

CHAPTER

2

Segment Measure and Coordinate Graphing

What You'll Learn

Key Ideas

- Find the distance between two points on a number line. *(Lesson 2-1)*
- Apply the properties of real numbers to the measures of segments. *(Lesson 2-2)*
- Identify congruent segments and find the midpoints of segments. *(Lesson 2-3)*
- Name and graph ordered pairs on a coordinate plane. *(Lesson 2-4)*
- Find the coordinates of the midpoint of a segment. *(Lesson 2-5)*

Key Vocabulary

coordinate plane (p. 68)

equation (p. 57)

graph (p. 68)

ordered pair (p. 68)

origin (pp. 52, 68)

Why It's Important

Science While most cats hunt by stalking and pouncing, a cheetah runs down its prey. Several characteristics make the cheetah built for speed. They have grooved pads on their feet for stopping, a flexible spine for quick turns, large nostrils and lungs for extra oxygen, and lightweight bones and small teeth for low body weight.

Segment measure and coordinate graphing are essential skills for the study of geometry. You will use a coordinate graph to analyze the speed of a cheetah and other animals in Lesson 2-4.



Study these lessons to improve your skills.

✓ Check Your Readiness

✓ Algebra Review, p. 722

✓ Algebra Review, p. 723

✓ Algebra Review, p. 724

Solve each equation. Check your solution.

- | | | |
|---------------------------|----------------------------|-------------------------------|
| 1. $5 + b = 9$ | 2. $12 + x = 21$ | 3. $34 = m + 15$ |
| 4. $a + 12 = 21$ | 5. $17 = q + 4$ | 6. $65 = d + 32$ |
| 7. $n - 6 = 3$ | 8. $16 = p - 5$ | 9. $42 - k = 27$ |
| 10. $10 - x = 4$ | 11. $16 - t = 5$ | 12. $2n - 4 = 8$ |
| 13. $3j + 2 = 23$ | 14. $14 - 2v = 6$ | 15. $11 + 7p = 39$ |
| 16. $5w + 2 = 27$ | 17. $26 = 3z + 2$ | 18. $36 - 5a = 1$ |
| 19. $\frac{x + 5}{2} = 9$ | 20. $\frac{y + 8}{2} = 10$ | 21. $\frac{y + (-1)}{2} = -3$ |
| 22. $c + 6 = 3c$ | 23. $6w = 3w + 12$ | 24. $2g = 5g - 6$ |
| 25. $8t + 6 = 10t - 4$ | 26. $37 - g = 4g + 2$ | 27. $18 - 3h = 26 - 5h$ |
| 28. $6f + 15 = 11f$ | 29. $q + 9 = 3q - 1$ | 30. $8r - 1 = 5r + 17$ |

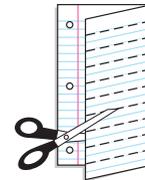
FOLDABLES™ Study Organizer

Make this Foldable to help you organize your Chapter 2 notes. Begin with a sheet of notebook paper.

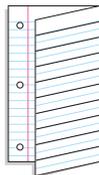
- 1 **Fold** lengthwise to the holes.



- 2 **Cut** along the top line and then cut 10 tabs.



- 3 **Label** each tab with a highlighted term from the chapter.



Reading and Writing Store your Foldable in a 3-ring binder. As you read and study the chapter, write definitions of important terms and examples under each tab.



Real Numbers and Number Lines

What You'll Learn

You'll learn to find the distance between two points on a number line.

Why It's Important

Weather

Meteorologists use the Ruler Postulate to determine the difference between temperatures on a thermometer. See Exercise 30.

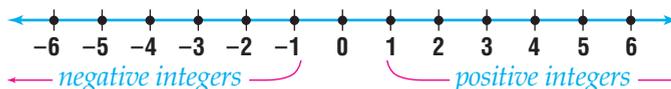
Numbers that share common properties can be *classified* or grouped into sets. Different sets of numbers can be shown on number lines.

Whole Numbers



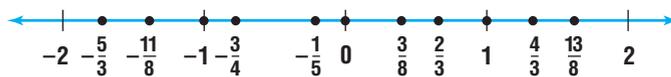
This figure shows the set of **whole numbers**. The whole numbers include 0 and the **natural**, or counting, **numbers**. The arrow to the right indicates that the whole numbers continue indefinitely. Zero is the least whole number.

Integers



A number line can be used to represent the set of **integers**. Integers include 0, the positive integers, and the negative integers. The arrows indicate that the numbers go on forever in both directions.

