{rel}}$ of samples $g(i\Delta t)$ of noisy signals as a ratio of the mean-root-square deviation σ_ε of the noise $\varepsilon(i\Delta t)$ to the mean-root-square deviation σ_g of the total signal $g(i\Delta t)$:

$$\overline{\lambda}_{\mathrm{rel}} = \frac{\sigma_\varepsilon}{\sigma_g} = \frac{\sqrt{D_\varepsilon}}{\sqrt{D_g}}. \tag{3.114}$$

Clearly, in this case the estimates of the average relative error of samples $g(i\Delta t)$ of the noisy signal both for the presense and the absence of the correlation between the legitimate signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ i.e., $r_{x\varepsilon} = 0$ and $r_{x\varepsilon} \neq 0$, with due regard to (3.39) and (3.69), can be determined as follows:

$$\overline{\lambda}_{\text{rel}} = \frac{\sqrt{\sum_{i=1}^{N}\left[\dot{g}^2(i\Delta t) + \dot{g}(i\Delta t)\dot{g}((i+2)\Delta t) - 2\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)\right]}}{\sqrt{\sum_{i=1}^{N}\dot{g}^2(i\Delta t)}}, \tag{3.115}$$

$$\overline{\lambda}_{\text{rel}} = \frac{\sqrt{\sum_{i=1}^{N}\left[\dot{g}^2(i\Delta t) - 3\,\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t) + 3\,\dot{g}(i\Delta t)\dot{g}((i+2)\Delta t) - \dot{g}(i\Delta t)\dot{g}((i+3)\Delta t)\right]}}{\sqrt{\sum_{i=1}^{N}\dot{g}^2(i\Delta t)}}. \tag{3.116}$$

3.9 Digital Technology for Determining Information Signs of Monitoring a Defect's Origin When the Classical Conditions Are Not Fulfilled

Based on the above algorithms of the analysis of the noise $\varepsilon(i\Delta t)$ and of the legitimate signal, one can form information technology of the following procedures:

1. According to the methods described in Section 2.3, one can determine the step of sampling of the noisy signal $g(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$.
2. The following conditions are checked:

$$\left|R_{gg}(\tau = \Delta t_\varepsilon) - R_{gg}(\tau = 2\Delta t_\varepsilon)\right| \le \Delta X,$$

$$\left|R_{gg}(\tau = 2\Delta t_\varepsilon) - R_{gg}(\tau = 3\Delta t_\varepsilon)\right| \le \Delta X.$$

   If the conditions are fulfilled, the step of sampling Δt_ε is adequate. If the conditions are not fulfilled, the obtained value of Δt_ε is not adequate, it is decreased, and the above conditions are checked again. Δt_ε, found at the moment of fulfilling the above conditions, is considered to be adequate.
3. According to the following expression:

$$m_\varepsilon \approx \frac{1}{N}\sum_{i=1}^{N}\operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]$$

$$\times\left\{\left|\dot g(i\Delta t)\left[\dot g(i\Delta t)+\dot g((i+2)\Delta t)-2\dot g((i+1)\Delta t)\right]\right.\right.$$
$$\left.\left.-\dot g(i\Delta t)\left[\dot g((i+1)\Delta t)+\dot g((i+3)\Delta t)-2\dot g((i+2)\Delta t)\right]\right|\right\}^{\frac{1}{2}},$$

a mathematical expectation of the noise $\varepsilon(i\Delta t)$ is determined.

4. According to the following expression:

$$D_\varepsilon \approx \frac{1}{N}\sum_{i=1}^{N}\left\{\dot g(i\Delta t)\left[\dot g(i\Delta t)+\dot g((i+2)\Delta t)-2\dot g((i+1)\Delta t)\right]\right.$$
$$\left.-\dot g(i\Delta t)\left[\dot g((i+1)\Delta t)+\dot g((i+3)\Delta t)-2\dot g((i+2)\Delta t)\right]\right\},$$

the estimation of the variance of noise is determined.

5. According to the following expression:

$$R_{xx}(\mu) \approx \frac{1}{N}\sum_{i=1}^{N}\left[\dot g(i\Delta t)-\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\right.$$
$$\times\left\{\left|\dot g(i\Delta t)\left[\dot g(i\Delta t)+\dot g((i+2)\Delta t)\right.\right.\right.$$
$$-2\dot g((i+1)\Delta t)\big]-\dot g(i\Delta t)\big[\dot g((i+1)\Delta t)+\dot g((i+3)\Delta t)$$
$$\left.\left.\left.-2\dot g\ (i+2)\Delta t\ \right]\right|\right\}^{\frac{1}{2}}\Bigg]\times\operatorname{sgn}\left[\varepsilon'(i\Delta t)\right.$$
$$\left.-\varepsilon''(i\Delta t)\right]\left\{\left|\dot g(i\Delta t)\left[\dot g(i\Delta t)+\dot g((i+2)\Delta t)\right.\right.\right.$$
$$-2\dot g((i+1)\Delta t)\big]-\dot g(i\Delta t)\big[\dot g((i+1)\Delta t)$$
$$\left.\left.+\dot g((i+3)\Delta t)-2\dot g((i+2)\Delta t)\right]\right|\Big\}^{\frac{1}{2}},$$

the estimation of the correlation function between the legitimate signal $\dot X(i\Delta t)$ and the noise $\dot\varepsilon(i\Delta t)$ is determined.

6. According to the expression

$$m_x \approx \frac{1}{N}\sum_{i=1}^{N}\left[\dot g(i\Delta t)-\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\right.$$
$$\times\left\{\left|\dot g(i\Delta t)\left[\dot g(i\Delta t)+\dot g((i+2)\Delta t)-2\dot g((i+1)\Delta t)\right]\right.\right.$$
$$\left.\left.\left.-\dot g(i\Delta t)\left[\dot g((i+1)\Delta t)+\dot g((i+3)\Delta t)-2\dot g((i+2)\Delta t)\right]\right|\right\}^{\frac{1}{2}}\right],$$

the estimation of the mathematical expectation of the legitimate signal $X(i\Delta t)$ is determined.

7. According to the expression

$$D_x \approx \frac{1}{N}\sum_{i=1}^{N}\Big[\dot{g}(i\Delta t) - \operatorname{sgn}\big[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\big] \times \Big\{\Big|\dot{g}(i\Delta t)\big[\dot{g}(i\Delta t) + \dot{g}((i+2)\Delta t) - 2\dot{g}((i+1)\Delta t)\big] - \dot{g}(i\Delta t)\big[\dot{g}((i+1)\Delta t) + \dot{g}((i+3)\Delta t) - 2\dot{g}((i+2)\Delta t)\big]\Big|\Big\}^{\frac{1}{2}}\Big]^2,$$

the estimate of the mean value of the legitimate signal $X(i\Delta t)$ is determined.

8. Determining the distribution law of the noise $\varepsilon(i\Delta t)$ when the coefficient of correlation is zero is realized as follows. A minimal magnitude of $\varepsilon_{\min}$ is given for all samples of approximate magnitudes $\varepsilon^*(i\Delta t)$ and the condition is checked:

$$[\varepsilon_{\min} + j\Delta x] \le \varepsilon^*(i\Delta t) \le [\varepsilon_{\min} + (j+1)\Delta x].$$

These conditions take the following form at $r_{x\varepsilon} = 0$:

$$\varepsilon_{\min} + j\Delta x \le \operatorname{sgn}\varepsilon'(i\Delta t)\Big\{\Big|\dot{g}(i\Delta t)\big[\dot{g}(i\Delta t) + \dot{g}((i+2)\Delta t) - 2\dot{g}((i+1)\Delta t)\big]\Big|\Big\}^2 \le \varepsilon_{\min} + (j+1)\Delta x.$$

In case $r_{x\varepsilon} \ne 0$, this condition may be transformed to the following form:

$$\varepsilon_{\min} + j\Delta x \le \operatorname{sgn}[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)] \times \Big\{\Big|\dot{g}(i\Delta t)\big[\dot{g}(i\Delta t) + \dot{g}((i+2)\Delta t) - 2\dot{g}((i+1)\Delta t)\big] - \dot{g}(i\Delta t)\big[\dot{g}((i+1)\Delta t) + \dot{g}((i+3)\Delta t) - 2\dot{g}((i+2)\Delta t)\big]\Big|\Big\}^{\frac{1}{2}} \le \varepsilon_{\min} + (j+1)\Delta x.$$

For determining the distribution law $W(\varepsilon^*)$ in succession for the values $j = 0$, $j = 1$, $j = 2$, …, $j = m$, one determines the number of samples N_0, N_1, N_2, …, N_m; the above conditions are fulfilled with this number of samples. It is clear that, by using them, one may form a curve of distribution law of speculated magnitudes of noise samples $W[\varepsilon(i\Delta t)]$. As the number of samples N increases, so this curve tends to the noise distribution law, i.e., $W(\varepsilon)$.

9. The estimate of the arithmetic mean of relative errors of samples $\dot{g}(i\Delta t)$ of noisy signal both in case $r_{x\varepsilon}=0$ and in case $r_{x\varepsilon}\neq 0$ is determined according to the following expression:

$$\overline{\lambda}^*_{\text{rel}} \approx \frac{1}{N}\sum_{i=1}^{N}\frac{\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}}{\dot{g}(i\Delta t)-\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}},$$

$$\overline{\lambda}^*_{\text{rel}} \approx \frac{1}{N}\sum_{i=1}^{N}\frac{\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)|}}{\dot{g}(i\Delta t)-\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)|}}.$$

In conclusion, we note that the modern PC allows one to realize the above technology in action easily.

3.10 Digital Identification of a Defect's Origin by Considering Noise as a Data Carrier

It is known [14, 27, 34, 37, 40, 41] that many problems of statistical diagnostics of technical objects may be reduced to the solution of an equation system, which has the following matrix form:

$$\vec{R}_{xy}(\mu)=\vec{R}_{xx}(\mu)\vec{W}(\mu), \tag{3.117}$$

where $\vec{R}_{xx}(\mu)$ is a matrix of the autocorrelation function $R_{xx}((i+\mu)\Delta t)$ of the following dimensional representation $\mu\times\mu$; $\vec{R}_{xy}(\mu)$ is a matrix-vector of the cross-correlation function $R_{xy}(\mu)((i+\mu)\Delta t)$; $\vec{W}$ is a matrix-vector of pulse transient functions

$$\vec{W}(\mu)=[W(0),W(\Delta t),W(2\Delta t),...,W(\mu\,\Delta t)]^T. \tag{3.118}$$

The statics problems are also solved by means of a matrix equation of the following form:

$$\vec{R}_{xy}(0)=\vec{R}_{xx}(0)\vec{B}, \tag{3.119}$$

where $\vec{R}_{xx}(0)$ is the matrix of auto- and cross-correlation functions $R_{x_ix_i}(0)$, $R_{x_ix_j}(0)$, $i, j=\overline{1,n}$, of input signals $X_1(i\Delta t)$, $X_2(i\Delta t)$, ..., $X_n(i\Delta t)$; $\vec{R}_{xy}(0)$ is a matrix-vector of the cross-correlation functions $R_{x_iy}(0)$, $i=\overline{1,n}$, between the input signals $X_1(i\Delta t)$, $X_2(i\Delta t)$, ...,

$X_n(i\Delta t)$; and the output signal $y(i\Delta t)$; $\vec{B} = [b_1, b_2, ..., b_n]$ is a matrix of the coefficients of the regression equation.

For real noisy input-output signals, Eqs. (3.117) and (3.119) have the following form:

$$\vec{R}_{g\eta}(\mu) = \vec{R}_{gg}(\mu)\vec{W}^*(\mu), \tag{3.120}$$

$$\vec{R}_{g\eta}(0) = \vec{R}_{gg}(0)\vec{B}^*. \tag{3.121}$$

Unfortunately, for the technological parameters $g(i\Delta t)$ and $\eta(i\Delta t)$, the estimates of correlation functions of legitimate signals $R_{xx}(\mu)$, $R_{xy}(\mu)$, $R_{yy}(\mu)$ and noisy signals $R_{gg}(\mu)$, $R_{g\eta}(\mu)$, $R_{\eta\eta}(\mu)$ do not coincide, i.e., the following inequalities hold:

$$\left.\begin{aligned} R_{xx}(0) &\neq R_{gg}(0) \\ R_{xy}(0) &\neq R_{g\eta}(0) \\ R_{xx}(\mu) &\neq R_{gg}(\mu) \\ R_{xy}(\mu) &\neq R_{g\eta}(\mu) \end{aligned}\right\}. \tag{3.122}$$

In turn, this results in the following inequalities:

$$\left.\begin{aligned} \vec{R}_{xy}(0) &\neq \vec{R}_{g\eta}(0) \\ \vec{R}_{xx}(0) &\neq \vec{R}_{gg}(0) \\ \vec{R}_{xy}(\mu) &\neq \vec{R}_{g\eta}(\mu) \\ \vec{R}_{xx}(\mu) &\neq \vec{R}_{gg}(\mu) \end{aligned}\right\}. \tag{3.123}$$

So, in spite of the sufficient development of methods of solving Eqs. (3.117) and (3.119), the obtained results for real objects working according to formulae (3.120) and (3.121) because of the errors arising due to the noises $\varepsilon(t)$ and $\varphi(t)$ turn out to be unsatisfactory [14, 34, 37, 40, 41].

At the same time, after separating approximate magnitudes of noise $\varepsilon^*(i\Delta t)$ from the total signal $g(i\Delta t)$, using the obtained magnitudes of samples of the legitimate the signal $X^*(i\Delta t)$, one may improve the correlation matrix stipulation. Here they may appear to be commensurable with the stipulations of correlation matrices of the legitimate signals $X(i\Delta t)$ and $y(i\Delta t)$. For this purpose in realizing correlation matrices, one may determine according to formulae (3.86) and (3.87) the approximate magnitudes of legitimate signal samples during the process of the processing of measurement information $g(i\Delta t)$ in the following cases:

1. in case the correlation coefficient is equal to zero, i.e., $r_{x\varepsilon} = 0$,

$$\dot{X}^*(i\Delta t) \approx \dot{g}(i\Delta t) - \dot{\varepsilon}^*(i\Delta t) = \dot{g}(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)$$

$$\times\sqrt{\left|\dot{g}^2(i\Delta t) + \dot{g}(i\Delta t)\dot{g}((i+2)\Delta t) - 2\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)\right|}; \quad (3.124)$$

2. in case there is the correlation between the noise $\varepsilon(i\Delta t)$ and the legitimate signal $X(i\Delta t)$, i. e., $r_{x\varepsilon} \neq 0$,

$$X^*(i\Delta t) \approx g(i\Delta t) - \operatorname{sgn}[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)]$$

$$\times\Big\{\Big|\dot{g}(i\Delta t)\big[\dot{g}(i\Delta t) + \dot{g}((i+2)\Delta t) - 2\dot{g}((i+1)\Delta t)\big]$$

$$-\dot{g}(i\Delta t)\big[\dot{g}((i+1)\Delta t) + \dot{g}((i+3)\Delta t)$$

$$-2\dot{g}((i+2)\Delta t)\big]\Big|\Big\}^{\frac{1}{2}}. \quad (3.125)$$

From the evidence of the validity of formulae (3.86) and (3.87), it follows that after separating the samples of the noise from the noisy signal when having obtained approximate magnitudes of samples of the legitimate signal $X^*(i\Delta t)$, one may form correlation matrix elements.

In this case one can suppose that the following equalities take place:

$$\vec{R}_{xx}(\mu) \approx R^*_{xx}(\mu),\ \vec{R}_{xx}(0) \approx \vec{R}^*_{xx}(0),$$

where

$$\vec{R}_{xx}(\mu) = \begin{vmatrix} \overline{\dot{X}_1(i\Delta t)\dot{X}_1((i+\mu)\Delta t)} & \overline{\dot{X}_1(i\Delta t)\dot{X}_2((i+\mu)\Delta t)} & \cdots & \overline{\dot{X}_1(i\Delta t)\dot{X}_n((i+\mu)\Delta t)} \\ \overline{\dot{X}_2(i\Delta t)\dot{X}_1((i+\mu)\Delta t)} & \overline{\dot{X}_2(i\Delta t)\dot{X}_2((i+\mu)\Delta t)} & \cdots & \overline{\dot{X}_2(i\Delta t)\dot{X}_n((i+\mu)\Delta t)} \\ \cdots & \cdots & \cdots & \cdots \\ \overline{\dot{X}_n(i\Delta t)\dot{X}_1((i+\mu)\Delta t)} & \overline{\dot{X}_n(i\Delta t)\dot{X}_2((i+\mu)\Delta t)} & \cdots & \overline{\dot{X}_n(i\Delta t)\dot{X}_n((i+\mu)\Delta t)} \end{vmatrix},$$

$$\vec{R}^*_{xx}(\mu) = \begin{vmatrix} \overline{\dot{X}^*_1(i\Delta t)\dot{X}^*_1((i+\mu)\Delta t)} & \overline{\dot{X}^*_1(i\Delta t)\dot{X}^*_2((i+\mu)\Delta t)} & \cdots & \overline{\dot{X}^*_1(i\Delta t)\dot{X}^*_n((i+\mu)\Delta t)} \\ \overline{\dot{X}^*_2(i\Delta t)\dot{X}^*_1((i+\mu)\Delta t)} & \overline{\dot{X}^*_2(i\Delta t)\dot{X}^*_2((i+\mu)\Delta t)} & \cdots & \overline{\dot{X}^*_2(i\Delta t)\dot{X}^*_n((i+\mu)\Delta t)} \\ \cdots & \cdots & \cdots & \cdots \\ \overline{\dot{X}^*_n(i\Delta t)\dot{X}^*_1((i+\mu)\Delta t)} & \overline{\dot{X}^*_n(i\Delta t)\dot{X}^*_2((i+\mu)\Delta t)} & \cdots & \overline{\dot{X}^*_n(i\Delta t)\dot{X}^*_n((i+\mu)\Delta t)} \end{vmatrix},$$

(3.126)

$$\vec{R}_{xx}(0)=\begin{vmatrix} \overline{\dot{X}_1(i\Delta t)\dot{X}_1(i\Delta t)} & \overline{\dot{X}_1(i\Delta t)\dot{X}_2(i\Delta t)} & \cdots & \overline{\dot{X}_1(i\Delta t)\dot{X}_n(i\Delta t)} \\ \overline{\dot{X}_2(i\Delta t)\dot{X}_1(i\Delta t)} & \overline{\dot{X}_2(i\Delta t)\dot{X}_2(i\Delta t)} & \cdots & \overline{\dot{X}_2(i\Delta t)\dot{X}_n(i\Delta t)} \\ \cdots & \cdots & \cdots & \cdots \\ \overline{\dot{X}_n(i\Delta t)\dot{X}_1(i\Delta t)} & \overline{\dot{X}_n(i\Delta t)\dot{X}_2(i\Delta t)} & \cdots & \overline{\dot{X}_n(i\Delta t)\dot{X}_n(i\Delta t)} \end{vmatrix}, \quad (3.127)$$

$$\vec{R}^*_{xx}(0)=\begin{vmatrix} \overline{\dot{X}^*_1(i\Delta t)\dot{X}^*_1(i\Delta t)} & \overline{\dot{X}^*_1(i\Delta t)\dot{X}^*_2(i\Delta t)} & \cdots & \overline{\dot{X}^*_1(i\Delta t)\dot{X}^*_n(i\Delta t)} \\ \overline{\dot{X}^*_2(i\Delta t)\dot{X}^*_1(i\Delta t)} & \overline{\dot{X}^*_2(i\Delta t)\dot{X}^*_2(i\Delta t)} & \cdots & \overline{\dot{X}^*_2(i\Delta t)\dot{X}^*_n(i\Delta t)} \\ \cdots & \cdots & \cdots & \cdots \\ \overline{\dot{X}^*_n(i\Delta t)\dot{X}^*_1(i\Delta t)} & \overline{\dot{X}^*_n(i\Delta t)\dot{X}^*_2(i\Delta t)} & \cdots & \overline{\dot{X}^*_n(i\Delta t)\dot{X}^*_n(i\Delta t)} \end{vmatrix}. \quad (3.128)$$

Even if one can suggest that in some cases (3.112) does not hold, Eq. (3.113) holds because under any conditions the following equalities hold:

$$\left.\begin{array}{c} P\left[\dot{X}_1(i\Delta t)\geq \dot{X}^*_1(i\Delta t)\right]=P\left[\dot{X}_1(i\Delta t)\leq \dot{X}^*_1(i\Delta t)\right], \\ P\left[\dot{X}_2(i\Delta t)\geq \dot{X}^*_2(i\Delta t)\right]=P\left[\dot{X}_2(i\Delta t)\leq \dot{X}^*_2(i\Delta t)\right], \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ P\left[\dot{X}_n(i\Delta t)\geq \dot{X}^*_n(i\Delta t)\right]=P\left[\dot{X}_n(i\Delta t)\leq \dot{X}^*_n(i\Delta t)\right], \\ P\left[\dot{X}_1((i+\mu)\Delta t)\geq \dot{X}^*_1((i+\mu)\Delta t)\right]=P\left[\dot{X}_1((i+\mu)\Delta t)\leq \dot{X}^*_1((i+\mu)\Delta t)\right], \\ P\left[\dot{X}_2((i+\mu)\Delta t)\geq \dot{X}^*_2((i+\mu)\Delta t)\right]=P\left[\dot{X}_2((i+\mu)\Delta t)\leq \dot{X}^*_2((i+\mu)\Delta t)\right], \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ P\left[\dot{X}_n((i+\mu)\Delta t)\geq \dot{X}^*_n((i+\mu)\Delta t)\right]=P\left[\dot{X}_n((i+\mu)\Delta t)\leq \dot{X}^*_n((i+\mu)\Delta t)\right]. \end{array}\right\} \quad (3.129)$$

Due to the above, the correlation matrices $\vec{R}^*_{xx}(\mu)$ and $\vec{R}^*_{xx}(0)$ may be considered to be identical to the matrices $\vec{R}_{xx}(\mu)$ and $\vec{R}_{xx}(0)$.

Thus, the stipulation of the correlation matrices $\vec{R}^*_{xx}(\mu)$ and $\vec{R}^*_{xx}(0)$ formed from the estimates calculated according to the approximate magnitudes of the legitimate signals $X^*(i\Delta t)$ may be considered to be the same with the stipulation of the correlation matrices of legitimate signals $\vec{R}_{xx}(\mu)$ and $\vec{R}_{xx}(0)$.

The analogous consideration may be given on respect to the vector columns $\vec{R}_{xy}(\mu)$ and $\vec{R}^*_{xy}(\mu)$, $\vec{R}_{xy}(0)$, and $\vec{R}^*_{xy}(0)$.

It follows that in the present variant one can reduce solving the problem of statistical diagnostics of the technical state of the object to solving the system of equations of the following form:

$$\vec{R}^*_{xy}(\mu) = \vec{R}^*_{xx}(\mu)\vec{W}^*(\mu), \tag{3.130}$$

$$\vec{R}^*_{xy}(0) = \vec{R}^*_{xx}(0)\vec{B}^*. \tag{3.131}$$

It opens real possibilities for solving a wide range of problems of monitoring and diagnosing various fields of science and technology.

4 Robust Correlation Monitoring of a Defect at Its Origin

4.1 Problem of Monitoring a Defect at Its Origin Using Technology of Correlation Analysis of Signal Received as the Output of Sensors

When using traditional technologies of signal analysis in solving a monitoring task, getting more or less acceptable results is possible only if the error has a salient character, the analyzed signals are stationary and are subjected to the normal distribution law, the correlation between the noise and the useful signal is equal to zero, and the noise represents white noise. However, even in this case, the errors from the obtained estimates depend on the change of the noise variance, the change of the correlation between the noise and the legitimate signal, or the change of their distribution law. Due to this, the adequacy of the description of many analyzed processes by means of probabilistic-statistical methods is not satisfied and we end up with wrong results in determining the origin of a defect. For these reasons, in the framework of classical theories, many problems of great importance are practically not solved nowadays. Thus, great possibilities are not realized, but if they were, we would solve a great number of problems having tremendous economical and social importance.

For example, eliminating the disadvantages of traditional technologies would allow us to increase the reliability of forecasting earthquakes and other natural disasters, improve disease diagnostics, increase the efficiency of prospecting mineral resources, increase the reliability of forecasting failures at heat and nuclear power stations, forecast failures in drilling, diagnose a technical plane state, allow us to realize adequate mathematical models, and so on. In this connection, among real applications of great potential of considered theories lies the necessity to revise traditional algorithms and create new technologies that provide the robustness of the obtained estimates in fulfilling classical conditions and in case there is a lack of obedience to classical conditions.

In this chapter, on the basis of the technology of noise analysis is a robust technology of correlation analysis. Due to this, the opportunity appears to eliminate serious obstacles by using the enormous potential of this technology for solving the most important tasks of monitoring an error at its origin.

4.2 Necessity of Providing Robustness of Correlation Monitoring of a Defect at Its Origin

In practice, data of the normal exploitation of the real objects show that, due to the unavoidable errors caused by the noise influence, the estimates of the auto- and cross-correlation functions of the input and output signals contain certain "pulsations." This is one of the reasons interfering with the wide application of the known probabilistic-statistical methods for solving the numerous applied problems. Because of this, the authenticity of the obtained results can be doubtful when solving the problem of monitoring the defect at its origin.

For eliminating the difficulty caused by these reasons, plenty of methods [14] have been suggested. In spite of the high theoretical level of these works, the experience of successfully applying them in practice in solving real problems of monitoring the error is not abundant.

Along this vein, a sharp necessity is observed in the exploitation of the new methods and algorithms oriented to eliminate the difficulty caused by the noise influence on the estimate of correlation functions. In order to explain the specific characteristics of this problem, first let us consider the specificity of correlation technologies in practice [10, 14, 34, 40].

It is known that the formula for determining estimates of the auto- and cross-correlation functions $R_{g_ig_i}(\tau)$, $R_{g_ig_j}(\tau)$, and $R_{g_i\eta}(\tau)$ between respectively noisy signals $g_i(t)$ and $g_j(t)$ (the noises are $\varepsilon_i(t)$ and $\varepsilon_j(t)$, respectively, for each signal), as well as between $g_i(t)$ and $\eta(t)$ noisy signals with the noise $\varphi(t)$, has the form

$$
\begin{aligned}
R_{g_ig_i}(\tau) &= \frac{1}{T}\int_0^T \dot{g}_i(t)\dot{g}_i(t+\tau)\,dt \\
&= \frac{1}{T}\int_0^T \left[\dot{x}_i(t)+\dot{\varepsilon}_i(t)\right]\left[\dot{x}_i(t+\tau)+\dot{\varepsilon}_i(t+\tau)\right]dt \\
&= R_{x_ix_i}(\tau)+\Lambda_{x_ix_i}(\tau),
\end{aligned}
\tag{4.1}
$$

$$R_{g_i g_j}(\tau) = \frac{1}{T}\int_0^T \dot{g}_i(t)\dot{g}_j(t+\tau)\,dt$$

$$= \frac{1}{T}\int_0^T \left[\dot{x}_i(t) + \dot{\varepsilon}_i(t)\right]\left[\dot{x}_j(t+\tau) + \dot{\varepsilon}_j(t+\tau)\right]dt$$

$$= R_{x_i x_j}(\tau) + \Lambda_{x_i x_j}(\tau), \tag{4.2}$$

$$R_{g_i \eta}(\tau) = \frac{1}{T}\int_0^T \dot{g}_i(t)\dot{\eta}(t+\tau)\,dt$$

$$= \frac{1}{T}\int_0^T \left[\dot{x}_i(t) + \dot{\varepsilon}_i(t)\right]\left[\dot{y}(t+\tau) + \dot{\varphi}(t+\tau)\right]dt$$

$$= R_{x_i y}(\tau) + \Lambda_{x_i y}(\tau), \tag{4.3}$$

where $\Lambda_{x_i x_i}(\tau)$, $\Lambda_{x_i x_j}(\tau)$, and $\Lambda_{x_i y}(\tau)$ are the values of errors of the estimates of the auto- and cross-correlation functions $R_{g_i g_i}(\tau)$, $R_{g_i g_j}(\tau)$, and $R_{g_i \eta}(\tau)$.

Taking into account that for real technological parameters

$$\frac{1}{T}\int_0^T \dot{\varepsilon}_i(t)\dot{\varepsilon}_j(t+\tau)\,dt \approx 0\,, \tag{4.4}$$

$$\frac{1}{T}\int_0^T \dot{\varepsilon}_i(t)\dot{\varphi}(t+\tau)\,dt \approx 0\,, \tag{4.5}$$

for the values of errors of the cross-correlation functions $R_{g_i g_j}(\tau)$, $R_{g_i \eta}(\tau)$, one has the expressions

$$\Lambda_{x_i x_j}(\tau) = \frac{1}{T}\int_0^T \left[\dot{x}_i(t)\dot{\varepsilon}_j(t+\tau) + \dot{\varepsilon}_j(t)\dot{x}_i(t+\tau)\right]dt\,, \tag{4.6}$$

$$\Lambda_{x_i y}(\tau) = \frac{1}{T}\int_0^T \left[\dot{x}_i(t)\dot{\varphi}(t+\tau) + \dot{\varepsilon}_i(t)\dot{y}(t+\tau)\right]dt\,. \tag{4.7}$$

The value of the error $\Lambda_{x_i x_i}(\tau)$ of estimates of the autocorrelation functions $R_{g_i g_i}(\tau)$ is determined by the expression

$$\Lambda_{x_i x_i}(\tau) = \frac{1}{T}\int_0^T \left[\dot{x}_i(t)\dot{\varepsilon}_i(t+\tau) + \dot{\varepsilon}_i(t)\dot{x}_i(t+\tau) + \dot{\varepsilon}_i(t)\dot{\varepsilon}_i(t+\tau)\right] dt. \quad (4.8)$$

However, taking into consideration that $\varepsilon_i(t)$ is white noise and the values $\varepsilon_i(t)$ and $\varepsilon_i(t+\tau)$ are not correlated at $\tau \neq 0$, that is,

$$\frac{1}{T}\int_0^T \dot{\varepsilon}_i(t)\dot{\varepsilon}_i(t+\tau)\,dt = 0 \quad \text{for} \quad \tau \neq 0, \quad (4.9)$$

then Eq. (4.8) holds only for values $\tau = 0$. When $\tau \neq 0$, the following equality holds:

$$\Lambda_{x_i x_i}(\tau) = \frac{1}{T}\int_0^T \left[\dot{x}_i(t)\dot{\varepsilon}_i(t+\tau) + \dot{\varepsilon}_i(t)\dot{x}_i(t+\tau)\right] dt.$$

With $\tau = 0$, the expression for the error $\Lambda_{x_i x_i}(\tau)$ has the form

$$\Lambda_{x_i x_i}(\tau) = \frac{2}{T}\int_0^T \dot{x}_i(t)\dot{\varepsilon}_i(t)\,dt + \frac{1}{T}\int_0^T \dot{\varepsilon}_i^2(t)\,dt.$$

Taking into consideration that the mean value of squares of values of noise is equal to the variance D_{ε_i} of the noise $\varepsilon_i(t)$, i.e.,

$$\frac{1}{T}\int_0^T \dot{\varepsilon}_i^2(t)\,dt = D_{\varepsilon_i},$$

with $\tau = 0$ one has

$$\Lambda_{x_i x_i}(\tau) = \frac{2}{T}\int_0^T \dot{x}_i(t)\dot{\varepsilon}_i(t)\,dt + D_{\varepsilon_i} = \Lambda'_{x_i x_i}(\tau) + D_{\varepsilon_i}, \quad (4.10)$$

where

$$\Lambda'_{x_i x_i}(\tau) = \frac{2}{T}\int_0^T \dot{x}_i(t)\dot{\varepsilon}_i(t)\,dt,$$

$$\vec{R}_{g\eta}(0) = \left\| R_{g_i\eta}(0) \right\|, \qquad i = \overline{1,n}. \quad (4.11)$$

From expressions (4.1)–(4.3), it is clear that the estimates $R_{g_ig_j}(0)$ differ from their true values $R_{x_ix_j}(0)$ by the value of the error $\Lambda_{x_ix_j}(0)$, and the estimates $R_{g_ig_i}(0)$, i.e., $i = j$, apart from the values of the error $\Lambda_{x_ix_i}(0)$, differ from the true values $R_{x_ix_i}(0)$ by the value of the variance D_{ε_i} of the noise $\dot{\varepsilon}_i(t)$. The estimates $R_{g_i\eta}(0)$ differ from their true values $R_{x_iy}(0)$ only by the value of the error $\Lambda_{x_iy}(0)$.

Thus, from expressions (4.1)–(4.11), it is clear that the reason for the decrease in the reliability of the obtained results is the presence of the errors $\Lambda_{x_ix_i}(0)$, $\Lambda_{x_ix_j}(0)$, and $\Lambda_{x_iy}(0)$ in each estimate as well as the presence of the variance D_{ε_i} of the noise $\varepsilon_i(t)$ in the estimates $R_{g_ig_i}(0)$.

From the technology of the correlation analysis, it is clear that because of the errors $\Lambda_{x_ix_i}(0)$, $\Lambda_{x_ix_j}(0)$, $\Lambda_{x_iy_i}(0)$, and the presence of the variance D_{ε_i} of the noise $\varepsilon_i(t)$ the estimates of signal significantly differ from their true values. In this connection, when solving the monitoring problem, there is no guarantee of receiving reliable results.

The performed research [10, 27, 34, 37, 40] showed that besides the influence of the errors themselves, $\Lambda_{x_ix_i}(0)$ and $\Lambda_{x_iy_i}(0)$, one of the important factors influencing the characteristics of correlation functions is the change of their value because of parametric vibrations, which leads to the essential change of the ratio spread ranges $D[\gamma_{x_ix_j}(0)]$, $D[\gamma_{x_iy}(0)]$ of the relative errors

$$\gamma_{x_ix_i}(0) = \Lambda_{x_ix_i}(0)/R_{x_ix_i}(0),\ \gamma_{x_ix_j}(0) = \Lambda_{x_ix_j}(0)/R_{x_ix_j}(0),$$

$$\gamma_{x_iy}(0) = R_{x_iy}(0)/\Lambda_{x_iy}(0)$$

to the spread ranges $D[\Lambda_{x_ix_j}(0)]$, $D[\Lambda_{x_iy}(0)]$ of the errors themselves, $\Lambda_{x_ix_i}(0)$, $\Lambda_{x_ix_j}(0)$, $\Lambda_{x_iy}(0)$.

As will be shown ahead, the independence of the error values on the changes of noise characteristics opens the opportunity for significant improvement of monitoring results, which, in turn, necessitates providing the robustness of determined estimate elements of the correlation matrix.

Thus, there arises a necessity in the creation of a robust technology allowing one, by means of accounting for the noise influence, to bring the estimates to a form where they maximally approach their true values.

4.3 Robust Method of Improving Estimates of Auto-Correlation Functions in Monitoring a Defect's Origin

In calculating auto- and cross-correlation function estimates, the realizations $g(t)$ and $\eta(t)$ of the useful signals $X(t)$ and $y(t)$ are in practice normally assumed to be random stationary ergodic functions with normal distribution law, and the interval T is chosen large enough; in view of these facts, the estimates $R_{xx}(\tau)$ and $R_{xy}(\tau)$ of correlation functions can be used rather than the correlation functions $R_{gg}(\tau)$ and $R_{g\eta}(\tau)$ themselves. However, in practice, estimates have appreciable errors because the realizations $g(t)$ and $\eta(t)$ contain a mixture of useful signals $X(t)$, $y(t)$ and noises $\varepsilon(t)$, $\varphi(t)$, whose distribution laws prove frequently to be different from normal ones; moreover, mathematical expectations of noises do not strictly equal zero, i.e., $m_\varepsilon \approx 0$, $m_\varphi \approx 0$; stronger stationarity is not observed in realizations $g(t)$, $\eta(t)$; the time interval T is restricted, and so on. In practice, the estimates $R_{gg}(\tau)$, $R_{g\eta}(\tau)$ are obtained with appreciable errors $\Lambda_{xx}(\tau)$, $\Lambda_{xy}(\tau)$. The values of the errors depend on the range of the noise change, that is, the obtained estimates besides errors have one more serious disadvantage: they do not meet robustness requirements. It lowers the degree of reliability of data obtained from primary signal processing and worsens correlation matrix stipulation; as a consequence, it does not provide due adequacy in solving identification problems. Thus, in order to raise and advance the efficiency of probabilistic-statistical methods, it is necessary that the errors $\Lambda_{xx}(\tau)$, $\Lambda_{xy}(\tau)$ be removed and that robustness in statistical characteristics of technological parameters be provided. One of the possible ways to provide proper robustness and to eliminate the error $\Lambda_{xx}(\tau)$ of the autocorrelation function $R_{gg}(\tau)$ [33, 34, 37, 40] is outlined ahead.

It is known that in calculating estimates of autocorrelation functions by using traditional algorithms, the following equalities occur:

$$R'_{xx}(\tau) = R'_{gg}(\tau),\ R_{xx}(\tau) = R_{gg}(\tau),$$

$$R_{gg}(\tau) = R'_{gg}(\tau),\ R_{x\varepsilon}(\tau) = 0, \tag{4.12}$$

where

$$R'_{gg}(\tau) = \frac{1}{T}\int_0^T g(t)g(t+\tau)dt - m_g^2$$

$$= \frac{1}{T}\int_0^T [X(t)+\varepsilon(t)][X(t+\tau)+\varepsilon(t+\tau)]dt - (m_x + m_\varepsilon)^2, \tag{4.13}$$

$$R'_{xx}(\tau) = \frac{1}{T}\int_0^T X(t)X(t+\tau)dt - m_x^2,$$

$$R_{x\varepsilon}(\tau) = \frac{1}{T}\int_0^T \dot{X}(t)\dot{\varepsilon}(t+\tau)dt.$$

However, in practice, due to the above-mentioned factors, these equalities do not take place, and in reality the following relations occur:

$$R'_{gg}(\tau) - R'_{xx}(\tau) = \Lambda_1(\tau), \tag{4.14}$$

$$R_{gg}(\tau) - R_{xx}(\tau) = \Lambda_2(\tau), \tag{4.15}$$

$$R'_{gg}(\tau=0) - R'_{xx}(\tau=0) = \Lambda_1(\tau=0) = \Lambda_2(\tau=0), \tag{4.16}$$

$$\left|\Lambda_1(\tau\neq 0) - \Lambda_2(\tau\neq 0)\right| = \lambda(\tau\neq 0), \tag{4.17}$$

$$\left|R'_{gg}(\tau) - R_{gg}(\tau)\right| = \lambda(\tau). \tag{4.18}$$

The main reason for the equalities is that the correlation between the useful signal $X(t)$ and the noise $\varepsilon(t)$ is different from zero, i.e., $R_{x\varepsilon}(\tau)\neq 0$, which involves errors caused by the noise $\varepsilon(t)$.

Consider the feasibility of determining total error of estimates $R_{gg}(\tau)$. For this the effect of noise on appearance of errors in the products of the noncentered noisy signals $g(t)g(t+\tau)$, the noncentered legitimate signals $X(t)X(t+\tau)$, the centered noisy signals $\dot{g}(t)\dot{g}(t+\tau)$, and the centered legitimate signals $\dot{X}(t)\dot{X}(t+\tau)$ should first be clarified by expressions (4.14) and (4.15):

$$\Lambda_1(\tau) = \frac{1}{T}\int_0^T g(t)g(t+\tau)dt - m_g^2 - \left[\frac{1}{T}\int_0^T X(t)X(t+\tau)dt - m_x^2\right]$$

$$= \frac{1}{T}\int_0^T \left[X(t)+\varepsilon(t)\right]\left[X(t+\tau)+\varepsilon(t+\tau)\right]dt - (m_x + m_\varepsilon)^2$$

$$-\left[\frac{1}{T}\int_0^T X(t)X(t+\tau)dt - m_x^2\right]$$

$$=\frac{1}{T}\int_0^T\left[\dot{X}(t)+m_x+\dot{\varepsilon}(t)+m_\varepsilon\right]\left[\dot{X}(t+\tau)+m_x+\dot{\varepsilon}(t+\tau)+m_\varepsilon\right]dt$$

$$-\frac{1}{T}\int_0^T\left[\dot{X}(t)+m_x\right]\left[\dot{X}(t+\tau)+m_x\right]dt\,.$$

With a glance to stationary random processes with normal distribution law, the mathematical expectations of products

$$\frac{1}{T}m_\varepsilon\int_0^T\dot{X}(t)dt\,,\ \frac{1}{T}m_\varepsilon\int_0^T\dot{\varepsilon}(t)dt\,,\ \frac{1}{T}m_\varepsilon\int_0^T\dot{X}(t+\tau)dt\,,$$

$$\frac{1}{T}m_\varepsilon\int_0^T\dot{\varepsilon}(t+\tau)dt\,,\ \frac{1}{T}m_x\int_0^T\dot{\varepsilon}(t)dt\,,\ \frac{1}{T}m_x\int_0^T\dot{\varepsilon}(t+\tau)dt\,,\ m_x m_\varepsilon\,,$$

can be taken equal to zero. In that case, the value of a noncentered signal when all products are positive is represented in the form

$$\Lambda_1(\tau)=\frac{1}{T}\int_0^T\left[\left|\dot{X}(t)\dot{\varepsilon}(t+\tau)\right|+\left|\dot{\varepsilon}(t)\dot{X}(t+\tau)\right|+\left|\dot{\varepsilon}(t)\dot{\varepsilon}(t+\tau)\right|\right]dt. \tag{4.19}$$

In a similar way the error of the centered signal is determined:

$$\Lambda_2(\tau)=\frac{1}{T}\int_0^T\left[\dot{g}(t)\dot{g}(t+\tau)-\dot{X}(t)\dot{X}(t+\tau)\right]dt$$

$$=\frac{1}{T}\int_0^T\left[\dot{X}(t)+\dot{\varepsilon}(t)\right]\left[\dot{X}(t+\tau)+\dot{\varepsilon}(t+\tau)\right]dt$$

$$-\frac{1}{T}\int_0^T\dot{X}(t)\dot{X}(t+\tau)dt$$

$$=\frac{1}{T}\int_0^T\left[\dot{X}(t)\dot{\varepsilon}(t+\tau)+\dot{\varepsilon}(t)\dot{X}(t+\tau)+\dot{\varepsilon}(t)\dot{\varepsilon}(t+\tau)\right]dt\,. \tag{4.20}$$

It follows from expressions (4.19) and (4.20) that because of the presence of the error $\varepsilon(t)$, estimates of the autocorrelation functions $R'_{gg}(\tau)$,

$R_{gg}(\tau)$ for both noncentered and centered signals are different from their true values $R'_{xx}(\tau)$, $R_{xx}(\tau)$ by values of $\Lambda_1(\tau)$, $\Lambda_2(\tau)$, respectively. As indicated previously, the estimate of the autocorrelation function $R_{gg}(\tau)$ of the centered signal is formed from the sum of a certain number of positive products $\dot{g}(t)\dot{g}(t+\tau)$ and a certain number of negative products $\dot{g}(t)\dot{g}(t+\tau)$. But the estimate of the autocorrelation function $R'_{gg}(\tau)$ of a noncentered signal according to expression (4.13) is only formed from the positive products $g(t)g(t+\tau)$. In this connection, in spite of the identity of expressions (4.19) and (4.20), the value $\Lambda_1(\tau)$ is formed from errors of the positive products $g(t)g(t+\tau)$, and the value $\Lambda_2(\tau)$ is formed from the difference of errors of the positive and negative products $\dot{g}(t)\dot{g}(t+\tau)$. Hence, the values $\Lambda_1(\tau)$ and $\Lambda_2(\tau)$ differ by the value $\lambda(\mu)$.