Rational Numbers



A number line can also show **rational numbers**. A rational number is any number that can be written as a fraction, $\frac{a}{b}$, where a and b are integers and b cannot equal 0. The number line above shows some of the rational numbers between -2 and 2 . In fact, there are infinitely many rational numbers between any two integers.

Rational numbers can also be represented by decimals.

$$\frac{3}{8} = 0.375 \qquad \frac{2}{3} = 0.666 \dots \qquad \frac{0}{5} = 0$$

Decimals may be **terminating** or **nonterminating**.

0.375 and 0.49 are terminating decimals.

0.666... and $-0.12345\dots$ are nonterminating decimals.

The three periods following the digits in the nonterminating decimals indicate that there are infinitely many digits in the decimal.



Read $0.\overline{17}$ as zero point one seven repeating.

Some nonterminating decimals have a repeating pattern.

$0.171717 \dots$ repeats the digits 1 and 7 to the right of the decimal point.

A bar over the repeating digits is used to indicate a repeating decimal.

$$0.171717 \dots = 0.\overline{17}$$

Each rational number can be expressed as a terminating decimal or a nonterminating decimal with a repeating pattern.

Irrational Numbers

Decimals that are nonterminating *and* do not repeat are called **irrational numbers**.

$6.028716 \dots$ and $0.101001000 \dots$ appear to be irrational numbers.

Real Numbers



Real numbers include both rational and irrational numbers. The number line above shows some real numbers between -2 and 2 .

Postulate 2-1 Number Line Postulate

Each real number corresponds to exactly one point on a number line. Each point on a number line corresponds to exactly one real number.

Examples

For each situation, write a real number with ten digits to the right of the decimal point.

- 1** a rational number less than 10 with a 3-digit repeating pattern

Sample answer:
 $5.1231231231 \dots$

- 2** an irrational number between -4 and -2

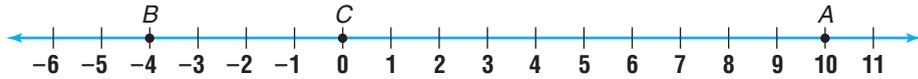
Sample answer:
 $-2.6366366636 \dots$

Your Turn

- a. a rational number greater than -10 with a 2-digit repeating pattern
- b. an irrational number between 1 and 2



The number that corresponds to a point on a number line is called the **coordinate** of the point. On the number line below, 10 is the coordinate of point A . The coordinate of point B is -4 . Point C has coordinate 0 and is called the **origin**.



The distance between two points A and B on a number line is found by using the Distance and Ruler Postulates.

**Postulate 2-2
Distance
Postulate**

Words: For any two points on a line and a given unit of measure, there is a unique positive real number called the **measure** of the distance between the points.



**Postulate 2-3
Ruler
Postulate**

Words: Points on a line are paired with the real numbers, and the measure of the distance between two points is the positive difference of the corresponding numbers.



Suppose you want to find the distance between points R and S on the number line below.



**Reading
Geometry**

RS represents the measure of the distance between points R and S .

The measure of the distance between points R and S is the positive difference $11 - 3$, or 8. The notation for the measure of the distance between two points is indicated by the capital letters representing the points. Since the measure from point S to point R is the same as from R to S , you can write $RS = 8$ or $SR = 8$.

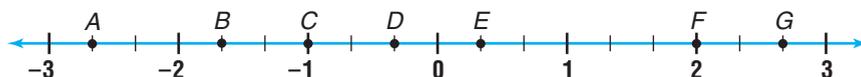
Another way to calculate the measure of the distance is by using **absolute value**. The absolute value of a number is the number of units a number is from zero on the number line. In symbols, the absolute value is denoted by two vertical slashes.

$$\begin{aligned} SR &= |11 - 3| & RS &= |3 - 11| \\ &= |8| & &= |-8| \\ &= 8 & &= 8 \end{aligned}$$



Example

3 Use the number line to find BE .



Algebra Review
Operations with
Fractions, p. 721

The coordinate of B is $-1\frac{2}{3}$, and the coordinate of E is $\frac{1}{3}$.

$$BE = \left| -1\frac{2}{3} - \frac{1}{3} \right| \quad \text{Subtract to find distance.}$$

$$= |-2| \quad \text{or } 2 \quad \text{Absolute values are nonnegative.}$$

Your Turn

Use the number line above to find each measure.

c. CF

d. AD

e. BG



Example Travel Link

4 Jamal traveled on I-71 from Grove City to Washington Courthouse. The Grove City entrance to I-71 is at the 100-mile marker, and the Washington Courthouse exit is at the 66-mile marker. How far did Jamal travel on I-71?



$$|100 - 66| = |34| \quad \text{or } 34 \quad \text{Ruler Postulate}$$

Jamal traveled 34 miles on I-71.

Check for Understanding

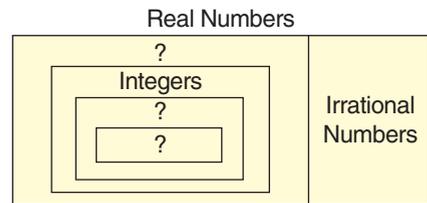
Communicating Mathematics

1. Explain why a number line has arrows at each end.
2. Write a problem that can be solved by finding $|9 - 17|$. What is the value of $|9 - 17|$?
3. Consider 0.34 , $0.\overline{34}$, and $0.\overline{3}4$.
 - a. How are these numbers alike? How are they different?
 - b. Which is greatest?
 - c. How would you read each number?

Vocabulary

whole numbers
natural numbers
integers
rational numbers
terminating decimals
nonterminating decimals
irrational numbers
real numbers
coordinate
origin
measure
absolute value

4. **Writing Math** Copy and complete the diagram at the right. Give two examples of each type of number represented in the large rectangle. Write a paragraph describing how this diagram shows the relationship among different sets of numbers.



Guided Practice
Examples 1 & 2

For each situation, write a real number with ten digits to the right of the decimal point.

5. an irrational number between 1 and 2
6. a rational number greater than 10 with a 2-digit repeating pattern

Example 3

Use the number line to find each measure.



7. CD
8. BF
9. EG

Example 3

10. **Geography** In the Netherlands, the higher region of the Dunes protects the lower region of the Polders from the sea. The Dunes rise to 25 feet above sea level. The lowest point of the Polders is 22 feet below sea level.
 - a. Represent these two numbers on a number line.
 - b. Find the distance between these two points on the number line.



The Netherlands

Exercises

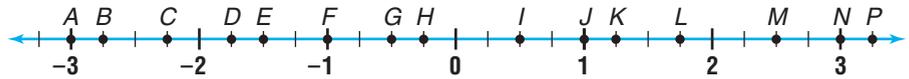
Practice

For each situation, write a real number with ten digits to the right of the decimal point.

11. a rational number less than 0 with a 2-digit repeating pattern
12. an irrational number between 5 and 6
13. a rational number greater than 3 with a 4-digit repeating pattern
14. a rational number between -3.5 and -4 with a 3-digit repeating pattern
15. two irrational numbers between 0 and 1
16. an irrational number between -7 and -6.8

Homework Help	
For Exercises	See Examples
11–16	1, 2
17–28, 31	3
29, 30	4
Extra Practice	
See page 728.	

Use the number line to find each measure.



17. AJ 18. AN 19. EG 20. IM 21. JK
 22. IN 23. FK 24. AP 25. CK 26. HM

27. Find the measure of the distance between B and J .
 28. What is the measure of the distance between D and L ?

Applications and Problem Solving

29. **Sports** Hatsu is practicing on a rock-climbing range. Markers on the wall indicate the number of feet she has climbed. When Hatsu started, she reached for a handhold at the 6-foot marker. She is now reaching for the 22-foot marker.
- How much higher is the current handhold than the first one?
 - If the highest handhold is at the 35-foot marker, how far does she need to climb?

interNET CONNECTED

Data Update For the latest information on the weather, visit www.geomconcepts.com

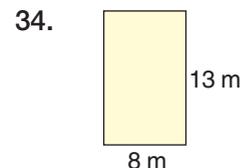
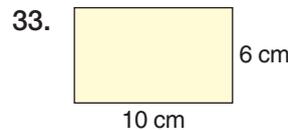
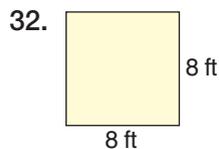
30. **Weather** The normal high and low temperatures for four cities for January are given in degrees Celsius. Find the measure of the difference between the two temperatures.

- a. Boston, 2°C , -6°C b. San Francisco, 13°C , 6°C
 c. Chicago, -2°C , -11°C d. Houston, 16°C , 4°C

31. **Critical Thinking** Name two points that are 7 units from -5 on the number line. (*Hint: Use a number line.*)

Mixed Review

Find the perimeter and area of each rectangle. (*Lesson 1–6*)



Name the tool needed to draw each figure. (*Lesson 1–5*)

35. circle 36. straight line

Standardized Test Practice

- (A) (B) (C) (D)

37. **Multiple Choice** Monsa purchased shoes that were originally priced at \$84.00. On that day, the store was having a 10% off sale. The sales tax was 7%. How much did Monsa pay for the shoes?
 (*Percent Review*)

- (A) \$70.31 (B) \$80.89 (C) \$85.93 (D) \$98.87

2-2

Segments and Properties of Real Numbers

What You'll Learn

You'll learn to apply the properties of real numbers to the measure of segments.