If we divide the time interval $[0, T]$ into the line segments $[t_k, t_{k+1}]$, $k = i_{1^+(\tau)}, i_{2^+(\tau)}, \ldots, i_{N^+(\tau)}$, where the products of noisy centered signals $\dot{g}(t)\dot{g}(t+\tau)$ take the positive values and line segments $[t_k, t_{k+1}]$, $k = i_{1^-(\tau)}, i_{2^-(\tau)}, \ldots, i_{N^-(\tau)}$, where the products $\dot{g}(t)\dot{g}(t+\tau)$ take the negative values, the equality (4.20) takes the following form:

$$\Lambda_2(\tau) = \frac{1}{T}\int_{T^+(\tau)} \left[\dot{X}(t)\dot{\varepsilon}(t+\tau) + \dot{\varepsilon}(t)\dot{X}(t+\tau) + \dot{\varepsilon}(t)\dot{\varepsilon}(t+\tau)\right]dt$$

$$-\frac{1}{T}\int_{T^-(\tau)} \left[\dot{X}(t)\dot{\varepsilon}(t+\tau) + \dot{\varepsilon}(t)\dot{X}(t+\tau) + \dot{\varepsilon}(t)\dot{\varepsilon}(t+\tau)\right]dt, \quad (4.21)$$

where $T^+(\tau)$ is the total length of the interval of integration inside of which $\dot{g}(t)\dot{g}(t+\tau) > 0$; $T^-(\tau)$ is the total length of the interval of integration where $\dot{g}(t)\dot{g}(t+\tau) < 0$ takes place and, in addition, where $T = T^+(\tau) + T^-(\tau)$.

Here, Eq. (4.18) can be transformed as follows:

$$\lambda(\tau = \Delta t) = \Lambda_1(\tau = \Delta t) - \Lambda_2(\tau = \Delta t).$$

Therefore, taking into account Eqs. (4.19) and (4.21), we have the following:

$$\lambda(\tau = \Delta t) = \left|\Lambda_1(\tau = \Delta t) - \Lambda_2(\tau = \Delta t)\right|$$

$$= \left|\frac{1}{T}\int_0^T \left[\left|\dot{X}(t)\dot{\varepsilon}(t+\Delta t)\right|\right.\right.$$

$$\left.\left. + \left|\dot{\varepsilon}(t)\dot{X}(t+\Delta t)\right| + \left|\dot{\varepsilon}(t)\dot{\varepsilon}(t+\Delta t)\right|\right]dt\right.$$

$$-\frac{1}{T}\int_{T^+(\tau=\Delta t)}\left[\dot{X}(t)\dot{\varepsilon}(t+\Delta t)+\dot{\varepsilon}(t)\dot{X}(t+\Delta t)+\dot{\varepsilon}(t)\dot{\varepsilon}(t+\Delta t)\right]dt$$

$$\left.+\frac{1}{T}\int_{T^-(\tau=\Delta t)}\left[\dot{X}(t)\dot{\varepsilon}(t+\Delta t)+\dot{\varepsilon}(t)\dot{X}(t+\Delta t)+\dot{\varepsilon}(t)\dot{\varepsilon}(t+\Delta t)\right]dt\right|.$$

Here, taking into account the following:

$$\frac{1}{T}\int_0^T\left[\left|\dot{X}(t)\dot{\varepsilon}(t+\Delta t)\right|+\left|\dot{\varepsilon}(t)\dot{X}(t+\Delta t)\right|+\left|\dot{\varepsilon}(t)\dot{\varepsilon}(t+\Delta t)\right|\right]dt$$

$$-\frac{1}{T}\int_{T^+(\tau=\Delta t)}\left[\dot{X}(t)\dot{\varepsilon}(t+\Delta t)+\dot{\varepsilon}(t)\dot{X}(t+\Delta t)+\dot{\varepsilon}(t)\dot{\varepsilon}(t+\Delta t)\right]dt$$

$$=\left|\frac{1}{T}\int_{T^-(\tau=\Delta t)}\left[\dot{X}(t)\dot{\varepsilon}(t+\Delta t)+\dot{\varepsilon}(t)\dot{X}(t+\Delta t)+\dot{\varepsilon}(t)\dot{\varepsilon}(t+\Delta t)\right]dt\right|,$$

we have

$$\lambda(\tau=\Delta t)\approx\left|\frac{2}{T}\int_{T^-(\tau=\Delta t)}\left[\dot{X}(t)\dot{\varepsilon}(t+\Delta t)+\dot{\varepsilon}(t)\dot{X}(t+\Delta t)+\dot{\varepsilon}(t)\dot{\varepsilon}(t+\Delta t)\right]dt\right|\cdot \tag{4.22}$$

At the same time, according to Eq. (4.18) without resorting to formula (4.22) to determine the estimates $R'_{gg}(\tau=\Delta t)$, $R_{gg}(\tau=\Delta t)$, the value

$$\left|R_{gg}(\tau=\Delta t)-R_{\dot{g}\dot{g}}(\tau=\Delta t)\right|=\lambda(\tau=\Delta t)$$

can be calculated by realization of the signal $g(t)$.

Thus, in calculating the correlation function $R_{gg}(\tau)$ of the centered signals with $\tau=\Delta t$ from the total value of the integral of products $\dot{g}(t)\dot{g}(t+\tau)$, integration over the interval $[0,T]$ in comparison with the estimate $R'_{gg}(\tau)$ for a noncentered signal $g(t)$, the integral of negative products $\dot{g}(t)\dot{g}(t+\Delta t)$ over integrating the interval of the length

$$T^-(\tau=\Delta t)=\sum_{k=i_{1^-(\tau=\Delta t)}}^{i_{N^-(\tau=\Delta t)}}\left[t_k,t_{k+1}\right],$$

one may determine the value $\lambda(\tau = \Delta t)$, which actually represents the sum of errors of these negative productions. Thus, Eq. (4.18) occurs in practice. Then the mean error $\langle \Delta\lambda(\tau = \Delta t)\rangle$ in a unit time shift $\tau = \Delta t$ is determined by

$$\langle \Delta\lambda(\tau = \Delta t)\rangle = \left[1/T^{-}(\tau = \Delta t)\right]\lambda(\tau = \Delta t). \tag{4.23}$$

It is obvious that one can calculate the robustness value $\lambda_{xx}^{R}(\tau)$, being aware of $\langle \Delta\lambda(\tau = \Delta t)\rangle$, which will improve the estimate $R_{gg}(\tau)$ through a different time shift τ between $\dot{g}(t)$ and $\dot{g}(t+\tau)$, by the expression

$$\lambda_{xx}^{R}(\tau) = \left[T^{+}(\tau) - T^{-}(\tau)\right]\langle \Delta\lambda(\tau = \Delta t)\rangle. \tag{4.24}$$

By doing so, the following procedure to determine the robustness value $\lambda_{xx}^{R}(\tau)$ can be formulated for estimates of the autocorrelation functions $R_{gg}(\tau)$.

1. Calculate the autocorrelation function values $R'_{gg}(\tau)$ for all the values τ for a noncentered signal $g(t)$ by the formula

$$R'_{gg}(\tau) = \frac{1}{T}\int_0^T g(t)g(t+\tau)dt - m_g m_g, \text{ where } m_g = \frac{1}{T}\int_0^T g(t)dt.$$

2. Calculate the autocorrelation function values $R_{gg}(\tau)$ for all the values τ for a centered signal $\dot{g}(t)$ by the formula

$$R_{gg}(\tau) = \frac{1}{T}\int_0^T \dot{g}(t)\dot{g}(t+\tau)dt.$$

3. Determine the error value $\lambda(\tau = \Delta t)$ with unit time shift $\tau = 1\cdot\Delta t$ by the expression

$$\lambda(\tau = \Delta t) = \left|R'_{gg}(\tau = \Delta t) - R_{gg}(\tau = \Delta t)\right|.$$

4. Determine the value of the mean error $\langle \Delta\lambda(\tau = \Delta t)\rangle$ for the negative product $\dot{g}(t)\dot{g}(t+\Delta t)$ with the unit time shift $\tau = 1\cdot\Delta t$ by the formula

$$\langle \Delta\lambda(\tau = \Delta t)\rangle = \left[\frac{1}{T^{-}(\tau = \Delta t)}\right]\lambda(\tau = \Delta t).$$

5. Calculate the robustness value $\lambda_{xx}^{R}(\tau)$ of the estimates $R_{gg}(\tau)$ with different time shift τ between $\dot{g}(t)$ and $\dot{g}(t+\tau)$ by the formula
$$\lambda_{xx}^{R}(\tau)=\left[T^{+}(\tau)-T^{-}(\tau)\right]\langle\Delta\lambda(\tau=\Delta t)\rangle .$$
6. Determine the robust estimates of the autocorrelation function $R_{gg}(\tau)$ by the expression
$$R_{gg}^{R}(\tau)=\begin{cases} R_{gg}(\tau)-\left[\lambda_{xx}^{R}(\tau)+D_{\varepsilon}\right] & \text{for} \quad \tau=0, \\ R_{gg}(\tau)-\lambda_{xx}^{R}(\tau) & \text{for} \quad \tau\neq 0. \end{cases}$$

But the autocorrelation function normally has to be determined in practice by processing experimental data representing a record or realizations of the studied random process $g(t)$. The procedure and digital algorithms for determining the robustness $\lambda_{xx}^{R}(\mu)$ of the autocorrelation function estimates $R_{gg}(\mu)$ of the sampled random signal $g(i\Delta t)$, consisting of a mixture of the useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$, will be considered in detail in Section 4.5.

4.4 Robust Method of Improving Cross-Correlation Function Estimates

The present section proposes an algorithm for improvement of estimates of the cross-correlation function between the signals $g(t)=X(t)+\varepsilon(t)$ and $\eta(t)=y(t)+\varphi(t)$ consisting of the mixture of the legitimate signals $X(t)$, $y(t)$ and the corresponding noises $\varepsilon(t)$, $\varphi(t)$ [8, 10, 14, 34, 37].

It is known that for stationary ergodic random signals $g(t)$, $\eta(t)$ with normal distribution law consisting of the useful signals $X(t)$, $y(t)$, where the noise $\varepsilon(t)$, $\varphi(t)$ is not correlated with useful signals and is presented in the form of "white noise" with the mathematical expectations $m_{\varepsilon}\approx 0$, $m_{\varphi}\approx 0$, the following equalities occur in calculating estimates of cross-correlation functions by using traditional methods:

$$\left.\begin{aligned} R'_{xy}(\tau)&=R'_{g\eta}(\tau)\\ R_{xy}(\tau)&=R_{g\eta}(\tau)\\ R_{g\eta}(\tau)&=R'_{g\eta}(\tau)\\ R'_{xy}(\tau)&=R_{xy}(\tau) \end{aligned}\right\}, \tag{4.25}$$

$$\left.\begin{aligned} R_{x\varphi}(\tau) &= 0 \\ R_{\varepsilon y}(\tau) &= 0 \\ R_{\varepsilon\varphi}(\tau) &= 0 \end{aligned}\right\}, \tag{4.26}$$

where

$$R'_{g\eta}(\tau) = \frac{1}{T}\int_0^T g(t)\eta(t+\tau)\,dt - m_g m_\eta$$

$$= \frac{1}{T}\int_0^T [X(t)+\varepsilon(t)][y(t+\tau)+\phi(t+\tau)]dt - (m_x + m_\varepsilon)(m_y + m_\phi), \tag{4.27}$$

$$R_{g\eta}(\tau) = \frac{1}{T}\int_0^T \dot{g}(t)\dot{\eta}(t+\tau)\,dt$$
$$= \frac{1}{T}\int_0^T \left[\dot{X}(t)+\dot{\varepsilon}(t)\right]\left[\dot{y}(t+\tau)+\dot{\varphi}(t+\tau)\right]dt, \tag{4.28}$$

$$R'_{xy}(\tau) = \frac{1}{T}\int_0^T X(t)y(t+\tau)\,dt - m_x m_y,$$

$$R_{xy}(\tau) = \frac{1}{T}\int_0^T \dot{X}(t)\,\dot{y}(t+\tau)\,dt, \tag{4.29}$$

$$R_{x\varphi}(\tau) = \frac{1}{T}\int_0^T \dot{X}(t)\dot{\varphi}(t+\tau)\,dt,$$

$$R_{\varepsilon y}(\tau) = \frac{1}{T}\int_0^T \dot{\varepsilon}(t)\,\dot{y}(t+\tau)\,dt,$$

$$R_{\varepsilon\varphi}(\tau) = \frac{1}{T}\int_0^T \dot{\varepsilon}(t)\dot{\varphi}(t+\tau)\,dt.$$

m_g, m_η, m_x, and m_y are mathematical expectations of the signals $g(t)$, $\eta(t)$, $X(t)$, and $y(t)$.

But because of factors mentioned earlier, Eq. (4.26) does not hold in practice and inequalities therefore occur:

$$R_{x\varphi}(\tau) \neq 0, \ R_{\varepsilon y}(\tau) \neq 0, \ R_{\varepsilon\varphi}(\tau) \neq 0. \tag{4.30}$$

For that reason, Eq. (4.25) does not hold, and often the following relations occur:

$$R'_{g\eta}(\tau) - R'_{xy}(\tau) = \Lambda_1(\tau), \tag{4.31}$$

$$R_{g\eta}(\tau) - R_{xy}(\tau) = \Lambda_2(\tau), \tag{4.32}$$

$$\left|\Lambda_1(\tau) - \Lambda_2(\tau)\right| = \lambda(\mu), \tag{4.33}$$

$$\left|R'_{g\eta}(\tau) - R_{g\eta}(\tau)\right| = \lambda(\mu). \tag{4.34}$$

The effect of the noise $\varepsilon(t)$ and $\varphi(t)$ on the appearance of errors $\Lambda_1(\tau)$ and $\Lambda_2(\tau)$ can be revealed from expressions (4.31) and (4.32):

$$\Lambda_1(\tau) = \frac{1}{T}\int_0^T g(t)\eta(t+\tau)dt - m_g m_\eta - \left[\frac{1}{T}\int_0^T X(t)y(t+\tau)dt - m_x m_y\right]$$

$$= \frac{1}{T}\int_0^T [X(t)+\varepsilon(t)][y(t+\tau)+\varphi(t+\tau)]dt - (m_x + m_\varepsilon)(m_y + m_\varphi)$$

$$-\frac{1}{T}\int_0^T X(t)y(t+\tau)dt - m_x m_y$$

$$= \frac{1}{T}\int_0^T \left[\dot{X}(t) + m_x + \dot{\varepsilon}(t) + m_\varepsilon\right]\left[\dot{y}(t+\tau) + m_y + \dot{\varphi}(t+\tau) + m_\varphi\right]dt$$

$$-(m_x + m_\varepsilon)(m_y + m_\phi)$$

$$-\left[\frac{1}{T}\int_0^T \left[\dot{X}(t) + m_x\right]\left[\dot{y}(t+\tau) + m_y\right]dt - m_x m_y\right].$$

Taking into account that for random processes the mathematical expectations

$$\frac{1}{T}m_\varphi\int_0^T \dot{X}(t)\,dt\,,\ \frac{1}{T}m_x\int_0^T \dot{\varphi}(t)\,dt\,,\ \frac{1}{T}m_y\int_0^T \dot{\varepsilon}(t)\,dt\,,$$

$$\frac{1}{T}m_\varphi\int_0^T \dot{\varepsilon}(t)\,dt\,,\ \frac{1}{T}m_\varepsilon\int_0^T \dot{y}(t+\tau)\,dt\,,\ \frac{1}{T}m_\varepsilon\int_0^T \dot{\varphi}(t+\tau)\,dt\,,$$

$$m_x m_\varphi,\ m_\varepsilon m_y,\ \frac{1}{T}\int_0^T \dot{\varepsilon}(t)\dot{\varphi}(t+\tau)\,dt$$

may be equal to zero, the error value for a noncentered signal is presented in the form

$$\Lambda_1(\tau)=\frac{1}{T}\int_0^T\Big[\big|\dot{X}(t)\dot{\varphi}(t+\tau)\big|+\big|\dot{\varepsilon}(t)\dot{y}(t+\tau)\big|\Big]dt\,. \tag{4.35}$$

The centered signal error is determined analogously:

$$\Lambda_2(\tau)=\frac{1}{T}\int_0^T \dot{g}(t)\dot{\eta}(t+\tau)\,dt-\int_0^T \dot{x}(t)\dot{y}(t+\tau)\,dt=\frac{1}{T}\int_0^T\big[\dot{x}(t)+\dot{\varepsilon}(t)\big]\big[\dot{y}(t+\tau)$$

$$+\dot{\varphi}(t+\tau)\big]-\int_0^T \dot{X}(t)\dot{y}(t+\tau)\,dt$$

$$=\frac{1}{T}\int_0^T\big[\dot{X}(t)\dot{\varphi}(t+\tau)+\dot{\varepsilon}(t)\dot{y}(t+\tau)\big]dt\,. \tag{4.36}$$

It follows from expressions (4.35) and (4.36) that, due to the presence of the noise $\varepsilon(t)$ and $\varphi(t)$, the estimates of cross-correlation functions for both centered and noncentered signals are different from their true values $R'_{xy}(\tau)$ and $R_{xy}(\tau)$ by the values $\Lambda_1(\tau)$ and $\Lambda_2(\tau)$, respectively. According to expression (4.28), the estimates of cross-correlation functions $R_{g\eta}(\tau)$ of centered signals are formed from the sum of a certain number of positive products $\dot{g}(t)\dot{\eta}(t+\tau)$ and a certain number of negative products $\dot{g}(t)\dot{\eta}(t+\tau)$. But estimates of the cross-correlation functions $R_{g\eta}(\tau)$ of noncentered signals are, according to expression (4.27), formed only from the positive products $g(t)\eta(t+\tau)$. In this connection, despite the identity of expressions (4.35) and (4.36), the value $\Lambda_1(\tau)$ is formed from the sum of errors of the positive products $g(t)\eta(t+\tau)$, and the value $\Lambda_2(\tau)$ is formed from the errors of the positive and negative products $\dot{g}(t)\dot{\eta}(t+\tau)$. Thus, the values $\Lambda_1(\tau)$ and $\Lambda_2(\tau)$ differ by the value $\lambda(\mu)$.

Considering the above intervals of integration, Eq. (4.36) can be represented as follows:

$$\Lambda_2(\tau) = \frac{1}{T}\int_{T^+(\tau)} \left[\dot{X}(t)\dot{\varphi}(t+\tau) + \dot{\varepsilon}(t)\dot{y}(t+\tau)\right]dt$$

$$-\frac{1}{T}\int_{T^-(\tau)} \left[\dot{X}(t)\dot{\varphi}(t+\tau) + \dot{\varepsilon}(t)\dot{y}(t+\tau)\right]dt\,. \tag{4.37}$$

Here, as it follows from expressions (4.35) and (4.36), Eq. (4.34) can be transformed in the following way:

$$\lambda(\tau) = \left|R'_{g\eta}(\tau) - R_{g\eta}(\tau)\right| = \left|R'_{xy}(\tau) + \Lambda_1(\tau) - R_{xy}(\tau) - \Lambda_2(\tau)\right|.$$

Therefore,

$$\lambda(\tau=0) = \left|\Lambda_1(\tau=0) - \Lambda_2(\tau=0)\right|$$

$$= \left|\frac{1}{T}\int_0^T \left[\left|\dot{X}(t)\dot{\varphi}(t)\right| + \left|\dot{\varepsilon}(t)\dot{y}(t)\right|\right]dt\right.$$

$$-\frac{1}{T}\int_{T^+(\tau=0)} \left[\dot{X}(t)\dot{\varphi}(t) + \dot{\varepsilon}(t)\dot{y}(t)\right]dt$$

$$\left.+\frac{1}{T}\int_{T^-(\tau=0)} \left[\dot{X}(t)\dot{\varphi}(t) + \dot{\varepsilon}(t)\dot{y}(t)\right]dt\right|.$$

Taking into account that

$$\frac{1}{T}\int_0^T \left[\left|\dot{X}(t)\dot{\varphi}(t+\tau)\right| + \left|\dot{\varepsilon}(t)\dot{y}(t+\tau)\right|\right]dt$$

$$-\frac{1}{T}\int_{T^+(\tau)} \left[\dot{X}(t)\dot{\varphi}(t+\tau) + \dot{\varepsilon}(t)\dot{y}(t+\tau)\right]dt$$

$$= \left|\frac{1}{T}\int_{T^-(\tau)} \left[\dot{X}(t)\dot{\varphi}(t+\tau) + \dot{\varepsilon}(t)\dot{y}(t+\tau)\right]dt\right|, \tag{4.38}$$

we get

$$\lambda(\tau=0) = \left|\frac{2}{T}\int_{T^-(\tau=0)} \left[\dot{X}(t)\dot{\varphi}(t) + \dot{\varepsilon}(t)\dot{y}(t)\right]dt\right|. \tag{4.39}$$

Now it is obvious that in calculating the cross-correlation functions $R_{g\eta}(\tau)$ of the centered signals $\dot{g}(t)$ and $\dot{\eta}(t)$ with $\tau = 0$, the integral of negative productions $\dot{g}(t)\dot{\eta}(t)$ in the integration interval with length $T^{-}(\tau = 0)$ determines the value $\lambda(\tau = 0)$, which is the sum of the errors of these negative products. Thus, Eq. (4.34) holds in practice. It is obvious that the formula for finding the mean error $\langle \Delta\lambda(\tau = 0)\rangle$ at time shift $\tau = 0$ can be represented in the form

$$\langle \Delta\lambda(\tau = 0)\rangle = \left[1/T^{-}(\tau = 0)\right]\lambda(\tau = 0), \tag{4.40}$$

and the formula for finding the robustness value of the estimates $R_{g\eta}(\tau)$ with different time shift τ between $\dot{g}(t)$ and $\dot{\eta}(t+\tau)$ will be

$$\lambda_{xy}^{R}(\tau) = \left[T^{+}(\tau) - T^{-}(\tau)\right]\langle \Delta\lambda(\tau = 0)\rangle. \tag{4.41}$$

Based on the above-mentioned results, one can formulate the following procedure for determining values of the robustness of the cross-correlation functions $R_{g\eta}(\tau)$.

1. Calculate values of the cross-correlation functions $R'_{g\eta}(\tau)$ for all the values τ for the noncentered signals $g(t)$, $\eta(t)$ by

$$R'_{g\eta}(\tau) = \frac{1}{T}\int_0^T g(t)\eta(t+\tau)dt - m_g m_\eta \text{, where } m_\eta = \frac{1}{T}\int_0^T \eta(t)dt.$$

2. Calculate values of the cross-correlation functions $R_{g\eta}(\tau)$ for all the values τ for the centered signals $\dot{g}(t)$, $\dot{\eta}(t)$ by

$$R_{g\eta}(\tau) = \frac{1}{T}\int_0^T \dot{g}(t)\dot{\eta}(t+\tau)dt.$$

3. Determine the error value $\lambda(\tau = 0)$ at time shift $\tau = 0\cdot\Delta t$ by

$$\lambda(\tau = 0) = \left|R'_{g\eta}(\tau = 0) - R_{g\eta}(\tau = 0)\right|.$$

4. Determine the mean error value with time shift $\tau = 0\cdot\Delta t$ by

$$\langle \Delta\lambda(\tau = 0)\rangle = \left[1/T^{-}(\tau = 0)\right]\lambda(\tau = 0).$$

5. Calculate the robustness value $\lambda_{xy}^{R}(\tau)$ of the estimates $R_{g\eta}(\tau)$ at different time shift τ between $\dot{g}(t)$ and $\dot{\eta}(t+\tau)$ by

$$\lambda_{xy}^{R}(\tau) = \left[T^{+}(\tau) - T^{-}(\tau)\right]\langle \Delta\lambda(\tau = 0)\rangle.$$

6. Determine the robustness estimates of the cross-correlation functions $R_{g\eta}(\tau)$ by

$$R^R_{g\eta}(\tau) = R_{g\eta}(\tau) - \lambda^R_{xy}(\tau).$$

However, in practice, the correlation function normally has to be determined with the help of experimental data processing presenting a record or realizations of the studied random processes $g(t)$, $\eta(t)$. The robust technology and digital algorithms for determining cross-correlation function estimates with the help of up-to-date processing means are given in Section 4.5.

4.5 Technology of Determining the Value of Providing the Robustness of Estimates of Auto- and Cross-Correlation Functions

In practice, when the measurement information obtained in the process of normal operation of appropriate objects is used for determining the estimates of the correlation functions, the value of the obtained signal consists of the sum of the real signal of a technological parameter and the noise. For many stochastic processes, their cross-correlation is equal to zero. However, for a number of industrial objects, their cross-correlation differs from zero; for these cases, the estimates of the correlation functions have large errors. As shown by the research in solving the corresponding problems, this special feature of the real objects is not taken into account. In this connection, the obtained results of the analysis of the experimental data are unsatisfactory. Thus, the application of the probabilistic-statistical methods in the experimental research is limited. For example, as stated above, the practical realization of the probabilistic-statistical methods in the development of the mathematical models of both statics and dynamics of the real industrial objects is not great, yet there is a great need for a solution to that problem [2, 28, 31, 44, 45, 52]. The efficiency of the application of the statistical methods of monitoring, diagnostics, and control of many modern technological processes where the number of the controlled technological parameters is about 1,000 is not great either.

Thus, it is expedient to develop algorithms that allow one—even when the experimental information consists of the signal and considerable noise and the correlation between them differs from zero—to improve the estimates of the auto- and cross-correlation functions.

The realized analysis of publications showed that the task of providing the necessary accuracy of the estimates of statistic characteristics for these cases isn't investigated sufficiently [10, 37].

In the known literature, it is assumed that the objects of control are linear and the realization $g(t)$, $\eta(t)$ of the random processes $X(t)$, $y(t)$ are the stationary ergodic processes with normal distribution law and the interval of time of the observation T is great enough; thus, instead of the correlation functions $R_{xx}(\mu)$, $R_{xy}(\mu)$, we can use their estimates $R_{gg}(\mu)$, $R_{g\eta}(\mu)$. However, in practice, for many experimental objects, the technological parameters $g(t)$ consist of the useful signal $X(t)$ and the noise $\varepsilon(t)$. In calculating the estimates $R_{gg}(\mu)$ and $R_{g\eta}(\mu)$, it is assumed that $\varepsilon(t)$ does not exert an influence on the useful signal $X(t)$ and that the cross-correlation between the useful signal and the noise is equal to zero. Theoretically, this holds for stationary random processes and for many real objects. At the same time, for most of the industrial objects as shown by Eqs. (4.22) and (4.39), this assumption does not apply because the cross-correlation between the useful signal $X(t)$ and the noise $\varepsilon(t)$ differs from zero. The realized analysis of the literature shows that the problem of providing the necessary accuracy of the estimates of the statistical characteristics for these cases has been insufficiently explored [10, 37].

It is known that for a sampled random signal $g(i\Delta t)$ with normal distribution law consisting of the signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ with the mathematical expectation $m_\varepsilon \approx 0$, the following equalities apply in calculating the estimates of the correlation functions by means of the application of the traditional methods:

$$\left.\begin{aligned} R'_{xx}(\mu) &= R'_{gg}(\mu) \\ R_{xx}(\mu) &= R_{gg}(\mu) \\ R_{gg}(\mu) &= R'_{gg}(\mu) \\ R_{x\varepsilon}(\mu) &= 0 \end{aligned}\right\}, \tag{4.42}$$

where

$$R'_{xx}(\mu) = \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)X((i+\mu)\Delta t) - m_x^2, \tag{4.43}$$

$$R_{xx}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \dot{X}(i\Delta t)\dot{X}((i+\mu)\Delta t), \tag{4.44}$$

$$R_{x\varepsilon}(\mu)=\frac{1}{N}\sum_{i=1}^{N}\dot{X}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t)\,, \tag{4.45}$$

$$R'_{gg}(\mu)=\frac{1}{N}\sum_{i=1}^{N}g(i\Delta t)g((i+\mu)\Delta t)-m_g^2$$

$$=\frac{1}{N}\sum_{i=1}^{N}\left[\dot{X}((i+\mu)\Delta t)+\dot{\varepsilon}((i+\mu)\Delta t)\right]-(m_x+m_\varepsilon)^2\,, \tag{4.46}$$

$$R_{gg}(\mu)=\frac{1}{N}\sum_{i=1}^{N}\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)$$

$$=\frac{1}{N}\sum_{i=1}^{N}\left[\dot{X}(i\Delta t)+\dot{\varepsilon}(i\Delta t)\right]\left[\dot{X}((i+\mu)\Delta t)+\dot{\varepsilon}((i+\mu)\Delta t)\right], \tag{4.47}$$

where

$$\dot{g}(i\Delta t)=g(i\Delta t)-m_g\,,$$

$$\dot{X}(i\Delta t)=X(i\Delta t)-m_x\,,$$

$$\dot{\varepsilon}(i\Delta t)=\varepsilon(i\Delta t)-m_\varepsilon$$

are the centered values of the sampled signals $g(i\Delta t)$, $X(i\Delta t)$, and $\varepsilon(i\Delta t)$; and m_g, m_x, and m_ε are the mathematical expectations of the signals $g(i\Delta t)$, $X(i\Delta t)$, and $\varepsilon(i\Delta t)$.

In practice, as mentioned above, for a certain group of objects these equalities really take place. Simultaneously, for some objects these equalities do not take place, in which case we get the following inequalities:

$$R'_{gg}(\mu)\neq R'_{xx}(\mu)\,, \tag{4.48}$$

$$R_{gg}(\mu)\neq R_{xx}(\mu)\,, \tag{4.49}$$

$$R_{x\varepsilon}\neq 0\,. \tag{4.50}$$

An algorithm for determining the values of robustness is considered below. As mentioned in Section 4.3, for many objects of the modern technological processes, the following conditions apply in practice:

$$R'_{gg}(\mu) - R'_{xx}(\mu) = \Lambda_1(\mu)\,, \tag{4.51}$$

$$R_{gg}(\mu) - R_{xx}(\mu) = \Lambda_2(\mu)\,, \tag{4.52}$$

$$R'_{gg}(\mu = 0) - R'_{xx}(\mu = 0) = \Lambda_1(\mu = 0) = \Lambda_2(\mu = 0)\,, \tag{4.53}$$

$$\left|\Lambda_1(\mu \neq 0) - \Lambda_2(\mu \neq 0)\right| = \lambda(\mu \neq 0)\,, \tag{4.54}$$

$$\left|R'_{gg}(\mu) - R_{gg}(\mu)\right| = \lambda(\mu)\,. \tag{4.55}$$

These equalities are caused by the fact that the value of the correlation between the useful signal $X(t)$ and the noise $\varepsilon(t)$ differs from zero and leads to the appearance of errors contained in each separate product $\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)$ and caused by the noise $\dot{\varepsilon}(i\Delta t)$. We shall call the value of the error in one separate product $\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)$ as the microerror and denote it as $\Delta\lambda(\mu)$.

Let us consider the opportunity of reducing the total error of the estimates $R_{gg}(\mu)$ to zero by means of balancing the microerrors of the positive and negative products. With this in mind, first we must determine the influence of the noise on the appearance of microerrors in the products $g(i\Delta t)g((i+\mu)\Delta t)$, $X(i\Delta t)X((i+\mu)\Delta t)$ and $\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)$, $\dot{X}(i\Delta t)\dot{X}((i+\mu)\Delta t)$:

$$\Lambda_1(\mu) = \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)g((i+\mu)\Delta t) - m_g^2 - \left[\frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)X((i+\mu)\Delta t)\right.$$

$$\left. - m_x^2\right] = \frac{1}{N}\sum_{i=1}^{N}\left[X(i\Delta t) + \varepsilon(i\Delta t)\right]\left[X((i+\mu)\Delta t) + \varepsilon((i+\mu)\Delta t)\right]$$

$$-(m_x + m_\varepsilon)^2 - \left[\frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)X((i+\mu)\Delta t) - m_x^2\right]$$

$$=\frac{1}{N}\sum_{i=1}^{N}\Big\{\Big[\dot{X}(i\Delta t)+m_x+\dot{\varepsilon}(i\Delta t)+m_\varepsilon\Big]\Big[\dot{X}((i+\mu)\Delta t)$$

$$+m_x+\dot{\varepsilon}((i+\mu)\Delta t)+m_\varepsilon\Big]-(m_x+m_\varepsilon)^2$$

$$-\Big[\dot{X}(i\Delta t)+m_x\Big]\Big[\dot{X}((i+\mu)\Delta t)+m_x\Big]\Big\}.$$

For the stationary random processes with normal distribution law, the mathematical expectations of the products $\dot{X}(i\Delta t)m_\varepsilon$, $\dot{\varepsilon}(i\Delta t)m_\varepsilon$, $\dot{X}((i+\mu)\Delta t)m_\varepsilon$, $\dot{\varepsilon}((i+\mu)\Delta t)m_\varepsilon$, $\dot{\varepsilon}(i\Delta t)m_x$, $\dot{\varepsilon}((i+\mu)\Delta t)m_x$, and $m_\varepsilon m_x$ can be taken equal to zero. At the same time, the value of the error of the noncentered signal for which all products are positive can be represented as

$$\Lambda_1(\mu)=\frac{1}{N}\sum_{i=1}^{N}\Big(\Big|\dot{X}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t)\Big|+\Big|\dot{\varepsilon}(i\Delta t)\dot{X}((i+\mu)\Delta t)\Big|$$

$$+\Big|\dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t)\Big|\Big). \tag{4.56}$$

The error of the centered signal is determined similarly:

$$\Lambda_2(\mu)=\frac{1}{N}\sum_{i=1}^{N}\Big(\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)-\dot{X}(i\Delta t)\dot{X}((i+\mu)\Delta t)\Big)$$

$$=\frac{1}{N}\sum_{i=1}^{N}\Big[\Big(\dot{X}(i\Delta t)+\dot{\varepsilon}(i\Delta t)\Big)\Big(\dot{X}((i+\mu)\Delta t)+\dot{\varepsilon}((i+\mu)\Delta t)\Big)$$

$$-\dot{X}(i\Delta t)\dot{X}((i+\mu)\Delta t)\Big]=\frac{1}{N}\sum_{i=1}^{N}\Big(\dot{X}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t)$$

$$+\dot{\varepsilon}(i\Delta t)\dot{X}((i+\mu)\Delta t)+\dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+\mu)\Delta t)\Big). \tag{4.57}$$

If, for the positive products $\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)$, one can use the summing indices $i=i_{1^+(\mu)}$, $i_{2^+(\mu)}$, ..., $i_{N^+(\mu)}$, where $N^+(\mu)$ is the number of positive products at time shift μ, and if, for the negative products

$\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)$, one can take indices $i = i_{1^-(\mu)}$, $i_{2^-(\mu)}$, ..., $i_{N^-(\mu)}$, where $N^-(\mu)$ is the number of negative products at time shift μ, then $\Lambda_2(\mu)$ may be represented as follows:

$$\Lambda_2(\mu) = \frac{1}{N} \sum_{i=i_{1^+(\mu)}}^{i_{N^+(\mu)}} \left(\dot{X}(i\Delta t)\dot{\varepsilon}\left((i+\mu)\Delta t\right) + \dot{\varepsilon}(i\Delta t)\dot{X}\left((i+\mu)\Delta t\right) \right.$$

$$\left. + \dot{\varepsilon}(i\Delta t)\dot{\varepsilon}\left((i+\mu)\Delta t\right)\right)$$

$$-\frac{1}{N} \sum_{i=i_{1^-(\mu)}}^{i_{N^-(\mu)}} \left(\dot{X}(i\Delta t)\dot{\varepsilon}\left((i+\mu)\Delta t\right) + \dot{\varepsilon}(i\Delta t)\dot{X}\left((i+\mu)\Delta t\right) \right.$$

$$\left. + \dot{\varepsilon}(i\Delta t)\dot{\varepsilon}\left((i+\mu)\Delta t\right)\right). \quad (4.58)$$

Here, taking into account

$$\lambda(\mu = 1) = \left| R'_{gg}(\mu = 1) - R_{gg}(\mu = 1) \right|, \quad (4.59)$$

$$R'_{xx}(\mu = 1) = R_{xx}(\mu = 1),$$

we can write the following:

$$\lambda(\mu = 1) = \left|\Lambda_1(\mu = 1) - \Lambda_2(\mu = 1)\right| = \left| \frac{1}{N} \sum_{i=1}^{N} \left(\left| \dot{X}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t) \right| \right.\right.$$

$$+ \left| \dot{\varepsilon}(i\Delta t)\dot{X}((i+1)\Delta t) \right| + \left| \dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t) \right|$$

$$-\frac{1}{N} \sum_{i=i_{1^+(\mu)}}^{i_{N^+(\mu)}} \left(\dot{X}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t) \right.$$

$$\left. + \dot{\varepsilon}(i\Delta t)\dot{X}((i+1)\Delta t) + \dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)\right)$$

$$+\frac{1}{N} \sum_{i=i_{1^-(\mu)}}^{i_{N^-(\mu)}} \left(\dot{X}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t) \right.$$

$$\left.\left. + \dot{\varepsilon}(i\Delta t)\dot{X}((i+1)\Delta t) + \dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)\right)\right|.$$

Taking into account the fact that

$$\frac{1}{N} \sum_{i=1}^{N} \left(\left| \dot{X}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t) \right| + \left| \dot{\varepsilon}(i\Delta t)\dot{X}((i+1)\Delta t) \right| + \left| \dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t) \right| \right)$$

$$-\frac{1}{N}\sum_{i=i_{1^+(\mu)}}^{i_{N^+(\mu)}}\left(\dot{X}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)+\dot{\varepsilon}(i\Delta t)\dot{X}((i+1)\Delta t)+\dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)\right)$$

$$=\left|\frac{1}{N}\sum_{i=i_{1^-(\mu)}}^{i_{N^-(\mu)}}\left(\dot{X}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)+\dot{\varepsilon}(i\Delta t)\dot{X}((i+1)\Delta t)\right.\right.$$

$$\left.\left.+\dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t)\right)\right|,$$

we derive

$$\lambda(\mu=1)\approx\left|\frac{2}{N}\sum_{i=i_{1^-(\mu)}}^{i_{N^-(\mu)}}\dot{X}(i\Delta t)\dot{\varepsilon}\big((i+1)\Delta t\big)+\dot{\varepsilon}(i\Delta t)\dot{X}\big((i+1)\Delta t\big)\right.$$

$$\left.+\dot{\varepsilon}(i\Delta t)\dot{\varepsilon}((i+1)\Delta t))\right|. \tag{4.60}$$

It follows that in calculating the correlation function $R_{gg}(\mu)$, the appearance of $\lambda(\mu=1)$ is caused by the number $N^-(\mu=1)$ of products $\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)$ with the negative sign (where N is the number of all products $\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)$). Here, the average microerror $\langle\Delta\lambda(\mu=1)\rangle$, which is detected in the one negative product, can be determined by means of the following formula:

$$\langle\Delta\lambda(\mu=1)\rangle=\left[\frac{1}{N^-(\mu=1)}\right]\lambda(\mu=1). \tag{4.61}$$

The value of providing the robustness $\lambda_{xx}^R(\mu)$ of estimates of the autocorrelation function is determined as the difference of microerrors of the estimates $R_{gg}(\mu)$ at different time shifts μ between $\dot{g}(i\Delta t)$ and $\dot{g}((i+1)\Delta t)$. It is determined as follows:

$$\lambda_{xx}^R(\mu)\approx\left[N^+(\mu)-N^-(\mu)\right]\langle\Delta\lambda(\mu=1)\rangle. \tag{4.62}$$

It is clear that the value of providing the robustness $\lambda_{xy}^R(\mu)$ of the cross-correlation function $R_{g\eta}(\mu)$ between the signals $g(i\Delta t)=X(i\Delta t)+\varepsilon(i\Delta t)$ and $\eta(i\Delta t)=y(i\Delta t)+\varphi(i\Delta t)$ consisting of the mixtures of the legitimate signals $X(i\Delta t)$ and $y(i\Delta t)$ and the appropriate noises $\varepsilon(i\Delta t)$ and $\varphi(i\Delta t)$ is determined according to the formula

$$\lambda_{xy}^{R}(\mu) \approx \left[N^{+}(\mu) - N^{-}(\mu)\right]\left\langle \Delta\lambda(\mu = 0)\right\rangle , \tag{4.63}$$

where

$$\left\langle \Delta\lambda(\mu = 0)\right\rangle = \frac{1}{N^{-}(\mu = 0)}\lambda(\mu = 0) ,$$

$$\lambda(\mu = 0) = \left|R'_{g\eta}(\mu = 0) - R_{g\eta}(\mu = 0)\right| ,$$

$$R'_{g\eta}(\mu) = \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)\eta((i + \mu)\Delta t) - m_g m_\eta ,$$

$$R_{g\eta}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \dot{g}(i\Delta t)\dot{\eta}((i + \mu)\Delta t) ,$$

$$\dot{g}(i\Delta t) = g(i\Delta t) - m_g ,$$

$$\dot{\eta}(i\Delta t) = \eta(i\Delta t) - m_\eta ,$$

where m_g and m_η are mathematical expectations of the signals $g(i\Delta t)$ and $\eta(i\Delta t)$, respectively.