Why It's Important

Auto Repair Auto mechanics use measurement when repairing cars. See Exercise 29.

Given three collinear points on a line, one point is always between the other two points. In the figure below, point B is between points A and C .

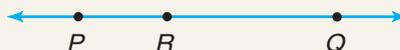


Point B lies to the right of point A and to the left of point C . **Betweenness** is also defined in terms of distances.

Definition of Betweenness

Words: Point R is between points P and Q if and only if R , P , and Q are collinear and $PR + RQ = PQ$.

Model:



Symbols:

$$PR + RQ = PQ$$

If and only if means that both the statement and its converse are true. Statements that include this phrase are called biconditionals.

Unless stated otherwise, betweenness and collinearity of points may be assumed if they are given in a figure.

Example

1 Points A , B , and C are collinear. If $AB = 12$, $BC = 48$, and $AC = 36$, determine which point is between the other two.

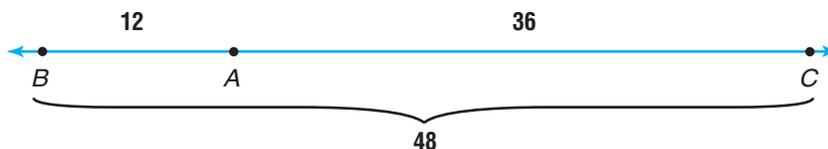
Check to see which two measures add to equal the third.

$$12 + 36 = 48$$

$$BA + AC = BC$$

Therefore, A is between B and C .

Check: You can check by modeling the distances on a number line. Let 12 units = 1 inch.



The solution checks.

Your Turn

a. Points R , S , and T are collinear. If $RS = 42$, $ST = 17$, and $RT = 25$, determine which point is between the other two.

Segment measures are real numbers. Let's review some of the properties of real numbers relating to equality.

Properties of Equality for Real Numbers	
Reflexive Property	For any number a , $a = a$.
Symmetric Property	For any numbers a and b , if $a = b$, then $b = a$.
Transitive Property	For any numbers a , b and c , if $a = b$ and $b = c$, then $a = c$.
Addition and Subtraction Properties	For any numbers a , b , and c , if $a = b$, then $a + c = b + c$ and $a - c = b - c$.
Multiplication and Division Properties	For any numbers a , b , and c , if $a = b$, then $a \cdot c = b \cdot c$, and if $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Substitution Property	For any numbers a and b , if $a = b$, then a may be replaced by b in any equation.

A statement that includes the symbol $=$ is an **equation** or equality. You can use equations to solve problems in geometry.

Example

Algebra Link

Algebra Review
Solving One-Step Equations, p. 722

2 If $QS = 29$ and $QT = 52$, find ST .



$$\begin{array}{ll}
 QS + ST = QT & \text{Definition of betweenness} \\
 29 + ST = 52 & \text{Substitution Property} \\
 29 + ST - 29 = 52 - 29 & \text{Subtraction Property} \\
 ST = 23 & \text{Substitution Property}
 \end{array}$$

Your Turn

b. Refer to the line above. If $PR = 27$ and $PT = 73$, find RT .

Measurements, such as 10 centimeters and 4 inches, are composed of two parts: the measure and the **unit of measure**. The measure of a segment gives the number of units. When only measures are given in a figure in this text, you can assume that all of the measures in the figure have the same unit of measure.



The measure of \overline{AB} is 9, and $AB = 9$. The unit of measure is the centimeter. So, the measurement of \overline{AB} is 9 centimeters.



The measurement of a segment is also called the *length* of the segment.

Example

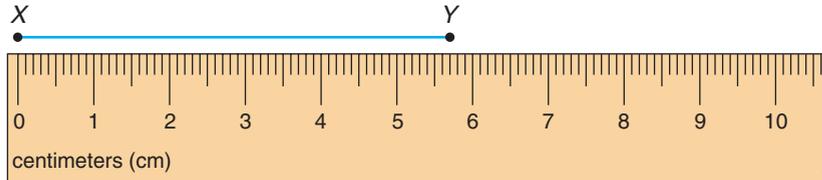
3 Find the length of \overline{XY} in centimeters and in inches.