Thus, the value of providing robustness of the estimates of the cross-correlation functions is also calculated as a difference of the microerrors of the estimates $R_{g\eta}(\mu)$ at various time shifts μ between $\dot{g}(i\Delta t)$ and $\dot{\eta}((i + \mu)\Delta t)$.

It is obvious that, by using λ_{xx}^{R} and λ_{xy}^{R}, one can balance the errors of the decision estimates.

4.6 Robust Algorithms of Improving Estimates of Auto- and Cross-Correlation Functions

From the algorithms described in the last section, it is obvious that by having variances D_ε and D_φ of the noises $\dot{\varepsilon}(i\Delta t)$ and $\dot{\phi}(i\Delta t)$ of the signals $g(i\Delta t)$, $\eta(i\Delta t)$ and the values $\Lambda_{xx}^{*}(\mu)$, $\Lambda_{xy}^{*}(\mu)$, we can provide the robustness of the estimates of auto- and cross-correlation functions $R_{gg}(\mu)$ and $R_{g\eta}(\mu)$ according to the expressions

$$R_{gg}^{R}(\mu)=\begin{cases} R_{gg}(\mu)-[\lambda_{xx}^{R}(\mu)+D_{\varepsilon}] & \text{for } \mu=0, \\ R_{gg}(\mu)-\lambda_{xx}^{R}(\mu) & \text{for } \mu\neq 0, \end{cases} \tag{4.64}$$

$$R_{xy}^{R}(\mu)=R_{g}(\mu)-\lambda_{xy}^{R}(\mu)\,. \tag{4.65}$$

To control the efficiency of robust technology, the appropriate computer experiments were realized. In these experiments for the determination of the variance of the noise D_{ε} and the values of providing the robustness $\lambda_{xx}^{R}(\mu)$ and $\lambda_{xy}^{R}(\mu)$, the formulae (4.39), (4.62), and (4.63) were applied. Both standard and random signals also are formed as the sum of the harmonic signals and the corresponding noise. In this case, the balancing is performed by means of the obtained values of the positive and negative microerrors. The residual errors and the variance of the noise of both the standard and random signals are also determined by means of the obtained values of the positive and negative microerrors. Here, the number of the negative products $N^{-}(\mu)$ by means of which the average microerror $\langle\Delta\lambda(\mu)\rangle$ and the variance D_{ε} of the noise $\varepsilon(t)$ are determined exerts considerable influence on the efficiency of balancing for the number of samples $N=8{,}192$. The results of the experiments showed that when the quantity $N^{-}(\mu)$ is increased, the value of the residual errors $\Lambda_{x_i x_j}(\mu)_{\text{rem}}$, and $\Lambda_{x_i y}(\mu)_{\text{rem}}$ tends to zero after balancing for all values μ:

$$\lambda_{x_i x_j}^{R}(\mu)\approx\Lambda_{x_i x_j}(\mu)\,,\quad \lambda_{x_i y}^{R}(\mu)\approx\Lambda_{x_i y}(\mu)$$

or

$$\Lambda_{x_i x_j}(\mu)_{\text{rem}}\approx\lambda_{x_i x_j}^{R}(\mu)-\Lambda_{x_i x_j}(\mu)\approx 0\,,$$

$$\Lambda_{x_i y}(\mu)_{\text{rem}}=\lambda_{x_i y}^{R}(\mu)-\Lambda_{x_i y}(\mu)\approx 0\,, \tag{4.66}$$

where $\Lambda_{x_i x_j}(\mu)$ and $\Lambda_{x_i y}(\mu)$ are the true values of the errors of the estimates $R_{g_i g_j}(\mu)$ and $R_{g_i \eta}(\mu)$; $\lambda_{x_i x_j}^{R}(\mu)$ and $\lambda_{x_i y}^{R}(\mu)$ are the values for providing the robustness of the estimates $R_{g_i g_j}(\mu)$ and $R_{g_i \eta}(\mu)$.

Thus, between the robust estimates of the correlation functions and their real values, the following equalities are true:

$$R_{x_i x_j}^{*}(\mu)\approx R_{x_i x_j}(\mu)\,,\ R_{x_i y}^{*}(\mu)\approx R_{x_i y}(\mu)\,. \tag{4.67}$$

In increasing the variance D_ε of the noise $\varepsilon(t)$ up to the value $D(\varepsilon) \le 0.1R_{xx}(0)$, the signals for which the equality of the relative residual errors of the estimates occurs after balancing are formed, i.e.,

$$\gamma_{x_i x_j}(\mu)_{\text{rem}} \approx \gamma_{x_k x_l}(\mu)_{\text{rem}} \approx \gamma \,, i, j, k, l = \overline{1, n}\,;$$

$$\gamma_{x_1 y}(\mu)_{\text{rem}} \approx \gamma_{x_2 y}(\mu)_{\text{rem}} \approx \ldots \approx \gamma_{x_n y}(\mu)_{\text{rem}} \approx \gamma\,,$$

$$D[\gamma_{x_i x_j}(\mu)_{\text{rem}}] \approx 0\,,\; D[\gamma_{x_i y}(\mu)_{\text{rem}}] \approx 0\,, \tag{4.68}$$

where

$$\gamma_{x_i x_j}(\mu)_{\text{rem}} \approx \Lambda_{x_i x_j}(\mu)_{\text{rem}} \big/ \Lambda_{x_i x_j}(\mu)\,,$$

$$\gamma_{x_i y}(\mu)_{\text{rem}} \approx \Lambda_{x_i y}(\mu)_{\text{rem}} \big/ \Lambda_{x_i y}(\mu)\,,$$

$D[\gamma_{x_i x_j}(\mu)_{\text{rem}}]$, $D[\gamma_{x_i y}(\mu)_{\text{rem}}]$ are the range of the spread of the relative errors after applying the balancing algorithms.

Thus, for experimental data, the values of the variance of the noise and the microerrors caused by the noise exert considerable influence on the estimates of their auto- and cross-correlation functions.

In conclusion, we note that it is expedient to realize the robust technology of improving estimates of autocorrelation functions as the following sequence:

1. The correlation function of the centered $\dot{g}(i\Delta t)$ and noncentered $g(i\Delta t)$ signals is determined correspondingly by means of the algorithms

$$R_{gg}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)\,, \tag{4.69}$$

$$R'_{gg}(\mu) = \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)g((i+\mu)\Delta t) - m_g^2\,. \tag{4.70}$$

2. The average microerror $\langle \Delta\lambda(\mu=1)\rangle$ of the product $\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)$ is determined by means of formula (4.61), i.e.,

$$\langle \Delta\lambda(\mu=1)\rangle = \left[1/N^-(\mu=1) \right] \lambda(\mu=1)\,. \tag{4.71}$$

In turn, the value $\lambda(\mu=1)$ is determined by means of formula (4.59), i.e.,

$$\lambda(\mu=1)=\left|R'_{gg}(\mu=1)-R_{gg}(\mu=1)\right|. \tag{4.72}$$

3. The value of providing the robustness can be determined by formula (4.62) as follows:

$$\lambda^R_{xx}(\mu)\approx\left[N^+(\mu)-N^-(\mu)\right]\langle\Delta\lambda(\mu=1)\rangle .$$

4. The variance of the noise is determined by means of formula (3.22) as follows:

$$D_\varepsilon=\frac{1}{N}\sum_{i=1}^{N}\left(\dot g(i\Delta t)\dot g(i\Delta t)+\dot g(i\Delta t)\dot g((i+2)\Delta t)-2\dot g(i\Delta t)\dot g((i+1)\Delta t)\right). \tag{4.73}$$

5. Robust estimates of the correlation functions can be determined by means of formula (4.64), i.e.,

$$R^R_{gg}(\mu)=\begin{cases}R_{gg}(\mu)-\left[\lambda^R_{xx}(\mu)+D_\varepsilon\right] & \text{for} \quad \mu=0,\\ R_{gg}(\mu)-\lambda^R_{xx}(\mu) & \text{for} \quad \mu\neq 0.\end{cases} \tag{4.74}$$

It is expedient to realize the robust technology of improving estimates of the cross-correlation function as follows:

1. By means of the algorithms

$$R_{g\eta}(\mu)=\frac{1}{N}\sum_{i=1}^{N}\dot g(i\Delta t)\dot\eta((i+\mu)\Delta t),$$

$$R'_{g\eta}(\mu)=\frac{1}{N}\sum_{i=1}^{N}g(i\Delta t)\eta((i+\mu)\Delta t),$$

the correlation functions of the centered $\dot g(i\Delta t)$ and $\dot\eta(i\Delta t)$ and noncentered $g(i\Delta t)$ and $\eta(i\Delta t)$ signals are determined.
2. The average microerror $\langle\Delta\lambda(\mu=0)\rangle$ of a product $\dot g(i\Delta t)\dot\eta((i+\mu)\Delta t)$ at $\mu=0$ is determined, i.e.,

$$\langle\Delta\lambda(\mu=0)\rangle=\left[1/N^-(\mu=0)\right]\lambda(\mu=0),$$

where $\lambda(\mu=0)$ is determined according to the formula

$$\lambda(\mu = 0) = \left| R'_{g\eta}(\mu = 0) - R_{g\eta}(\mu = 0) \right|.$$

3. The value of providing robustness is determined as follows:

$$\lambda_{xy}^{R}(\mu) \approx \left[N^{+}(\mu) - N^{-}(\mu) \right] \langle \Delta\lambda(\mu = 0) \rangle .$$

4. Robust estimates of a cross function are determined as follows:

$$R_{g\eta}^{R}(\mu) = R_{g\eta}(\mu) - \lambda_{xy}^{R}(\mu) .$$

Thus, in monitoring the defects by means of determining and accounting for the robustness value, the opportunity to eliminate the influence of the external factors on the received estimates appears. Thus, when even the insignificant change of the value of the estimates of the correlation functions appears by the influence of the defect's origin, these defects are considered to be detected in time.

5 Spectral Monitoring of a Defect's Origin

5.1 Necessity of Providing Robustness During Spectral Monitoring of a Defect's Origin

The spectral analysis of random processes or the measurement of the value of spectral functions that are the frequency distribution of the energy characteristics of the process is the most important part of the statistical measurements. At first, spectral analysis was used for solving the problem of investigating the characteristics of deterministic processes in contrast to the analysis of the distribution functions and correlation analysis, which were formed directly as a type of statistical measurement. Spectral analysis became an independent branch only after the role of measurement theory of the probability characteristics of random processes as well as the need for apparatus analysis of random processes had increased.

At present, both the theory and the practice of measuring the spectral function values of the random processes are highly developed. In our literature, the questions of spectral analysis of random processes are systematically discussed; a number of monographs are devoted to the methods of description and measurements of the spectral functions [2, 14, 52, 54, 58].

The spectral density of power, the spectral function, the width of the spectrum of a random process, the places and the values of the maximums of the spectral density of power, the border frequencies, and other topics are determined in the spectral analysis of the results of experiments. The spectral density of power $G_x(f)$ characterizes the frequency properties of the signal $X(t)$ received from the corresponding pickup devices and its intensity for various frequencies, i.e., average power per unit of the frequency band. A picture of the distribution of the average power of a random process in frequencies is called the spectrum of power. In this connection, the method of obtaining the above-mentioned characteristics and the approximation of the spectral densities of the power of measuring signals are an important problem of experimental analysis.

It will be shown ahead that in spectral analysis, when the measurement information consists of the useful signal and the noise, the error of the required estimates depends on the difference between the sum of errors of positive and negative products of samples of the total signal and samples of cosinusoids and sinusoids, respectively [14].

In practice, the principle of superposition of signals can be used during the analysis of the operation of linear elements and systems. This principle is based on the following. If the input signal is represented as the sum of two signals, then the output signal is determined as the sum of the output signals, which we would have at the output of the system if each of the input signals acted separately. It is the easiest method of determining the reaction of the linear system or the linear part of the nonlinear system to the input signal with unspecified form. Here the harmonic analysis is applied. Because of this fact, spectral analysis methods and algorithms are widely applied in experimental works [12, 14]. As shown earlier, in spectral analysis the analyzed signal is represented as the sum of harmonic components' sinusoids and cosinusoids, whose ordinate sum at each moment t gives the magnitude of the function:

$$X(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t), \tag{5.1}$$

where $a_0/2$ is the average value of the function $X(t)$ for the period T, and a_n and b_n are the amplitudes of the sinusoid and cosinusoid with frequency $n\omega$.

The following inequality must take place to provide sufficient accuracy of representation of the signal $X(t)$ as the sum of sinusoids and cosinusoids:

$$\sum_{i=1}^{n} \lambda_i^2 \le S, \tag{5.2}$$

where λ_i^2 are the squares of deviations between the sum of the right-hand side of Eq. (5.1) and samples of the signal $X(t)$ at the moments of sampling t_0, t_1, …, t_i, …, t_m with sampling step Δt; S is the permissible value of the mean-root-square deviation.

In formula (5.1) in decomposing the function $X(t)$ in trigonometric Fourier series, the value ω is equal to $2\pi / T$ and the coefficients a_n and b_n are determined as

$$a_n = \frac{2}{T}\int_0^T X(t)\cos n\omega t dt \qquad \text{for} \quad n = 1, 2, ..., \tag{5.3}$$

$$b_n = \frac{2}{T}\int_0^T X(t)\sin n\omega t dt \qquad \text{for} \quad n = 1, 2, ... \tag{5.4}$$

Here the first harmonic has frequency $2\pi / T$, and its period and the period T of the function $X(t)$ are the same. The coefficients a_1 and b_1, a_2 and b_2, and a_3 and b_3 are the amplitudes of cosinusoids and sinusoids obtained for $n = 1$, $n = 2$, $n = 3$, etc.

Theoretically, condition (5.2) for the given value S holds for the periodic signals $X(t)$ without the noise $\varepsilon(t)$. But in reality, the useful signal $X(t)$ is accompanied by a certain noise $\varepsilon(t)$, i.e., it is the sum $g(t) = X(t) + \varepsilon(t)$. Because of this, condition (5.2) does not always hold. Nevertheless, many important problems are successfully solved by means of the application of algorithms (5.3) and (5.4) when the noise changes in certain limits. But when the noise has a considerable value and inequality (5.2) does not hold, solving the problems by means of spectral methods seems to be impossible.

In practice, when the analyzed signal $g(t)$ is the sum of the useful random signal $X(t)$ and the noise $\varepsilon(t)$, i.e.,

$$g(t) = X(t) + \varepsilon(t), \tag{5.5}$$

formulae (5.3) and (5.4) can be represented as follows:

$$a_n = \frac{2}{T}\int_0^T [X(t) + \varepsilon(t)]\cos \; n\omega t \, dt$$

$$= \frac{2}{T}\left[\int_0^T X(t)\cos n\omega t dt + \int_0^T \varepsilon(t)\cos n\omega t dt\right], \tag{5.6}$$

$$b_n = \frac{2}{T}\int_0^T [X(t) + \varepsilon(t)]\sin n\omega t \, dt$$

$$= \frac{2}{T}\left[\int_0^T X(t)\sin n\omega t dt + \int_0^T \varepsilon(t)\sin n\omega t dt\right]. \tag{5.7}$$

In this case, the fulfillment of condition (5.2) can be real when

$$\sum_{i=1}^{N^+} \int_{t_i}^{t_{i+1}} \varepsilon(t) \cos n\omega t dt = \sum_{i=1}^{N^-} \int_{t_{i+1}}^{t_{i+2}} \varepsilon(t) \cos n\omega t dt, \tag{5.8}$$

$$\sum_{i=1}^{N^+} \int_{t_i}^{t_{i+1}} \varepsilon(t) \sin n\omega t\, dt = \sum_{i=1}^{N^-} \int_{t_{i+1}}^{t_{i+2}} \varepsilon(t) \sin n\omega t\, dt. \tag{5.9}$$

Here N^+, t_i, and t_{i+1} are the quantity, the beginning, and the end of the positive half-periods of the $\cos n\omega t$ observed in time T, respectively; N^-, t_{i+1}, and t_{i+2} are the quantity, the beginning, and the end of the negative half-periods of the $\cos n\omega t$ observed in time T, respectively.

Otherwise, when that equality does not occur, the differences

$$\lambda_{an} = \sum_{i=1}^{N^+} \int_{t_i}^{t_{i+1}} \varepsilon(t) \cos n\omega t dt - \sum_{i=1}^{N^-} \int_{t_{i+1}}^{t_{i+2}} \varepsilon(t) \cos n\omega t dt, \tag{5.10}$$

$$\lambda_{bn} = \sum_{i=1}^{N^+} \int_{t_i}^{t_{i+1}} \varepsilon(t) \sin n\omega t\, dt - \sum_{i=1}^{N^-} \int_{t_{i+1}}^{t_{i+2}} \varepsilon(t) \sin n\omega t\, dt \tag{5.11}$$

lead to the error of the estimates of the coefficients a_n and b_n, respectively. At the same time, as follows from expressions (5.10) and (5.11), the differences λ_{an} and λ_{bn} increase in increasing the variance of the $\varepsilon(t)$. The differences λ_{an} and λ_{bn} also increase if there is a correlation between the useful signal $X(t)$ and the noise $\varepsilon(t)$ and when the distribution law of the analyzed signal $g(t)$ differs from the normal. From this point of view, in some cases the errors of the estimates λ_{an} and λ_{bn} can be commensurable with the required coefficients a_n and b_n.

In this connection, it is necessary to develop the algorithms that allow us to provide the inequalities $\lambda_{an} << S_n$ and $\lambda_{bn} << S_n$ and condition (5.2) by eliminating the cause of the errors λ_{an} and λ_{bn} for increasing the reliability of the results of the analysis of the experimental data. At the same time, it is necessary that these algorithms be robust, i.e., they must allow us to eliminate the connection between the values λ_{an} and λ_{bn} and the variance of the error $\varepsilon(t)$. It is also necessary that the estimate's error not depend on the change of the form of the distribution law of the analyzed signal or on the coefficient of the correlation between the useful signal $X(t)$ and the noise $\varepsilon(t)$, etc.

5.2 Reasons the Difference Between Positive and Negative Errors Caused by Noises Appears When Spectral Analysis of Signals Obtained from the Sensors Is Used

Let us assume that the time T of observing the realization of the signal $g(t) = X(t) + \varepsilon(t)$, consisting of the useful signal $X(t)$ and the noise $\varepsilon(t)$, is great enough. Here, assuming that the function $X(t)$ is a sampled ergodic stationary centered random signal $\dot{X}(i\Delta t)$ with normal distribution law and the noise $\dot{\varepsilon}(t)$ is a sampled centered random signal $\dot{\varepsilon}(i\Delta t)$ with the mathematical expectation equal to zero, i.e., $m_\varepsilon = 0$, then the formula for determining the coefficient a_n is represented as

$$
\begin{aligned}
a_n^* &= \frac{2}{N}\sum_{i=1}^{N}\left[\dot{X}(i\Delta t) + \dot{\varepsilon}(i\Delta t)\right]\cos n\omega(i\Delta t)\\
&= \frac{2}{N}\sum_{i=1}^{N^+}\left[\dot{X}(i\Delta t) + \dot{\varepsilon}(i\Delta t)\right]\cos n\omega(i\Delta t)\\
&\quad + \frac{2}{N}\sum_{i=1}^{N^-}\left[\dot{X}(i\Delta t) + \dot{\varepsilon}(i\Delta t)\right]\cos n\omega(i\Delta t),
\end{aligned}
\tag{5.12}
$$

$$
\begin{aligned}
b_n^* &= \frac{2}{N}\sum_{i=1}^{N}\left[\dot{X}(i\Delta t) + \dot{\varepsilon}(i\Delta t)\right]\sin n\omega(i\Delta t)\\
&= \frac{2}{N}\sum_{i=1}^{N^+}\left[\dot{X}(i\Delta t) + \dot{\varepsilon}(i\Delta t)\right]\sin n\omega(i\Delta t)\\
&\quad + \frac{2}{N}\sum_{i=1}^{N^-}\left[\dot{X}(i\Delta t) + \dot{\varepsilon}(i\Delta t)\right]\sin n\omega(i\Delta t).
\end{aligned}
\tag{5.13}
$$

Here for stationary random processes with normal distribution, the result without error occurs only when the errors of positive and negative products,

$$\left[\dot{X}\left(i\Delta t\right) + \dot{\varepsilon}\left(i\Delta t\right)\right]\cos n\omega\left(i\Delta t\right),$$

are balanced and the following equations are fulfilled:

$$\frac{2}{N}\sum_{i=1}^{N^+}\dot{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t)=\frac{2}{N}\sum_{i=1}^{N^-}\left|\dot{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t)\right|, \tag{5.14}$$

$$\frac{2}{N}\sum_{i=1}^{N^+}\dot{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t)=\frac{2}{N}\sum_{i=1}^{N^-}\left|\dot{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t)\right|. \tag{5.15}$$

In practice, the positive and negative errors are compensated to some degree.

Thus, as mentioned earlier, many important problems can be solved when a high accuracy of the obtained results is not required and Eqs. (5.14) and (5.15) do not hold. But due to Eqs. (5.6) and (5.7), the noise has a considerable influence on the result of the analysis for a wide class of objects. At the same time, the result of the calculation has considerable errors. That causes the appearance of a difference between the sums of the positive and negative errors of the pair products, i.e.,

$$\frac{2}{N}\sum_{i=1}^{N^+}\dot{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t)\neq\frac{2}{N}\sum_{i=1}^{N^-}\left|\dot{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t)\right|, \tag{5.16}$$

$$\frac{2}{N}\sum_{i=1}^{N^+}\dot{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t)\neq\frac{2}{N}\sum_{i=1}^{N^-}\left|\dot{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t)\right|, \tag{5.17}$$

and their difference is the errors of obtained estimates

$$\lambda_{an}=\frac{2}{N}\sum_{i=1}^{N^+}\dot{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t)-\frac{2}{N}\sum_{i=1}^{N^-}\left|\dot{\varepsilon}(i\Delta t)\cos n\omega(i\Delta t)\right|, \tag{5.18}$$

$$\lambda_{bn}=\frac{2}{N}\sum_{i=1}^{N^+}\dot{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t)-\frac{2}{N}\sum_{i=1}^{N^-}\left|\dot{\varepsilon}(i\Delta t)\sin n\omega(i\Delta t)\right|. \tag{5.19}$$

According to the value of the errors λ_{an} and λ_{bn}, one can evaluate the influence of the noise $\varepsilon(t)$ on the obtained results.

They can be considerably greater than the given value S and in some cases can be commensurable with the required value:

$$\sum_{n=1}^{r}(\lambda_{an}+\lambda_{bn})\geq S. \tag{5.20}$$

It is obvious that, in practice, solving the numerous problems by means of the spectral method is not satisfactory [14]. One should especially notice that for this reason monitoring the beginning of the defect's origin by using spectral methods does not provide the necessary degree of reliability.

5.3 Algorithms for Calculating Errors of Coefficients of Fourier Series of Signals Obtained from Sensors

As follows, for the considered cases it is necessary to determine the difference between the sums of the negative and positive microerrors and to provide condition (5.20) by means of balancing the microerrors for obtaining a satisfactory result of the spectral analysis. Taking into account formula (5.19), the determination of the errors λ_{an} and λ_{bn} of the estimates a_n and b_n can be represented as follows:

$$\lambda_{an} = \frac{2}{N}\left[\sum_{i=1}^{N^{++}} \dot{\varepsilon}(i\Delta t)\cos^{+} n\omega(i\Delta t) + \sum_{i=1}^{N^{--}} \dot{\varepsilon}(i\Delta t)\cos^{-} n\omega(i\Delta t)\right]$$

$$-\frac{2}{N}\left[\sum_{l=1}^{N^{+-}} \dot{\varepsilon}(i\Delta t)\cos^{-} n\omega(i\Delta t) + \sum_{i=1}^{N^{-+}} \dot{\varepsilon}(i\Delta t)\cos^{+} n\omega(i\Delta t)\right], \quad (5.21)$$

$$\lambda_{bn} = \frac{2}{N}\left[\sum_{i=1}^{N^{++}} \dot{\varepsilon}(i\Delta t)\sin^{+} n\omega(i\Delta t) + \sum_{i=1}^{N^{--}} \dot{\varepsilon}(i\Delta t)\sin^{-} n\omega(i\Delta t)\right]$$

$$-\frac{2}{N}\left[\sum_{l=1}^{N^{+-}} \dot{\varepsilon}(i\Delta t)\sin^{-} n\omega(i\Delta t) - \sum_{i=1}^{N^{-+}} \dot{\varepsilon}(i\Delta t)\sin^{+} n\omega(i\Delta t)\right]. \quad (5.22)$$

Here $\cos^{+} n\omega(i\Delta t)$, $\sin^{-} n\omega(i\Delta t)$, $\cos^{-} n\omega(i\Delta t)$, and $\sin^{+} n\omega(i\Delta t)$ are the samples of the positive and negative half-periods of the nth cosinusoid and sinusoid, respectively; N^{++}, N^{--}, N^{-+}, and N^{+-} are the quantities of errors having sign ++, −−, −+, and +−, respectively.

As follows from expressions (5.21) and (5.22), it is necessary to determine the absolute errors $\lambda_g(i\Delta t)$ of the samples $g(i\Delta t)$ for balancing the errors of the pair products $\dot{g}(i\Delta t)\cdot\cos n\omega t(i\Delta t)$ [14]:

$$\lambda_g(i\Delta t) = \overline{\lambda}_{\text{rel}}\dot{g}(i\Delta t)\,. \quad (5.23)$$

Here the arithmetic mean value of the relative errors $\overline{\lambda}_{\text{rel}}$ of the samples $g(i\Delta t)$ is determined as follows [14]:

$$\overline{\lambda}_{\text{rel}} = \frac{\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}(i\Delta t)}{\frac{1}{N}\sum_{i=1}^{N}\dot{g}(i\Delta t)}. \tag{5.24}$$

It is clear that determining the relative error $\overline{\lambda}_{\text{rel}}$ according to that formula is practically impossible. The reasons are that only the samples $g(i\Delta t)$ are known, the values of the noise $\varepsilon(i\Delta t)$ are unknown, and its immediate determination is impossible. One can determine the value of the relative error of the sample $\lambda_{\text{rel}}(i\Delta t)$ according to the following formula:

$$\lambda_{\text{rel}}(i\Delta t) = \frac{\dot{\varepsilon}(i\Delta t)}{\dot{g}(i\Delta t)} = \frac{\sqrt{\dot{\varepsilon}^2(i\Delta t)}}{\sqrt{\dot{g}^2(i\Delta t)}}. \tag{5.25}$$

If, in that expression, $\dot{\varepsilon}^2(i\Delta t)$ and $\dot{g}^2(i\Delta t)$ are substituted by their arithmetic mean, then the expression for determining the arithmetic mean of the relative error can be transformed to the following form:

$$\overline{\lambda}_{\text{rel}} = \frac{1}{N}\sum_{i=1}^{N}\lambda_{\text{rel}}(i\Delta t) \approx \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\dot{\varepsilon}^2(i\Delta t)}}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\dot{g}^2(i\Delta t)}}. \tag{5.26}$$

However, according to expression (5.26), for determining the relative error $\overline{\lambda}_{\text{rel}}$, it is necessary to calculate the variance D_ε of the noise $\dot{\varepsilon}(i\Delta t)$. For this purpose one can use methods and algorithms from Chapters 3 and 4. As a result, $\overline{\lambda}_{\text{rel}}$ is determined as follows:

$$\overline{\lambda}_{\text{rel}} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\Big(\dot{g}(i\Delta t)\dot{g}(i\Delta t) + \dot{g}\big((i+2)\Delta t\big)\dot{g}(i\Delta t) - 2\dot{g}(i\Delta t)\dot{g}\big((i+1)\Delta t\big)\Big)}}{\sqrt{\frac{1}{N}\sum_{i=1}^{N}\dot{g}^2(i\Delta t)}}. \tag{5.27}$$

Consequently, the arithmetic mean of the relative error $\overline{\lambda}_{\text{rel}}$ of the samples $g(i\Delta t)$ is determined as a ratio of the noise variance D_ε and the total signal D_g, i.e.,

$$\overline{\lambda}_{\text{rel}} = \frac{D_\varepsilon}{D_g}. \tag{5.28}$$

It is clear that knowing $\lambda_g(i\Delta t)$ by formula (5.23) makes it easy to determine the magnitudes of microerrors by means of the expressions

$$\lambda_{an}(i\Delta t) = \lambda_g(i\Delta t) \cdot \cos n\omega(i\Delta t), \tag{5.29}$$

$$\lambda_{bn}(i\Delta t) = \lambda_g(i\Delta t) \cdot \sin n\omega(i\Delta t), \tag{5.30}$$

both for sinusoids and cosinusoids.

Here the possibility to determine the magnitudes of noise errors appears:

$$\lambda_a^R = \lambda_{an}^+ - \lambda_{an}^- = \left[\sum_{i=i_1^{++}}^{N^{++}} \lambda_{an}^{++}(i\Delta t) + \sum_{i=i_1^{--}}^{N^{--}} \lambda_{an}^{--}(i\Delta t)\right] - \left[\sum_{i=i_1^{+-}}^{N^{+-}} \lambda_{an}^{+-}(i\Delta t) + \sum_{i=i_1^{-+}}^{N^{-+}} \lambda_{an}^{-+}(i\Delta t)\right], \tag{5.31}$$

$$\lambda_b^R = \lambda_{bn}^+ - \lambda_{bn}^- = \left[\sum_{i=i_1^{++}}^{N^{++}} \lambda_{bn}^{++}(i\Delta t) + \sum_{i=i_1^{--}}^{N^{--}} \lambda_{bn}^{--}(i\Delta t)\right] - \left[\sum_{i=i_1^{+-}}^{N^{+-}} \lambda_{bn}^{+-}(i\Delta t) + \sum_{i=i_1^{-+}}^{N^{-+}} \lambda_{bn}^{-+}(i\Delta t)\right], \tag{5.32}$$

where i^{++}, i^{--}, i^{+-}, i^{-+}, N^{++}, N^{--}, N^{+-}, and N^{-+} are the indices of the summing and the quantity of the multipliers having signs + +, − +, + −, − −.

Thus, the robust formulas for determining the coefficients of the Fourier series can be represented as follows:

$$a_n^R = \frac{2}{N}\left\{\sum_{i=1}^{N}\left[\dot{g}(i\Delta t)\cos n\omega(i\Delta t) - \lambda_{an}^R\right]\right\}, \tag{5.33}$$

$$b_n^R = \frac{2}{N}\left\{\sum_{i=1}^{N}\left[\dot{g}(i\Delta t)\sin n\omega(i\Delta t) - \lambda_{bn}^R\right]\right\}. \tag{5.34}$$

It is clear that for realization of expressions (5.31) and (5.32), it is necessary to determine the sign of the errors of samples preliminarily. For that purpose, one can have information about the sign of the noise. It is impossible to get such information during calculations. That is why for practical use of robust algorithms it is necessary to transform them to the form where the mentioned disadvantage is absent. Additionally, those algorithms can be realized on modern–day personal computers easily.

5.4 Algorithms for Determining Robust Estimations of Fourier Series Coefficients in Spectral Monitoring of a Defect's Origin

It is obvious from expressions (5.31)–(5.34) that the value of the error caused by the noise in the obtained estimates a_n and b_n in analyzing signals collected from sensors is considerable. That is the reason why determining the defect's origin is practically impossible. One of the possible variants of solving this problem by means of traditional technologies is shown next.

It is shown in [14] that if equalities hold,

$$\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)} = \frac{1}{N}\sum_{i=1}^{N} \dot{g}(i\Delta t)\cos n\omega(i\Delta t)\,, \tag{5.35}$$

$$\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)} = \frac{1}{N}\sum_{i=1}^{N} \dot{g}(i\Delta t)\sin n\omega(i\Delta t)\,, \tag{5.36}$$

and taking into account the signs of samples of the signal $g(i\Delta t)$ and the signs of cosinusoids $\cos n\omega(i\Delta t)$, one can determine the difference of microerrors of coefficients a_n of Fourier series according to the expression

$$\begin{aligned}\lambda_a^R &= \lambda_{an}^+ - \lambda_{an}^- \\ &= \left[N_{an}^{++}\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)} + N_{an}^{--}\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)} \right] \\ &\quad - \left[N_{an}^{+-}\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)} + N_{an}^{-+}\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)} \right].\end{aligned} \tag{5.37}$$

With due regard to the following equality:

$$N^{+} = N^{++} + N^{--}\,, \tag{5.38}$$

$$N^{-} = N^{+-} + N^{+-}\,, \tag{5.39}$$

one can represent this difference in the following form:

$$\lambda_{an}^R = (N_{an}^+ - N_{an}^-)\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}\,, \tag{5.40}$$

$$\lambda_{bn}^{R} = (N_{bn}^{+} - N_{b_n}^{-})\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)}\,. \tag{5.41}$$

In monitoring the beginning of the defect's origin for receiving reliable results, it is expedient to create technologies of robust spectral analysis. These technologies allow one to obtain more precise estimates a_n and b_n, unlike those obtained from formulas (5.40) and (5.41).

It is connected with the fact that in expressions (5.40) and (5.41) part of the error is not taken into account. It is caused by the inequality $\Pi^{+} \neq \Pi^{-}$. To take them into account in realizing traditional algorithms during the process of calculating the sum

$$\sum_{i=1}^{N} \dot{g}(i\Delta t)\cos n\omega(i\Delta t)\ ,$$

it is expedient to determine the average value of all products:

$$\Pi = \overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}\,,$$

and the average value of positive products:

$$\Pi^{+} = \overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{\,+}\,,$$

and that of negative products:

$$\Pi^{-} = \overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{\,-}\,,$$

as well as their numbers, i.e., N, N^{+}, and N^{-}.

At the same time, in case $N^{+} = N^{-}$ and $\Pi^{+} \neq \Pi^{-}$, the error not taken into account is equal to

$$\frac{1}{2}N\left[\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{\,+} - \overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{\,-}\right].$$

It's obvious that the analogous error at $N^{+} > N^{-}$ and $\Pi^{+} > \Pi^{-}$ is equal to

$$\frac{1}{2}\left[N - \left(N_{an}^{+} - N_{an}^{-}\right)\right]\left[\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{\,+} - \overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{\,-}\right].$$

When $N^{+} < N^{-}$ and $\Pi^{+} < \Pi^{-}$, the mentioned error is equal to

$$r_{x\varepsilon}\frac{1}{2}\left[N - \left(N_{an}^{-} - N_{an}^{+}\right)\right]\left[\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{\,+} - \overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{\,-}\right].$$

Consequently, taking into account the mentioned errors, formula (5.40) for determining λ_{an}^{R} can be represented as follows:

$$\lambda_{an}^{R}=\begin{cases} 0 \quad \text{for } N^{+}=N^{-} \text{ and } \Pi^{+}=\Pi^{-}, \\ r_{x\varepsilon}\dfrac{1}{4}(N_{an}^{+}-N_{an}^{-})\,\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)} \\ +r_{x\varepsilon}\dfrac{1}{2}N\left[\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+}-\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right] \\ \text{for } N^{+}=N^{-} \text{ and } \Pi^{+}\neq\Pi^{-}, \\ r_{x\varepsilon}\dfrac{1}{4}\left(N_{an}^{+}-N_{an}^{-}\right)\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+} \\ +r_{x\varepsilon}\dfrac{1}{2}\left[N-\left(N_{an}^{+}-N_{an}^{-}\right)\right]\left[\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+}\right. \\ \left.-\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right] \\ \quad \text{for } N^{+}>N^{-} \text{ and } \Pi^{+}>\Pi^{-}, \\ r_{x\varepsilon}\dfrac{1}{4}\left(N_{an}^{-}-N_{an}^{+}\right)\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-} \\ +r_{x\varepsilon}\dfrac{1}{2}\left[N-\left(N_{an}^{-}-N_{an}^{+}\right)\right]\left[\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+}\right. \\ \left.-\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right] \\ \quad \text{for } N^{+}<N^{-} \text{ and } \Pi^{+}<\Pi^{-}. \end{cases} \tag{5.42}$$

5.5 Technology of Spectral Analysis of Noise in Monitoring a Defect's Origin

The possibility of determining approximate values of the noise $\varepsilon^{*}(i\Delta t)$ according to the technology described in Chapter 3 opens wide possibilities for improving spectral analysis results according to traditional information technology. That is necessary for monitoring the defect at the beginning of its origin [19–22]. In this vein, we first consider the problem of determining spectral noise characteristics. It is clear that having the approximate values of the noise $\varepsilon^{*}(i\Delta t)$, the formulae for determining the Fourier series coefficients $a_{n\varepsilon}$ and $b_{n\varepsilon}$ of the noise at $r_{x\varepsilon}=0$ between $X(t)$ and $\varepsilon(i\Delta t)$ can be represented as follows:

$$a_{n\varepsilon} \approx \frac{2}{T}\int_0^T \dot{\varepsilon}^*(t)\cos n\omega t dt \approx \frac{2}{N}\sum_{i=1}^{N}\dot{\varepsilon}^*(i\Delta t)\cos n\omega(i\Delta t)$$

$$= \frac{2}{N}\left[\sum_{i=1}^{N}\left[\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}\right]\cos n\omega(i\Delta t)\right] = \frac{2}{N}\sum_{i=1}^{N}\left[\operatorname{sgn}\varepsilon'(i\Delta t)\right.$$

$$\left.\times\sqrt{\left|\dot{g}(i\Delta t)\left[\dot{g}(i\Delta t)+\dot{g}((i+2)\Delta t)-2\dot{g}((i+1)\Delta t)\right]\right|}\right]\cos n\omega(i\Delta t), \tag{5.43}$$

$$b_{n\varepsilon} \approx \frac{2}{T}\int_0^T \dot{\varepsilon}^*(t)\sin n\omega t dt \approx \frac{2}{N}\sum_{i=1}^{N}\dot{\varepsilon}^*(i\Delta t)\sin n\omega(i\Delta t)$$

$$= \frac{2}{N}\left[\sum_{i=1}^{N}\left[\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}\right]\sin n\omega(i\Delta t)\right] = \frac{2}{N}\sum_{i=1}^{N}\left[\operatorname{sgn}\varepsilon'(i\Delta t)\right.$$

$$\left.\times\sqrt{\left|\dot{g}(i\Delta t)\left[\dot{g}(i\Delta t)+\dot{g}((i+2)\Delta t)-2\dot{g}((i+1)\Delta t)\right]\right|}\right]\sin n\omega(i\Delta t). \tag{5.44}$$

These formulae at $r_{x\varepsilon} \neq 0$ can be represented as follows:

$$a_{n\varepsilon} \approx \frac{2}{N}\int_0^T \dot{\varepsilon}^*(t)\cos n\omega t dt = \frac{2}{N}\sum_{i=1}^{N}\dot{\varepsilon}^*(i\Delta t)\cos n\omega(i\Delta t)$$

$$= \frac{2}{N}\sum_{i=1}^{N}\left[\operatorname{sgn}\left[\varepsilon'(i\Delta t)+\varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right|}\right]\cos n\omega(i\Delta t)$$

$$= \frac{2}{N}\sum_{i=1}^{N}\left[\operatorname{sgn}\left[\varepsilon'(i\Delta t)+\varepsilon''(i\Delta t)\right]\left\{\left|\dot{g}(i\Delta t)\left[\dot{g}(i\Delta t)\right.\right.\right.\right.$$

$$+\dot{g}((i+2)\Delta t)-2\dot{g}((i+1)\Delta t)\Big]+\dot{g}(i\Delta t)\Big[\dot{g}((i+1)\Delta t)+\dot{g}((i+3)\Delta t)$$

$$\left.\left.\left.\left.-2\dot{g}((i+2)\Delta t)\right]\right|\right\}^{\frac{1}{2}}\cos n\omega(i\Delta t), \tag{5.45}$$

$$b_{n\varepsilon} \approx \frac{2}{N}\int_{0}^{T} \dot{\varepsilon}^{*}(t)\sin n\omega t dt = \frac{2}{N}\sum_{i=1}^{N} \dot{\varepsilon}^{*}(i\Delta t)\sin n\omega(i\Delta t)$$

$$= \frac{2}{N}\sum_{i=1}^{N}\left[\operatorname{sgn}\left[\varepsilon'(i\Delta t)+\varepsilon''(i\Delta t)\right]\right]\sqrt{\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]}\sin n\omega(i\Delta t)$$

$$= \frac{2}{N}\sum_{i=1}^{N}\left[\operatorname{sgn}\left[\varepsilon'(i\Delta t)+\varepsilon''(i\Delta t)\right]\left\{\left|\dot{g}(i\Delta t)\left[\dot{g}(i\Delta t)\right.\right.\right.\right.$$

$$+\dot{g}((i+2)\Delta t)-2\dot{g}((i+1)\Delta t)\Big]+\dot{g}(i\Delta t)\Big[\dot{g}((i+1)\Delta t)+\dot{g}((i+3)\Delta t)$$

$$\left.\left.\left.-2\dot{g}((i+2)\Delta t)\Big]\right|\right\}^{\frac{1}{2}}\sin n\omega(i\Delta t). \tag{5.46}$$

According to the analogous expressions, by means of the analysis of the values of samples $X(i\Delta t)$, the estimates a_n and b_n of the legitimate signal are determined. It is obvious that the realization of these algorithms during the process of the spectral monitoring of the defect requires sufficiently complicated calculations for determining approximate magnitudes of the noise $\varepsilon^{*}(i\Delta t)$. However, present-day computers have enough resources for realization algorithms (5.43)–(5.46).

In order to provide a high degree of the reliability for the results of monitoring, one can duplicate the process of analyzing the signals $g(i\Delta t)$ by using robust algorithms (5.42):

$$a_{n\varepsilon} = (N^{+}_{a_{n_\varepsilon}} - N^{-}_{a_{n_\varepsilon}})\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)} - \lambda'_{an}, \tag{5.47}$$

$$b_{n\varepsilon} = (N^{+}_{b_{n_\varepsilon}} - N^{-}_{b_{n_\varepsilon}})\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)} - \lambda'_{bn}, \tag{5.48}$$

where λ'_{an} and λ'_{bn} are parts of the error that are not taken into account.