Use a metric ruler to measure the segment. Put the 0 point at point X.

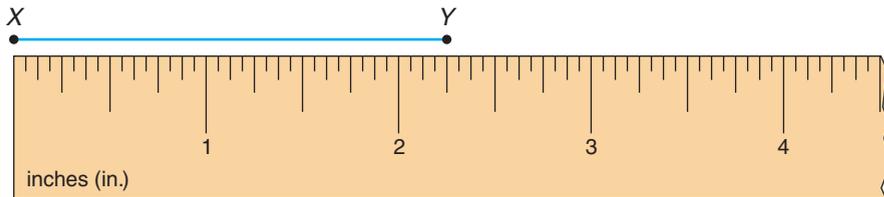
Caution: This point may not be at the end of the ruler.

Then measure the distance to Y on the metric scale.



The length of \overline{XY} is 5.7 centimeters.

Use a customary ruler to measure \overline{XY} in inches. Put the 0 point at X and measure the distance to Y.



The length of \overline{XY} is $2\frac{1}{4}$ inches.

The **precision** of a measurement depends on the smallest unit used to make the measurement. The **greatest possible error** is half the smallest unit used to make the measurement. The **percent of error** is found by comparing the greatest possible error with the measurement itself.

$$\text{percent of error} = \frac{\text{greatest possible error}}{\text{measurement}} \times 100\%$$

Compare the two measurements of \overline{XY} in Example 3.

Centimeters	Inches
measurement: 5.7 cm or 57 mm	measurement: $2\frac{1}{4}$ (or 2.25) in.
precision: 1 mm	precision: $\frac{1}{16}$ in.
greatest possible error: 0.5 mm	greatest possible error: $\frac{1}{32}$ (or 0.03125) in.
percent of error: $\frac{0.5}{57} \times 100\%$ or about 0.88%	percent of error: $\frac{0.03125}{2.25} \times 100\%$ or about 1.39%



Check for Understanding

Communicating Mathematics

1. Write a sentence that explains the difference between the *measure* and the *measurement* of a segment.
2. Name some units of measure for length.
3. **You Decide?** Jalisa says that the most precise measurement for a can of corn would be 2 pounds. Joseph says that 34 ounces is more precise. Who is correct, and why?

Vocabulary

betweenness
equation
measurement
unit of measure
precision
greatest possible error
percent of error

Guided Practice

Example 1

Three segment measures are given. The three points named are collinear. Determine which point is between the other two.

4. $TM = 21$, $MH = 37$, $TH = 16$
5. $XZ = 36$, $YZ = 17$, $XY = 19$

Example 2

Refer to the line for Exercises 6–7.



6. If $AB = 23$ and $AD = 51$, find BD .
7. If $CD = 19$ and $AC = 38$, find AD .

Example 3

Find the length of each segment in centimeters and in inches.

8. _____
9. _____

Example 2

10. **Travel** Emilio drives on Route 40 from Little Rock to Nashville. He stops in Memphis for lunch. The distance from Little Rock to Memphis is 139 miles, and the distance from Little Rock to Nashville is 359 miles. How far does Emilio need to travel after lunch to reach Nashville?

Exercises

Practice

Three segment measures are given. The three points named are collinear. Determine which point is between the other two.

11. $AD = 25$, $ED = 33$, $AE = 58$
12. $RS = 45$, $TS = 19$, $RT = 26$
13. $GH = 44$, $HK = 87$, $GK = 43$
14. $PQ = 34$, $QR = 71$, $PR = 37$
15. $AB = 32$, $BC = 13.8$, $AC = 18.2$
16. $WV = 27.6$, $VZ = 35.8$, $WZ = 8.2$



Homework Help

For Exercises	See Examples
11–16	1
17–22, 31	2
23–29	3
Extra Practice	
See page 728.	

Refer to the line for Exercises 17–22.



17. If $RS = 19$ and $RV = 71$, find SV .
18. If $UV = 17$ and $SU = 38$, find SV .
19. If $VX = 13$ and $SX = 30$, find SV .
20. If $TW = 81$ and $VW = 35$, find TV .
21. If $SW = 44.5$ and $SV = 37.1$, find VW .
22. If $TU = 15.9$ and $UW = 28.3$, find TW .

Find the length of each segment in centimeters and in inches.

23. _____
24. _____
25. _____
26. _____
27. _____
28. _____

Applications and Problem Solving

29. **Auto Mechanics** Lucille Treganowan is a grandmother with a weekly TV show on auto repair. She uses a socket wrench to tighten and loosen bolts on cars. Measure the distance across the head of each bolt in millimeters to find the size of socket needed for the bolt.



- a.
- b.
- c.

30. **Clothing** The sizes of men's hats begin at $6\frac{1}{4}$ and go up by $\frac{1}{8}$ inch. How precise are the hat sizes?
31. **Critical Thinking** If $AB = 5$, $BD = 14$, $CE = 19$, and $AE = 35$, find BC , CD , and DE .



Mixed Review

Use the number line to find each measure. (Lesson 2-1)



32. BG

33. AF

34. DE

35. **Photography** The outer edges of a picture frame are 21 inches by 15 inches. The sides of the frame are 2 inches wide. (Lesson 1-6)

- Draw a picture to represent the frame. Label all the information presented in the problem.
- Find the area of a picture that will show in this frame.



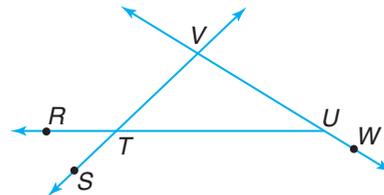
Standardized Test Practice

- (A) (B) (C) (D)

36. **Short Response** Describe the intersection of two planes. (Lesson 1-3)

37. **Multiple Choice** Which point is collinear with T and U ? (Lesson 1-2)

- (A) R (B) S
 (C) V (D) W



Quiz 1

Lessons 2-1 and 2-2

- Write a rational number between 4 and 5 with a 3-digit repeating pattern. Name ten digits to the right of the decimal point. (Lesson 2-1)
- Use the number line to find PQ . (Lesson 2-1)



- Points R , S , and T are collinear. If $RS = 71$, $ST = 55$, and $RT = 16$, determine which point is between the other two. (Lesson 2-2)
- Refer to the line below. If $AB = 28$ and $AC = 44$, find BC . (Lesson 2-2)



- Find the length of the line segment in centimeters and in inches. (Lesson 2-2)



2-3

Congruent Segments

What You'll Learn

You'll learn to identify congruent segments and find the midpoints of segments.

Why It's Important

Construction

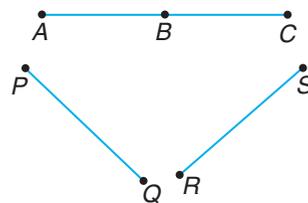
Builders use congruent segments to frame houses. See Exercise 1.

In geometry, two segments with the same length are called **congruent segments**.

Definition of Congruent Segments

Two segments are congruent if and only if they have the same length.

In the figures at the right, \overline{AB} is congruent to \overline{BC} , and \overline{PQ} is congruent to \overline{RS} . The symbol \cong is used to represent congruence.



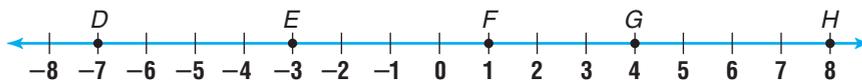
$$\overline{AB} \cong \overline{BC} \text{ and } \overline{PQ} \cong \overline{RS}$$

From the definition of congruent segments, we can also say $AB = BC$ and $PQ = RS$.

Example

- 1 Use the number line to determine whether the statement is *true* or *false*. Explain your reasoning.

\overline{DF} is congruent to \overline{EG} .



Because $DF = 8$ and $EG = 7$, $DF \neq EG$. So, \overline{DF} is not congruent to \overline{EG} , and the statement is false.

Your Turn

- a. Is the statement $\overline{EG} \cong \overline{FH}$ true or false? Explain your reasoning.



Read $\overline{EG} \cong \overline{FH}$ as segment EG is congruent to segment FH .

Since congruence is related to the equality of segment measures, there are properties of congruence that are similar to the corresponding properties of equality. These statements are called **theorems**. Theorems are statements that can be justified by using logical reasoning.



We know that $AB = AB$. Therefore, $\overline{AB} \cong \overline{AB}$ and we can see that congruence is reflexive. You can make similar arguments to show congruence is symmetric and transitive.

Theorem	Words	Symbols
2-1	Congruence of segments is reflexive.	$\overline{AB} \cong \overline{AB}$
2-2	Congruence of segments is symmetric.	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.
2-3	Congruence of segments is transitive.	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example

- 2 Determine whether the statement is *true* or *false*. Explain your reasoning.

\overline{JK} is congruent to \overline{KJ} .