It is possible to show that with the help of these algorithms one can obtain sufficiently exact estimates $a_{n\varepsilon}$ and $b_{n\varepsilon}$. This is connected with the fact that in expressions (5.37) and (5.38) part of the error is not taken into account. It is caused by the inequality $\Pi^{+} \neq \Pi^{-}$. To take them into account, in realizing traditional algorithms during the process of calculating the sum

$$\sum_{i=1}^{N}\dot{g}(i\Delta t)\cos n\omega(i\Delta t)\ ,$$

it is expedient to determine the average value of all products:

$$\Pi = \overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)},$$

and the average value of positive products:

$$\Pi^+ = \overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^+,$$

and that of negative products:

$$\Pi^- = \overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^-,$$

as well as their numbers, i.e., N, N^+, and N^-.

The analysis of the realization of algorithms (5.37) and (5.38) shows that in case $N^+ = N^-$ and $\Pi^+ \neq \Pi^-$, the error not taken into account is equal to

$$r_{x\varepsilon} N\left[\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^+ - \overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^-\right].$$

It's obvious that the analogous error at $N^+ > N^-$ and $\Pi^+ > \Pi^-$ is equal to

$$r_{x\varepsilon}\left[N - \left(N_{an}^+ - N_{an}^-\right)\right]\left[\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^+ - \overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^-\right].$$

When $N^+ < N^-$ and $\Pi^+ < \Pi^-$, the mentioned error is equal to

$$r_{x\varepsilon}\left[N - \left(N_{an}^- - N_{an}^+\right)\right]\left[\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^+ - \overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^-\right].$$

Consequently, one can determine the error for determining $a_{n\varepsilon}$.

Thus, taking into account the mentioned errors, the formulae for determining $a_{n\varepsilon}$ may be represented as follows:

1. According to the technology suggested in Chapter 2, the approximate value of the step of the sampling of the noise $\varepsilon(i\Delta t)$ is determined as follows:

$$\Delta t_\varepsilon = \Delta t .$$

2. The estimates $R_{gg}(\mu = \Delta t)$, $R_{gg}(\mu = 2\Delta t)$, and $R_{gg}(\mu = 3\Delta t)$ are determined, and the following conditions are checked:

$$R_{gg}(\mu = \Delta t) - R_{gg}(\mu = 2\Delta t) \geq \Delta x ,$$

$$R_{gg}(\mu = 2\Delta t) - R_{gg}(\mu = 3\Delta t) \geq \Delta x ,$$

where Δx is an amplitude quantization step. For the lack of obedience to these conditions, the step of the sampling Δt_ε of the noise decreases, and the procedure for checking conditions is repeated.

3. According to expressions (3.39) and (3.69), the noise variance D_ε is determined for both $r_{x\varepsilon} = 0$ and $r_{x\varepsilon} \neq 0$. According to expressions (3.39) and (3.69), we determine the variances of the noise D_ε for both $r_{x\varepsilon} = 0$ and $r_{x\varepsilon} \neq 0$.
4. According to expressions (1.83) and (1.90), we determine the correlation coefficient $r_{x\varepsilon}$ between the noise and the legitimate signal.
5. At $r_{x\varepsilon} = 0$, we determine the coefficients $a_{n\varepsilon}$, a_{nx} and $b_{n\varepsilon}$, b_{nx} according to expressions (5.43) and (5.44). At $r_{x\varepsilon} \neq 0$, the coefficients are determined according to expressions (5.45) and (5.46).
6. The estimations of the magnitudes λ_{an}^R and λ_{bn}^R are determined in the following order:

6.1. The average value of the relative error of samples $\overline{\lambda}_{\text{rel}}$ and the regression coefficient $r_{x\varepsilon}$ are determined.

6.2. The values Π^+, Π^-, N^+, and N^- are determined.

6.3. The conditions $N^+ = N^-$ and $\Pi^+ = \Pi^-$ under which the noise coefficients $a_{n\varepsilon}$ and $b_{n\varepsilon}$ equal zero are checked.

6.4. Under the conditions $N^+ \neq N^-$ and $\Pi^+ = \Pi^-$, the formulae for determining the estimates $a_{n\varepsilon}$ and $b_{n\varepsilon}$ are used in the following form:

$$a_{n\varepsilon} = \left(N_{n\varepsilon}^+ - N_{n\varepsilon}^-\right)\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)},$$

$$b_{n\varepsilon} = \left(N_{n\varepsilon}^+ - N_{n\varepsilon}^-\right)\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)}.$$

6.5. Under the conditions $N^+ > N^-$ and $\Pi^+ \neq \Pi^-$, the estimates $a_{n\varepsilon}$ and $b_{n\varepsilon}$ are determined by the expressions

$$a_{n\varepsilon} = \left[\left(N_{an}^+ - N_{an}^-\right)\right]\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+} + r_{x\varepsilon}\left[N - \left(N_{an}^+ - N_{an}^-\right)\right]$$

$$\times\left[\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+} - \overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right],$$

$$b_{n\varepsilon} = \left[\left(N_{bn}^+ - N_{bn}^-\right)\right]\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)}^{+} + r_{x\varepsilon}\left[N - \left(N_{bn}^+ - N_{bn}^-\right)\right]$$

$$\times\left[\overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)}^{+} - \overline{\lambda}_{\text{rel}}\,\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)}^{-}\right].$$

6.6. If $N^+ < N^-$ and $\Pi^+ \neq \Pi^-$ hold, then the estimates $a_{n\varepsilon}$ and $b_{n\varepsilon}$ are determined according to the expressions

$$a_{n\varepsilon} = \left[\left(N_{a_n}^- - N_{a_n}^+\right)\right]\overline{\lambda}_{\text{rel}}\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^- + r_{x\varepsilon}\left[N - \left(N_{an}^- - N_{an}^+\right)\right]$$

$$\times\left[\overline{\lambda}_{\text{rel}}\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^+ - \overline{\lambda}_{\text{rel}}\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^-\right],$$

$$b_{n\varepsilon} = \left[\left(N_{bn}^- - N_{bn}^+\right)\right]\overline{\lambda}_{\text{rel}}\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)}^- + r_{x\varepsilon}\left[N - \left(N_{bn}^- - N_{bn}^+\right)\right]$$

$$\times\left[\overline{\lambda}_{\text{rel}}\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)}^+ - \overline{\lambda}_{\text{rel}}\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)}^-\right].$$

6.7. Under the conditions $N^+ = N^-$ and $\Pi^+ \neq \Pi^-$, the desired estimates are determined according to the following formulae:

$$a_{n\varepsilon} = r_{x\varepsilon}N\left[\overline{\lambda}_{\text{rel}}\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^+ - \overline{\lambda}_{\text{rel}}\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^-\right],$$

$$b_{n\varepsilon} = r_{x\varepsilon}N\left[\overline{\lambda}_{\text{rel}}\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)}^+ - \overline{\lambda}_{\text{rel}}\overline{\dot{g}(i\Delta t)\sin n\omega(i\Delta t)}^-\right].$$

Thus, the spectral analysis of noise may be performed according to either steps 1–5 or steps 6.1–6.7, depending on the requirements of accuracy and operating time.

5.6 Algorithms for Determining Coefficients of Fourier Series of a Legitimate Signal in Using Spectral Monitoring of a Defect's Origin

As stated earlier, with the use of spectral analysis of noisy signals due to the lack of obedience to classical conditions, the obtained results have perceived errors. Even under classical conditions, the accuracy of analysis results is not satisfactory in many cases because of the difference between the number of positive and negative errors. That is why, when using the technology of spectral analysis in solving problems of monitoring the beginning of the defect's origin, it is necessary to create methods and algorithms allowing one to eliminate the errors of processing mentioned in Sections 5.2–5.4. The solution to this problem comes in two variants, as discussed next.

In the first variant, with the help of the information technology for determining approximate values of the samples $\varepsilon^*(i\Delta t)$ of the noise, the noise influence on the legitimate signal $X^*(i\Delta t)$ is eliminated and the required estimates are determined.

In the second variant, according to formula (5.47), the coefficients of the Fourier series are determined and used for correction of the results of traditional technologies of spectral analysis of noisy signals.

Consider the first variant. The expression for determining the coefficients Fourier series of a legitimate signal at $r_{x\varepsilon} = 0$ may be represented as follows:

$$a_n^* = \frac{2}{T}\int_0^T \dot{X}(t)\cos n\omega t dt = \frac{2}{T}\int_0^T \left[\dot{g}(t) - \dot{\varepsilon}(t)\right]\cos n\omega t dt$$

$$= \frac{2}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t) - \dot{\varepsilon}(i\Delta t)\right]\cos n\omega(i\Delta t) \approx \frac{2}{N}\sum_{i=1}^{N}\left[g(i\Delta t) - \varepsilon^*(i\Delta t)\right]\cos n\omega(i\Delta t)$$

$$= \frac{2}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\varepsilon'(i\Delta t)}\right]\cos n\omega(i\Delta t) = \frac{2}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\right.$$

$$\left.\times\sqrt{\left|\dot{g}(i\Delta t)\left[\dot{g}(i\Delta t) + \dot{g}((i+2)\Delta t) - 2\dot{g}((i+1)\Delta t)\right]\right|}\right]\cos n\omega(i\Delta t), \tag{5.47}$$

$$b_n^* = \frac{2}{T}\int_0^T \dot{X}(t)\sin n\omega t dt = \frac{2}{T}\int_0^T \left[\dot{g}(t) - \dot{\varepsilon}(t)\right]\sin n\omega t dt$$

$$= \frac{2}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t) - \dot{\varepsilon}(i\Delta t)\right]\sin n\omega(i\Delta t)$$

$$\approx \frac{2}{N}\sum_{i=1}^{N}\left[g(i\Delta t) - \varepsilon^*(i\Delta t)\right]\sin n\omega(i\Delta t)$$

$$= \frac{2}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\varepsilon'(i\Delta t)}\right]\sin n\omega(i\Delta t)$$

$$= \frac{2}{N}\sum_{i=1}^{N}\left\{\left[\dot{g}(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\left[\left|\dot{g}(i\Delta t)\left[\dot{g}(i\Delta t) + \dot{g}((i+2)\Delta t)\right.\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\left.-2\dot{g}((i+1)\Delta t)\right]\right|\right]^{\frac{1}{2}}\right\}\sin n\omega(i\Delta t). \tag{5.48}$$

At $r_{x\varepsilon} \neq 0$ between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$, these formulae have the following forms:

$$a_n = \frac{2}{T}\int_0^T X(t)\cos n\omega t dt \approx \frac{2}{T}\int_0^T \left[\dot{g}(t) - \dot{\varepsilon}(t)\right]\cos n\omega t dt$$

$$= \frac{2}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t) - \dot{\varepsilon}^*(i\Delta t)\right]\cos n\omega(i\Delta t)$$

$$= \frac{2}{N}\sum_{i=1}^{N}\Big\{\Big[\dot{g}(i\Delta t) - \operatorname{sgn}\left[\varepsilon'(i\Delta t) + \varepsilon''(i\Delta t)\right]$$

$$\times\Big[\Big|\dot{g}(i\Delta t)\left[\dot{g}(i\Delta t) + \dot{g}((i+2)\Delta t) - 2\dot{g}((i+1)\Delta t)\right]\Big|$$

$$+\Big|\dot{g}(i\Delta t)\left[\dot{g}((i+1)\Delta t) + \dot{g}((i+3)\Delta t) - 2\dot{g}((i+2)\Delta t)\right]\Big|\Big]^{\frac{1}{2}}\Big\}\cos n\omega(i\Delta t), \tag{5.49}$$

$$b_n = \frac{2}{T}\int_0^T X(t)\sin n\omega t dt = \frac{2}{T}\int_0^T \left[\dot{g}(t) - \dot{\varepsilon}(t)\right]\sin n\omega t dt$$

$$\approx \frac{2}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t) - \dot{\varepsilon}^*(i\Delta t)\right]\sin n\omega(i\Delta t) = \frac{2}{N}\sum_{i=1}^{N}\Big\{\Big[\dot{g}(i\Delta t)$$

$$-\operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\Big[\Big|\dot{g}(i\Delta t)\Big[\dot{g}(i\Delta t) + \dot{g}((i+2)\Delta t)$$

$$-2\dot{g}((i+1)\Delta t)\Big]\Big| - \Big|\dot{g}(i\Delta t)\Big[\dot{g}((i+1)\Delta t) + \dot{g}((i+3)\Delta t)$$

$$-2\dot{g}((i+2)\Delta t)\Big]\Big|\Big]^{\frac{1}{2}}\Big\}\sin n\omega(i\Delta t). \tag{5.50}$$

By means of analysis of the expressions (5.47) and (5.48), one can show that by $r_{x\varepsilon} = 0$ the equality holds:

$$P\left[X(i\Delta t)\cos n\omega(i\Delta t) > \left[g(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}\right]\cos n\omega(i\Delta t)\right]$$

$$\approx P\left[X(i\Delta t)\cos n\omega(i\Delta t) < \left[g(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}\right]\cos n\omega(i\Delta t)\right], \tag{5.51}$$

$$P\left[X(i\Delta t)\sin n\omega(i\Delta t) > \left[g(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}\right]\sin n\omega(i\Delta t)\right]$$

$$\approx P\left[X(i\Delta t)\sin n\omega(i\Delta t) < \left[g(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}\right]\sin n\omega(i\Delta t)\right]. \tag{5.52}$$

Analogous equalities hold at $r_{x\varepsilon} \neq 0$.

Expressions (5.51) and (5.52) show that the estimates a_n and b_n calculated for the legitimate signal $X(i\Delta t)$ coincide with the estimates a_n^* and b_n^* calculated for the approximate magnitudes $X^*(i\Delta t)$ of the legitimate signal.

In effect, expressions (5.43)–(5.46) and (5.47)–(5.50) open extended possibilities for creating a number of effective information technologies of spectral analysis, allowing one to solve the problem of monitoring the defect at the beginning of its origin. In conclusion, next we represent the sequence of procedures for spectral analysis of the legitimate signal $X(i\Delta t)$ while monitoring the defect.

1. The sampling steps $g(i\Delta t)$ and $\varepsilon(i\Delta t)$ are determined according to the technology of noise analysis.
2. According to expressions (1.83) and (1.90), the correlation coefficient $r_{x\varepsilon}$ between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$ is determined.
3. At $r_{x\varepsilon} = 0$, the samples of the legitimate signal $X(i\Delta t)$ during the process of spectral analysis are changed with its approximate value $X^*(i\Delta t)$, i.e., with the difference

$$\left[\dot{g}(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}\right].$$

At $r_{x\varepsilon} \neq 0$, the samples $X(i\Delta t)$ are changed with the value

$$X^*(i\Delta t) = \left[g(i\Delta t) - \operatorname{sgn}[\varepsilon'(i\Delta t) + \varepsilon''(i\Delta t)]\sqrt{|\varepsilon'(i\Delta t) + \varepsilon''(i\Delta t)|}\right].$$

4. The estimates a_n^* and b_n^* are determined at $r_{x\varepsilon} = 0$ according to expressions (5.47) and (5.48).
5. The estimates a_n^* and b_n^* are determined at $r_{x\varepsilon} \neq 0$ according to expressions (5.49) and (5.50).
6. In conclusion, we note that taking into consideration the importance of providing the reliability of results of monitoring in the suggested technology, the spectral monitoring of the beginning of the defect's origin is doubled.

Thus, the coefficients a_n^* and b_n^* of Fourier series of the legitimate signal are determined by means of subtracting from the results of spectral analysis obtained according to the traditional technologies, and the coefficients of Fourier series of the noise $\varepsilon(i\Delta t)$ are determined according to the algorithms of spectral analysis of the noise. Here, in case there is no correlation between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$, one uses algorithms (5.43) and (5.44). In case the correlation between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$ differs from zero, the algorithms (5.45) and (5.46) or (5.33) and (5.34) are used. Thus, determining a_n^* and b_n^* for the legitimate signal is reduced to the following expressions:

$$a_n^* = a_n - a_{n\varepsilon}, \tag{5.54}$$

$$b_n^* = b_n - b_{n\varepsilon}, \tag{5.55}$$

where a_n^* is a corrected estimate of the legitimate signal, and a_n is the estimate of the total noisy signal obtained by traditional technology.

6 The Digital Technology of Forecasting Failures by Considering Noise as a Data Carrier

6.1 The Problem of Digital Forecasting of Failures by Considering Noise as a Data Carrier

In the past, the errors in forecasting such objects as oil-chemical complexes, deep-water stationary sea platforms and communications, hydraulic works, etc. were assumed to be connected with meteorological and assurance characteristics of the element base of information measurement systems. Now they have been improved, but the probability of a failure has remained the same. Analysis shows that the main reason for an inadequate decision from a diagnostic system is connected with the impossibility of detecting the initial state of arising defects with the known methods of analyzing noisy signals [14, 15, 17, 56].

In the literature, a "fault" is defined as the inability of a system to realize required functions [55–57]. The fault is the initial state of the failure, which means the inability of work on a substantial scale. In this case, the fault of certain elements of a system leads to faults of general parts of an object and finally to complete destruction. In this case, the future use of an object is impossible or requires major repairs [55–57].

In most cases, fatigue damages such as vibration efforts and working conditions are the reason for a fault. For example, vibrations and cyclic bend deformations caused by waves are the reason for damage to bases of sea stationary platforms and bridges, elements of drilling devices, and compression stations [55–57].

The events leading to a system fault are casual, and that is the reason why forecasting fault is difficult. For example, the appearance of cracks begins with the appearance of microcracks, and it is almost impossible to detect the appearance of microcracks with the currently known methods [14, 56].

In practice, with the leading oil companies, in most cases the problem of forecasting failures is solved intuitively by relying on the experience of specialists. For example, the problem of drill column sticking is difficult, because the known methods of analysis of measurement information do not allow one to forecast this failure beforehand, and diagnostic systems signal a failure when it is almost impossible to prevent it [31]. At the same time, it is obvious that a process is reflected on the noise of measurement signals, and this can be used in forecasting drill column sticking.

As usual, forecasting earthquakes is belated, too. In this case, the significant part of the useful information is contained in the noise of weak seismic signals preceding the quake and suppressed by filtration [6, 16]. In addition, the impossibility of forecasting the fault of an airplane before flight leads to an inefficiency of forecasting failure during the flight. The above-mentioned examples show that the possibility of using connections between microchanges to the state of the object and their representation as noises is of great theoretical and practical importance in forecasting failures.

It is known that all elements of objects are subject to continuous qualitative changes [14, 56]. The set of internal properties of an object at a certain moment of time T defines its state. When they take a clearly defined form, changes to the state of an object are reflected in changes to signals that hold diagnostic information. This corresponds to the case when the state of the object is represented as signals $X_1(t)$, $X_2(t)$, …, $X_n(t)$ noisy by the noises $\varepsilon_1(t)$, $\varepsilon_2(t)$, …, $\varepsilon_n(t)$, which differ from "white noise." Analyzed signals are the sum of the legitimate signal and noise in this case, i.e.,

$$g_1(t) = X_1(t) + \varepsilon_1(t),\ g_2(t) = X_2(t) + \varepsilon_2(t),\ldots,$$

$$g_n(t) = X_n(t) + \varepsilon_n(t)\,. \tag{6.1}$$

These signals in digital form can be represented as follows:

$$g_1(i\Delta t) = X_1(i\Delta t) + \varepsilon_1(i\Delta t),\ g_2(i\Delta t) = X_2(i\Delta t) + \varepsilon_2(i\Delta t),\ \ldots,$$

$$g_n(i\Delta t) = X_n(i\Delta t) + \varepsilon_n(i\Delta t)\,. \tag{6.2}$$

The regular record of signals, the determination of information signs, and their comparison with sample values provide the basis for controlling the change to an object's characteristics. For example, it is assumed that the obtained estimates of signals represent corresponding changes to the technical state of equipment in solving many problems of diagnostics and forecasting by means of the theory of casual processes, the theory of recognition of patterns, etc. [12, 56]. For this purpose, the estimates of the statistical characteristics of the signals obtained from the diagnosed object are compared with sample estimates of the signals representing the typical state of the object, which should be different while diagnosing. In these cases, forecasting of faults or failures is realized via results of diagnostics based on extrapolation methods. Tendencies of changes to the object's state are determined by past data in accordance with extrapolation methods. Forecasting of faults by means of diagnostics data can be also reduced to application of known statistical methods of determination of reliability and the estimate of the remaining resources of the diagnosed objects [55–57].

In most cases, the obtained results of diagnostics are sufficient and allow one to forecast possible tendencies of future changes to the state of the object. However, often microchanges represented as the noises $\varepsilon_1(i\Delta t)$, $\varepsilon_2(i\Delta t)$, ..., $\varepsilon_n(i\Delta t)$ only take place on real objects. Despite the fact that they are the reason for future catastrophic failures, information signs obtained from corresponding signals do not change for a long time. In this case, results from diagnostics are late because only characteristics of the noises $\varepsilon_1(i\Delta t)$, $\varepsilon_2(i\Delta t)$, ..., $\varepsilon_n(i\Delta t)$ are changed when microchanges appear. In this connection, taking into account that these cases are the most dangerous and inflict the biggest total damage during failure, it is necessary to create a corresponding theory and technology of noise based on forecasting the change to the state of technological objects.

According to the suggested theory of noise based on forecasting, in most cases there is a connection between the noise of measurement information and the hidden microchanges preceding the beginning of failure on real objects. It is possible to use this relationship for forecasting the change to an object's state into the emergency state. For this purpose, the set of information signs of an initial state is formed on the basis of analysis of noise using the above-mentioned connection. And identification of microchanges and forecasting possible failures can be realized during the exploitation of the object while detecting the difference between the current corresponding state from the analogue sets [15, 16].

Algorithms of noise based on forecasting failures are represented below. The opportunity for revealing the initial stage of changes to the state of the object has been shown in the literature [3, 4, 7, 11, 18, 24, 29, 30, 35, 39, 47–51].

6.2 Algorithms for Forecasting the Transition of an Object into the Failure State by Considering Noise as a Data Carrier

There are great opportunities for using information technology in the analysis of noise for forecasting possible failures when using the connection between microchanges of an object's state and the change to the noise's value [15, 16]. For detection of the initial stage of microchanges by means of characteristics of the noises $\varepsilon_1(i\Delta t)$, $\varepsilon_2(i\Delta t)$, …, $\varepsilon_n(i\Delta t)$ of noisy signals $g_1(i\Delta t)$, $g_2(i\Delta t)$, …, $g_n(i\Delta t)$, it is advisable to analyze them by means of the following [16]:

$$m_\varepsilon = \begin{cases} \dfrac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|} & \text{for } r_{x\varepsilon} = 0, \\ \dfrac{1}{N}\sum_{i=1}^{N} \operatorname{sgn}[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)]\sqrt{|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)|} & \text{for } r_{x\varepsilon} \neq 0, \end{cases} \tag{6.3}$$

$$D_\varepsilon = \begin{cases} \dfrac{1}{N}\sum_{i=1}^{N} \varepsilon'(i\Delta t) & \text{for } \quad r_{x\varepsilon} = 0, \\ \dfrac{1}{N}\sum_{i=1}^{N} [\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)] & \text{for } \quad r_{x\varepsilon} \neq 0, \end{cases} \tag{6.4}$$

$$R_{x\varepsilon}^{(\mu)} = \begin{cases} \dfrac{1}{N}\sum_{i=1}^{N} \left[g(i\Delta t) - \operatorname{sgn} \varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|}\right] \operatorname{sgn} \varepsilon'(i\Delta t)\sqrt{|\varepsilon'(i\Delta t)|} & \text{for } r_{x\varepsilon} = 0, \\ \dfrac{1}{N}\sum_{i=1}^{N} \left[g(i\Delta t) - \operatorname{sgn}[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)]\sqrt{|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)|}\right] & \\ \times \operatorname{sgn}[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)]\sqrt{|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)|} & \text{for } r_{x\varepsilon} \neq 0. \end{cases} \tag{6.5}$$

Microchange to the state of the control objects also leads to a change to the estimate $R_{x\varepsilon}(\mu)$ of the cross-correlation function between $X(i\Delta t)$ and $\varepsilon(i\Delta t)$, which can be determined by the value of robustness, by the difference of the quantity of the positive $N^+(\mu)$ and negative $N^-(\mu)$ products $\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)$ according to the formula

$$\lambda_{xx}^R(\mu) \approx \begin{cases} \left[N^+(\mu) - N^-(\mu)\right]\left\langle \lambda(\mu = 1)\right\rangle + D_\varepsilon & \text{for } \mu = 0, \\ \left[N^+(\mu) - N^-(\mu)\right]\left\langle \lambda(\mu = 1)\right\rangle & \text{for } \mu \neq 0, \end{cases} \tag{6.6}$$

where

$$\left\langle \Delta\lambda(\mu = 1)\right\rangle = [1/N^-(\mu = 1)]\lambda(\mu = 1), \tag{6.7}$$

$$\left|R'_{gg}(\mu = 1) - R_{gg}(\mu = 1)\right| = \lambda(\mu = 1). \tag{6.8}$$

When detecting the initial stage of microchanges as one of the important information signs, it is advisable to use estimates of coefficients of the correlation $r_{x\varepsilon}$ between noise and a signal, which can be represented as

$$r_{x\varepsilon} = \frac{R_{x\varepsilon}(0)}{\sqrt{R_{xx}(0)R_{\varepsilon\varepsilon}(0)}} = \frac{R_{x\varepsilon}(0)}{\sqrt{D_x D_\varepsilon}}$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left[\left[\dot{g}(i\Delta t) - \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\right]\right.$$

$$\left.\times \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\right]$$

$$\times\left\{\left[\frac{1}{N}\sum_{i=1}^{N}\left[\dot{g}(i\Delta t) - \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\right]^2\right]\right.$$

$$\left.\times\left[\frac{1}{N}\sum_{i=1}^{N}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\right]\right\}^{-\frac{1}{2}}. \tag{6.9}$$

In some cases, it is possible to use formulae (1.90) for determination of the correlation coefficient, i.e.,

$$r_{x\varepsilon} \approx r_{x\varepsilon}^* = \frac{1}{N}\sum_{i=1}^{N}\operatorname{sgn}\dot{x}^*(i\Delta t)\operatorname{sgn}\dot{\varepsilon}^*(i\Delta t)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\operatorname{sgn}\left[\dot{g}(i\Delta t) - \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\right.$$

$$\left.-\varepsilon'(i\Delta t)\ \sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\right]$$

$$\times \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}. \tag{6.10}$$

It is understood that microchanges to the object's state also lead to changes to the estimates of spectral characteristics of the noises $a_{n\varepsilon}$ and $b_{n\varepsilon}$ of corresponding signals. It is advisable to determine them by the following expressions for $r_{x\varepsilon} = 0$:

$$a_{n\varepsilon} \approx \begin{cases} \dfrac{2}{N}\sum\limits_{i=1}^{N}\left[\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}\right]\cos n\omega(i\Delta t) & \text{for } \mathrm{r}_{\mathrm{x}\varepsilon}=0, \\ \dfrac{2}{N}\sum\limits_{i=1}^{N}\left\{\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right|}\right\}\cos n\omega(i\Delta t) & \text{for } \mathrm{r}_{\mathrm{x}\varepsilon}\neq 0, \end{cases} \tag{6.11}$$

$$b_{n\varepsilon} \approx \begin{cases} \dfrac{2}{N}\sum\limits_{i=1}^{N}\left[\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}\right]\sin n\omega(i\Delta t) & \text{for } \mathrm{r}_{\mathrm{x}\varepsilon}=0, \\ \dfrac{2}{N}\sum\limits_{i=1}^{N}\left\{\operatorname{sgn}\left[\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t)-\varepsilon''(i\Delta t)\right|}\right\}\sin n\omega(i\Delta t) & \text{for } \mathrm{r}_{\mathrm{x}\varepsilon}\neq 0. \end{cases} \tag{6.12}$$

It is possible to use the following expression for purposes we previously described:

$$a_{n\varepsilon} = \begin{cases} 0 \quad \text{for } N_{an}^{+}=N_{an}^{-} \text{ and } \Pi_{an}^{+}=\Pi_{an}^{-}, \\ r_{x\varepsilon}(N_{an}^{+}-N_{an}^{-})\,\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)} \\ +\dfrac{1}{2}N\left[\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+}-\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right] \\ \qquad \text{for } N_{an}^{+}=N_{an}^{-} \text{ and } \Pi_{an}^{+}\neq\Pi_{an}^{-}, \\ r_{x\varepsilon}\left(N_{an}^{+}-N_{an}^{-}\right)\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+} \\ +\dfrac{1}{2}\left[N-\left(N_{an}^{+}-N_{an}^{-}\right)\right]\left[\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+}-\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right] \\ \qquad \text{for } N_{an}^{+}>N_{an}^{-} \text{ and } \Pi_{an}^{+}>\Pi_{an}^{-}, \\ r_{x\varepsilon}\left(N_{an}^{-}-N_{an}^{+}\right)\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-} \\ +\dfrac{1}{2}\left[N-\left(N_{an}^{-}-N_{an}^{+}\right)\right]\left[\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+}-\overline{\lambda}_{\mathrm{rel}}\,\overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-}\right] \\ \qquad \text{for } N_{an}^{+}<N_{an}^{-} \text{ and } \Pi_{an}^{+}<\Pi_{an}^{-}, \end{cases} \tag{6.13}$$

where

$$\overline{\lambda}_{\text{rel}} = \frac{\sqrt{D_{\varepsilon}}}{\sqrt{D_g}};\ \Pi_{a_n} = \overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)};$$

$$\Pi_{an}^{+} = \overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{+};\ \Pi_{an}^{-} = \overline{\dot{g}(i\Delta t)\cos n\omega(i\Delta t)}^{-};$$

and N, N_{an}^{+}, and N_{an}^{-} are the quantities of the respective products.

6.3 Positional-Binary Technology of Detecting Initial Stage of Change to the Technical State of Objects

Let us consider an opportunity of forecasting by means of connection between microchanges to the state of the object and the value of noise through the use of positional-binary technology [14, 44–46].

It is known that while continuous signals are coded, the values of the binary codes of the corresponding digits q_k of the samples x_i, of the signal $g(t)$ in each sampling step Δt, initially are accepted equal to

$$X_{ocm(n-1)}(i\Delta t) = X(i\Delta t),$$

where

$$g(i\Delta t) > 2^n;\ g_{\text{rem}(n-1)}(i\Delta t) = g(i\Delta t).$$

Then the signals $q_k(i\Delta t)$ are formed with iterations as a code 1 or 0. Thus, in the first step, the samples $g(i\Delta t)$ are compared with the value $2^{n-1}\Delta g$.

For $g(i\Delta t) \geq 2^{n-1}\Delta g$, the value $q_{n-1}(i\Delta t)$ can be taken equal to unit and the value of the remaining $g_{\text{rem}(n-2)}(i\Delta t)$ is determined by the difference

$$g(i\Delta t) - 2^{n-1}\Delta g = g_{\text{rem}(n-2)}(i\Delta t). \tag{6.14}$$

The sequence of these signals $q_k(i\Delta t)$ is represented as positional-binary-impulse signals whose sum will be equal to an initial signal, i.e.,

$$X(i\Delta t) \approx q_{n-1}(i\Delta t) + q_{n-2}(i\Delta t) + \cdots + q_1(i\Delta t) + q_0(i\Delta t) = X^*(i\Delta t). \tag{6.15}$$

Microchanges to the state of the object are reflected on the noise $\varepsilon(t)$ of the noisy signal $g(i\Delta t) = X(i\Delta t) + \varepsilon(i\Delta t)$ as short-term impulses with duration that is repeatedly lower than positional signals $q_k(i\Delta t)$, i.e.,

$$q_{\varepsilon k}(i\Delta t) = \begin{cases} 1, & \text{for } q_k((i-1)\Delta t) = 0,\ q_k(i\Delta t) = 1,\ q_k((i+1)\Delta t) = 0, \\ 0, & \text{for } q_k((i-1)\Delta t) = q_k(i\Delta t) = q_k((i+1)\Delta t), \\ -1, & \text{for } q_k((i-1)\Delta t) = 1,\ q_k(i\Delta t) = 0,\ q_k((i+1)\Delta t) = 1. \end{cases} \tag{6.16}$$

Due to this, the positional noises $q_{\varepsilon k}(i\Delta t)$ of the noisy signal $g(i\Delta t)$ are formed by means of formula (4.14) by expression (4.16) while coding every positional signal, i.e.,

$$\varepsilon(i\Delta t) = q_{\varepsilon 0}(i\Delta t) + q_{\varepsilon 1}(i\Delta t) + q_{\varepsilon 2}(i\Delta t)$$

$$+ \cdots + q_{\varepsilon k}(i\Delta t) + \cdots + q_{\varepsilon(m-1)}(i\Delta t) = \sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t). \tag{6.17}$$

By the difference

$$X(i\Delta t) = g(i\Delta t) - \varepsilon(i\Delta t) = g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t), \tag{6.18}$$

it is possible to determine samples of the legitimate signal $X(i\Delta t)$.

In this case, the estimates of the variance and spectral characteristics of noise, the cross-correlation function, and the coefficient of correlation between the noise and a legitimate signal can be determined by the following expressions:

$$D_\varepsilon = \frac{1}{N}\sum_{i=1}^{N} \varepsilon^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]^2, \tag{6.19}$$

$$R_{x\varepsilon}(0) = \frac{1}{N}\sum_{i=1}^{N}\left[g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]\left[\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right],$$

$$r_{x\varepsilon} = \frac{R_{x\varepsilon}(0)}{\sqrt{D_\varepsilon R_{xx}(0)}} = \frac{\sum_{i=1}^{N}\left[g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]\left[\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]}{\sqrt{\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]^2 \sum_{i=1}^{N}\left[g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]^2}},$$

$$R_{g\varepsilon}(0) = \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)\varepsilon(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\left[g(i\Delta t)\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right], \tag{6.20}$$

$$r_{g\varepsilon} = \frac{R_{g\varepsilon}(0)}{\sqrt{D_\varepsilon R_{gg}(0)}} \approx \frac{\sum_{i=1}^{N}\left[g(i\Delta t)\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]}{\sqrt{\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]^2 \sum_{i=1}^{N} \dot{g}^2(i\Delta t)}}, \tag{6.21}$$

$$a_{n\varepsilon} = \frac{2}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\cos n\omega(i\Delta t) \approx \frac{2}{N}\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]\cos n\omega(i\Delta t), \tag{6.22}$$

$$b_{n\varepsilon} = \frac{2}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\sin n\omega(i\Delta t) \approx \frac{2}{N}\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]\sin n\omega(i\Delta t). \tag{6.23}$$

So the microchanges to the state of the object are reflected on the least significant position-binary-impulse signals $q_{\varepsilon 0}(i\Delta t)$, $q_{\varepsilon 1}(i\Delta t)$, $q_{\varepsilon 2}(i\Delta t)$, …, $q_{\varepsilon k}(i\Delta t)$, and their effect to estimates of the noise $\varepsilon(i\Delta t)$ is defined by expressions (6.19)–(6.23). Thus, it is possible to forecast the tendency of changes leading to the object's failure by means of a connection between microchanges and noises using estimates obtained by position-binary technology D_ε, $R_{x\varepsilon}(0)$, $R_{g\varepsilon}(0)$, $r_{x\varepsilon}$, $r_{g\varepsilon}$, $a_{n\varepsilon}$, and $b_{n\varepsilon}$.

It is possible to show many directions when the use of traditional methods does not bring satisfactory results. When forecasting the transition of an object into the failure state in such a case, it is advisable to use the relationship between the initial stage of a microchange to the object's state and the change to the noise characteristics obtained by the positional-binary technology. For example, analysis of seismic signals obtained while drilling or compressor station operation, etc. shows that noises carry significant useful information defined by microchanges to the state of an object, and they can be used in forecasting. As another example it is possible to show the use of this theory in medicine. Research shows that in most cases the initial state of various illnesses is not reflected in both corresponding signals and estimates of their correlation and spectral characteristics. At the same time, considerably before the change to the signals and their estimates, certain noises appear in electrocardiograms, in electroencephalograms, and in other signals. That opens wide possibilities to use relationships between noise and the organism's state for forecasting corresponding illnesses.

6.4 Technology of Forecasting Failures by Considering Noise as a Data Carrier

Analysis of various technological objects shows that characteristics of the legitimate signal $\dot{X}(t)$ do not change before failures when the object's state is assumed to be stable. At the same time, there is a certain change to the parameters of the noise $\dot{\varepsilon}(t)$ formed at the initial state of the failure [5, 6, 8, 9, 14–17, 25].

In most cases of real objects, there are certain difficulties with forecasting changes to the object's state at the initial failure state by means of known methods. At best, they only give a possibility of detecting explicit failures [10, 27, 33]. Analysis of the appearance of failures with real objects shows that hidden microfailures represented as microwears, microbeats, microvibrations, and/or microcracks of some elements of objects always precede failures. Well-timed detection of these microfailures gives the possibility to forecast changes to the object's state that can be used for prevention of serious failures. For this purpose in the structure of modern information systems, it is possible to create a subsystem of noise based on forecasting failures on the basis of analyzing the noises $\dot{\varepsilon}_1(i\Delta t)$, $\dot{\varepsilon}_2(i\Delta t)$, …, $\dot{\varepsilon}_n(i\Delta t)$ of the noisy signals $\dot{g}_1(i\Delta t)$, $\dot{g}_2(i\Delta t)$, …, $\dot{g}_n(i\Delta t)$ considering the noise as a carrier of useful information [9, 15, 17, 25, 28].

In general, the process of forecasting can be represented as the sum of the three following parts:

1. the set W formed by the estimates D_{ε_1}, D_{ε_2}, …, D_{ε_n}; $R_{\varepsilon_1\varepsilon_1}(\mu)$, $R_{\varepsilon_2\varepsilon_2}(\mu)$, …, $R_{\varepsilon_n\varepsilon_n}(\mu)$; $R_{x_1\varepsilon_1}(\mu)$, $R_{x_2\varepsilon_2}(\mu)$, …, $R_{x_n\varepsilon_n}(\mu)$; $r_{x_1\varepsilon_1}$, $r_{x_2\varepsilon_2}$, …, $r_{x_n\varepsilon_n}$; $a_{0\varepsilon_1}$, $a_{1\varepsilon_1}$, …, $a_{m\varepsilon_1}$, $b_{0\varepsilon_1}$, $b_{1\varepsilon_1}$, …, $b_{m\varepsilon_1}$; $a_{0\varepsilon_2}$, $a_{1\varepsilon_2}$, …, $a_{m\varepsilon_2}$, $b_{0\varepsilon_2}$, $b_{1\varepsilon_2}$, …, $b_{m\varepsilon_2}$; …; $a_{0\varepsilon_n}$, $a_{1\varepsilon_n}$, …, $a_{m\varepsilon_n}$, $b_{0\varepsilon_n}$, $b_{1\varepsilon_n}$, …, $b_{m\varepsilon_n}$, corresponding to each ith state from all k possible states of an object;
2. the set V formed by current similar information signs carrying information about the current state of the object;
3. the identification rule F mapping each element of set W to an element from set V, and vice versa, each element of set V to an element from set W. The set of elements of sets W and V are information support for the subsystem [15, 17, 28, 56].

In the first stage, the subsystem works in teaching mode using the signals $\dot{g}_1(i\Delta t)$, $\dot{g}_2(i\Delta t)$, …, $\dot{g}_n(i\Delta t)$ obtained from sensors by means of expressions (1.45)–(1.49), (1.78)–(1.96), and (2.41)–(2.51). Information signs, variance, correlation, and spectral characteristics are determined for each ith state, and k standard sets W_i are created. They can be represented as follows:

$$W_i = \begin{bmatrix} D_{\varepsilon_1} & D_{\varepsilon_2} & \cdots & D_{\varepsilon_n} \\ R_{\varepsilon_1\varepsilon_1}(\mu) & R_{\varepsilon_2\varepsilon_2}(\mu) & \cdots & R_{\varepsilon_n\varepsilon_n}(\mu) \\ R_{x_1\varepsilon_1}(\mu) & R_{x_2\varepsilon_2}(\mu) & \cdots & R_{x_n\varepsilon_n}(\mu) \\ r_{x_1\varepsilon_1} & r_{x_2\varepsilon_2} & \cdots & r_{x_n\varepsilon_n} \\ a_{0\varepsilon_1} & a_{0\varepsilon_2} & \cdots & a_{0\varepsilon_n} \\ b_{0\varepsilon_1} & b_{0\varepsilon_2} & \cdots & b_{0\varepsilon_n} \\ a_{1\varepsilon_1} & a_{1\varepsilon_2} & \cdots & a_{1\varepsilon_n} \\ b_{1\varepsilon_1} & b_{1\varepsilon_2} & \cdots & b_{1\varepsilon_n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m\varepsilon_1} & a_{m\varepsilon_1} & \cdots & a_{m\varepsilon_n} \\ b_{m\varepsilon_1} & b_{m\varepsilon_2} & \cdots & b_{m\varepsilon_n} \end{bmatrix}^{[i]} . \tag{6.24}$$

In the second stage, the identification problem is solved by means of current combinations of estimates, i.e., by means of information signs of the set V and the set W_i. The decision of whether the change to the object's state takes place is made on the basis of the obtained results. At the same time, the teaching process goes on. If new combinations of information signs are detected, corresponding information is formed as the new standard set W_{k+1}.

The third stage differs from the second one. During the third stage, the teaching process stops and the state of the object is identified by the obtained current combination of estimates in each cycle. If the combination V of the current estimates of the state differs from the sample set W, the information about the change to the object's state is formed similarly to the second stage. The obtained results are used for making decisions about future exploitation of an object, future repairs, or technical servicing [14, 17, 28].

Summing up, let us discuss the identification rule F mapping each element of the set W to elements of the set V by means of recognition methods in more detail.

The following classification of methods of recognition of patterns is used in this process:

- methods based on principles of separation and comparison with prototype;
- statistical methods;
- methods based on "potential functions";
- methods of calculation of estimates;
- heuristic methods.

Methods based on principles of separation and comparison with prototype are accepted if recognized classes are mapped into sign space by compact geometric groups.

Let V be a point of some sign space. W_i is the sign space characterizing one of the possible object states. The nearest sample class in space W_i is found to classify an unknown state formed by means of the vector V. The separation plane in space is found by means of a method based on separation. It is possible to determine which space contains the vector V by means of the obtained function.

In methods based on comparison with a prototype, the center of a geometric group or class or the nearest to the center of space W_i is chosen as the point-prototype. The nearest prototype is found for classification of an unknown state, and the vector V corresponds to the same space as the prototype. It is obvious that generalized patterns of classes are not formed in this method. Various types of metrics can be used as a measure of proximity [16].