Congruence of segments is reflexive, so $\overline{JK} \cong \overline{JK}$. We know that \overline{KJ} is another name for \overline{JK} . By substitution, $\overline{JK} \cong \overline{KJ}$. The statement is true.

Your Turn

- b. If $\overline{AB} \cong \overline{CD}$ and $\overline{DC} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

There is a unique point on every segment called the **midpoint**. On the number line below, M is the midpoint of \overline{ST} . What do you notice about SM and MT ?



Definition of Midpoint

Words: A point M is the midpoint of a segment \overline{ST} if and only if M is between S and T and $SM = MT$.

Model:

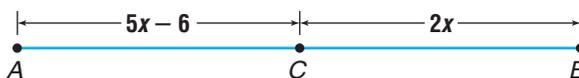
Symbols: $SM = MT$

The midpoint of a segment separates the segment into two segments of equal length. So, by the definition of congruent segments, the two segments are congruent.



Example**Algebra Link**

3 In the figure, C is the midpoint of \overline{AB} . Find the value of x .



Explore You are given a segment and its midpoint. You want to find the value of x .

Plan Since C is the midpoint of \overline{AB} , $AC = CB$. Use this information to write an equation involving x , and solve for x .

Solve $AC = CB$ *Definition of Midpoint*

$$5x - 6 = 2x \quad \textit{Substitute.}$$

$$5x - 6 - 5x = 2x - 5x \quad \textit{Subtract } 5x \textit{ from each side.}$$

$$-6 = -3x \quad \textit{Simplify.}$$

$$\frac{-6}{-3} = \frac{-3x}{-3} \quad \textit{Divide each side by } -3.$$

$$2 = x \quad \textit{Simplify.}$$

Examine Replace x with 2 to find AC and CB .

$$AC = 5x - 6 \quad \textit{Original equation} \quad CB = 2x$$

$$= 5(2) - 6 \quad \textit{Substitution Property} \quad = 2(2)$$

$$= 4 \quad \textit{Simplify.} \quad = 4$$

Since $AC = CB$, C is the midpoint of \overline{AB} , and the answer is correct.

Algebra Review

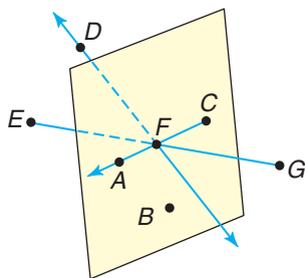
Solving Multi-Step Equations, p. 723

Your Turn

c. In the figure below, W is the midpoint of \overline{XY} . Find the value of a .



To **bisect** something means to separate it into two congruent parts. The midpoint of a segment bisects the segment because it separates the segment into two congruent segments. A point, line, ray, segment, or plane can also bisect a segment.



Point F bisects \overline{EG} .

\overline{FD} bisects \overline{EG} .

\overline{FA} bisects \overline{EG} .

\overline{AC} bisects \overline{EG} .

Plane ABC bisects \overline{EG} .

The midpoint of the segment must be found to separate a segment into two congruent segments. If the segment is part of a number line, you can use arithmetic to find the midpoint. If there is no number line, you can use a construction to find the midpoint.

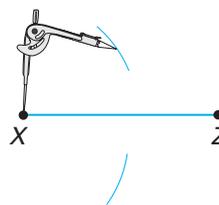
Hands-On Geometry Construction

Materials:  compass  straightedge

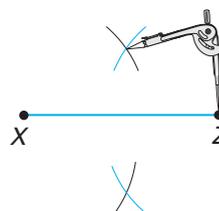
Step 1 Use a straightedge to draw the segment you wish to bisect. Name it \overline{XZ} .



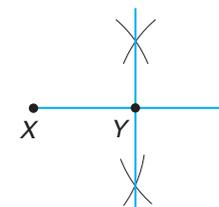
Step 2 Place the compass at point X. Use any compass setting greater than one half of \overline{XZ} . Draw an arc above and below \overline{XZ} .



Step 3 Using the same compass setting, place the compass at point Z. Draw an arc above and below \overline{XZ} . These arcs should intersect the ones previously drawn.



Step 4 Use a straightedge to align the two intersections. Draw a segment that intersects \overline{XZ} . Label the point of intersection Y.



Try These

1. Measure \overline{XY} and \overline{YZ} . What can you conclude about point Y?
2. Fold \overline{XZ} so that Z is over X. Does this confirm your conclusion in Exercise 1?
3. Can you make any other conjectures about the line segment that intersects \overline{XZ} ?

Check for Understanding

Communicating Mathematics

1. Draw two diagrams or find two photographs that illustrate the use of congruent segments when building houses in the area where you live.