Cluster analysis is used for dividing a set of objects into a given or unknown number of classes by means of some mathematical criterion of quality of classification:

- the objects must be tightly connected to each other inside groups;
- objects of different groups must be far away each from other.

The most important moment of cluster analysis is the choice of a metric or measurement of the proximity of objects. The choice is individual for each problem, taking into account the main targets of research and physical and statistical features of the signs used [16].

The method of potential functions is the most popular method of pattern recognition. It is widely used in designing models based on neuron nets. The method requires dividing two patterns with an empty intersection into the patterns W_1 and W_2. It means that there is at least one function that fully separates sets corresponding to the patterns W_1 and W_2. This function must be positive at points corresponding to the objects from the pattern W_1 and negative at points of the pattern W_2. That function must be built in a teaching process. Sometimes it is required to build the best function in some sense. Let us assume that the teaching sequence of objects corresponds to the sequence of vectors V_1, V_2, …, V_n in a space of patterns connected with the sequence $U(V,V_1)$, $U(V,V_2)$, …, $U(V,V_n)$ of potential functions used for building. The functions $f(V,V_1,V_2,...,V_n)$ f must tend to one of the separating functions while increasing the number of objects. Potential functions for each pattern (state) can be built as the result of a teaching process:

$$U_1(V) = \sum_{V_i \in W_1} U(V, V_1), \tag{6.25}$$

$$U_2(V) = \sum_{V_i \in W_2} U(V, V_2). \tag{6.26}$$

The separation function $f(V)$ can be represented as follows:

$$f(V) = U_1(V) - U_2(V) \tag{6.27}$$

and is positive for the objects of one kind and negative for the objects of the other. Let us consider the following function as a potential function:

$$U(V, V_i) = \sum_{j=1}^{\infty} \lambda_j^2 \varphi_j(V) \varphi_j(V_i) = \sum_{j=1}^{\infty} \phi_j(V) \phi_j(V_i), \tag{6.28}$$

where $\varphi_j(V)$ is a linear, independent system of functions; the λ_j are real numbers different from zero for each $j = 1,2,...$; V_i is a point corresponding to the i th object in a teaching sequence. The teaching sequence is represented in the teaching process, and the approximation $f_n(V)$ determined by following the recurrent procedure is built on each n th step of teaching:

$$f_{n+1}(V) = q_n f(V) + r_n U(V_{n+1}, V). \tag{6.29}$$

As usual, the values q_n and r_n are chosen as follows:

$$q_n \equiv 1,$$

$$r_n \equiv \gamma_n (S(f_n(V_{n+1}), f(V_{n+1}))), \tag{6.30}$$

where $S(f_n, f)$ are non-increasing functions and

$$\begin{cases} S(f_n, f) \equiv 0, \\ S(f_n, f) \le 0, \quad f_n \ge f, \\ S(f_n, f) > 0, \quad f_n < f. \end{cases} \tag{6.31}$$

The coefficients γ_n are represented as a nonnegative sequence of numbers depending on the number n only. γ_n can be taken as follows:

$$\gamma_n = 1/n. \tag{6.32}$$

Several versions of algorithms for potential functions have been developed. The difference between them is in the choice of the laws of correction of the separation function step by step, i.e., in the choice of the coefficients r_n.

The methods of calculating estimates are described in detail in [12, 14, 16]; the need for heuristic methods appears only in special cases when it is difficult to formalize signs and expert intuition is required.

The process of diagnosing and forecasting is repeated. If the same result appears, more precise determination of the object's state by highly qualified specialists is needed.

A recognition block works in two modes. The second one directly solves the problem of recognition. The result of identification gives the possibility for making decisions. The following cases can take place here:

1. identification was performed successfully; diagnosis and forecasting are unique;
2. identification is not unique, but it is possible to obtain diagnosis and forecasting by methods described in the block;
3. identification is not unique, and there is not sufficient information to make decisions. The process of diagnostics and forecasting is repeated; if the same result occurs, recommendations about the state of the object will be given to specialists.

6.5 Diagnosing and Forecasting the Change to the State of Sea Platforms

Sea deep-water stationary platforms (SDWSP) are the most important objects of a sea oil-gas refinery. SDWSP are very complicated and valuable constructions. There are many serious difficulties in solving problems of diagnostics of these objects in practice. From this point of view, despite the fact that their building costs hundred millions of dollars, in most cases reliable diagnostics is not provided. At the same time, the state of the objects is changed with time. It is obvious that future exploitation of an object can be impossible if that process is not controlled and if actions for their recovery are not performed [16, 31].

Taking into the account all mentioned above it is obvious that organization of continuous control of the state of the objects of sea oil-gas constructions can be assumed to be one of the most actual problems [52].

Analysis of failure situations on sea constructions shows that hidden faults caused by certain changes represented as wear, cracks, bends, vibrations, beats, etc. of some elements of objects [15, 28] always precede the appearance of serious faults. Their well-timed forecasting can be used to prevent serious failures [15, 28, 52].

Today information systems to control the state of sea constructions solve a wide range of problems with measuring, recording, and processing of various information, as well as problems of recognition, identification, diagnostics, forecasting, etc. The system must provide not only the control of the current object's state but also its identification and forecasting of future behavior. As stated above, solving this problem by means of well-known algorithms has some difficulties. In the suggested system (Fig. 6.1), one must use methods, algorithms, and information technology to analyze noise so that the above problems are solved [52].

In connection with the above, it is expedient to use algorithms allowing one, by means of noise analysis, to forecast hidden faults in the control of an object's state. This corresponds with the fact that the beginning of hidden microchanges may lead to changes to all characteristics of the noise $\varepsilon(i\Delta t)$.

The system consist of the following blocks: 1 is the block of the analysis of the information about visual observation, sonar, underwater research of divers obtained from the specialists; 2 is the block of the analysis of the information from the sensors; 3 is the block of the analysis of the signals of the inclinometers and accelerometers fixing angles of slope and twisting; 4 is the block of the analysis of the signals of the level indicators determining linear shifts of the platform; 5 is the block of the analysis of the signals of the vibration sensors' measurements of frequencies of the oscillations of the elements of the construction; 6 is the block of the analysis of signals of the acoustic emission system with piezoconverters; 7 is the block of analysis of the signals of strain sensors of deformation of the construction elements; 8 is the block of the analysis of the signals of the sensors of the velocity and direction of the wind; 9 is the block of the analysis of the signals of the pressure sensors in the elements of the construction; 10 is the block of the

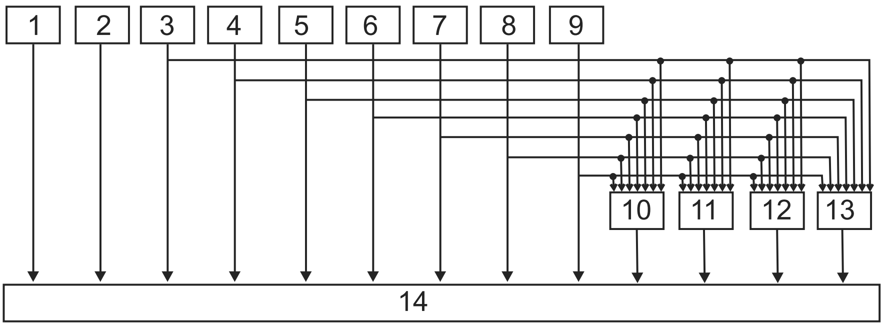

Fig. 6.1. Block scheme of the hybrid system of diagnostics.

diagnostics by means of the algorithms of robust correlation analysis; 11 is the block of the diagnostics by means of the algorithms of robust spectral analysis; 12 is the block of the diagnostics by means of the algorithms of the position-binary-impulse analysis; 13 is the block of the forecasting of the measurement of the state of the object by means of the analysis of the noise from the sensors of the technological parameters; 14 is the block of the signaling and representation of the information to the specialists.

Blocks 1–9 work in the regime of control of the corresponding technological parameters. If the results received at the time of measurement are in the given range of the corresponding operating conditions of the object, then the state is considered to be normal. All blocks of the analysis and control (blocks 1–9 in Fig. 6.1) work identically. At the same time, the signals $X_3(i\Delta t)$, $X_4(i\Delta t)$, …, $X_n(i\Delta t)$ from those blocks (besides blocks 1 and 2) are given to blocks 10–13 in parallel.

Initially, blocks 10–13 work in the teaching mode and information signs, robust, variance, correlation, and spectral characteristics are determined by means of information technology described in Chapter 4 for various states by measurement information $g(i\Delta t)$, and the corresponding robust sample sets W^R are formed.

$$W^R = \begin{bmatrix} D_3^R & D_4^R & \dots & D_9^R \\ R_{x_3}^R(\mu) & R_{x_4}^R(\mu) & \dots & R_{x_9}^R(\mu) \\ 0 & R_{x_3x_4}^R(\mu) & \dots & R_{x_3x_9}^R(\mu) \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & R_{x_8x_9}^R(\mu) \\ 0 & r_{x_3x_4}^R & \dots & r_{x_3x_9}^R \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & r_{x_8x_9}^R \\ a_{0x_3} & a_{0x_4} & \dots & a_{0x_9} \\ \dots & \dots & \dots & \dots \\ a_{mx_3} & a_{mx_4} & \dots & a_{mx_9} \\ b_{0x_3} & b_{0x_4} & \dots & b_{0x_9} \\ \dots & \dots & \dots & \dots \\ b_{mx_3} & b_{mx_4} & \dots & b_{mx_9} \end{bmatrix}. \qquad (6.33)$$

At the same time, the characteristics of the noise are determined, and the sample set W^{ε} is formed from these information signs by means of technology described in Chapters 1–3.

$$W^{\varepsilon} = \begin{bmatrix} D_{\varepsilon_3} & D_{\varepsilon_4} & \dots & D_{\varepsilon_8} & D_{\varepsilon_9} \\ R_{\varepsilon_3}(\mu) & R_{\varepsilon_4}(\mu) & \dots & R_{\varepsilon_8}(\mu) & R_{\varepsilon_9}(\mu) \\ R_{\varepsilon_3 x_3}(\mu) & R_{\varepsilon_4 x_4}(\mu) & \dots & R_{\varepsilon_8 x_8}(\mu) & R_{\varepsilon_9 x_9}(\mu) \\ \dots & \dots & \dots & \dots & \dots \\ r_{\varepsilon_3 x_3} & r_{\varepsilon_4 x_4} & \dots & r_{\varepsilon_8 x_8} & r_{\varepsilon_9 x_9} \\ a_{0\varepsilon_3} & a_{0\varepsilon_4} & \dots & a_{0\varepsilon_8} & a_{0\varepsilon_9} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m\varepsilon_3} & a_{m\varepsilon_4} & \dots & a_{m\varepsilon_8} & a_{m\varepsilon_9} \\ b_{0\varepsilon_3} & b_{0\varepsilon_4} & \dots & b_{0\varepsilon_8} & b_{0\varepsilon_9} \\ \dots & \dots & \dots & \dots & \dots \\ b_{m\varepsilon_3} & b_{m\varepsilon_4} & \dots & b_{m\varepsilon_8} & b_{m\varepsilon_9} \end{bmatrix}. \qquad (6.34)$$

In the second stage, the opportunity to accept the decision about the presence of the change to the state q of the object is considered, and the problem of identifying the object is solved by the combinations of robust estimates, i.e., by information signs. Also, the teaching process goes on. When detecting a new combination of information signs, corresponding information is formed as the new sample sets V^k and W^{ε}.

The third stage differs from the second one. At that stage, the teaching process stops and the state of the object is identified by the current combination of estimates in each cycle. If a new combination of estimates of the current state V^R, V^{ε} appears and does not coincide with the standard set W, corresponding information about changes to the state of the object is formed and the identification problem is solved. Diagnostics problems are solved by means of W^R. Problems with forecasting are solved by means of W^{ε}.

It is obvious that these values are stable for the stable state of the object. Certain changes to total noises occur at the beginning of hidden faults, and that is reflected on mentioned values remembered as corresponding sample sets. The results obtained from diagnostics and forecasting are given to the specialists, who use them for making decisions about future exploitation, repair, or technical servicing. It is thus possible to forecast and inform specialists about a threat of failure before failures appear in the process of exploitation.

Using robust estimates in W^R in the considered system allows one to avoid diagnostics error to a certain degree. During the stable state of the object, the corresponding information signs do not change. But if the signal characteristics $g(i\Delta t)$ are changed under the influence of the external factors, the authenticity of the diagnostics of the controlled objects increases due to the simultaneous work of blocks 10–13 and the improvement in robustness of determined noises. The property of the noise $\varepsilon(i\Delta t)$ to react to the microchanges in the control objects leads to changes to the elements of W^ε and to the revelation, early in operation, of the hidden changes occurring in the SDWSP state. Thus, this system allows one to forecast the possibility of changes to the state of the object beforehand. It is obvious from the work scheme in Fig. 6.1 that given information technology can be easily realized as a hybrid information system.

6.6 Telemetric Information System of Forecasting Accidents During Drilling by Considering Noise as a Data Carrier

The probability of an accident's origin during drilling is connected with such features as multiple-factor vagueness of the mechanism of the accident's origin, their regional specificity, the transience, the difficulties of accessing the instrumental control, and the blur and the ambiguity of the observed symptoms. The methods of preventing and eliminating the accidents used in practice insufficiently affect the decrease of the accident risk during the drilling. And these methods are considered to be improved over earlier methods.

Research has shown that errors that appear in the estimates of the spectral and correlation characteristics of the measured information often lead to mistaken forecasts of the accidents during drilling. At the same time, the opportunity to use the noises containing some technological parameters of the drilling as a data carrier is not taken into account. The estimates of the signals obtained from the sensors in the process of drilling are made with great errors. Besides, their stability, i.e., the robustness, is not provided, and that is the basic reason for the decrease of the reliability of forecasting the accidents in the existing systems.

Here we consider the use of the robust technology of forming the statistical knowledge bases in the telemetric intellectual systems of the control and management of the oil-well drilling in order to increase the reliability of forecasting accidents.

The necessity to form sample sets of the estimates of the statistical characteristics of the signals, which are used as the knowledge base for making the decisions, appears during the creation and the application of the intellectual system of forecasting the accidents in the process of drilling. The signals obtained from the sensors of the following controlled drilling parameters are the basic data for such system: the axial load of the instrument $g_1(t)$; the rotational moment of the spindle of the rotation head of the boring machine $g_2(t)$; the rotational moment of the rotor of the boring machine $g_3(t)$; the penetration speed $g_4(t)$; the fluid washover consumption on the output of the circulating system $g_5(t)$; the pressure of the washover fluid on the output of the circulating system $g_6(t)$; the qualitative parameters of the washover fluid $g_7(t)$, etc.

Research has shown that, in practice, various sufficient noises $\varepsilon_1(t)$, $\varepsilon_2(t)$, …, $\varepsilon_6(t)$ appear during the measurement of these parameters, and these parameters are the total signals $g_1(t) = X_1(t) + \varepsilon_1(t)$, $g_2(t) = X_2(t) + \varepsilon_2(t)$, …, $g_6(t) = X_6(t) + \varepsilon_6(t)$ consisting of the useful signal $X(t)$ and the noise $\varepsilon(t)$. The specificity of the considered process also differs in that the sufficient changes of the values of the noises of the measured parameters precede the most dangerous accidents, for example, sticking the plunger, and these changes appear long before the origin of the accident itself. First, these changes are reflected by the axial load on the instrument $g_1(t)$, the rotational moment of the spindle of the rotation head of the boring machine $g_2(t)$, the rotational moment of the rotor of the boring machine $g_3(t)$, and the penetration speed $g_4(t)$. That is why it is first necessary to estimate the variances D_{ε_1}, D_{ε_2}, D_{ε_3}, D_{ε_4} of the noises of the total signals $g_1(t)$, $g_2(t)$, $g_3(t)$, $g_4(t)$ for analysis of the mentioned controlled drilling parameters and, on their basis, to form the set of the robust sample indications W_D, the elements of which are the correlations $D_{\varepsilon 1}/D_{g1}$, $D_{\varepsilon 2}/D_{g2}$, $D_{\varepsilon 3}/D_{g3}$, $D_{\varepsilon 4}/D_{g4}$ containing the following greatly useful information:

$$W_D = \begin{Bmatrix} D_{\varepsilon 1} & D_{\varepsilon 2} & D_{\varepsilon 3} & D_{\varepsilon 4} \\ \dfrac{D_{\varepsilon 1}}{D_{g1}} & \dfrac{D_{\varepsilon 2}}{D_{g2}} & \dfrac{D_{\varepsilon 3}}{D_{g3}} & \dfrac{D_{\varepsilon 4}}{D_{g4}} \end{Bmatrix}, \tag{6.36}$$

where D_{g1}, …, D_{g4} are the variances of the total signals of the $g_1(t)$, …, $g_4(t)$, respectively; $D_{\varepsilon 1}$, …, $D_{\varepsilon 4}$ are the variances of the noises of the same signals, determined by the following formula:

$$D_{\varepsilon} = \sigma^2(\varepsilon) \approx \frac{1}{N}\sum_{i=1}^{N}\Big[\dot{g}^2(i\Delta t) + \dot{g}(i\Delta t)\dot{g}\big((i+2)\Delta t\big)$$

$$-2\dot{g}(i\Delta t)\dot{g}\big((i+2)\Delta t\big)\Big], \tag{6.37}$$

where $\dot{g}(i\Delta t)$ is the total centered signal consisting of the useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ with the average of distribution m_{ε}, which is close to zero; and Δt is the sampling step.

The robust set is formed by using the robust technology [12, 14] of the estimates of coefficients of the Fourier series of the signals $g_1(t)$, …, $g_4(t)$ obtained from the sensors in the process of drilling:

$$W_{ab}^{R} = \begin{Bmatrix} a_{11}^{R}b_{11}^{R} & a_{12}^{R}b_{12}^{R} & \ldots & a_{14}^{R}b_{14}^{R} \\ a_{21}^{R}b_{21}^{R} & a_{22}^{R}b_{22}^{R} & \ldots & a_{24}^{R}b_{24}^{R} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1}^{R}b_{n1}^{R} & a_{n2}^{R}b_{n2}^{R} & \ldots & a_{n4}^{R}b_{n4}^{R} \end{Bmatrix}, \tag{6.38}$$

where $a_{11}^{R}b_{11}^{R}$, ..., $a_{14}^{R}b_{14}^{R}$, ..., $a_{n1}^{R}b_{n1}^{R}$, ..., $a_{n4}^{R}b_{n4}^{R}$ are the robust coefficients of the Fourier series of the signals $g_1(t)$, …, $g_4(t)$ determined by the following formulae:

$$a_n^{R} = \frac{2}{N}\left\{\sum_{i=1}^{N}\Big[\dot{g}(i\Delta t)\cos n\omega(i\Delta t) - \lambda_{an}^{R}\Big]\right\}, \tag{6.39}$$

$$b_n^{R} = \frac{2}{N}\left\{\sum_{i=1}^{N}\Big[\dot{g}(i\Delta t)\sin n\omega(i\Delta t) - \lambda_{bn}^{R}\Big]\right\}. \tag{6.40}$$

The value of the improvement to the robustness of the coefficients a_n, b_n is calculated by formulae represented in Chapter 4.

The set of the robust estimates of the correlation functions of such drilling parameters as the consumption $g_5(t)$ and the pressure of the washover fluid on the input of the circulating system $g_6(t)$ is formed along with the estimate of the spectral characteristics of the signals $g_1(t)$, $g_2(t)$, $g_3(t)$, $g_4(t)$:

$$W_{R_{gg}}^{R} = \left\{R_{gg1}^{R}, R_{gg2}^{R}, R_{gg3}^{R}, \ldots, R_{gg6}^{R}\right\}, \tag{6.41}$$

where R^R_{gg1}, R^R_{gg2}, R^R_{gg3}, …, R^R_{gg6} are the robust estimates of the correlation functions of the total signals $g_1(t)$, $g_2(t)$, ..., $g_6(t)$ determined by the following formula:

$$R^R_{gg}(\mu)=\begin{cases} R_{gg}(\mu)-\left[\lambda^R_{gg}(\mu)+D_\varepsilon\right] & \text{for} \quad \mu=0, \\ R_{gg}(\mu)-\lambda^R_{gg}(\mu) & \text{for} \quad \mu\neq 0, \end{cases} \tag{6.42}$$

where $R_{gg}(\mu)$ is the correlation function of the centered signal, D_ε is the variance of the noise, and

$$\lambda^R_{gg}(\mu)\approx\left[n^+(\mu)-n^-(\mu)\right]\left\langle\Delta\lambda(\mu=1)\right\rangle,$$

where $n^+(\mu)$ and $n^-(\mu)$ are the number of the products $\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)$ of the positive and the negative signs, respectively;

$$\left\langle\Delta\lambda(\mu=1)\right\rangle=\left[1/n^-(\mu=1)\right]\lambda(\mu=1),$$

where

$$\lambda(\mu=1)=\left|R'_{gg}(\mu=1)-R_{gg}(\mu=1)\right|$$

is the average error of one product $\dot{g}(i\Delta t)\dot{g}((i+1)\Delta t)$.

The robust estimates of the cross-correlation functions of the signals are determined similarly; the robust set of the informative indications is also formed on their basis as follows:

$$W^R_{R_g}=\begin{Bmatrix} R^R_{g12} & R^R_{g13} & \ldots & R^R_{g16} \\ R^R_{g21} & R^R_{g23} & \ldots & R^R_{g26} \\ \ldots & \ldots & \ldots & \ldots \\ R^R_{g51} & R^R_{g52} & \ldots & R^R_{g56} \end{Bmatrix}. \tag{6.43}$$

So the formed sets (6.36), (6.38), (6.41), and (6.43) are the sample robust value bases for making decisions during drilling. The telemetric intellecttual system of forecasting the accidents by considering the noise as a data carrier can be created on their basis.

The analysis of the tendencies of the development of the intellectual systems of forecasting the accidents shows that the creation of such systems not for the individual oil well but for the whole oil and gas field is advisable. In this case, even the expensive intellectual forecasting systems become more justified.

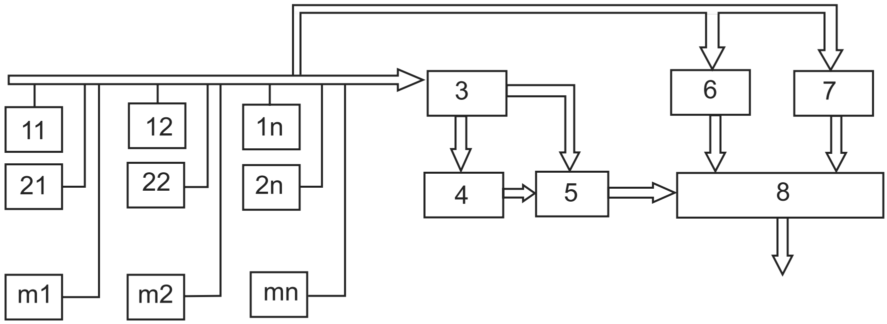

Fig. 6.2. The telemetric intellectual system of forecasting the accidents by considering the noise as a data carrier.

The block scheme of the telemetric system of robust forecasting accidents of oil wells is represented in Fig. 6.2. The block of the analysis of the signals by the traditional methods and the algorithms along with the robust technology of forming the knowledge bases are present in this block scheme.

The system consists of the following blocks: 11, 12, ..., *mn* are the sensors of the controlled drilling parameters; 2 is the interface; 3 is the block of forming the robust informative indications; 4 is the sample robust knowledge base; 5 is the block of the robust identification; 6 is the block of the analysis by traditional methods and algorithms; 7 is the base of the technological and geological data; 8 is the block of the analysis and decision making representing the expert system.

The procedure of forecasting accidents during drilling is reduced to the following. The system works in the training mode in the first stage or for the first oil well of the oil and gas field, and the robust sets of the sample estimates of these signals are formed in block 3 by expressions (6.36), (6.38), (6.41), and (6.43) by the results of the analysis of the signals obtained from the sensors 11, 12, ..., *mn* in the process of drilling. These estimates are contained in block 4 and correspond to the accident-free conditions of the drilling process. The corresponding informative indications are accepted as the elements of the sample sets preceding the emergency conditions during trouble in the drilling process. In block 5 of the robust identification, the current estimates are compared with the sets of the estimates obtained when measuring and processing the same signals in the current moment. These signals are also analyzed by the traditional methods, and algorithm 6 in the analysis block and then with the basis of the technological and the geological data in 7 are transmitted to the analysis and decision-making block 8, to which the results of the robust identification also arrive. The maintenance staff makes the decision about the change to the object's state (whether the change takes place or not) using the obtained informative indications. At the same time, the training process goes on if the new estimates differing from the elements of the sample set but not affecting the state of the drilling are received. The obtained new combination of the estimates is included in the set of the sample elements. The process of adapting the intellectual system to the concrete drilling conditions, taking into account the knowledge of the experts and technological, geological, and other data, ends with that.

The cumulative knowledge bases adapted for the first oil well of the field are used in future drilling. The new obtained robust informative indications are compared with the elements of the knowledge base in the identification block. If even one of the elements of these informative indications does not coincide with the elements of the knowledge base, that is the evidence of changes to the state of the controlled object. Then the information about it is immediately represented to the experts, who use it for making decisions about the future exploitation of the oil well. The telemetric intellectual system performs the following functions: detecting emergency and dangerous situations; detecting and diagnosing accidents; making recommendations about the localization and elimination of emergency and dangerous situations; providing the audio-visual-preventive information and the recommendations on the display of a boring master.

6.7 Technology and System of Monitoring a Defect's Origin in the Most Vulnerable Modules of Objects of Thermoelectric Power Stations and Nuclear Power Plants

High-powered steam-turbine plants that have reached a high degree of perfection are considered as the base of modern power engineering. They work on the supercritical steam pressure and have an advanced heat scheme.

Combined plants using modern high-temperature gas turbines are very effective, with a coefficient of efficiency above 55–60%.

The state of these objects is characterized by a great number of technological parameters. Among them the following parameters are the indicators of the steam-and-gas plant: the initial gas temperature; the air consumption on the entrance to the compressor; the degree of the pressure increase; the capacity of the gas-turbine plant (GTP); the gas temperature beyond the gas turbine; the steam conditions under high pressure; the consumption; the steam conditions under low pressure; the coefficient of efficiency of the combined plant; the specific consumption of the standard fuel, etc.

At present, the control of the current state of thousands of technological parameters is performed by the informational systems of the control and management of the objects of the thermoelectric power stations and the nuclear power plants. Besides the wide range of the problems connected with measuring, the registration, various information processing, the solutions to problems of recognition, the identification, the diagnostics, the optimization, the management, etc. are also performed by these systems.

Among all these problems, providing reliability when monitoring the technical state of the object is of great primary importance both for the thermoelectric power stations and for the nuclear power plants. Let us assume that the object of monitoring consists of n modules: M_1, M_2, ..., M_n. At the same time, the state of the object Q_0 is determined by the state of these modules, i.e.,

$$Q_0 = F\left(Q_{M_1}, Q_{M_2}, ..., Q_{M_p}\right).$$

Let us suppose that the state of each of these modules is characterized by the input signals

$$\begin{array}{c} g_{1M_1}(t), g_{2M_1}(t), ..., g_{iM_1}(t), ..., g_{mM_1}(t) \\ g_{1M_2}(t), g_{2M_2}(t), ..., g_{iM_2}(t), ..., g_{mM_2}(t) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ g_{1M_N}(t), g_{2M_N}(t), ..., g_{iM_N}(t), ..., g_{mM_N}(t) \end{array}$$

and the output signals

$$\begin{array}{c} \eta_{1M_1}(t), \eta_{2M_1}(t), ..., \eta_{iM_1}(t), ..., \eta_{mM_1}(t) \\ \eta_{1M_2}(t), \eta_{2M_2}(t), ..., \eta_{iM_2}(t), ..., \eta_{mM_2}(t) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \eta_{1M_N}(t), \eta_{2M_N}(t), ..., \eta_{iM_N}(t), ..., \eta_{mM_N}(t). \end{array}$$

At the same time, it is necessary to provide the calculations for estimates of the statistical characteristics of the useful signals of the mentioned technological parameters for solving the problems of monitoring the technical states of these modules. Determining the estimates of the statistical characteristics allows one to estimate the state and the quality of the work of the system for a certain time interval (for example, for a shift, for a day, etc.). It is assumed in the literature that the realizations $g(t)$, $\eta(t)$ of the technological parameters $X(t)$, $y(t)$ of power engineering are stationary ergodic with normal distribution law, and the time interval T is set sufficiently large. Also, the estimates $R_{gg}(\mu)$ and $R_{g\eta}(\mu)$ are usually used instead of the correlation functions $R_{xx}(\mu)$ and $R_{xy}(\mu)$ of the useful signals $X(t)$ and $y(t)$. These and other estimates of the technological parameters are commonly used when solving the problem of control, diagnostics, identification, and management.

All these estimates adequately reflect the technical state of the mentioned modules during their exploitation in normal conditions for the working time T_0. But a change to the found estimate is considered as a change to the useful signal during the rise of the defect's origin in one of the mentioned modules. Due to this, on the one hand, the chance of early monitoring of the defect is lost; on the other hand, the wrong decision is made by the management system.

As research has shown, at present exactly this feature of modeling the signal reflecting the process of the defect's origin in the given objects of the power engineering is not taken into account when solving the problems of control and diagnostics. That leads to sufficient errors in processing results and, in turn, leads to tardiness of the monitoring results.

Due to this, the efficiency of using modern methods of monitoring decreases for a large number of objects of power engineering. For example, the practical realization of methods of monitoring the technical state of the basic models of the real systems of the thermoelectric power stations is not too difficult despite the fact that in practice there is a great need to solve such problems [14, 33]. The use of methods to monitor the defect is not effective also for the vulnerable modules of the nuclear power plants, where the number of controlled technological parameters is in the thousands. In this connection, the creation of technologies of early detection of the origin of defects in the basic modules of the objects of power engineering is of a certain interest.

The specificity of these objects requires the determination of the moment of the defect's origin on the stage of macro- and mini-changes to the state of the most vulnerable modules in the process of exploitation. That, in turn, requires a high sensitivity of the informative indications to various micro-changes, which usually precede serious failures in the thermoelectric power stations and the nuclear power plants. From this point of view besides the traditional estimates, it is expedient to provide the calculations of the estimates of the noise $\varepsilon(t)$, the useful signal $X(t)$, the determination of the robust correlation functions, the spectral characteristics of the correlation coefficients, the values of the robustness, their position-binary components, etc. during monitoring for input-output signals of each module, i.e.,

$$a_1^R, b_1^R; a_2^R, b_2^R; \ldots; a_n^R, b_n^R,$$

$$R_{x_1x_1}^R(\mu), R_{x_2x_2}^R(\mu); \ \ldots; \ R_{x_nx_n}^R(\mu),$$

$$D_{\varepsilon_{x1}}, D_{\varepsilon_{x2}}, \ldots, D_{\varepsilon_{xn}}; D_{\varepsilon_{y1}}, D_{\varepsilon_{y2}}, \ldots, D_{\varepsilon_{ym}},$$

$$f_{0_{x1}}, f_{1_{x1}}, \ldots, f_{m_{x1}}; f_{0_{x2}}, f_{1_{x2}}, \ldots, f_{m_{x2}}; \ldots; f_{0_{xm}}, f_{1_{xm}}, \ldots, f_{m_{xn}},$$

$$\tau_{x_1y_1} \pm \Delta\tau_{x_1y_1}, \tau_{x_1y_2} \pm \Delta\tau_{x_1y_2}; \ldots; \tau_{x_1y_m} \pm \Delta\tau_{x_1y_m},$$

$$K_1(\eta), K_2(\eta), K_3(\eta), \ldots, K_n(\eta).$$

At the same time, by using them as the informative indications, it is possible to form the sample sets corresponding to their normal state both for each module particularly and for the electric power station as a whole. For example, the following sets can be determined for the state of the module M_1 by using robust algorithms:

$$W_{QM_1}^{ab} = \begin{Bmatrix} a_{11}^R b_{11}^R & a_{12}^R b_{12}^R & \ldots & a_{1n}^R b_{1n}^R \\ a_{21}^R b_{21}^R & a_{22}^R b_{22}^R & \ldots & a_{2n}^R b_{2n}^R \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1}^R b_{m1}^R & a_{m2}^R b_{m2}^R & \ldots & a_{mn}^R b_{mn}^R \end{Bmatrix},$$

$$W_{QM_1}^{R} = \begin{Bmatrix} R_1^R(\mu) & R_2^R(\mu) & \ldots & R_m^R(\mu) \\ R_{12}^R(\mu) & R_{31}^R(\mu) & \ldots & R_{1m}^R(\mu) \\ \ldots & \ldots & \ldots & \ldots \\ R_{m1}^R(\mu) & R_{m2}^R(\mu) & \ldots & R_{(n-1)m}^R(\mu) \end{Bmatrix},$$

$$W_{QM_1}^{D} = \begin{Bmatrix} D_{\varepsilon 1}^R & D_{\varepsilon 2}^R & \ldots & D_{\varepsilon m}^R \\ D_1 & D_2 & \ldots & D_m \end{Bmatrix},$$

$$W_{QM_1}^{\tau} = \begin{Bmatrix} \tau_{11} & \tau_{12} & \ldots & \tau_{1m} \\ \tau_{21} & \tau_{22} & \ldots & \tau_{2m} \\ \ldots & \ldots & \ldots & \ldots \\ \tau_{m1} & \tau_{m2} & \ldots & \tau_{mm} \end{Bmatrix},$$

$$W_{QM_1}^{f} = \begin{Bmatrix} f_{10} & f_{11} & f_{12} & \cdots & f_{1n} \\ f_{20} & f_{21} & f_{22} & \cdots & f_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ f_{m0} & f_{m1} & f_{m2} & \cdots & f_{mn} \end{Bmatrix},$$

$$W_{QM_1}^{K} = \begin{Bmatrix} K_{11}(\eta) & K_{12}(\eta) & \ldots & K_{1m}(\eta) \\ K_{21}(\eta) & K_{22}(\eta) & \ldots & K_{2m}(\eta) \\ \cdots & \cdots & \cdots & \cdots \\ K_{m1}(\eta) & K_{m2}(\eta) & \ldots & K_{mn}(\eta) \end{Bmatrix}.$$

Similar sets are formed for the diagnostics of other modules. As a whole, the diagnostics system contains the following sets:

$$W_{QM1}^{ab},\ W_{QM2}^{ab},\ \ldots,\ W_{QMP}^{ab};\ W_{QM1}^{R},\ W_{QM2}^{R},\ \ldots,\ W_{QMP}^{R};$$

$$W_{QM1}^{K},\ W_{QM2}^{K},\ \ldots,\ W_{QMP}^{K}.$$

The procedure of monitoring is reduced to the following. In the process of exploiting the object, for the signals obtained from the output of the sensors installed on the corresponding modules M_1, M_2, …, M_n, the above-mentioned estimates are determined and are compared with the elements of the sample sets corresponding to the normal state. While detecting the difference, exceeding the given range even for one of the informative indications, this information is given to the maintenance staff, which is responsible for the safety of the exploitation of the object. In the future, after analysis of this information, the experts make the final decision about the state of the controlled object. At the same time, sufficient effectiveness of monitoring is provided due to the opportunity to detect the defects on the stage of their origin in contrast with the estimates obtained by traditional algorithms.

7 The Technology of Monitoring a Defect's Origin by Considering Noise as a Data Carrier

7.1 Specific Properties of the Technology of Monitoring a Defect's Origin by Considering Noise as a Data Carrier

It is known that monitoring and diagnostics by vibration are commonly used for controlling the technical state of the most important equipment for airplanes, helicopters, tankers, compressor stations, electric power stations, main oil-and-gas pipelines, deep-sea platforms, and so on, and especially for objects with rotating equipment, for example compressor stations (http://www.rotatingequip.com/). However, all technological parameters obtained from the output of sensors as the signals $g_1(i\Delta t)$, $g_2(i\Delta t)$, …, $g_m(i\Delta t)$ are analyzed in modern information systems provide reliable results during monitoring. At the same time, the used measuring tools and the information systems detect changes to the technical state of the equipment only after a series of significant defects has appeared [12, 14, 56, 57]. Unfortunately, in some cases, this detection occurs not long before an accident [15, 56]. Methods and technologies of detecting the defects at their origin have been worked out recently [16, 19–23]. Simultaneously, for example, for the above-mentioned objects, it is considered that weak vibrations appear in the initial moment in the spot of the defect's origin. However, they quickly damp during the spread. They are represented as noise with a high-frequency spectrum in the signals obtained from the vibration sensors. For example, a change to the properties of the frictional forces and caused by vibrations are the basic indications of the defects in the bearings used in many vulnerable places of technical objects. Their extraction and sub–sequent analysis can give the opportunity to detect this defect at its origin—in some cases, sufficiently before an accident [15, 21, 22, 56].

At present, the problem of extracting the corresponding high-frequency components $\varepsilon_j(i\Delta t)$ from the vibration signal $x_j(i\Delta t)$ is solved by means of the respective band filters [15, 55, 57]. This technology is most effective for objects with rotating equipment working in the same mode for a long duration of time. The bands of the spectra of the high-frequency $\varepsilon_j(i\Delta t)$ and the low-frequency $x_j(i\Delta t)$ components of the signal $g(i\Delta t)$ change in the permissible limits and can be determined using a priori information. But in practice, in many cases it is rather difficult to determine the spectral bands of the noises $\varepsilon_j(i\Delta t)$ of the signals $g_j(i\Delta t)$. That is why it is often impossible to be sure that the spectra of the band's filters coincide with the spectra of vibration signals in the initial stage of the defect's origin. In addition, the spectral distribution of the high-frequency components of the total signal continuously changes from the beginning of the origin of a defect through the detection of the defect's salient character. For example, during the micro crack in the main oil pipelines, at first the faint squeak is heard in that place, then the squeak grows, turns into a whistle, and at last turns into the snore. It is obvious that in this case the spectra $\varepsilon_j(i\Delta t)$ for corresponding time intervals are sufficiently different from each other. This is exactly why not all the defects are detected in the initial stage of their origin by means of the band filters in many modern systems of diagnostics by vibrations based on the high-frequency and low-frequency spectra of the components of the signal $g(i\Delta t)$.

The following conditions were taken into account when detecting the defect's origin in the considered objects when the digital technologies of the analysis of the signals $g(i\Delta t)$ were obtained from the output of the sensors:

1. The objects of monitoring by considering the noise as a data carrier can work in a great number of the various modes. During their changeover from one mode to another, both spectra of the useful signal and spectra of the noise also change correspondingly. At the same time, when the defect is present, the noise spectra will be of a high frequency in comparison with the spectra of the useful signal in all modes.
2. It is necessary to provide the reliability and adequacy of the results of the detection of the defect's origin during every possible change to the work mode of the object.
3. The spectrum of the noise $\varepsilon(i\Delta t)$ in the beginning of the defect's origin intersects with the spectrum of the useful signal $X(i\Delta t)$, i.e., there is a certain correlation between $\varepsilon(i\Delta t)$ and $X(i\Delta t)$.
4. The spectrum and the variance of the vibration signal $\varepsilon_j(i\Delta t)$ gradually change through the evolution of the process of the defect's formation.

5. The stable high-frequency spectra of the noise $\varepsilon(i\Delta t)$ of the signal $g(i\Delta t)$ are formed in the period when they gain their salient character. It is obvious that the digital technologies, assigned for monitoring the defect at its origin, must support the corresponding specific properties.

First, let us consider the problem of extracting the high-frequency signal $\varepsilon_j(i\Delta t)$ from the total signal $g_j(i\Delta t)$. Let us assume that the high-frequency signal $\varepsilon_j(i\Delta t)$ reflected on the total signal $g_j(i\Delta t)$ obtained from the output of the vibration sensor S is formed as the result of the defect's origin. In the works [14, 16, 19–23] and in Chapter 3, it was shown that the approximate value $\varepsilon_j^*(i\Delta t)$ of the noise of the high-frequency spectrum can be determined by the samples of the total signal $g_j(i\Delta t)$ by the expression

$$\dot{\varepsilon}_j^*(i\Delta t) \approx \operatorname{sgn}\left[\varepsilon_j'(i\Delta t) - \varepsilon_j''(i\Delta t)\right]\sqrt{\left|\varepsilon_j'(i\Delta t) - \varepsilon_j''(i\Delta t)\right|}, \tag{7.1}$$

where

$$\varepsilon_j'(i\Delta t) = \dot{g}_j^2(i\Delta t) + \dot{g}_j(i\Delta t)\dot{g}_j((i+2)\Delta t) - 2\dot{g}_j(i\Delta t)\dot{g}_j((i+1)\Delta t), \tag{7.2}$$

$$\varepsilon_j''(i\Delta t) = \dot{g}_j(i\Delta t)\dot{g}_j((i+1)\Delta t) + \dot{g}_j(i\Delta t)\dot{g}_j((i+3)\Delta t)$$

$$-2\dot{g}_j(i\Delta t)\dot{g}_j((i+2)\Delta t). \tag{7.3}$$

At the same time, the approximate values of the samples of the useful signal $X_j(i\Delta t)$ can be determined by means of the samples $\varepsilon_j^*(i\Delta t)$, i.e.,

$$X_j^*(i\Delta t) = g_j(i\Delta t) - \varepsilon_j^*(i\Delta t). \tag{7.4}$$

As mentioned above, in this case it is advisable to consider the total signals $g_j(i\Delta t)$ as the random processes, because the signals $\varepsilon(i\Delta t)$ and $X(i\Delta t)$, besides the periodical components, are also affected by other external factors that appear in the process of the normal exploitation of the considered objects. In this connection, solving the problem of monitoring the defect at its origin is first reduced to the determination of the approximate estimates of the samples of the noise $\varepsilon^*(i\Delta t)$ and the useful signal $X^*(i\Delta t)$ and their analysis by the technologies represented in Chapters 2–5.

The performed analysis of the defect's origin shows that, for example, the coefficients of the correlation between the noises $\varepsilon_j(i\Delta t)$ and $\varepsilon_{j+1}(i\Delta t)$ of the signals $g_j(i\Delta t)$ and $g_{j+1}(i\Delta t)$ obtained from the output of the neighbor sensors also hold certain information during the vibration control. These coefficients can be determined by the following expression:

$$r_{\varepsilon_j \varepsilon_{j+1}} \approx \frac{1}{N} \sum_{i=1}^{N} \operatorname{sgn} \dot{\varepsilon}_{\nu}^{*} \left(i\Delta t \right) \operatorname{sgn} \dot{\varepsilon}_{\nu+1}^{*} \left(i\Delta t \right). \tag{7.5}$$

The time shifts $\mu_{j,(j+1)}\Delta t$ between the neighbor vibration signals, which can be determined by the extremal values of the cross-correlation functions, have a certain informational potential at the defect's origin.

The opportunity to use the correlation between the process of forming the defect and the appearance of the noise, which can be detected by the analysis of the vibration signals by using position-binary technology, has great importance when the defect's origin is monitored [14, 44]. According to this technology, the position signals $q_k \left(i\Delta t \right)$ are formed from the values of the binary codes of the corresponding digits q_k of the samples $g(i\Delta t)$ of the signal $g(t)$ when the continuous signals are coded.

According to the literature [14, 44, 32], if the state of the object is stable, the value $\left\langle T_{k_0} \right\rangle$ can be accepted as the non-random value for all realizations of the same signal $g(i\Delta t)$. The values of the periods of the lower position-binary-impulse signals (PBIS) are sufficiently sensitive informative indications. They rapidly change at the beginning of the defect's origin, for example, due to the appearance of even a weak high-frequency vibration. Besides, the combinations of the estimates of the average time intervals $\left\langle T_{q_0} \right\rangle$, $\left\langle T_{q_1} \right\rangle$, …, $\left\langle T_{q_{n-1}} \right\rangle$ of the components $q_k(i\Delta t)$ of the signal $g_i(i\Delta t)$ are also the non-random values for the sufficiently long observation time T. They change at the defect's origin in any vulnerable place, which explains why they are important informative indications.

Additionally, as shown in Chapter 2, the position noises $q_{\varepsilon k}\left(i\Delta t \right)$ of the vibration signal $g(i\Delta t)$ are formed as the short-term impulses because of the influence of the defect's origin when each position signal is coded.

It was shown in the literature [14, 19, 20, 22, 23, 44, 46] and in Chapter 2 that if the state of the object is stable, the correlation of the quantity $N_{\varepsilon k}$ of the signals $q_{\varepsilon k}(i\Delta t)$ to the total quantity N_{qk} of the position-binary-impulse signals $q_k(i\Delta t)$ for time T is a non-random value. At the same time, the quantity $N_{\varepsilon k}$ for time T increases in all position signals at the moment of the defect's origin. Therefore, the values of the coefficients K_{q_0}, K_{q_1}, K_{q_2}, ..., $K_{q_{m-1}}$ also change from this time. This allows one to use them as another informative indication when solving the problem of detecting the defect's origin.

Considering all this, let us note that solving the problem of monitoring the defect's origin by considering the noise as a data carrier reduces to the analysis of the signals $g_1(i\Delta t)$, $g_2(i\Delta t)$, ..., $g_m(i\Delta t)$ by the position-binary technology, the technology of the analysis of the noise, and the robust correlation and spectral analysis. These technologies are considered in detail in Chapters 2–4. They allow one to detect the weak components $\varepsilon(i\Delta t)$ of the signal $g(i\Delta t)$ on a background of the strong $X(i\Delta t)$ ones by means of the determination of the estimates D_ε, $r_{x\varepsilon}$, $R_{x\varepsilon}^R(\mu)$, $R_{gg}^R(\mu)$, $a_{n\varepsilon}$, $b_{n\varepsilon}$, a_n^R, b_n^R $r_{\varepsilon_j\varepsilon_{j+1}}$, $\mu_{g_jg_{j+1}}$, $\mu_{x_jx_{j+1}}$, $\mu_{\varepsilon_j\varepsilon_{j+1}}$, $\langle T_{q_0}\rangle$, $\langle T_{q_1}\rangle$, $\langle T_{q_2}\rangle$, …, $\langle T_{q_{m-1}}\rangle$, $\langle f_{q_0}\rangle$, K_{q_0}, K_{q_1}, K_{q_2}, ..., $K_{q_{m-1}}$ allowing one to detect the appearance of the defect at its origin.

7.2 Digital System of Monitoring a Defect's Origin by Considering Noise as a Data Carrier

The reliability of received decisions is the basic system requirement during digital monitoring of the defect. It is necessary to create sufficiently simple, reliable, and inexpensive technologies and systems for monitoring the defect, taking into account the wide area of application of such systems. Let us consider some of the possible variants of the solution to this problem [19–23].

The offered system of monitoring on the basis of considering the noise as a data carrier is created by the below-mentioned scheme and is intended for determination of the place and time of the defect's origin. Due to the simplicity of the realization of the worked-out digital technologies, the process of monitoring the beginning of the defect's origin is realized simultaneously in several variants. Besides, after the appearance of the first alarm signals, the results of monitoring are under extra analysis, with the exception of every possible fault. And this process is duplicated many times by the change to the sampling step Δt_ε. Thus, the reliability of monitoring the defect's origin increases.

The system consists of the sensors S_1, S_2, S_3, …, S_m, interface 1 with the multichannel analog-code converter and serial controllers 2, 3 (Fig. 7.1). The number of sensors is determined by the requirement connected with the specificity of the object of monitoring and depends on the permissibility of time intervals between the cycles of monitoring, the availability of the vulnerable places for measuring the vibration, the distance between the placed sensors, etc. In the simplest case, it is possible to perform necessary monitoring by moving the same handheld apparatus with the sensor through all vulnerable places of the equipment. But in this case, by the time of defect monitoring increases correspondingly with an increase in the quantity of these points. In some cases, for example, during preflight monitoring of the technical state of the airplanes, the time is limited. For such systems, when the quantity of the sensors increases, it is advisable to minimize the monitoring time. In this case, the sensors S_1, S_2, S_3, ..., S_m are set in all vulnerable places of the object. The "narrow places" and the statistics of the corresponding defects are well known for specialists of objects such as airplanes, tankers, platforms, compressor stations, etc. The vibration sensors are set exactly in these places or, at least, in the transition points of the high-frequency vibration to the other units of the object that allow the measurement of the vibration.

The estimates of the variances, the correlation functions, the correlation coefficients, and the spectral characteristics of the noise of the vibration signals are determined in the self-training stage in block 2 for each cycle by the algorithms represented in Chapters 2–5. For example, in an airplane the removable sensors S_1, S_2, S_3, ..., S_m are set in corresponding "vulnerable places" of the body, the wing, the undercarriage, the engine, etc., for preflight monitoring of the defect's origin. The vibration signals obtained from these sensors are converted to the digital codes $g_1(i\Delta t)$, $g_2(i\Delta t)$, ..., $g_j(i\Delta t)$, ..., $g_m(i\Delta t)$ in each cycle. The current estimates D_ε, D_x, $a_{n\varepsilon}$, $b_{n\varepsilon}$, a_{nx}, b_{nx}, $r_{x\varepsilon}$, $R_{gg}^R(\mu)$, $\mu^*_{g_\nu g_{\nu+1}}$, $\mu^*_{x_\nu x_{\nu+1}}$, $\mu^*_{\varepsilon_\nu \varepsilon_{\nu+1}}$, $r^*_{\varepsilon_\nu \varepsilon_{\nu+1}}$ are determined many times for the obviously good state of the airplane for each point, i.e., for each sensor during several cycles. And they are remembered as the sample values in block 3.

The ranges of their deviations $\pm\Delta D_\varepsilon$, $\pm\Delta D_x$, $\pm\Delta a_{n\varepsilon}$, $\pm\Delta b_{n\varepsilon}$, $\pm\Delta a_{nx}$, $\pm b_{nx}$, $\pm\Delta r_{x\varepsilon}$, $\pm\Delta\mu^*_{g_\nu g_{\nu+1}}$, $\pm\Delta\mu^*_{x_\nu x_{\nu+1}}$, $\pm\Delta\mu^*_{\varepsilon_\nu \varepsilon_{\nu+1}}$, $\pm\Delta r^*_{\varepsilon_\nu \varepsilon_{\nu+1}}$ are also determined on this stage during normal exploitation of the object. Besides, the combinations of the average time intervals $\langle T_{q_0}\rangle$, $\langle T_{q_1}\rangle$, $\langle T_{q_2}\rangle$, ..., $\langle T_{q_{m-1}}\rangle$ of the position-impulse signals, the combinations of the coefficients K_{q_0}, K_{q_1}, K_{q_2}, ..., $K_{q_{m-1}}$ and the average frequency $\langle f_{q_0}\rangle$ of the lower position-binary signals of all vibration signals $g_j(i\Delta t)$ are also determined. The values of the ranges of their deviations are determined at the same time. That ends the self-training process of the system for this object.

Then the system passes into the monitoring stage by using the above-mentioned technology to detect the defect's origin by considering noise as a data carrier. At the same time, for example, the current estimates D_ε, D_x, $a_{n\varepsilon}$, $b_{n\varepsilon}$, a_{nx}, b_{nx}, $\mu_{g_j g_{j+1}}$, $R_{gg}^R(\mu)$, $r_{x\varepsilon}$, $\mu_{x_j x_{j+1}}$, $\mu_{\varepsilon_j \varepsilon_{j+1}}$, $R_{\varepsilon_j \varepsilon_{j+1}}(\mu)$

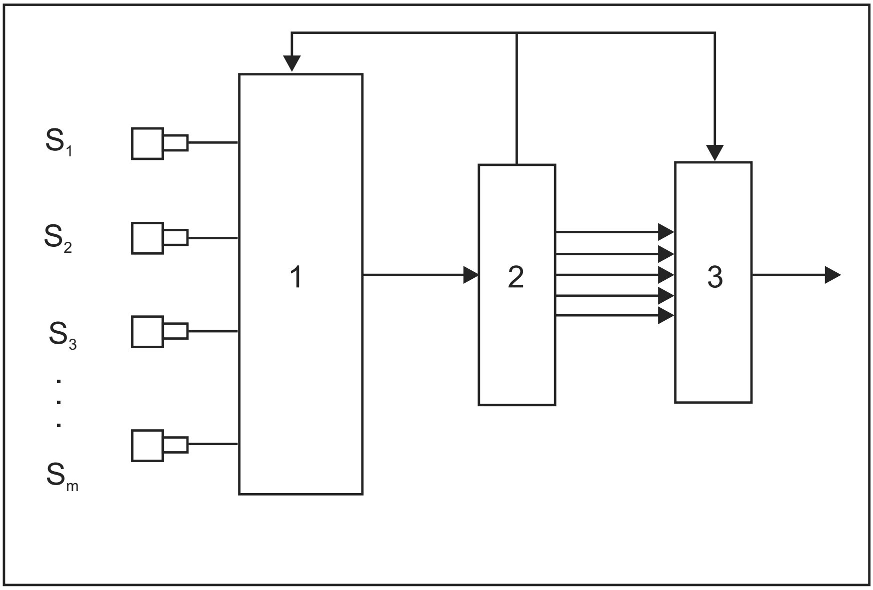

Fig. 7.1. The digital system of monitoring the defect's origin by considering the noise as a data carrier.

are determined for the airplanes in the process of their exploitation similar to the self-training stage in each cycle. Then the ranges of the deviations of the found current estimates from the sample estimates are determined. After completing this process, the current deviations of the mentioned estimates are compared with the sample ranges of the corresponding estimates in each cycle for each signal $g_1(i\Delta t)$, $g_2(i\Delta t)$, ..., $g_j(i\Delta t)$, ..., $g_m(i\Delta t)$. If they do not exceed the sample ranges recorded in the self-training process, it is considered that no defect in the point corresponds to the number of the sensor of the analyzed signal. Otherwise, if even one of the estimates turns out to be more or less than the corresponding sample range, the time of the beginning of the cycle is recorded as the beginning of the defect's origin in this unit of the object.

After that the estimates $\langle T_{q_0} \rangle$, $\langle T_{q_1} \rangle$, $\langle T_{q_2} \rangle$, ..., $\langle T_{q_{m-1}} \rangle$, $\langle f_{q_0} \rangle$, K_{q_0}, K_{q_1}, K_{q_2}, ..., $K_{q_{m-1}}$ of all vibration signals are determined by the algorithms represented in Chapter 2. They are compared with the corresponding sample informative indications. At the same time, if their values are in the sample ranges of the deviations, it is considered that there are no serious reasons for making emergency decisions. But from this moment, the alarm signal remains during a certain period of time and the analysis of monitoring based on considering the noise as a data carrier is repeated in each cycle. For the case when the signals about the defect's origin obtained by the above-mentioned technologies are confirmed, then the place and time of the defect's origin are determined, the corresponding information is formed, and the warning signal settles down by the results obtained from the position-binary technology by the number of the sensors and the cycle time.

The system can be used in three variants. Its use in the stationary variant is advisable in those cases when the establishment of the required quantity of the sensors does not worsen the service properties of the object. For example, it is advisable to use these systems in the stationary variant for compressor stations, deep-sea platforms and communications, tankers, electric power stations, etc. It is advisable to use the offered system at the portable variant for airplanes, helicopters, etc., where the establishment of the many sensors on the wings, the undercarriage, the body, etc. can worsen their exploitation characteristics. The use of the system can be the combination of the stationary variant with the portable variant for such objects as the main gas-oil pipelines, the railways, the offshore piers, and other communications.

7.3 Robust Information System for Forecasting Accidents on Compressor Stations of Main Gas-Oil Pipelines

Compressor stations (CS) are the basic objects of the main gas-oil pipelines and the important industrial objects of the large petrochemical complexes. That is why it is necessary to create reliable information systems for the diagnostics and forecasting of accidents on the compressor stations for providing trouble-free and effective work of the oil-processing and petrochemical industries.

In the past, the faults of forecasting the accidents on the compressor stations were considered to be connected with the meteorological and the reliable characteristics of the elemental basis of the informational-measuring systems for control of the vibratory state. They have been greatly improved, but the probability of the accident's appearance has not decreased. The performed analysis shows that the impossibility of detecting the initial stage of the defect's origin through known methods is often the basic reason behind inadequate decisions by the informational systems when emergency conditions appear [12, 14]. In practice, the solution to the problem of forecasting accidents on the compressor stations is connected with major difficulties because the analysis of the measured information does not allow one to forecast an accident beforehand, and the diagnostic systems give signals at the beginning of the failure—when it is practically impossible to prevent it [12].

It follows that the opportunity to use the correlation between the microchanges to the state of the basic units of the equipment and the rotational movement machinery and their representation in the signals as the noise for forecasting accidents on the compressor stations is of great theoretical and practical importance.

All the elements are subject to the continuous qualitative changes during the exploitation of the compressor stations [13]. The totality of the inner properties of these elements determines the state of the stations during the time T. Changes to the states of these objects are reflected in the changes to the signals, which are the diagnostic—data carriers. This corresponds to the case when the object's state is described by the signals $x_1(t)$, $x_2(t)$, …, $x_n(t)$ given by the corresponding noises $\varepsilon_1(t)$, $\varepsilon_2(t)$, …, $\varepsilon_n(t)$ that differ from white noise. At the same time, the analyzed signals are the sum of the useful signal and the noise, i.e., $g_1(t) = x_1(t) + \varepsilon_1(t)$, $g_2(t) = x_2(t) + \varepsilon_2(t)$, …, $g_n(t) = x_n(t) + \varepsilon_n(t)$, which can be represented in the discrete form as follows: $g_1(i\Delta t) = x_1(i\Delta t) + \varepsilon_1(i\Delta t)$, $g_2(i\Delta t) = x_2(i\Delta t) + \varepsilon_2(i\Delta t)$, …, $g_n(i\Delta t) = x_n(i\Delta t) + \varepsilon_n(i\Delta t)$. Determining the informative indications and comparing them with the sample values provides the control of the change to the characteristics of the compressor stations. It is assumed that the obtained estimates of the signals reflect the corresponding changes to the technical state of the equipment when solving the problems of the diagnostics [13, 14]. For that purpose, the estimates of the statistical characteristics of the signals obtained from the corresponding sensors are compared with the sample estimates of the signals corresponding to the typical normal states of the object. At the same time, forecasting failures or accidents is performed with the results of the diagnostics.

In many cases, the results of the diagnostics appear to be satisfactory and allow one to forecast the possible directions of future changes to the object's state. But microchanges take place in the compressor stations quite often. These microchanges are the reason for future disastrous accidents. They are reflected only on the changes to the noises $\varepsilon_1(i\Delta t)$, $\varepsilon_2(i\Delta t)$, …, $\varepsilon_n(i\Delta t)$. Despite this, however, the informative indications obtained from the signals themselves do not change during the long period of exploitation. The results of the diagnostics appear to be belated due to the fact that only characteristics of the noises $\varepsilon_1(i\Delta t)$, $\varepsilon_2(i\Delta t)$, …, $\varepsilon_n(i\Delta t)$ change during the microchanges' origin. Along those lines, taking into account that compressor stations are characterized by accidents, which are inadmissible from the viewpoint of the total damage, it is necessary to forecast the change to the state of the CS together with the control and diagnostics.

For that purpose, it is necessary to create digital technologies to detect the defects of basic units of the equipment that appear in the early stages. Research has shown that it is advisable to use the information technology for the analysis of the noise represented in Chapter 3 when forecasting the possible accidents by using the correlation between the microchanges of the vibratory state of the CS and the value of the noise. The variance of the noise D_ε and the cross-correlation function $R_{g\varepsilon}(0)$ between the noise $\dot{\varepsilon}(i\Delta t)$ and the noisy signal $\dot{g}(i\Delta t)$ appear to be the most effective estimates for detecting the beginning of the latent changes.

The correlation coefficients $r_{g\varepsilon}$ and $r_{x\varepsilon}$ between the noise and the corresponding signals obtained from the vibration sensors are also good informative indications.

The spectral analysis of the noises obtained from the primary vibration measurement devices also gives many opportunities for research of the vibration signals reflecting the technical state of the equipment of the CS. It is connected with the fact that microchanges to the vibratory state of the vibration signals affect the estimates of the spectral characteristics λ_{an} and λ_{bn} of the noise, which can be determined with the technologies described in Chapter 5.

Figure 7.2 represents the block scheme of the offered informational system of forecasting CS accidents by considering the noise as a data carrier. The interface transmitting the corresponding signals $g_1(i\Delta t)$, $g_2(i\Delta t)$, …, $g_i(i\Delta t)$, …, $g_n(i\Delta t)$ obtained from the sources (sensors) s_1, s_2, …, s_j, s_n established on the equipment of the CS to the input of the blocks of the traditional, robust, correlation, and spectral analyses and the analysis of the noise is used for receiving the source information about the technical state of the diagnosed machinery.

Special attention is given in the system to the systematization, processing, and analysis of the vibration metrical data. The common use of the vibration signals as the informative indications reflected by the mechanical oscillation of the object in the control points allows one to get the various characteristics of the vibratory state of such basic units and details of the CS as the body and various rotational apparatus. The failures of these units and details can cause breakage of the basic units and the details of the CS.

In contrast with traditional systems in the considered variant, the parallelization of the processes of forecasting by considering the noise as a data carrier and the diagnostics of the change to the state of the CS by the informational system is performed to exclude the risk of possible accidents. For that purpose, the signals $g_1(i\Delta t)$, $g_2(i\Delta t)$, …, $g_n(i\Delta t)$

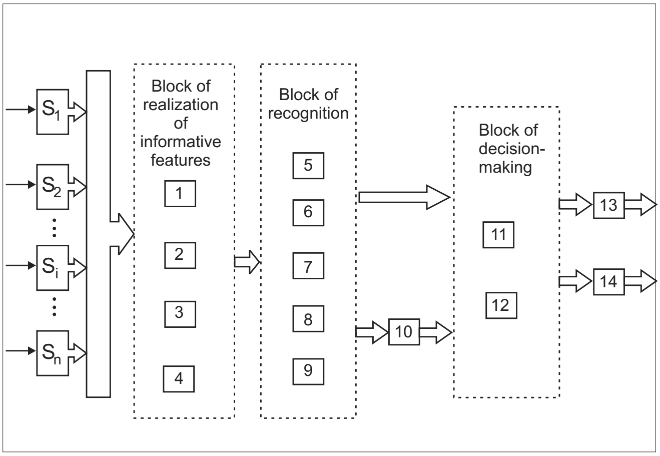

Fig. 7.2. The block scheme of the offered informational system of forecasting CS accidents by considering the noise as a data carrier.

obtained from the sources s_1, s_2, …, s_n are transmitted simultaneously to the input of all blocks. The estimates of the characteristics of the noise, i.e., D_ε, $R_{g\varepsilon}(0)$, $R_{g\varepsilon}(\mu)$, $r_{g\varepsilon}$, λ_{an}, λ_{bn}, D_ε^R, $R_{g\varepsilon}^R(0)$, $R_{g\varepsilon}^R(\mu)$, $r_{g\varepsilon}^R$, λ_{an}^R, and λ_{bn}^R, are determined by the input of the block of the analysis of the noise. In the blocks of the correlation and the spectral analyses, these estimates are used to provide robustness to the statistical characteristics of the signals obtained by traditional algorithms, i.e., $D_g^R = D_g - D_\varepsilon$, $R_g^R(0) = R_g(0) - R_{g\varepsilon}(0)$, …, $a_n^R = a_n - \lambda_{an}$, $b_n^R = b_n - \lambda_{bn}$ [1, 14, 58]. When the dependence of these estimates on the noise is removed, the reliability of the diagnostic data increases. It is reasonable that these values are stable during the stable state of the CS. They also remain stable at the beginning of the processes leading to the microchange to the state of the station. At the same time, the estimates of the summary noise change.

Because the given technology is in contrast with traditional methods, the opportunity to diagnose and forecast the transition of the CS to the new state beforehand arises. In general, this process can be represented as the totality of three components: the set of the possible states of the object W; the set of the informative indications V giving information about the state; and the identification rule F comparing each element of the set W with the element of the set V, and vice versa. The identification rule F is the direct and inverse functions. According to the offered technology, the set W consists of the subsets W_g^* of the robust estimates of the statistical characteristics of the signals $g_1(i\Delta t)$, $g_2(i\Delta t)$, …, $g_n(i\Delta t)$, and the subsets W_ε

of the robust estimates of the noises $\varepsilon_1(i\Delta t)$, $\varepsilon_2(i\Delta t)$, …, $\varepsilon_n(i\Delta t)$ determined in the corresponding blocks. The elements of the sets W and V are the vector values and are characterized by several numerical parameters.

In the first stage, the signals $g_1(i\Delta t)$, $g_2(i\Delta t)$, …, $g_n(i\Delta t)$ obtained from the corresponding sensors must be received by the blocks of the traditional algorithms, the analysis of the noise, and the robust, correlation, and spectral analyses during the work of the informational system to get the necessary informative indications and to form the current set V. According to the offered technology, in the beginning the system works in the training mode and the informative indications, i.e., the variance, the correlation, and the spectral characteristics of the noisy signals and their noises, are determined by the corresponding algorithms in the forecast blocks for the various states of the CS. The sample subsets W_g and W_ε are created on their basis.

In the second stage, the opportunity to detect the change to the state of the CS is considered, and the problem of identifying the state of the CS is solved by the combinations of the current estimates and the informative indications of the subsets W_g. In this case, the presence of the microchanges is determined by the differences in the combinations of the current estimates of the noises $\varepsilon_1(i\Delta t)$, $\varepsilon_2(i\Delta t)$, …, $\varepsilon_n(i\Delta t)$ from the corresponding sample elements W_ε, and the results of forecasting by considering the noise as a data carrier are formed on the basis of the state identification. The training process also goes on during the detection of the new informative indications. The information about it is entered as the corresponding sample elements of the subsets W_g and W_ε.

The third stage of the work of the system differs from the second in that the training process stops and the combinations of the estimates of the noises and the noisy signals obtained through identification of the sample elements of the subsets W_g and W_ε forecasting the microchanges by considering the noise as a data carrier or predicting the diagnostics of the salient changes are made in each cycle if the state of the diagnosed object is considered to be unstable. The intellectual methods are used in the block of making the decision on the basis of the expert systems, allowing one to identify the state of the CS and solve the problem of diagnostics and forecasting by several variants in the corresponding blocks with the obtained results. As the result, the corresponding recommendations about the continuation of the exploitation, the performance of the preventive repair works, or the break of the CS are formulated in the decision-making block by means of both the separate data and their various combinations.

7.4 Digital City System of Monitoring the Technical State of Socially Important Objects by Considering Noise as a Data Carrier

In countries situated in seismically active zones, regular monitoring of the technical state of residential dwellings and strategic structures is required to ensure public safety. The importance of the problem grows several-fold for cases in which, besides seismic danger, there is also the probability of landslides. Consequently, for cities located in such regions, there is a critical need to create a citywide system for monitoring the technical state of the housing stock and building structures and a system of signals to warn of the onset of anomalous seismic processes. The many failures that have led to catastrophic consequences in these countries in recent years point to the need to solve the following urgent problems:

1. Creation of a digital technology and system for implementing regular, successive monitoring of changes to the technical state of important building structures.
2. Creation of a digital technology for implementing regular monitoring of the technical state of a group of structures situated in landslide zones.
3. Creation of a digital technology and an information system for implementing citywide monitoring and warnings of the initial stage of anomalous seismic processes.
4. Creation of an intelligent information technology and system for combining the citywide monitoring system for important building structures with the short-term prediction of the onset of possible dangerous anomalous seismic processes.

The research showed that simultaneous change to the estimates of statistical characteristics of the noises of the signals obtained from the vibration-acoustic sensors established on socially important objects and situated considerable distances from each other in the seismic regions can be used as the indicator of the beginning of abnormal seismic processes. A single change to the estimates is connected with the beginning of the change to the technical state of the corresponding object. According to the literature [19, 20, 22, 23], with these specific characteristics of seismic regions it is possible to suggest a digital technology and a citywide system of monitoring the technical state of housing objects and signaling the beginning of abnormal seismic processes by considering the noise as a data carrier.

The scheme of a system of monitoring by considering the noise as a data carrier is represented in Fig. 7.3.

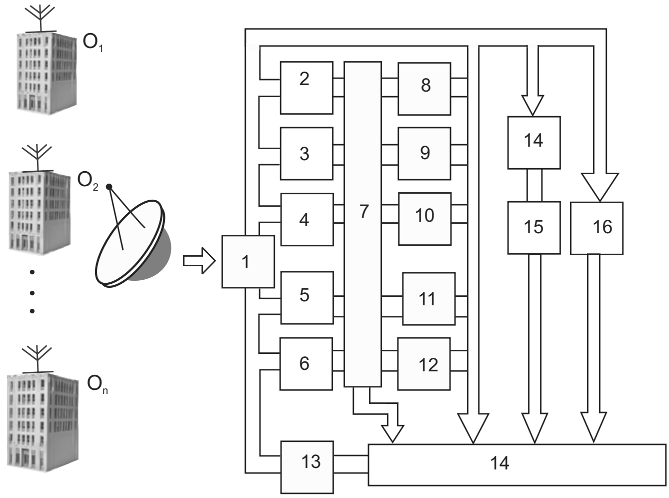

Fig. 7.3. Scheme of the system of monitoring by considering the noise as a data carrier.

In creating the system, each of the structures O_1, O_2, …, O_N is equipped with a local block with a black box on the base of a controller and corresponding vibro-, seismo-, pyezo-, and tenzosensors D_1, D_2, …, D_m mounted on the most vulnerable parts of the structure. Following primary processing in the local system controller, the signals $g_1(i\Delta t)$ $g_2(i\Delta t)$, …, $g_n(i\Delta t)$ from each structure are transmitted to the modem of the central noise monitoring system through the modems and radio communication facilities. In the course of system operation, block 1 receives signals from the monitored structures through the modem; these signals are represented in digital form and transmitted by the block in turn to the inputs of blocks 2–6, 8–12, and 13, 14, and 16 by radio communication.

The system functions in four modes in accordance with the noise monitoring. In the first, second, and third modes, training is performed at the initial stage. For this purpose, estimators of the mathematical expectation m_ε of the noise, the variance D_ε of the noise, the correlation coefficient $r_{x\varepsilon}$ between the useful signal and the noise, and the coefficients a_ε and b_ε of the Fourier series of the noise in block 2 are made. This block is responsible for the analysis of the noise $\varepsilon_j(i\Delta t)$ of the signals $g_j(i\Delta t)$ that have been received from the corresponding sensors of the monitored structures. The estimators are stored as reference quantities. Ranges of possible minimal, mean, and maximal deviations $\Delta m_{\varepsilon\,\min}$, $\Delta m_{\varepsilon\,\mathrm{mean}}$, $\Delta m_{\varepsilon\,\max}$; $\Delta D_{\varepsilon\,\min}$, $\Delta D_{\varepsilon\,\mathrm{mean}}$, $\Delta D_{\varepsilon\,\max}$; $\Delta a_{n\varepsilon\,\min}$, $\Delta b_{n\varepsilon\,\min}$,

$\Delta a_{n\varepsilon \text{mean}}$, $\Delta b_{n\varepsilon \text{mean}}$, $\Delta a_{n\varepsilon \max}$, $\Delta b_{n\varepsilon \max}$; $\Delta r_{x\varepsilon \min}$, $\Delta r_{x\varepsilon \text{mean}}$, $\Delta r_{x\varepsilon \max}$ are also established for the estimators. Moreover, for each signal distribution, the histograms $W[\varepsilon(i\Delta t)]$ of the noise $\varepsilon_j(i\Delta t)$ and the magnitudes of their potential deviations are determined. These are also stored as reference information attributes. For this purpose, the desired histogram curve is constructed in time $T = N\Delta t$ from the approximate values of the readings

$$\varepsilon^*(i\Delta t) = \operatorname{sgn} \varepsilon'(i\Delta t)\sqrt{\varepsilon'(i\Delta t)},$$

which are specified at equal intervals $\Delta\dot{\varepsilon}$ in the range from 0 to $\varepsilon_{\max}$ from the number of readings N_1, N_2, ..., N_m. When the correlation between the useful signal $\dot{X}(i\Delta t)$ and the noise $\dot{\varepsilon}(i\Delta t)$ is nonzero, the histogram of the distribution law of the noise is constructed from the number of readings N_1, N_2, ..., N_m of the quantities

$$\dot{\varepsilon}^*(i\Delta t) = \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]},$$

situated in the corresponding ranges $\dot{\varepsilon}(i\Delta t)$. With increasing $N \to \infty$, the computed estimators N_1, N_2, ..., N_m of the histograms tend to the desired values of the distribution law of the noise $W[\varepsilon(i\Delta t)]$.

After a training stage, the system enters the monitoring mode. While the system functions in the first mode, the values of the current estimators of the mathematical expectation, variance, correlation coefficient, Fourier coefficients, and noise histogram are successively determined by means of block 2. These values are compared with the corresponding standard values established in the course of training. If their respective differences do not exceed the adopted minimal ranges, it is assumed that the technical state of the corresponding structure O_1, O_2, ..., O_N has not undergone any change. Otherwise, the differences that have been obtained are transmitted to decision block 17, where a signal indicating the onset of a change to the technical state of the corresponding structure is generated. The degree of seriousness of the situation that exists is determined from the magnitude of the difference in the range of deviation.

Unlike the first mode, in the second mode a warning is generated indicating the start of a landslide only where a deviation is simultaneously detected; note that this deviation must exceed the minimal range of estimators of the noise of signals that are received from several closely situated groups of structures.

The third mode differs from the second mode by the fact that a warning indicating the onset of anomalous seismic processes is created upon detection of a deviation that exceeds the minimal range of estimators of noise from several groups of structures situated at significant distances from each other. In block 17 the degree of danger that the particular seismic situation represents is estimated on the basis of the number of groups, especially in terms of the magnitude of the range of the deviation. For cases in which the magnitudes of the deviation exceed the maximal threshold levels, block 17 generates a warning signal that indicates the threatening nature of the incipient anomalous processes.

To increase the reliability of the results of operations in the system, a solution to the monitoring problem according to traditional statistical methods and robust methods and on the basis of position-binary and heuristic technologies, respectively, is envisaged together with use of the noise analysis algorithm by means of blocks (7.10)–(7.15). At the same time, the analysis of the results is performed in all three modes by the estimates of the signals $g_j(i\Delta t)$ obtained in block 17 similarly to above (e.g., in the operation of the system in the fourth mode). In the course of training, reference sets that have been obtained through the use of the information technologies are generated in blocks 2–6. Estimators of the current information attributes of the reference set, which are obtained through use of the above technologies, are generated in blocks 8–12. Alternate recognition of the technical state of the corresponding structures O_1, O_2, ..., O_N is performed according to the current information attributes in block 7. As a result of this process, if the current information attributes differ from the reference sets, information to this effect is transmitted to block 17, where analysis is performed and a decision arrived at in accordance with the above procedure [1, 23, 33].

In the fourth mode, analysis of the signals that have been received from seismic sensors mounted on the controlled structures is performed in block 14 according to technologies that are widely used by seismologists. A basis of seismic, geological, geophysical, statistical, and other information attributes is generated in block 13. In block 15 a base of the seismological reference information attributes of the reference sets is generated on the basis of information that has been obtained from the city's seismological service. Then, using all possible estimators of the signals $g_j(i\Delta t)$ at the current time, analysis and recognition of the seismic situation at the controlled sites are performed. Ultimately, the set of information that has been obtained as a result of the operation of blocks 2–6, 8–12, 13, 15, and 16 is used in decision block 17 to achieve systems analysis and determination of the probability and approximate estimation of the time for the onset of dangerous seismic processes [1, 2, 33, 23, 59].

Note that the system of noise monitoring is to be created in several stages. In the first stage, local systems on the basis of modern controllers are created for each structure. These systems function independently and monitor the technical state of the structures. In the second stage, radio communication facilities and system hardware are created in accordance with the flowchart presented in Fig. 7.2; the flowchart also enables the system to function in the first three modes. Finally, in the third stage, information technology that enables the system to function in the fourth mode is realized.

In addition, the control of operational imperfection on the objects O_1, O_2, ..., O_N is provided in the system. For example, during elevator accidents, gas leakage, short-circuits in power supplies, etc., in the monitored structures, a corresponding warning signal will be generated in block 17 together with an indication as to the nature of the breakdown and the number of the structure.

In conclusion, let us note some factors showing the significance and urgency of the exploitation and the creation of the above-mentioned system:

1. With the use of traditional technologies to monitor the technical state of building structures, results of changes to these structures are established when they have acquired a pronounced form. At the same time, the use of a digital technology for the analysis of noise for this purpose makes it possible to detect these changes initially. As a result, through the early discovery of minor defects, it becomes possible to organize timely preventive measures and forestall the occurrence of serious defects, thus making it possible to significantly reduce the total volume of repair operations and expenditures and to decrease the number of sudden collapses.
2. The proposed citywide system of noise monitoring will make it possible to detect the initial stages of landslides whenever changes in the range of deviations of the corresponding estimators of noise from a group of structures situated in a particular area of the city occur. As a result, it will become possible to generate warning signals and thus alert the proper city services of the potential for a very dangerous and destructive ecological process.
3. The creation of a citywide noise monitoring system is both critically necessary and feasible. This is largely due to the fact that if the range of deviation of the magnitudes of the corresponding estimators of the noise of signals received from sensors of structures located at many different sites in the city exceeds the established threshold values, it will be possible to detect sufficiently reliably the initial stages of anomalous seismic processes and to adopt a decision about an appropriate warning signal.
4. Once a citywide noise monitoring system has been created, it will be possible to increase the reliability of short-term analysis of seismic

processes through the combined use of the technologies employed in the analysis of noise with the technologies used in the analysis of information that has been received from the seismic service.

In conclusion, let us note that in the considered system both the technology given in Chapter 2 and the positional-binary technology are used to determine the sampling frequency and detect the origin of abnormal seismic processes.

7.5 Digital Technology and the System of Receiving Seismic Information from Deep Beds of the Earth and Monitoring the Origin of Anomalous Seismic Processes by Considering Noise as a Data Carrier

Traditionally, the supervision of the safety of strategic objects and high-rise buildings as a rule has been performed by systematic visual inspections. In some cases, geophysical and seismic research is performed. The conclusions and the reports represented on a paper are the results of these inspections. It is very difficult to analyze them for many years of the exploitation of the constructions. However, it is more difficult to diagnose the behavior of the constructions and the structures and to give the recommendations about their future exploitation. In this connection lately for such kind of objects, the systems of monitoring by methods of the system analysis of the signals obtained from the measuring instruments located on the controlled constructions were created [23]. In this case, the current state of the object is collected, stored, processed, and compared with the limit of permissible values given to the system beforehand, and the diagnosis is performed on the basis of these operations. However, in this case, monitoring the origin of microchanges caused by micro damages or micro cracks is not provided for the controlled object. Besides, it is necessary to monitor the origin of anomalous seismic processes and to provide short-term forecasting of earthquakes for the regions. In the previous section, the common principles of creating the system of seismically active regions are given. Combining the solutions to the mentioned problems in the single system has great importance for providing the safety of the population of big cities located in the seismically active area, and monitoring the technical state of the building objects in the seismically active regions is offered. The technology and the monitoring system of the origin of anomalous seismic processes to receive information about seismic processes from the deep beds of the earth and the working principles of the local systems of monitoring the technical state of the building objects by considering the noise as a data carrier are offered below. The principle of building and describing the work of the distributed hybrid intellectual city system created on their basis is also given.

The performed analysis shows that the basic reasons causing the change to the technical state of the socially important objects and the hydraulic structures in the seismically active regions are

1. the faults made in the design and in laying the foundation and the deviations from the seismic requirements during building and assembling,
2. washing out the foundation by the subterranean, rain, and waste waters,
3. the influence of the seismic and landslide processes, hurricane winds, rock vibrations caused by various construction works, the movement of transport vehicles, airplane, and helicopter flights,
4. the lack of adherence of the quality of the construction materials to the required norms and standards.

That is why regular control of the change to the technical state of socially important objects is performed in seismically active regions for the prevention of disastrous effects from the appearance of the anomalous seismic processes. The most commonly used variants of the solution to this problem are as follows:

1. if the seismic vibrations exceed the fixed threshold levels, the seismic signals are registered by means of the seismic apparatus, and the analysis of the technical states of the controlled objects is performed by means of the spectral methods;
2. the earth vibrations are created by means of the heavy load thrown down from a great height or by special detonation, and the analysis of their technical states is performed by registering and processing the signals obtained from the sensors located on the controlled objects;
3. the registration of the obtained signals is performed by the passive observation of the process of the normal exploitation during a sufficiently long period of time by means of the vibration-acoustical sensors and other apparatus. The estimates of the obtained signals are determined by use of the various methods. Then the study is performed for the various situations, and the corresponding sample sets are formed. The stage of monitoring begins after that by comparing the current estimates of the signals with the estimates of the sample sets.

But despite the certain effects of these methods, they are evidently insufficient [23]. The results of many earthquakes with huge human and pecuniary costs show that it is necessary to create new, more effective informational technologies and systems of monitoring the origin of anomalous seismic processes and the technical states of the objects, allowing one to receive the following information regularly:

1. It is necessary to perform short-term forecasting for a time that would be enough for people to leave their homes after receiving seismic information from the deep beds of the earth.
2. It is necessary to receive information about the technical state of the controlled objects.
3. The information obtained from the results of monitoring must allow one to perform a comparative analysis of the technical states of the various objects and determine the groups of constructions situated in the most unfortunate state.
4. It is necessary to give the opportunity for the synchronous detection of the change to the technical state of the group of closely located objects by the result of monitoring for signaling about the process of a landslide's origin.
5. It is necessary to give the opportunity of the synchronous detection of the change to the technical state of many objects located in the different city districts by the results of monitoring for signaling about the anomalous seismic process's origin.

In Fig. 7.4, block 1 consists of the vibration-acoustic sensor and the sound sensor established on the well head 2 at the depth of 3–6 km. Block 3 is the standard seismic apparatus that allows one to estimate the strength of the seismic vibrations exceeding the threshold level. Block 4 monitors the anomalous seismic processes and performs short-term forecasting of earthquakes. The temporarily closed oil wells 2 remaining unused after the depletion of oil formation are used as the communication channel for receiving information from the deep (3–6 km) seismic processes.

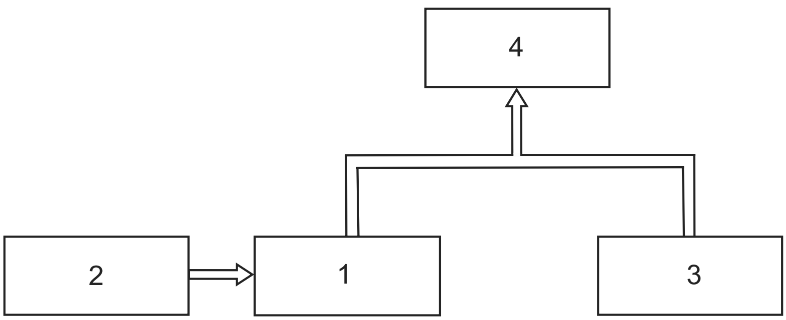

Fig. 7.4. The block scheme of the system SP receiving the seismic information from the deep beds of the earth and short-term forecasting of the earthquakes.

The statistic estimates of the variance D_g of the noisy signal $g(i\Delta t)$, the estimates of the variance D_ε of the noises $\varepsilon(i\Delta t)$, the mutual correlation functions $R_{x\varepsilon}(\mu)$ between the useful signals $X(i\Delta t)$ and the noises $\varepsilon(i\Delta t)$, the estimates of the averages of distribution m_x of the useful signals $X(i\Delta t)$, and the estimates of the variances D_x of the useful signals $X(i\Delta t)$ calculated by means of the algorithms described in Chapters 2–5 are used in the system SP as the information indications about the origin of anomalous seismic processes in the deep beds of the earth.

The position-binary technology can also be used in the system of monitoring the origin of seismic processes and short-term forecasting of earthquakes [14, 19, 20, 22, 23]. The corresponding changes to the combinations of the average time intervals of the position signals take place at the beginning of the change to the technical state of the object. They can be used as the informative indications during monitoring. Besides, the position noises $q_{\varepsilon k}(i\Delta t)$ of the noisy signal $g(i\Delta t)$ are generated as short-term impulses under the influence of the dynamics of the origin of factors causing these changes when each position signal $q_k(i\Delta t)$ is formed. It has been shown [14, 19, 20, 22, 23] that if the technical state of the object is stable, the coefficients K_{q_0}, K_{q_1}, …, $K_{q_{m-1}}$ of the ratio of the quantities $N_{\varepsilon k}$ of the signals $q_{\varepsilon k}(i\Delta t)$ to the total quantities N_{qk} of the position-impulse signals $q_k(i\Delta t)$ for time T,

$$K_{q_0} = \frac{N_{\varepsilon_0}}{N_{q_0 k}},\ K_{q_1} = \frac{N_{\varepsilon_1}}{N_{q_1 k}},\ \ldots,\ K_{q_{m-1}} = \frac{N_{\varepsilon(m-1)}}{N_{q(m-1)k}}, \tag{7.6}$$

are non-random values. These coefficients change during the micro-changes to the state of the controlled object and also during the appearance of anomalous seismic processes.

The training mode is performed in the initial stage in block 1 of the system SP. For this purpose, the estimates of the sound and vibration signals D_g, D_ε, $D_{x\varepsilon}$, m_x, D_x, T_k, T_{k1}, T_{k0}, K_{q_0}, K_{q_1}, …, $K_{q_{m-1}}$, obtained from the corresponding sensors of block 1, are determined by the corresponding technologies. The corresponding sample sets are formed and recorded in block 4 in the initial stage of the work when the anomalous seismic processes are absent for a long period of time. Later the obtained current estimates are compared with these sample sets during the work of the system in each cycle for time T by the results of the analysis. If the difference does not exceed the given range, it is assumed that they do not differ from the samples and their quantity is recorded. In this case, the seismic processes are also considered as the normal ones in the ground-based seismic apparatus 3 as long as they do not exceed the given threshold level. The system turns to the next cycle after time T. This process goes on until the current estimates of the signals obtained from the corresponding sensors differ from the values of the estimates of the sample sets by values greater than the given range. Time is the recorder in this case. If the current estimates also differ from the estimates of the sample set by values greater than the given range in the following cycles, it is noted in block 4 in the beginning of the seismic processes. At the same time, the corresponding information is transmitted to the server S of the central system. So the information about the origin of anomalous seismic processes is formed in the system during the reaction of the sound and vibrations sensors located on the steel shaft collar of the oil well passing at a depth of 3–6 km. Also, the reaction of the standard ground-based apparatus 3 at the appearance of anomalous seismic processes is sufficiently later because the mechanical spread of the seismic vibrations from the depth of 3–6 km until the ground surface requires sufficiently longer time. At the same time, the strength of the seismic vibration is determined in apparatus 3 by the Richter scale. The information about it is transmitted to block 4, where the difference between the times of the registration of the origin of the anomalous seismic processes is also determined. It can be assumed that their difference is the short-term forecasting time. Besides, in block 4 the approximate value of the strength of the forecasted earthquake is set by means of self-training, by the value of the difference between the sample sets and the current sets at the moment of the detection of the anomalous seismic processes, based on the strength of the seismic vibration obtained in block 3. If periodical low-powered seismic vibrations appear in the process of the exploitation, the system is adapted to the identification of these vibrations in the process of self-training. When a certain time has passed, the system will be able to determine the approximate value of the strength and the time of the possible earthquake by the value of the difference between the sample and the current sets and also by the difference of the reaction times in blocks 1 and 3.

Due to the intellectualization, the system SP has the possibility to give warnings of strong earthquake vibrations. In this case, the time of forestalling depends on two factors. First, during the origin of anomalous seismic processes, their influence is transmitted by the steel pipe of the oil well from a depth of 5–6 km with the sound speed and is picked up at first by means of block 1 by the sound sensor and some time later by the vibration sensor. At the same time, the mechanical vibrations of the seismic processes reach the ground surface with many times lesser speed, which explains why they are registered by the sensors of the ground-based apparatus sufficiently later. Second, the use the technology of considering the noise as a data carrier and the position-binary technology allows one to detect the origin of anomalous seismic processes that usually precede the strong destructive seismic vibrations by a sufficiently long time. Due to these factors, receiving information from the system SP gives the opportunity to warn the population of the city about an earthquake in enough time so that people have time to leave their homes.

Figure 7.4 represents the block scheme of the intellectual hybrid system consisting of the local system's "black boxes" besides blocks 1–3 of the system represented in Fig. 7.3. It consists of the sensors D_1, D_2, D_3, …, D_m, interface 1 with the multichannel analog-code converter and serial controllers 2 and 3. The quantity of the sensors is determined by the requirements arising from the specificity of monitoring the object, and they are located in its most vulnerable places.

The principles of the creation of the systems of Figs. 7.2 and 7.4 are similar.

In the self-training mode for each cycle, the estimates of the characteristics of the noise of the signals obtained from the sensors D_1, D_2, D_3, …, D_m by the corresponding algorithms are determined in block 2 while the system works. The sample sets are formed from the calculated estimates, and the permissible ranges of their deviations are also determined in the normal exploitation of the object.

After that the system passes into the monitoring stage by using the above-mentioned technology to detect the origin of the change to the technical state of an object by considering the noises as a data carrier. At the same time, for example, for high-rise buildings, the current estimates of the analyzed signals are determined both in the process of their exploitation in each cycle and in the self-training stage. Then the ranges of the deviations of the found current estimates from the sample are determined. The current deviations of the mentioned estimates are compared with the sample ranges of the corresponding estimates in each cycle in the end of this process for each signal $g_1(i\Delta t)$, $g_2(i\Delta t)$, …, $g_j(i\Delta t)$, …, $g_m(i\Delta t)$. If they do not exceed the sample sets created as the result of self-training, it is assumed that there is no defect in the controlled areas corresponding to the number of the sensor of the analyzed signal. Otherwise, even if one of the estimates turns out to be more or less than the corresponding sample range, the time of the cycle's beginning is recorded in the process of a defect's origin in the corresponding areas of the controlled object.

During the creation of the system (Fig. 7.5), each of the socially important objects O_1, O_2, …, O_m is provided with the local system L_{11}, L_{12}, …, L_{1m}, L_{21}, L_{22}, …, L_{2m}, L_{N1}, L_{N2}, …, L_{Nm} of monitoring by considering the noise as a data carrier by the scheme represented in Fig. 7.2.

The signals $g_1(i\Delta x)$, $g_2(i\Delta t)$, ..., $g_m(i\Delta t)$ obtained from each object are analyzed in the controller of the local system, and the obtained results are transmitted to the modem of the server S of the central system by the modems and radio network devices as the result of the work of the local system. The results of monitoring and short-term forecasting of the earthquake are also transmitted to the server of the central system from the system SP.

The technologies presented in Chapters 2–5 are used for the solution of the problem of monitoring the technical state of the controlled objects and short-term forecasting of anomalous seismic processes and for the system analysis of the measured information in the server S of the city system.

Besides, the bar charts of the distributions $W[\varepsilon(i\Delta t)]$ of the noises $\varepsilon_j(i\Delta t)$ and the values of their possible deviations are determined for each signal. For that purpose, the curve of the required bar chart is easily made for time $T = N\Delta t$ by the approximate values of the samples

$$\dot{\varepsilon}^*(i\Delta t) = \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\varepsilon'(i\Delta t)}\,,$$

given for the range from 0 through $\varepsilon_{\max}$ by the equal segments $\Delta\varepsilon$ by the quantity of the samples N_1, N_2, ..., N_m. When the correlation between the useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ is nonzero, the bar chart of the distribution law of the noise is made by the quantity of the samples N_1, N_2, ..., N_m of the values

$$\dot{\varepsilon}^*(i\Delta t) = \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]}\,,$$

located in the corresponding ranges $\varepsilon(i\Delta t)$. The calculated estimates N_1, N_2, ..., N_m of the bar chart tend to the required values of the distribution law of the noise $W[\varepsilon(i\Delta t)]$ with an increase of $N \to \infty$. The bar charts are also recorded as the sample informative indications.

The system works in four modes. The self-training is performed in the initial stage while the system works. At the same time, the corresponding estimates are determined by expressions (7.23)–(7.35), by means of analyzing the signals $g_j(i\Delta t)$ and their noises $\varepsilon_j(i\Delta t)$ obtained from the corresponding sensors of the objects. They are recorded as the sample values. Also, the ranges of their minimal, average, and maximum deviations are also determined.

After the self-training stage, the system passes into the monitoring stage. The values of the current estimates, $R_{gg}(\mu)$, $R^R_{gg}(\mu)$, a_{n_g}, b_{n_g}, a_{n_ε}, b_{n_ε}, a_{n_x}, b_{n_x}, $r_{g\varepsilon}$, and the bar chart of the noise $W[\varepsilon(i\Delta t)]$ are determined during the work of the system in the first mode. They are

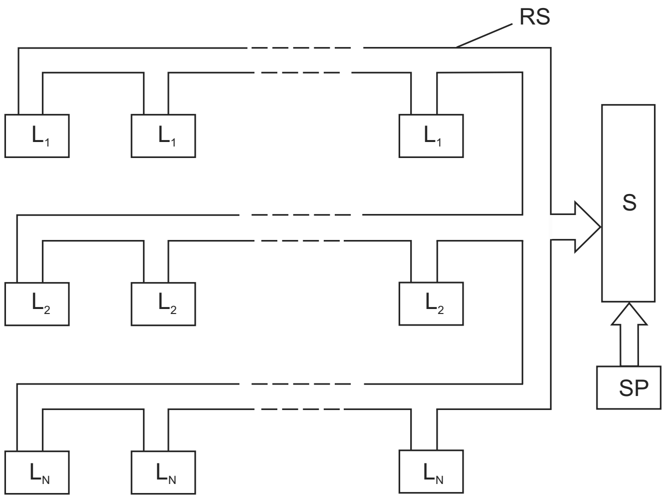

Fig. 7.5. The intellectual hybrid system of monitoring anomalous seismic processes and the technical state of the objects by considering the noise as a data carrier.

compared with the corresponding sample values and recorded during the self-training. If their difference during this does not exceed the accepted minimum ranges, it is assumed that the technical state of the corresponding object O_1, O_2, …, O_m did not change. Otherwise, the received differences are used for making the decisions and forming the signal showing the beginning of the change to the technical state of the corresponding object. At the same time, the importance level of the arisen situation is determined by the value of the difference of the deviation range.

In the second mode, in contrast with the first one, the signaling about the landslide's origin is only formed during synchronous detection of the deviation exceeding the minimal range of the listed estimates of the signals obtained from the closely located groups of objects.

The server S of the system turns into the third mode during the synchronous registration of the deviations of the current estimates above the minimal range from several groups of the objects located far apart. The identification of the obtained results is simultaneously performed by all used technologies. The corresponding sample sets are formed by the obtained estimates of the signals $g_j(i\Delta t)$ as well as in the first and second modes in the initial stage, similarly to what was described earlier, in the self-training process. Their difference from the sample situations is recorded as the alarm state. At the same time, the level of the danger of the seismic situation is estimated by the quantity of the groups, especially by the value of the ranges of the deviations. When the values of the deviations exceed the maximum threshold levels, the server signals about the precarious situation of the origin of anomalous processes.

At last the system passes into the fourth mode by receiving the alarming information from SP. At the same time, the system works similarly as in the third mode. Besides, the identification of the seismic situation is simultaneously performed in the server S and in the system SP to increase the reliability of the final results. All technologies of processing and analyzing the signals are also used for that purpose.

The control of breaking the rules of accident prevention on the objects O_1, O_2, ..., O_m is performed both in the local systems (black boxes) L_{11}, L_{12}, ..., L_{N1}, L_{N2}, ..., L_{Nm} and in the server *S*. For example, during a breakdown of an elevator, a gas escape, a short-circuit in the power supply, a conflagration, etc., the corresponding signal specifying the nature of the failure and the number of the object is formed both in the objects of monitoring and in the corresponding services.

The technology used in the system, the technology of receiving the information from the deep beds of the earth by means of the "seismic informative oil well" in combination with monitoring by considering the noise as a data carrier, gives the opportunity to perform short-term forecasting of anomalous seismic processes and an earthquake's origin. The system has a higher sensitivity to seismic processes, due to the measured information it receives from the black boxes of all high-rise buildings located far apart. It also allows one to combine the monitoring of the technical state of the building with the alerts about all possible emergency states. Due to these features, it is possible to perform the series of important actions for preventing the unexpected, fatal, or otherwise disastrous processes by the obtained results of the work of the considered system. So the stems of the temporarily closed oil wells of the depleted oil fields are used as the "phonendoscope" in the offered system for receiving sound signals from the deep beds of the earth. The informative indications of the anomalous seismic processes registered by the standard ground-based seismic apparatus some time later are determined by analysis of the noise. They are compared, and the intellectual technology of their identification is formed by means of self-training. This process goes on during the exploitation of the system, and the necessary degree of adequacy of forecasting the strength and time of the dangerous seismic vibrations is provided. The offered system can be used for the following purposes:

1. To determine the origin of anomalous seismic processes and the level of the danger of the forecasted earthquake and, if necessary, to warn the corresponding services of the state as the result of the use of the

technology of considering the noise as a data carrier and the position-binary technology by the obtained signals from the "seismic informative oil well."
2. To give the information to the corresponding city services about the landslide or the origin of anomalous seismic processes during the synchronous signaling from the black boxes about the changes of the technical state of many socially important objects.
3. To change the direction of the subterranean waters changing the technical state of the object by washing out the foundation.
4. To stop the leak of the rain and waste waters under the foundation, to remove the reasons for worsening the technical state of the object by means of the major repairs of the asphalt coats, the hatches, and the sewerage system.
5. To stop the evolution of landslide processes by building special anti-landslide installations.
6. To minimize the vibration processes leading to microchanges to the technical state of the controlled object by changing the route of moving the heavy carriers.

7.6 The Technology of Monitoring the Origin of Vascular Pathology of the Human Organism

Vascular pathology takes a premier place among all diseases of the human organism. The vessels affected occur in various anatomical levels and cover various strata of society and turn into a problem of first and foremost importance. The difficulty of solving this problem is redoubled by the imperfection of the methods of diagnosing the affected vessels in the early stages, when the affected vessels reach a considerable degree of problems, but medicine does not have methods to diagnose subclinical forms during a prophylactic inspection.

One of the possible variants of the data interpretation of the reovasographical research allowing one to solve the problem fairly is considered below.

The character of the method is the use of modern diagnostic criteria in clinical practice, allowing one to monitor ischemic diseases at their origin.

The biophysical fundamentals of the method are the following. Abstracting the process of the blood filling, let us consider the model of the vessel of abnormal patency, assuming that the blood vessel is the rigid tube, the abnormality of the patency of which is imitated by the local hemodynamic resistance. Let us denote as S_0 the cross-sectional area of the vessel in the area of the local resistance, and as S_1 the unaffected districts. It is clear that in practice the form and the geometry of the local resistances can vary, but they are mostly adequate from the viewpoint of the hydrodynamics.

The Reynolds number Re characterizing the blood flow in the vessel must be selected in the supposition that the blood is the non-Newtonian liquid, for which one has to use the values of the effective (seeming) viscosity. At the same time, in the modern literature devoted to the problems of the hemodynamics, the value of the number Re corresponding to the effectiveness of the blood viscosity was not discovered by the authors. So it is quite difficult to select the number Re uniquely for the offered model. Taking into account everything mentioned above and also that the flow in the vessels is laminar, we assume that $\mathrm{Re} < 25$ in the considered model.

Our research has shown that in this case arterial abnormalities cause a decrease in the consumption of the blood by the following value:

$$\frac{S_1}{S_0}\sqrt{1+\frac{25.2\sqrt{\dfrac{S_0}{S_1}}}{\mathrm{Re}}} > 1\,.$$

Let us note that the influence of the postarterial parts is not taken into account during the representation of the mentioned dependence. Actually, the capillaries and the veins exert a significant influence on the nature of the blood flow in the arteries. Therefore, the dependence determines only the primary factors affecting the consumption of the blood in the affected vessels.

That is why on the output of the sensor the value of the reovasosignal containing information about the state of all parts of the bloodstream depends on the nature of the blood-filling process, i.e., on the influence of the factor S_0 on the volumetric blood flow. In other words, the arterial inflow of the affected vessels is the function of the lumen area S_0 of the vessel mapping the local hemodynamic resistance.

The experimental research for a great number of patients has shown that the arterial inflow and the venous outflow really decrease during such functional disorders. It leads not only to a change in the amplitude of the signal but to the presence of the time shift between the peaks of the signals, corresponding to the affected and unaffected vessels. In addition, the higher the degree of the abnormalities, the greater the time shift is between the peaks. Thus, monitoring the origin of the vascular pathology of a human organism can be reduced to determining the time shift between the reovasosignals obtained from the symmetrical extremities of a human. Figure 7.6 shows the technology of the diagnostics of the functional state of the vessels in the extremities (7.9) by the delay time $\Delta\tau$ between the signals $X(t)$ and $Y(t)$, taken by two pairs of electrodes from the corresponding points of the symmetrical parts of the patient's extremities. The cross-correlation function $R(\mu\Delta t)$ of the peak by time is determined by the obtained reovasosignals. The degree of the disease in a patient's vessels is determined by the delay. The procedure is performed in several reverse cycles to get a well-defined estimate, and $t_{\min}$ (the minimal value of t) is selected from the obtained results.

The block scheme of the system performing the offered method is represented in Fig. 7.6. The registration of the reovasosignals obtained from the corresponding points situated on the symmetrical parts of the patient's extremities is performed with the sensors for the realization of the block scheme. The signals are obtained by two pairs of the conducting electrodes: the first pair is used for transmitting the actuating signal from the high-frequency generator, and the second is used for measuring the received reovasosignals. The obtained reovasosignals are amplified and converted into the digital codes, and the samples $X(i\Delta t)$ and $Y(i\Delta t)$ are transmitted to the input of the processing apparatus (a computer).

Experimental research has shown that the known classical conditions, especially the normality of the distribution law and the absence of correlation between the useful signal and the noise, are quite often broken in the realizations of the reovasosignals.

The robust technology of the analysis of the signals is used by the expression

$$R_{xy}(\mu\Delta t) = \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)Y((i+\mu)\Delta t) - \left[n^{+}(\mu) - n^{-}(\mu)\right]\Delta\lambda(\mu = 1).$$

except for the microerrors of the products $X(i\Delta t)Y((i+\mu)\Delta t)$ when the estimates of the cross-correlation functions between $X(i\Delta t)$ and $Y((i+\mu)\Delta t)$ are calculated.

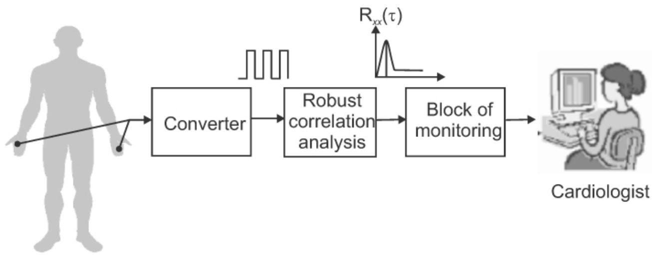

Fig. 7.6. The system of monitoring the origin of ischemic diseases.

Due to that, the influence of the noise on the obtained results is removed, greatly increasing the accuracy of the results. Monitoring the origin of ischemic diseases of the cardiovascular system is provided.

The estimates of the correlation functions $R_{xy}(\mu = 0\Delta t)$, $R_{xy}(\mu = 1\Delta t)$, $R_{xy}(\mu = 2\Delta t)$, …, $R_{xy}(\mu = m\Delta t)$, showing the change to the value of these estimates depending on time $\mu \cdot \Delta t$, are received as the result of processing and analyzing the reovasosignals for $\mu = 0, 1, 2, 3, \ldots$. If the patient is healthy, $R_{xy}(\mu = 0)$ takes its maximal value. Otherwise, the higher μ is, during which $R_{xy}(\mu = 0)$ takes the maximal value, the more dangerous a patient's state is.

The following estimates:

$$R_{xy}(0) = r_{XY}^* \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \Delta\dot{X}(i\Delta t) \operatorname{sgn} \Delta\dot{Y}(i\Delta t),$$

$$r_{X\varepsilon}^* \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \Delta\dot{X}(i\Delta t) \operatorname{sgn} \dot{X}(i\Delta t), \tag{7.7}$$

$$D_\varepsilon = \frac{1}{N}\sum_{i=1}^{N} \dot{X}(i\Delta t)\left[\dot{X}(i\Delta t) + \dot{X}((i+2)\Delta t) - 2\dot{X}(i\Delta t)\right], \tag{7.8}$$

$$K_\varepsilon = \frac{D_\varepsilon}{D_X}, \tag{7.9}$$

can be used for monitoring the ischemic diseases during a mass clinical examination of the population.

The conditions $r_{xy}^* \approx 1$, $r_{x\varepsilon}^* \approx 0$, $D_\varepsilon \approx \Delta\varepsilon$, $K_\varepsilon \approx 0$ take place if the patient is healthful. Otherwise, he belongs to the risk group. Due to the fact that diseases of the vessels of the extremities, especially the lower extremities, are a difficult problem of both general and vascular surgery, in practice, the given method, which does not require expensive equipment for application, can be used for a mass clinical prophylactic examination of the population.

7.7 Correlation Indicators of a Defect's Origin

The digital technologies of robust monitoring of a defect's origin are described in Sections 5.1–5.5. Their analysis shows that, in solving the problem of monitoring the defect's origin, the technology of calculation of the robustness value is of extremely great importance. The performed experimental investigations showed that in most cases such classical conditions as normality of the distribution law, stationarity, etc. do not take place for the signals received as the output of the corresponding sensors when the defect appears. Due to this, the unknown estimates are determined with great errors when using traditional technologies. Thus, the given data do not allow one to detect the origin of the defect. This disadvantage can be eliminated by means of the robustness value. Its application allows one to determine the unknown estimates with sufficient accuracy.

Taking into account everything mentioned so far, the specific characteristics of the robustness value and the results of calculating experiments showing the influence of the microerror on the products that appeared in the process of traditional processing of the obtained results are now considered in more detail.

Signal and noise databases were created for this purpose. They include

1. a database of useful signals in the form of sinusoidal curves and a sum of sinusoidal and cosinusoidal curves with variable frequencies generated with different digitization steps,
2. a database of simulated physical random useful signals with different distribution laws obtained as a result of the interpolation of a decimated random sequence,
3. a database of different types of noise with different distribution laws obtained by means of a random number generator,
4. a database of simulated physical random noise with different distribution laws obtained as a result of interpolation of a decimated random sequence,
5. a database of correlated simulated random useful signals and noise,
6. a database of simulated noisy signals consisting of a sum of different useful signals and different types of noise with different distribution laws and different "useful signal-to-noise" ratios.

These signals simulating the physical random noisy signals obtained from the sensors of the technological processes were created to perform the computational experiments, to certify the reliability of the proposed methods of analysis of signals and noises [12, 14, 60], and, in particular, to check the efficiency of the method used to calculate the variance of the noise in the noisy signal.

In addition, simulated signals are used to carry out computational experiments designed to verify the efficiency of methods of computing other characteristics of noisy signals that have not been previously subjected to extensive study, in particular, for the case in which classical conditions are never satisfied.

The following aims were set in creating the software and the system of experimental research:

1. the development of theoretical bases for calculating the mean value of the microerror of products obtained with the use of the traditional technology of computing estimators of auto- and cross-correlation functions and analysis of the properties of the methodology;
2. the development of a methodology for computing an estimator of the magnitude of the robustness of a noisy signal;
3. the creation of a program, the performance of a computational experiment, and the identification of features of the robustness factor as a new characteristic of a random noisy signal with different violations of the classical conditions.

Later in this chapter we will analyze the basic factors that make it possible to identify the mean value of a microerror with different violations of the classical conditions and present algorithms for computing the robustness factor as a new characteristic of a noisy signal [12, 14, 60].

It is known that, in actual practice, when computing estimators of auto- and cross-correlation functions, it is usually supposed that the following classical conditions hold:

1. the realizations of a noisy signal $g(t)$ and of a useful signal $X(t)$ are random stationary functions with normal distribution law;
2. the noise $\varepsilon(t)$ obeys a normal distribution law with mathematical expectation $m_\varepsilon \approx 0$;
3. the correlation between the useful signal $X(t)$ and the noise $\varepsilon(t)$ is absent, that is, $m_{x\varepsilon} \approx 0$, and so on.

As a consequence, in practice, the following equalities [12, 14, 60] must hold when computing estimators of the auto- and cross-correlation functions of a sampled noisy signal $g(i\Delta t)$, useful signal $X(i\Delta t)$, and noise $\varepsilon(i\Delta t)$ when using traditional algorithms:

$$R_{xx}(\mu) = R'_{xx}(\mu), \tag{7.10}$$

$$R_{gg}(\mu) = R'_{gg}(\mu), \tag{7.11}$$

$$R_{xx}(\mu) = R_{gg}(\mu), \tag{7.12}$$

$$R'_{xx}(\mu) = R'_{gg}(\mu), \tag{7.13}$$

$$R_{x\varepsilon}(\mu) \approx R'_{x\varepsilon}(\mu) \approx 0\,, \tag{7.14}$$

$$R_{\varepsilon x}(\mu) \approx R'_{\varepsilon x}(\mu) \approx 0\,, \tag{7.15}$$

where

$$R_{xx}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \dot{X}(i\Delta t)\,\dot{X}\big((i+\mu)\Delta t\big), \tag{7.16}$$

$$R'_{xx}(\mu) = \frac{1}{N}\sum_{i=1}^{N} X(i\Delta t)X\big((i+\mu)\Delta t\big) - m_x^2\,, \tag{7.17}$$

$$R_{gg}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \dot{g}(i\Delta t)\,\dot{g}\big((i+\mu)\Delta t\big), \tag{7.18}$$

$$R'_{gg}(\mu) = \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)g\big((i+\mu)\Delta t\big) - m_g^2\,, \tag{7.19}$$

$$R_{x\varepsilon}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \dot{X}(i\Delta t)\,\dot{\varepsilon}\big((i+\mu)\Delta t\big), \tag{7.20}$$

$$R'_{x\varepsilon}(\mu) = \frac{1}{N}\sum_{i=1}^{N} x(i\Delta t)\varepsilon\big((i+\mu)\Delta t\big) - m_x m_\varepsilon\,, \tag{7.21}$$

$$R_{\varepsilon x}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \dot{\varepsilon}(i\Delta t)\,\dot{X}\big((i+\mu)\Delta t\big), \tag{7.22}$$

$$R'_{\varepsilon x}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)X\big((i+\mu)\Delta t\big) - m_\varepsilon m_x\,, \tag{7.23}$$

$$\dot{g}(i\Delta t) = g(i\Delta t) - m_g\,,$$

$$\dot{X}(i\Delta t) = X(i\Delta t) - m_x\,,$$

$$\dot{\varepsilon}(i\Delta t) = \varepsilon(i\Delta t) - m_\varepsilon\,,$$

where m_g, m_x, m_ε are the mathematical expectations $g(i\Delta t)$, $X(i\Delta t)$, $\varepsilon(i\Delta t)$.

However, in practice, the classical conditions are not satisfied for a definite class of physical objects. This means that

1. stationarity is not satisfied in realizations of a noisy signal $g(i\Delta t)$ and of a useful signal $X(i\Delta t)$, and the distribution laws of realizations of a noisy signal $g(i\Delta t)$, useful signal $X(i\Delta t)$, and noise $\varepsilon(i\Delta t)$ often turn out to differ from a normal distribution;
2. the mathematical expectation of the noise is nonzero, that is,

$$m_\varepsilon \neq 0; \tag{7.24}$$

3. the correlation between the useful signal and the noise is nonzero, that is,

$$r_{x\varepsilon} \neq 0. \tag{7.25}$$

As a consequence, the estimators $R_{gg}(\mu)$ are obtained in practical applications with noticeable errors $\lambda(\mu)$ and the following relations [12, 14] hold:

$$R'_{gg}(\mu) - R'_{xx}(\mu) = \lambda_1(\mu), \tag{7.26}$$

$$R_{gg}(\mu) - R_{xx}(\mu) = \lambda_2(\mu), \tag{7.27}$$

$$R'_{gg}(\mu = 0) - R'_{xx}(\mu = 0) = \lambda_1(\mu = 0) = \lambda_2(\mu = 0), \tag{7.28}$$

$$\left|\lambda_1(\mu \neq 0) - \lambda_2(\mu \neq 0)\right| = \lambda(\mu \neq 0), \tag{7.29}$$

$$\left|R'_{gg}(\mu) - R_{gg}(\mu)\right| = \lambda(\mu). \tag{7.30}$$

Moreover, it should be noted that it is precisely as a result of applying (7.26)–(7.30) that it becomes possible to establish the mean value of the microerror of the products $\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)$ [1, 13–14, 58]:

$$\Delta\lambda(\mu = \Delta t) = \lambda(\mu = \Delta t) / N_\circ^-(\mu = \Delta t), \tag{7.31}$$

where $N_\circ^-(\mu = \Delta t)$ is the number of negative products $\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)$ if $\mu = \Delta t$.

Thus, if the classical conditions are violated, a microerror in the products arises that may then be used to calculate the estimator, called the robustness factor. With different time shifts μ it may be calculated by the formula

$$\Lambda^{R}(\mu)=\left[N_{\circ}^{+}(\mu)-N_{\circ}^{-}(\mu)\right]\Delta\lambda(\mu=\Delta t), \tag{7.32}$$

where $N_{\circ}^{+}(\mu)$ and $N_{\circ}^{-}(\mu)$ are the number of positive, respectively negative, products $\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)$ with time shift μ.

One can easily be convinced that upon monitoring of in-time failure origin, the robustness magnitude may be used as a new estimate of statistical characteristics of a random noisy signal when the classical conditions are broken. It is for this reason, for practical application of this technology for real conditions, when the classical conditions are broken that detailed investigations into the properties of the average value of the microerror $\Delta\lambda(\mu=\Delta t)$ and robustness magnitude $\Lambda^{R}(\mu)$ were made. Qualitative computational experiments using databases of simulated signals, verification that conditions (7.7)–(7.15) are satisfied, and analysis of the results obtained in accordance with Eqs. (7.26)–(7.30) for different violations of the classical conditions are needed.

The Matlab computer mathematics software tool was used to conduct these experimental investigations. The computational experiments were performed in the following way. A useful signal $X(i\Delta t)$ and noise $\varepsilon(i\Delta t)$ with specified characteristics were selected from the database of signals and a noisy signal $g(i\Delta t)=X(i\Delta t)+\varepsilon(i\Delta t)$ was formed. The following were then calculated:

1. the autocorrelation functions $R_{xx}(\mu)$ and $R'_{xx}(\mu)$ of the centered $\dot{X}(i\Delta t)$, respectively noncentered, useful signal $X(i\Delta t)$ using formulae (7.16) and (7.17);
2. the autocorrelation functions $R_{gg}(\mu)$ and $R'_{gg}(\mu)$ of the centered $\dot{g}(i\Delta t)$, respectively noncentered, noisy signal $g(i\Delta t)$ using formulae (7.18) and (7.19);
3. the cross-correlation functions $R_{x\varepsilon}(\mu)$ and $R'_{x\varepsilon}(\mu)$ between the centered useful signal $\dot{X}(i\Delta t)$ and noise $\dot{\varepsilon}(i\Delta t)$, respectively between the noncentered useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$, using formulae (7.20) and (7.21);
4. the cross-correlation functions $R_{\varepsilon x}(\mu)$ and $R'_{\varepsilon x}(\mu)$ between the centered noise $\dot{\varepsilon}(i\Delta t)$ and the useful signal $\dot{X}(i\Delta t)$, respectively and between the noncentered noise $\varepsilon(i\Delta t)$ and the useful signal $X(i\Delta t)$, using formulae (7.22) and (7.23);

5. the autocorrelation function $R_{\varepsilon\varepsilon}(\mu)$ of the centered noise $\dot{\varepsilon}(i\Delta t)$ and autocorrelation function $R'_{\varepsilon\varepsilon}(\mu)$ of the noncentered noise $\varepsilon(i\Delta t)$ according to the formulae

$$R_{\varepsilon\varepsilon}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \dot{\varepsilon}(i\Delta t)\,\dot{\varepsilon}((i+\mu)\Delta t), \tag{7.33}$$

$$R'_{\varepsilon\varepsilon}(\mu) = \frac{1}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\varepsilon((i+\mu)\Delta t) - m_{\varepsilon}^{2}; \tag{7.34}$$

6. the detected part of the error $\lambda(\mu = \Delta t)$ using formula (7.30);
7. the mean value of the microerror $\Delta\lambda(\mu = \Delta t)$ using formula (7.31);
8. the robustness factor $\Lambda^{R}(\mu)$ using formula (7.32);
9. the true magnitude of error

$$\Lambda(\mu) = R_{gg}(\mu = \Delta t) - R_{xx}(\mu = \Delta t); \tag{7.35}$$

10. the correlation coefficient between the useful signal and the noise $r_{s\varepsilon}$;
11. the variance of noise according to the traditional algorithm

$$D(\varepsilon) = (1/N)\sum_{i=1}^{N}\left[\dot{\varepsilon}(i\Delta t)\right]^{2}; \tag{7.36}$$

12. the variance of noise according to the nontraditional algorithm [1, 13–14, 58]

$$D^{*}(\varepsilon) = R_{gg}(0) - 2R_{gg}(\mu = 1\Delta t) + R_{gg}(\mu = 2\Delta t). \tag{7.37}$$

Four variant experiments were performed to calculate the robustness factor and analyze its properties.

First variant experiment. The useful signal is created in the form of a sinusoidal curve $X(i\Delta t) = 40\sin(i\Delta t) + 100$. The noise obeys a normal distribution law with mathematical expectation $m_{\varepsilon} \approx 0$ and standard deviation $\sigma_{\varepsilon} \approx 10$. Thus, an "ideal" useful signal and "ideal" noise have been created that satisfy virtually all the above classical conditions.

The following law manifests itself in all computational experiments of the first type that were carried out:

1. condition (7.10) is satisfied for all values of the time shift $\mu = 0$, Δt, $2\Delta t$, ...; that is, the estimators of the correlation function of the centered $\dot{X}(i\Delta t)$ and the noncentered $X(i\Delta t)$ useful signals,

represented in the form of a sinusoidal curve, coincide; this means that the useful signal satisfies the classical conditions;

2. conditions (7.14) and (7.15) are not satisfied; that is, even when the correlation between the useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ is absent, the values of the estimators of the cross-correlation functions $R_{x\varepsilon}(\mu)$, $R'_{x\varepsilon}(\mu)$, $R_{\varepsilon x}(\mu)$, and $R'_{\varepsilon x}(\mu)$ between the useful signal and the noise are nonzero, that is,

$$R_{x\varepsilon}(\mu)\neq 0,\ R'_{x\varepsilon}(\mu)\neq 0,\ R_{\varepsilon x}(\mu)\neq 0,\ R'_{\varepsilon x}(\mu)\neq 0; \tag{7.38}$$

this means that the presence of even negligible correlation $r_{x\varepsilon}\approx 0$, which according to the classical conditions is negligible, leads to noticeable values of the quantities $R_{x\varepsilon}(\mu)$, $R'_{x\varepsilon}(\mu)$, $R_{\varepsilon x}(\mu)$, and $R'_{\varepsilon x}(\mu)$;

3. conditions (7.12) and (7.13) are not satisfied; that is, the estimators of the correlation functions $R_{gg}(\mu)$ and $R'_{gg}(\mu)$ of the noisy signal $g(i\Delta t)$ are not equal to the estimators of the correlation $R_{xx}(\mu)$ and $R'_{xx}(\mu)$ functions of the useful signal $X(i\Delta t)$; this means that even if the classical conditions are satisfied, the presence of negligible correlation together with the fact that the mathematical expectation of the noise is approximately equal to zero are causes for the appearance of perceptible errors $\lambda_1(\mu)$ and $\lambda_2(\mu)$;
4. condition (7.11) is satisfied only if $\mu=0$ and is not satisfied if $\mu\neq 0$; that is, the estimators of the correlation functions $R_{gg}(\mu)$ and $R'_{gg}(\mu)$ of the noncentered $g(i\Delta t)$, respectively centered $\dot{g}(i\Delta t)$, noisy signal are not equal to $\mu\neq 0$; this means that the errors $\lambda_1(\mu)$ and $\lambda_2(\mu)$ similarly are not equal; hence, it is necessary to compute the mean value of the microerror $\Delta\lambda(\mu=\Delta t)$, and the robustness factor $\Lambda^R(\mu)$ is evident [60].

Second variant experiment. The useful signal is created in the form of a sinusoidal curve $X(i\Delta t)=40\sin(i\Delta t)+100$. The noise obeys an exponential distribution law with mathematical expectation $m_\varepsilon\approx 5$ and standard deviation $\sigma_\varepsilon\approx 5$. For this type of experiment, the classical condition, that the noise obey a normal distribution law and have mathematical expectation $m_\varepsilon\approx 0$, is also violated.

The following law manifests itself in all the computational experiments of the second type that were carried out:

1. condition (7.10) is satisfied for all values of the time shift $\mu=0$, Δt, $2\Delta t$, …; that is, the estimators of the correlation function of the centered $\dot{X}(i\Delta t)$ and the noncentered $X(i\Delta t)$ useful signals,

represented in the form of a sinusoidal curve, coincide; this means that the useful signal satisfies the classical conditions;

2. conditions (7.14) and (7.15) are not satisfied; that is, when the correlation between the useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ is absent, though at the same time the mathematical expectation of the noise is nonzero, $m_\varepsilon \neq 0$, the estimators of the cross-correlation functions $R_{x\varepsilon}(\mu)$, $R'_{x\varepsilon}(\mu)$, $R_{\varepsilon x}(\mu)$, and $R'_{\varepsilon x}(\mu)$ between the useful signal and the noise are nonzero; this means that the presence of even negligible correlation $r_{x\varepsilon} \approx 0$, which according to the classical conditions is negligible, along with nonzero mathematical expectation leads to noticeable values of the quantities $R_{x\varepsilon}(\mu)$, $R'_{x\varepsilon}(\mu)$, $R_{\varepsilon x}(\mu)$, and $R'_{\varepsilon x}(\mu)$;
3. conditions (7.12) and (7.13) are not satisfied; that is, the estimators of the correlation functions $R_{gg}(\mu)$ and $R'_{gg}(\mu)$ of the noisy signal $g(i\Delta t)$ are not equal to the estimators of the correlation $R_{xx}(\mu)$ and $R'_{xx}(\mu)$ functions of the useful signal $X(i\Delta t)$; this means that even if the classical conditions are satisfied by the useful signal $X(i\Delta t)$, violation of the normality of the distribution law together with the nonzero mathematical expectation of the noise are causes for the appearance of the noticeable errors $\lambda_1(\mu)$ and $\lambda_2(\mu)$;
4. condition (7.11) is satisfied only if $\mu = 0$ and is not satisfied if $\mu \neq 0$; that is, the estimators of the correlation functions $R_{gg}(\mu)$ and $R'_{gg}(\mu)$ of the noncentered $g(i\Delta t)$, respectively centered $\dot{g}(i\Delta t)$, noisy signal are not equal to $\mu \neq 0$; this means that the errors $\lambda_1(\mu)$ and $\lambda_2(\mu)$ similarly are not equal; consequently, here again it is necessary to compute the mean value of the microerror $\Delta\lambda(\mu = \Delta t)$ and the robustness factor $\Lambda^R(\mu)$ [60].

Third variant experiment. The useful signal is created as a result of a Fourier series interpolation of a decimated (with parameter $r = 100$) normally distributed random sequence. The noise obeys a normal distribution law with mathematical expectation $m_\varepsilon \approx 0$ and standard deviation $\sigma_\varepsilon \approx 10$. For this type of experiment, the condition that the mathematical expectation m_x of the useful signal $X(i\Delta t)$ on different time intervals T_1, T_2, ..., T_n be strictly constant is violated. This means that in place of the condition

$$m_x(T_1) = m_x(T_2) = \ldots = m_x(T_n) = m_x, \tag{7.39}$$

the following condition holds:

$$m_x(T_1) \approx m_x(T_2) \approx \ldots \approx m_x(T_n) \approx m_x. \tag{7.40}$$

The following law manifests itself in all the computational experiments of the third type that were carried out:

1. condition (7.10) is strictly satisfied only if $\mu = 0$, while if $\mu \neq 0$, it is satisfied inadequately strictly, that is, the estimators of the correlation functions $R_{xx}(\mu)$ and $R'_{xx}(\mu)$ of the centered $\dot{X}(i\Delta t)$, respectively noncentered $X(i\Delta t)$, useful signal are not equal, and the following approximate equality holds:
$$R_{xx}(\mu) \approx R'_{xx}(\mu) \text{ for } \mu \neq 0; \tag{7.41}$$
this means that the condition of strict stationarity is violated by the useful signal;
2. conditions (7.14) and (7.15) are not satisfied; that is, even when the correlation between the useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ is absent and the mathematical expectation of the noise is nonzero, $m_\varepsilon \neq 0$, the estimators of the cross-correlation functions $R_{x\varepsilon}(\mu)$, $R'_{x\varepsilon}(u)$, $R_{\varepsilon x}(\mu)$, and $R'_{\varepsilon x}(\mu)$ between the useful signal and the noise are nonzero; this means that the presence of even negligible correlation $r_{x\varepsilon} \approx 0$, which according to the classical conditions is negligible, leads to noticeable values of the quantities $R_{x\varepsilon}(\mu)$, $R'_{x\varepsilon}(\mu)$, $R_{\varepsilon x}(\mu)$, and $R'_{\varepsilon x}(\mu)$;
3. conditions (7.12) and (7.13) are not satisfied; that is, the estimators of the correlation functions $R_{gg}(\mu)$ and $R'_{gg}(\mu)$ of the noisy signal $g(i\Delta t)$ are not equal to the estimators of the correlation $R_{xx}(\mu)$ and $R'_{xx}(\mu)$ functions of the useful signal $X(i\Delta t)$; this means that even with "ideal" noise, violation of the condition that the mathematical expectation of the useful signal $X(i\Delta t)$ be strictly constant over the entire time interval T is a cause for the appearance of noticeable errors $\lambda_1(\mu)$ and $\lambda_2(\mu)$;
4. condition (7.11) is satisfied only if $\mu = 0$ and is not satisfied if $\mu \neq 0$; that is, the estimators of the correlation functions $R_{gg}(\mu)$ and $R'_{gg}(\mu)$ of the noncentered $g(i\Delta t)$, respectively centered $\dot{g}(i\Delta t)$, noisy signal are not equal if $\mu \neq 0$; this means that the errors $\lambda_1(\mu)$ and $\lambda_2(\mu)$ similarly are not equal; consequently, here again it is necessary to compute the mean value of the microerror $\Delta\lambda(\mu = \Delta t)$ and the robustness factor $\Lambda^R(\mu)$ [60].

Fourth variant experiment. The useful signal is obtained as a result of a cubic spline interpolation (with parameter $r = 100$) of a normally distributed random sequence. The noise obeys a normal distribution law with mathematical expectation $m_\varepsilon \approx 0$ and standard deviation $\sigma_\varepsilon \approx 10$.

For this type of experiment, the condition that the mathematical expectation of the useful signal $X(i\Delta t)$ on different time intervals T_1, T_2, …, T_n be constant is violated. This means that in place of condition (7.39) the following condition holds:

$$m_x(T_1) \neq m_x(T_2) \neq \ldots \neq m_x(T_n) \neq m_x . \tag{7.42}$$

The following law manifests itself in all the computational experiments of the fourth type that were carried out:

1. condition (7.10) is strictly satisfied only if $\mu = 0$, and is not satisfied if $\mu \neq 0$, that is, the estimators of the correlation function $R_{xx}(\mu)$ and $R'_{xx}(\mu)$ of the centered $\dot{X}(i\Delta t)$, respectively noncentered $X(i\Delta t)$, useful signal are not equal, and the following equality holds:

   $$R_{xx}(\mu) \neq R'_{\mathrm{xx}}(\mu) \text{ for } \mu \neq 0 ; \tag{7.43}$$

   this means that the condition of stationarity is violated for the useful signal;
2. conditions (7.14) and (7.15) are not satisfied; that is, even when the correlation between the useful signal $X(i\Delta t)$ and the noise $\varepsilon(i\Delta t)$ is absent and the mathematical expectation of the noise is nonzero, $m_\varepsilon \neq 0$, the estimators of the cross-correlation functions $R_{x\varepsilon}(\mu)$, $R'_{x\varepsilon}(\mu)$, $R_{\varepsilon x}(\mu)$, and $R'_{\varepsilon x}(\mu)$ between the useful signal and the noise are nonzero; this means that the presence of even negligible correlation $r_{x\varepsilon} \approx 0$, which according to the classical conditions is negligible, leads to noticeable values of the quantities $R_{x\varepsilon}(\mu)$, $R'_{x\varepsilon}(\mu)$, $R_{\varepsilon x}(\mu)$, and $R'_{\varepsilon x}(\mu)$;
3. conditions (7.12) and (7.13) are not satisfied; that is, the estimators of the correlation functions $R_{gg}(\mu)$ and $R'_{gg}(\mu)$ of the noisy signal $g(i\Delta t)$ are not equal to the estimators of the correlation $R_{xx}(\mu)$ and $R'_{xx}(\mu)$ functions of the useful signal $X(i\Delta t)$; this means that even with "ideal" noise, the condition that the mathematical expectation of the useful signal $X(i\Delta t)$ be stationary is a cause for the appearance of the noticeable errors $\lambda_1(\mu)$ and $\lambda_2(\mu)$;
4. condition (7.11) is satisfied only if $\mu = 0$ and is not satisfied if $\mu \neq 0$; that is, the estimators of the correlation functions $R_{gg}(\mu)$ and $R'_{gg}(\mu)$ of the noncentered $g(i\Delta t)$, respectively centered $\dot{g}(i\Delta t)$, noisy signal are not equal if $\mu \neq 0$; this means that the errors $\lambda_1(\mu)$ and $\lambda_2(\mu)$ similarly are not equal; consequently, here again it is necessary to compute the mean value of the microerror $\Delta\lambda(\mu = \Delta t)$ and the robustness factor $\Lambda^R(\mu)$ [60].

The value of robustness as a new characteristic of a noisy signal is analyzed by the results of computing experiments when various classical conditions do not take place. As a result of an analysis of the results obtained, we are led to the following conclusions:

1. In all four variant experiments, the estimators of the correlation functions $R_{gg}(\mu)$ and $R'_{gg}(\mu)$ of the noisy signal if $\mu \neq 0$, and correspondingly, the errors $\lambda_1(\mu)$ and $\lambda_2(\mu)$, differ; therefore, as a result of computations according to the algorithms (7.18), (7.19), and (7.30)–(7.32), it is possible to compute the mean value of the microerror $\Delta\lambda(\mu = \Delta t)$ and the robustness factor $\Lambda^R(\mu)$.
2. In all four variant computations, the robustness factor $\Lambda^R(\mu)$ constitutes an estimator of an additional characteristic that arises as a result of a violation of the classical conditions. By means of the robustness factor, the estimator of the correlation functions of noisy signals may be corrected according to the formula

$$R^*_{gg}(\mu) \approx R_{gg}(\mu) - \Lambda^R(\mu). \tag{7.44}$$

The corrected estimators thus obtained, $R^*_{gg}(\mu)$, of the noisy signal $\dot{g}(i\Delta t)$ prove to be closer in magnitude to the estimators of the correlation functions $R_{xx}(\mu)$ of the useful signal $\dot{X}(i\Delta t)$:

$$R^*_{gg}(\mu) \approx R_{xx}(\mu). \tag{7.45}$$

3. According to the classical conditions, if correlation between the useful signal and the noise is absent, that is, if condition (7.36) is satisfied, the following conditions must hold:

$$R_{x\varepsilon}(\mu) \approx 0, \; R_{\varepsilon x}(\mu) \approx 0. \tag{7.46}$$

However, the computational experiments showed that even in this case, conditions (7.46) are not satisfied. Then the robustness factor $\Lambda^R(\mu)$ allows us to arrive at a judgment as to an approximate estimator of the cross-correlation function between the useful signal and noise, though the assumptions

$$\Lambda^R(\mu) \approx R_{x\varepsilon}(\mu), \; \Lambda^R(\mu) \approx R_{\varepsilon x}(\mu) \tag{7.47}$$

are rather coarse.
4. The robustness factor $\Lambda^R(\mu)$ is an independent characteristic of the noisy signal $g(i\Delta t)$ for the case in which the classical conditions are violated. Therefore, the robustness factor may be used as yet one more new and supplementary characteristic of a noisy signal in the

solution of problems of monitoring a defect's origin, diagnostics, prediction, identification, etc. [60].

7.8 Technologies of Indication of a Defect's Origin by Considering Noise as a Data Carrier

Research shows that in most cases monitoring is commonly reduced to the indication of the process of the defect's origin. It is advisable to create the simple indication technologies of the defect's origin for this kind of case. From this point of view, it is advisable to use estimates of the signum correlation functions, which can be determined by means of the following algorithms, as the informative indications:

$$R^*_{x\varepsilon}(0) = \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \dot{x}(i\Delta t) \operatorname{sgn} \dot{\varepsilon}(i\Delta t), \tag{7.48}$$

$$R^*_{xx}(0) = \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \dot{x}(i\Delta t) \operatorname{sgn} \dot{x}(i\Delta t), \tag{7.49}$$

$$R^*_{\varepsilon\varepsilon}(0) = \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \dot{\varepsilon}(i\Delta t) \operatorname{sgn} \dot{\varepsilon}(i\Delta t), \tag{7.50}$$

$$r^*_{x\varepsilon} = \frac{R^*_{x\varepsilon}(0)}{\sqrt{R^*_{xx}(0) R^*_{\varepsilon\varepsilon}(0)}}. \tag{7.51}$$

Taking into account that the estimates $R^*_{xx}(0)$ and $R^*_{\varepsilon\varepsilon}(0)$ in the calculation by formulae (7.49) and (7.50) are equal to unit, i.e., $R^*_{xx}(0) = 1$ and $R^*_{\varepsilon\varepsilon}(0) = 1$, formula (7.51) can be represented as follows:

$$r^*_{x\varepsilon} \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \dot{x}(i\Delta t) \operatorname{sgn} \dot{\varepsilon}^*(i\Delta t). \tag{7.52}$$

At the same time, taking into account the following equality:

$$\operatorname{sgn}\ \dot{g}(i\Delta t) = \operatorname{sgn} \dot{x}(i\Delta t),$$

we can write

$$r_{x\varepsilon} = \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn}\left[\dot{g}(i\Delta t) - \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\,\right]$$

$$\times \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\,. \tag{7.53}$$

Similarly, the formula of the calculation of the coefficient of the correlation $r_{g\varepsilon}$ between the noisy signal $\dot{g}(i\Delta t)$ and the noise $\dot{\varepsilon}(i\Delta t)$ can be represented as follows:

$$r_{g\varepsilon} = \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \dot{g}(i\Delta t) \operatorname{sgn} \dot{\varepsilon}(i\Delta t) \tag{7.54}$$

or

$$r_{g\varepsilon} \approx r^{*}_{g\varepsilon} = \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \dot{g}(i\Delta t) \operatorname{sgn} \dot{\varepsilon}^{*}(i\Delta t)$$

$$= \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \dot{g}(i\Delta t) \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\,. \tag{7.55}$$

The estimates of the relay correlation functions can also be used for the indication of the defect's origin [14]:

$$R^{rl}_{x\varepsilon}(0) = \frac{1}{N}\sum_{i=1}^{N} \dot{x}(i\Delta t) \operatorname{sgn} \dot{\varepsilon}(i\Delta t)\,, \tag{7.56}$$

$$R^{rl}_{g\varepsilon}(0) = \frac{1}{N}\sum_{i=1}^{N} \dot{g}(i\Delta t) \operatorname{sgn} \dot{\varepsilon}(i\Delta t)\,, \tag{7.57}$$

or

$$R^{rl}_{x\varepsilon}(0) \approx R^{**}_{x\varepsilon}(0) = \frac{1}{N}\sum_{i=1}^{N} \dot{x}^{*}(i\Delta t) \operatorname{sgn} \dot{\varepsilon}^{*}(i\Delta t)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\Big[\dot{g}(i\Delta t) - \operatorname{sgn}\big[\varepsilon'(i\Delta t)$$

$$- \varepsilon''(i\Delta t)\big]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\Big]$$

$$\times \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\,, \tag{7.58}$$

$$R^{rl}_{g\varepsilon}(0) \approx R^{**}_{g\varepsilon}(0) = \frac{1}{N}\sum_{i=1}^{N} \dot{g}(i\Delta t) \operatorname{sgn} \dot{\varepsilon}^{*}(i\Delta t)$$

$$= \frac{1}{N} \sum_{i=1}^{N} \dot{g}(i\Delta t) \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right] \sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|} \,. \tag{7.59}$$

It is obvious from expressions (7.52)–(7.57) that, during the indication of the defect's origin, instead of the samples of the noise $\varepsilon(i\Delta t)$, their increase can be used. At the same time, solving the considered problem can be greatly simplified by the determination of the signs of the samples of the noise. It is intuitively clear that it is possible to determine the sign of the sample of the noise $\dot{\varepsilon}(i\Delta t)$ approximately by the sign of the increase of the samples:

$$\operatorname{sgn} \ \Delta g(i\Delta t) = \dot{g}(i\Delta t) - \dot{g}(i-1)\Delta t \,.$$

The analysis of forming the real technological parameters and the results of sampling of both their samples,

$$\dot{g}(i\Delta t) = \dot{X}(i\Delta t) + \dot{\varepsilon}(i\Delta t) \,,$$

and their increases,

$$\Delta g(i\Delta t) = \dot{g}(i\Delta t) - \dot{g}(i-1)\Delta t \,,$$

show that it is possible to provide the following approximate equality:

$$\lim_{\Delta t \to 0} \dot{X}(i\Delta t) \approx \lim_{\Delta t \to 0} \dot{X}(i-1)\Delta t \,, \tag{7.60}$$

by decreasing the quantization step by time Δt until the value Δt_ε, corresponding to the necessary sampling step of the noise $\dot{\varepsilon}(t)$.

At the same time, the formula for determining the sign $\varepsilon(i\Delta t)$ by the value of the increase $\Delta g(i\Delta t)$ can be represented as follows:

$$\Delta g(i\Delta t) = \lim_{\Delta t \to 0}\left[\dot{X}(i\Delta t) + \dot{\varepsilon}(i\Delta t)\right] - \lim_{\Delta t \to 0}\Big[\dot{X}\big((i-1)\Delta t\big)$$

$$+ \dot{\varepsilon}\big((i-1)\Delta t\big)\Big] = \dot{\varepsilon}(i\Delta t) - \dot{\varepsilon}\big((i-1)\Delta t\big). \tag{7.61}$$

Equations (7.60) and (7.61) show that the value of the increase $\Delta g(i\Delta t)$ is close to the value of the noise for time Δt_ε, i.e.,

$$\lim_{\Delta t \to 0} \Delta g(i\Delta t) = \begin{cases} \dot{\varepsilon}(i\Delta t) & \text{for } \ \dot{\varepsilon}(i-1)\Delta t < \Delta x, \\ \dot{\varepsilon}(i-1)\Delta t & \text{for } \ \dot{\varepsilon}(i\Delta t) < \Delta x. \end{cases} \tag{7.62}$$

Thus, it is possible to determine the sign of the noise $\dot{\varepsilon}(i\Delta t)$ by the sign of the increase $\Delta g(i\Delta t)$ of each sample $\dot{g}(i\Delta t)$ by the following expression:

$$\operatorname{sgn}\ \varepsilon^*(i\Delta t) \approx \operatorname{sgn}\ \Delta g(i\Delta t) = \begin{cases} + \text{ for } (\dot{g}(i\Delta t) - \dot{g}(i-1)\Delta t) \geq 0, \\ - \text{ for } (\dot{g}(i\Delta t) - \dot{g}(i-1)\Delta t) \leq 0, \end{cases}$$

when the condition of sampling the signal $\dot{g}(t)$ takes place for the quantization step Δt_ε.

Formula (7.51) for the determination of the coefficient of the correlation $r^*_{x\varepsilon}$ between the useful signal $\dot{X}(i\Delta t)$ and the noise $\dot{\varepsilon}(i\Delta t)$ can be represented as follows:

$$r^*_{x\varepsilon} = R^*_{\varepsilon g}(0) = \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn}\ \dot{\varepsilon}(i\Delta t)\ \operatorname{sgn}\ \dot{g}(i\Delta t). \tag{7.63}$$

If we assume that the equality $\operatorname{sgn} \varepsilon^*(i\Delta t) \approx \operatorname{sgn} \Delta g(i\Delta t)$ takes place when the sampling step $\Delta t = \Delta t_\varepsilon$ is chosen, formula (7.51) for determining $r_{x\varepsilon}$ can be represented as follows:

$$r^*_{x\varepsilon} \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \varepsilon^*(i\Delta t) \operatorname{sgn} g(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \Delta g(i\Delta t) g(i\Delta t), \tag{7.64}$$

$$r^*_{xy} \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \Delta x(i\Delta t) \operatorname{sgn} \Delta y(i\Delta t) \tag{7.65}$$

and can be used for the indication of the defect's origin.

7.9 Spectral Indicators of a Defect's Origin

It is advisable to use the spectral technology of monitoring the defect's origin, the algorithms for which are presented in Chapter 5, for the complicated objects such as deep-sea platforms, large-capacity tankers, airplanes, compressor stations, main oil-and-gas pipelines, etc. However, it is advisable to use simplified technologies for objects with lower requirements for monitoring the defect. A possible solution to this problem is offered ahead. It was shown [12, 19, 20, 22, 23] that the influence of the noise affects both the quantity of the positive and the negative products $g(i\Delta t)\cos n\omega(i\Delta t)$, $g(i\Delta t)\sin n\omega(i\Delta t)$ and the normal values of their sum. It can be shown that the distribution law of the noisy signal $g(i\Delta t)$, the change of its spectrum, the levels of the correlation between the useful signal and the noise, etc. affect these values. At the same time, these parameters of the signal do not change under normal conditions of the work. In most cases, they change because of the rise of certain failures in the objects. In this connection, it is possible to use the mentioned values as the informative indications for detecting the origins of these failures. One of the possible variants of the easily realized spectral indicators is as follows:

$$K_{zc} = \frac{\Delta N_{zc}}{N} = \frac{N_{zc}^{+} - N_{zc}^{-}}{N} = \frac{N_{zc}^{++} + N_{zc}^{--} - N_{zc}^{+-} - N_{zc}^{-+}}{N}, \tag{7.66}$$

$$K_{zs} = \frac{\Delta N_{zs}}{N} = \frac{N_{zs}^{+} - N_{zs}^{-}}{N} = \frac{N_{zs}^{++} + N_{zs}^{--} - N_{zs}^{+-} - N_{zs}^{-+}}{N}, \tag{7.67}$$

where N_{zc}^{++}, N_{zc}^{--}, N_{zc}^{+-}, N_{zc}^{-+} are the quantities of the products $g(i\Delta t)\cos\omega^{*}(i\Delta t)$, $g(i\Delta t)\sin\omega^{*}(i\Delta t)$ with the sign of multipliers ++, −−, +−, −+, respectively.

Here the coefficients K_{zc} and K_{zs} are only determined for the spectrum ω^{*} of the period T_{ω}, determined by the following formula:

$$T_{\omega} \approx (2 \div 5)\Delta t_{\varepsilon}. \tag{7.68}$$

The defect's origin is accompanied by noise with a frequency close to the mentioned one.

It is possible to use the values $\Delta\Pi_C$ and $\Delta\Pi_S$, which can be determined by the following formulae, as the another spectral indicator:

$$\Delta\Pi_C = \Pi_C^{+} - \Pi_C^{-} = \overline{\dot{g}(i\Delta t)\cos\omega^{*}(i\Delta t)}^{+} - \overline{\dot{g}(i\Delta t)\cos\omega^{*}(i\Delta t)}^{-}, \tag{7.69}$$

$$\Delta\Pi_s = \Pi_s^{+} - \Pi_s^{-} = \overline{\dot{g}(i\Delta t)\sin\omega^{*}(i\Delta t)}^{+} - \overline{\dot{g}(i\Delta t)\sin\omega^{*}(i\Delta t)}^{-}, \tag{7.70}$$

where

$$\Pi_C^{+} = \frac{1}{N}\sum_{i=1}^{N}\dot{g}(i\Delta t)\cos\omega^{*}(i\Delta t) = \overline{\dot{g}(i\Delta t)\cos\omega^{*}(i\Delta t)}^{+}, \tag{7.71}$$

$$\Pi_C^{-} = \frac{1}{N}\sum_{i=1}^{N}\dot{g}(i\Delta t)\cos\omega^{*}(i\Delta t) = \overline{\dot{g}(i\Delta t)\cos\omega^{*}(i\Delta t)}^{-}. \tag{7.72}$$

But for simplicity of the realization of monitoring the defect's origin, it is more comfortable to use the following expressions:

$$a_{\omega}^{*} = \frac{2}{N}\sum_{i=1}^{N}\operatorname{sgn} g(i\Delta t)\operatorname{sgn}\cos\omega^{*}(i\Delta t), \tag{7.73}$$

$$b_{\omega}^{*} = \frac{2}{N}\sum_{i=1}^{N} \operatorname{sgn} g(i\Delta t) \operatorname{sgn}\sin\omega^{*}(i\Delta t). \tag{7.74}$$

When the defect is being formed, if the change of the spectrum, the distribution law, the variance, and the other characteristics of the signal or the noise happen on the output of the sensor, the values of these spectral indicators K_{zc}, K_{zs}, $\Delta\Pi_C$, $\Delta\Pi_S$, a_{ω}^{*}, b_{ω}^{*} also change. Thus, it is possible to detect the initial stage of the defect's origin.

7.10 Position-Binary Indicators of a Defect's Origin

The possibilities of monitoring the defect's origin by using the position-binary technology were considered in detail in Chapter 2. Research has shown that even the simplest variants of this technology allow one to detect the beginning of the defect's origin with sufficient reliability. Two effective position-binary indicators, which can be widely used, are considered here.

It has been shown [1, 12, 13] that if the state of the object is stable, the ratio N_{ε_0}, N_{ε_1}, …, $N_{\varepsilon_{(m-1)}}$ of the quantity of the noise impulses $q_{\varepsilon_0}(i\Delta t)$, $q_{\varepsilon_1}(i\Delta t)$, …, $q_{\varepsilon(m-1)}(i\Delta t)$ to the total quantity $N_{q_0 k}$, $N_{q_1 k}$, …, $N_{q_{(m-1)}k}$ of the position-impulse signals $q_0(i\Delta t)$, $q_1(i\Delta t)$, …, $q_{(m-1)}(i\Delta t)$ and also the ratio of the quantity of the transfers from zero to unit state N_{q_0}, N_{q_1}, …, $N_{q_{m-1}}$ to the total number of the samples N for time T are the non-random values

$$K_{q_0} = \frac{N_{\varepsilon_0}}{N_{q_0 k}},\ K_{q_1} = \frac{N_{\varepsilon_1}}{N_{q_1 k}},\ \ldots,\ K_{q_{m-1}} = \frac{N_{\varepsilon(m-1)}}{N_{q_{(m-1)}k}}, \tag{7.75}$$

$$K'_{q_0} = \frac{N_{q_0}}{N},\ K'_{q_1} = \frac{N_{q_1}}{N},\ \cdots,\ K'_{q_{m-1}} = \frac{N_{q_{m-1}}}{N}, \tag{7.76}$$

where N_{ε_0}, N_{ε_1}, …, $N_{\varepsilon_{(m-1)}}$ are the quantities of the position noises, respectively $q_{\varepsilon_0}(i\Delta t)$, $q_{\varepsilon_1}(i\Delta t)$, …, $q_{\varepsilon(m-1)}(i\Delta t)$; $N_{q_0 k}$, $N_{q_1 k}$, …, $N_{q_{(m-1)}k}$ are the total amounts of the position-impulse signals $q_0(i\Delta t)$, $q_1(i\Delta t)$, …, $q_{(m-1)}(i\Delta t)$ for time T.

Let us note that the estimates of the average frequency $\langle f_{q_0}\rangle$, $\langle f_{q_1}\rangle$, $\langle f_{q_2}\rangle$, …, $\langle f_{q_{m-1}}\rangle$ of the position signals $q_0(i\Delta t)$, $q_1(i\Delta t)$, …, $q_{(m-1)}(i\Delta t)$ can be determined by the quantity of the transfers of the signals from zero to unit state for a unit of time, for example, for a second.

The coefficients of the correlation between the samples $g(i\Delta t)$, $X(i\Delta t)$, $\varepsilon^*(i\Delta t)$ and the position-binary-impulse signals (PBIS) $q_j(i\Delta t)$ also can be used as the easily realized indicators:

$$r^*_{q_j\varepsilon} \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \varepsilon^*(i\Delta t) q_j(i\Delta t), \tag{7.77}$$

$$r^*_{q_j g} \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} g(i\Delta t) q_j(i\Delta t), \tag{7.78}$$

$$r^*_{q_j x} \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \mathrm{X}(i\Delta t) q_j(i\Delta t). \tag{7.79}$$

For simplicity of realization of the most expedient variant for determining the coefficient of the correlation between $\varepsilon(i\Delta t)$ and $q_j(i\Delta t)$, one has the formulae

$$r^*_{gq_\varepsilon} \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} g(i\Delta t) q_\varepsilon(i\Delta t), \tag{7.80}$$

$$r^*_{q_j\varepsilon} \approx \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \Delta g(i\Delta t) q_j(i\Delta t). \tag{7.81}$$

The following estimates can also be used for the indication of the defect:

$$\left.\begin{aligned} a_{\varepsilon g} &\approx \frac{2}{N}\sum_{i=1}^{N} \mathrm{g}_j(i\Delta t) \operatorname{sgn} \cos \omega^*(i\Delta t) \\ b_{\varepsilon g} &\approx \frac{2}{N}\sum_{i=1}^{N} g_j(i\Delta t) \operatorname{sgn} \sin \omega^*(i\Delta t) \end{aligned}\right\}. \tag{7.82}$$

So the indication of the defect's origin can be performed by the change to the combination of the coefficient of the estimates K_{q_0}, K_{q_1}, K_{q_2}, …, $K_{q_{m-1}}$, K'_{q_0}, K'_{q_1}, K'_{q_2}, …, $K'_{q_{m-1}}$, $r^*_{q_j\varepsilon}$, $r^*_{q_j g}$, $r^*_{q_j x}$, $r^*_{\mathrm{gq}_\varepsilon}$, $a_{\varepsilon g}$, $b_{\varepsilon g}$.

7.11 Recommended Digital Technologies for Monitoring a Defect's Origin

Summing up the suggested digital technologies, in the result let us give them in brief. The noises of the signals received as the output of corresponding sensors often contain information about the defect's origin. And the variance, the distribution law, the correlation and spectral and the other noise characteristics, and their correlation with the useful signal at the initial stage of the defect's origin vary continuously. One of the possible variants of the signal model can be represented in the following form:

$$g(i\Delta t) \approx= \begin{cases} X(T_0 + i\Delta t) + \varepsilon(T_0 + i\Delta t), & D_\varepsilon < 0,05D_g; r_{x\varepsilon} = 0; \omega_{ng} > \omega_{ng} \\ X(T_0 + T_1 + i\Delta t) + \varepsilon(T_0 + T_1 + i\Delta t), & D_\varepsilon < 0,1D_g; r_{x\varepsilon} \neq 0; \omega_{ng} > \omega_{ng} \\ X(T_0 + T_1 + T_2 + i\Delta t) + \varepsilon(T_0 + T_1 + T_2 + i\Delta t), & D_\varepsilon \approx 0,1D_g; r_{x\varepsilon} \neq 0; \omega_{ng} \approx \omega_{ng} \\ X(T_0 + T_1 + T_2 + T_3 + i\Delta t) & \\ +\varepsilon(T_0 + T_1 + T_2 + T_3 + i\Delta t), & D_\varepsilon < 0,2D_g; r_{x\varepsilon} \neq 0,5; \omega_{ng} < \omega_{ng} \\ X(T_0 + T_1 + T_2 + T_3 + T_4 + i\Delta t) & \\ +\varepsilon(T_0 + T_1 + T_2 + T_3 + T_4 + i\Delta t), & D_\varepsilon > 0,2D_g; r_{x\varepsilon} \approx 0,5; \omega_{ng} \approx \omega_{ng} \end{cases} \tag{7.83}$$

The approximate magnitude of the samples $\varepsilon_j^*(i\Delta t)$ of the noise with high-frequency spectrum can be determined using the samples of the total signal $g_j(i\Delta t)$ according to the following expression:

$$\dot{\varepsilon}_j^*(i\Delta t) \approx \operatorname{sgn}\left[\varepsilon_j'(i\Delta t) - \varepsilon_j''(i\Delta t)\right]\sqrt{\left|\varepsilon_j'(i\Delta t) - \varepsilon_j''(i\Delta t)\right|}, \tag{7.84}$$

where

$$\varepsilon_j'(i\Delta t) = \dot{g}_j^2(i\Delta t) + \dot{g}_j(i\Delta t)\dot{g}_j((i+2)\Delta t) - 2\dot{g}_j(i\Delta t)\dot{g}_j((i+1)\Delta t), \tag{7.85}$$

$$\varepsilon_j''(i\Delta t) = \dot{g}_j(i\Delta t)\dot{g}_j((i+1)\Delta t) + \dot{g}_j(i\Delta t)\dot{g}_j((i+3)\Delta t) - 2\dot{g}_j(i\Delta t)\dot{g}_j((i+2)\Delta t). \tag{7.86}$$

Here, using the samples $\varepsilon_j^*(i\Delta t)$, one can determine the approximate magnitudes of the samples of a legitimate signal $X_j(i\Delta t)$ and also the estimations of the noise characteristics in the following way:

$$X_j^*(i\Delta t) = g_j(i\Delta t) - \varepsilon_j^*(i\Delta t), \tag{7.87}$$

$$m_\varepsilon = \begin{cases} \dfrac{1}{N}\sum\limits_{i=1}^{N} \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|} & \text{at } r_{x\varepsilon} = 0, \\ \dfrac{1}{N}\sum\limits_{i=1}^{N} \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|} & \text{at } r_{x\varepsilon} \neq 0, \end{cases} \tag{7.88}$$

$$D_{\varepsilon} = \begin{cases} \dfrac{1}{N}\sum_{i=1}^{N} \varepsilon'(i\Delta t) & \text{at} \quad r_{x\varepsilon} = 0, \\ \dfrac{1}{N}\sum_{i=1}^{N} \left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right] & \text{at} \quad r_{x\varepsilon} \neq 0, \end{cases} \tag{7.89}$$

$$R_{x\varepsilon}^{(\mu)} = \begin{cases} \dfrac{1}{N}\sum_{i=1}^{N}\left[g(i\Delta t) - \operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}\right]\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|} \quad \text{at } r_{x\varepsilon} = 0, \\ \dfrac{1}{N}\sum_{i=1}^{N}\left[g(i\Delta t) - \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\right] \\ \times \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|} \quad \text{at } r_{x\varepsilon} \neq 0, \end{cases} \tag{7.90}$$

$$a_{n\varepsilon} \approx \begin{cases} \dfrac{2}{N}\sum_{i=1}^{N}\left[\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}\right]\cos n\omega(i\Delta t) \quad \text{at } \mathrm{r}_{\mathrm{x}\varepsilon} = 0, \\ \dfrac{2}{N}\sum_{i=1}^{N}\left\{\operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\right\}\cos n\omega(i\Delta t) \quad \text{at } \mathrm{r}_{\mathrm{x}\varepsilon} \neq 0, \end{cases} \tag{7.91}$$

$$b_{n\varepsilon} \approx \begin{cases} \dfrac{2}{N}\sum_{i=1}^{N}\left[\operatorname{sgn}\varepsilon'(i\Delta t)\sqrt{\left|\varepsilon'(i\Delta t)\right|}\right]\sin n\omega(i\Delta t) \quad \text{at } \mathrm{r}_{\mathrm{x}\varepsilon} = 0, \\ \dfrac{2}{N}\sum_{i=1}^{N}\left\{\operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\right\}\sin n\omega(i\Delta t) \quad \text{at } \mathrm{r}_{\mathrm{x}\varepsilon} \neq 0, \end{cases} \tag{7.92}$$

$$\begin{aligned} r_{x\varepsilon} \approx r_{x\varepsilon}^{*} &= \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn} \dot{x}(i\Delta t) \operatorname{sgn} \dot{\varepsilon}^{*}(i\Delta t) \\ &= \frac{1}{N}\sum_{i=1}^{N} \operatorname{sgn}\Big[\dot{g}(i\Delta t) \\ &\quad - \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}\Big] \\ &\quad \times \operatorname{sgn}\left[\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right]\sqrt{\left|\varepsilon'(i\Delta t) - \varepsilon''(i\Delta t)\right|}. \end{aligned} \tag{7.93}$$

With the problem of monitoring, it is expedient to use the following robust technology of correlation and spectral analysis [27, 33]:

$$R_{gg}^{R}\left(0\right)=\frac{1}{n}\sum_{i=1}^{n}\dot{g}^{2}(i\Delta t)-D_{\varepsilon}-[n^{+}(\mu)-n^{-}(\mu)]\left\langle\Delta\lambda(\mu=1)\right\rangle, \tag{7.94}$$

$$R_{gg}^{R}(\mu)=\frac{1}{n}\sum_{i=1}^{n}\dot{g}(i\Delta t)\dot{g}((i+\mu)\Delta t)-[n^{+}(\mu)-n^{-}(\mu)]\left\langle\Delta\lambda(\mu=1)\right\rangle, \tag{7.95}$$

$$R_{g\eta}^{R}(\mu)=\frac{1}{n}\sum_{i=1}^{n}\dot{g}(i\Delta t)\dot{\eta}((i+\mu)\Delta t)-[n^{+}(\mu)-n^{-}(\mu)]\left\langle\Delta\lambda(\mu=1)\right\rangle, \tag{7.96}$$

$$\left\langle\Delta\lambda(\mu=1)\right\rangle=[1/N^{-}(\mu=1)]\lambda(\mu=1), \tag{7.97}$$

$$\left|R'_{gg}(\mu=1)-R_{gg}(\mu=1)\right|=\lambda(\mu=1), \tag{7.98}$$

$$\lambda_{xx}^{R}(\mu)\approx\begin{cases}\left[N^{+}(\mu)-N^{-}(\mu)\right]\left\langle\lambda(\mu=1)\right\rangle+D_{\varepsilon} & \text{at } \mu=0,\\ \left[N^{+}(\mu)-N^{-}(\mu)\right]\left\langle\lambda(\mu=1)\right\rangle & \text{at } \mu\neq 0.\end{cases} \tag{7.99}$$

Monitoring by means of positional-binary-impulse signals is also provided for monitoring the defect's origin:

$$g\left(i\Delta t\right)\approx q_{n-1}\left(i\Delta t\right)+q_{n-2}\left(i\Delta t\right)+\ldots+q_{1}\left(i\Delta t\right)+q_{0}\left(i\Delta t\right). \tag{7.100}$$

For each q_k th component of the signal $g_n(i\Delta t)$, one can determine the average value of the period $\left\langle T_k\right\rangle$ and the average value of the unit $\left\langle T_{k1}\right\rangle$ and zero $\left\langle T_{k0}\right\rangle$ half-periods of signals $q_k(i\Delta t)$ when having sufficient time of observation T according to the following formulae:

$$\left\langle T_{k1}\right\rangle=\frac{1}{N}\sum_{i=1}^{N}T_{K1_i}, \tag{7.101}$$

$$\left\langle T_{k0}\right\rangle=\frac{1}{N}\sum_{i=1}^{N}T_{K0_i}, \tag{7.102}$$

$$\left\langle T_{k}\right\rangle=\left\langle T_{k1}\right\rangle+\left\langle T_{k0}\right\rangle. \tag{7.103}$$

During the process of the defect's origin, in coding each positional signal, the positional noises $q_{\varepsilon k}\left(i\Delta t\right)$ are formed. They are positional noises of a noisy signal $g\left(i\Delta t\right)$, and they are represented as short time pulses:

$$q_{\varepsilon}(i\Delta t) = q_{\varepsilon n-1}(i\Delta t) + q_{\varepsilon n-2}(i\Delta t) + ... + q_{\varepsilon 1}(i\Delta t) + q_{\varepsilon 0}(i\Delta t), \tag{7.104}$$

$$D_{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N} \varepsilon^2(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]^2, \tag{7.105}$$

$$R'_{x\varepsilon}(0) = \frac{1}{N}\sum_{i=1}^{N}\left[g(i\Delta t) - \sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]\left[\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right], \tag{7.106}$$

$$R'_{g\varepsilon}(0) = \frac{1}{N}\sum_{i=1}^{N} g(i\Delta t)\varepsilon(i\Delta t) \approx \frac{1}{N}\sum_{i=1}^{N}\left[g(i\Delta t)\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right], \tag{7.107}$$

$$a_{n\varepsilon} = \frac{2}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\cos n\omega(i\Delta t) \approx \frac{2}{N}\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]\cos n\omega(i\Delta t), \tag{7.108}$$

$$b_{n\varepsilon} = \frac{2}{N}\sum_{i=1}^{N} \varepsilon(i\Delta t)\sin n\omega(i\Delta t) \approx \frac{2}{N}\sum_{i=1}^{N}\left[\sum_{k=0}^{m-1} q_{\varepsilon k}(i\Delta t)\right]\sin n\omega(i\Delta t). \tag{7.109}$$

As indicated above in monitoring for certain classes of objects, the application of expressions (7.87)–(7.96) and (7.101)–(7.109) may be not expedient because of the difficulties in their realization. For monitoring such objects as cars, speedboats, tractors, bulldozers, combines, and so on, one can use technologies of indication of the beginning stage of the defect's origin. Ahead we show some technologies that are easy to realize.

The frequency characteristics of the positional noises $q_{\varepsilon}(i\Delta t)$ can be used to indicate the defect's origin. If the state of the object is stable, then the ratio of the number N_{ε_0}, N_{ε_1}, …, $N_{\varepsilon_{(m-1)}}$ of impulses $q_{\varepsilon_0}(i\Delta t)$, $q_{\varepsilon_1}(i\Delta t)$, …, $q_{\varepsilon(m-1)}(i\Delta t)$ of noises to the total number $N_{q_0 k}$, $N_{q_1 k}$, …, $N_{q_{(m-1)}k}$ of positional-impulse signals $q_0(i\Delta t)$, $q_1(i\Delta t)$, …, $q_{(m-1)}(i\Delta t)$, and also the ratio of number of the transitions N_{q_0}, N_{q_1}, ⋯, $N_{q_{m-1}}$ to the total number of samples N for time T are non-random values:

$$K_{q_0} = \frac{N_{\varepsilon_0}}{N_{q_0 k}},\ K_{q_1} = \frac{N_{\varepsilon_1}}{N_{q_1 k}},\ ...,\ K_{q_{m-1}} = \frac{N_{\varepsilon_{(m-1)}}}{N_{q_{(m-1)}k}}, \tag{7.110}$$

$$K'_{q_0} = \frac{N_{q_0}}{N},\ K'_{q_1} = \frac{N_{q_1}}{N},\ ...,\ K'_{q_{m-1}} = \frac{N_{q_{m-1}}}{N}, \tag{7.111}$$

where N_{ε_0}, N_{ε_1}, …, $N_{\varepsilon_{(m-1)}}$ are the number of positional noises, respectively $q_{\varepsilon_0}(i\Delta t)$, $q_{\varepsilon_1}(i\Delta t)$, …, $q_{\varepsilon(m-1)}(i\Delta t)$; $N_{q_0 k}$, $N_{q_1 k}$, …, $N_{q_{(m-1)}k}$ are the total number of positional-impulse signals $q_0(i\Delta t)$, $q_1(i\Delta t)$, …, $q_{(m-1)}(i\Delta t)$ during time T; $N = T/\Delta t$, N_{q_0}, N_{q_1}, …, $N_{q_{m-1}}$ are the number of transitions $q_0(i\Delta t)$, $q_1(i\Delta t)$, $\cdots$, $q_{m-1}(i\Delta t)$ from the zero state to the unit state during time T.

One of the easy realized indicators for fixing the initial stage of the defect's origin is a sign correlation coefficient that can be determined according to the following expressions:

$$r^*_{x\varepsilon} \approx \frac{1}{N}\sum_{i=1}^{N} \mathrm{sgn}\left(\varepsilon^*(i\Delta t)\right)\mathrm{sgn}\left(g(i\Delta t)\right), \qquad (7.112)$$

$$r^*_{x\varepsilon} \approx \frac{1}{N}\sum_{i=1}^{N} \mathrm{sgn}\left(\Delta g(i\Delta t)\right)\mathrm{sgn}\left(g(i\Delta t)\right), \qquad (7.113)$$

$$r^*_{q_j\varepsilon} \approx \frac{1}{N}\sum_{i=1}^{N} \mathrm{sgn}\Delta g(i\Delta t)\, q_j(i\Delta t), \qquad (7.114)$$

$$r^*_{q_j\varepsilon} \approx \frac{1}{N}\sum_{i=1}^{N} \mathrm{sgn}\varepsilon^*(i\Delta t)\, q_j(i\Delta t), \qquad (7.115)$$

$$r^*_{xy} \approx \frac{1}{N}\sum_{i=1}^{N} \mathrm{sgn}\,\Delta x(i\Delta t)\Delta y(i\Delta t), \qquad (7.116)$$

where r^* is an approximate estimation of the sign correlation coefficient. As indicators, one can also use the following magnitudes:

$$\lambda^R(\mu) = \left[N^+(\mu) - N^-(\mu)\Delta\lambda(\mu = 1)\right], \qquad (7.117)$$

where $N^+(\mu)$ and $N^-(\mu)$ are the number of positive and negative products $\dot{g}(i\Delta t)\dot{g}(i+\mu)$ for the time shift $\mu = \Delta t$.

The indications also are

$$a^*_{\varepsilon q} \approx \frac{2}{N}\sum_{i=1}^{N} g_j(i\Delta t)\,\mathrm{sgn}\cos\omega^*(i\Delta t), \qquad (7.118)$$

$$b_{\varepsilon q}^{*} \approx \frac{2}{N}\sum_{i=1}^{N} g_j(i\Delta t)\,\mathrm{sgn}\sin\omega^{*}(i\Delta t)\,, \tag{7.119}$$

$$a_{\varepsilon q}^{\prime} \approx \frac{2}{N}\sum_{i=1}^{N} q_j(i\Delta t)\,\mathrm{sgn}\cos\omega^{*}(i\Delta t)\,, \tag{7.120}$$

$$b_{\varepsilon q}^{\prime} \approx \frac{2}{N}\sum_{i=1}^{N} q_j(i\Delta t)\,\mathrm{sgn}\sin\omega^{*}(i\Delta t)\,. \tag{7.121}$$

Here the coefficients $a_{\varepsilon q}^{*}$, $b_{\varepsilon q}^{*}$, $a_{\varepsilon q}^{\prime}$, $b_{\varepsilon q}^{\prime}$ are only determined for one frequency corresponding to ω^{*}, which is determined proceeding from the spectrum $\omega(i\Delta t)$ according to the following formula:

$$T_{\omega} \approx (2 \div 5)\Delta t_{\varepsilon}\,.$$

At the moment the defect arises when the spectrum of the signal $g(i\Delta t)$ is changed, the estimation of monitoring magnitudes m_{ε}, D_{ε}, $R_{x\varepsilon}^{\prime}(\mu)$, $a_{n\varepsilon}$, $b_{n\varepsilon}$, $R_{gg}^{R}(0)$, $R_{gg}^{R}(\mu)$, $\langle T_{k1}\rangle$, $\langle T_{k0}\rangle$, $\langle T_{k}\rangle$, the estimation of indication magnitudes $K_{q_0}^{\prime}$, $K_{q_1}^{\prime}$, $\ldots, K_{q_{m-1}}^{\prime}$; K_{q_0}, K_{q_1}, …, $K_{q_{m-1}}$; $\lambda^{R}(\mu)$; $r_{x\varepsilon}^{*}$, $r_{q_j\varepsilon}^{*}$, $a_{\varepsilon q}^{*}$, $b_{\varepsilon q}^{*}$, $a_{\varepsilon q}^{\prime}$, $b_{\varepsilon q}^{\prime}$ are varied at the output of the sensor.

The combination of suggested digital technologies of noise analysis, robust correlation and spectral technologies, and positional-binary technologies of analysis of signals received as the output of the sensors allows one to detect various changes to corresponding details of the objects at the initial stage. Due to this, the opportunity to exclude the lateness of monitoring the defects appears, which in some cases leads to catastrophic situations.

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