Vocabulary

congruent segments
theorem
midpoint
bisect



2. a. Explain why segment congruence is symmetric.
- b. Explain why segment congruence is transitive.

Guided Practice

Example 1

Use the number line to determine whether each statement is *true* or *false*. Explain your reasoning.



3. \overline{AB} is congruent to \overline{CD} .
4. D is the midpoint of \overline{CE} .

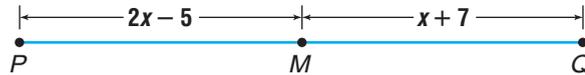
Example 2

Determine whether each statement is *true* or *false*. Explain your reasoning.

5. If $\overline{XY} \cong \overline{YZ}$, then Y is the midpoint of \overline{ZY} .
6. If $\overline{RS} \cong \overline{CD}$, then $\overline{CD} \cong \overline{RS}$.

Example 3

7. **Algebra** In the figure below, M is the midpoint of \overline{PQ} . Find the value of x .



Exercises

Practice

Use the number line to determine whether each statement is *true* or *false*. Explain your reasoning.



8. \overline{DG} is congruent to \overline{GJ} .
9. \overline{BF} is congruent to \overline{EI} .
10. \overline{AG} is congruent to \overline{DJ} .
11. F is the midpoint of \overline{BI} .
12. E is the midpoint of \overline{BH} .
13. D is the midpoint of \overline{CF} .

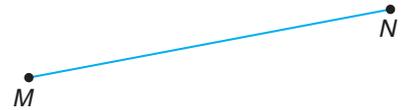
Determine whether each statement is *true* or *false*. Explain your reasoning.

14. If $XY = YZ$, then $\overline{XY} \cong \overline{YZ}$.
15. If $\overline{AB} \cong \overline{BC}$, $\overline{XY} \cong \overline{FG}$, and $\overline{BC} \cong \overline{FG}$, then $\overline{AB} \cong \overline{XY}$.
16. Every segment has only one bisector.
17. A plane can bisect a segment at an infinite number of points.
18. If $\overline{RS} \cong \overline{ST}$, then S is the midpoint of \overline{RT} .
19. If points D , E , and F are collinear and E is not between D and F , then F is between D and E .

Homework Help	
For Exercises	See Examples
8–10	1
11–13, 21, 22	3
15	2
Extra Practice	
See page 728.	

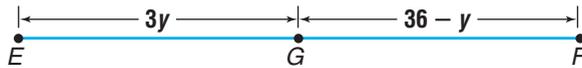


20. Draw a segment like \overline{MN} on your paper. Then use a compass and straightedge to bisect the segment.

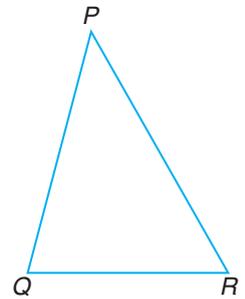


Applications and Problem Solving

21. **Algebra** In the figure below, G is the midpoint of \overline{EF} .



- Find the value of y .
 - Find EG and GF .
 - Find EF .
22. **Science** The center of mass of an object is the point where the object can be balanced in all directions. Draw the shape of a triangular object like the one at the right. Use the following steps to find its center of mass.
- Find the midpoint of each side of the triangle.
 - Draw a segment between the midpoint of \overline{QR} and P .
 - Draw a segment between the midpoint of \overline{PR} and Q .
 - Draw a segment between the midpoint of \overline{PQ} and R .
 - The center of mass is the point where these three segments intersect. Label the center of mass C .
23. **Critical Thinking** In the figure below, C is any point between A and B , E is the midpoint of \overline{AC} , and F is the midpoint of \overline{CB} . Write a ratio comparing AB to EF .



Mixed Review

Three segment measures are given. The three points named are collinear. Determine which point is between the other two. (Lesson 2-2)

24. $MN = 17$, $NP = 6.5$, $MP = 23.5$ 25. $RS = 7.1$, $TR = 2.9$, $TS = 4.2$
26. Write an irrational number between 0 and -2 that has ten digits to the right of the decimal point. (Lesson 2-1)

Standardized Test Practice

(A) (B) (C) (D)

27. **Grid In** A soccer field is a rectangle that is 100 meters long and 73 meters wide. Find the area of the soccer field in square meters. (Lesson 1-6)

28. **Multiple Choice** Solve $2y + 3 = 9$. (Algebra Review)

(A) 6 (B) 5
(C) 4 (D) 3



2-4

The Coordinate Plane

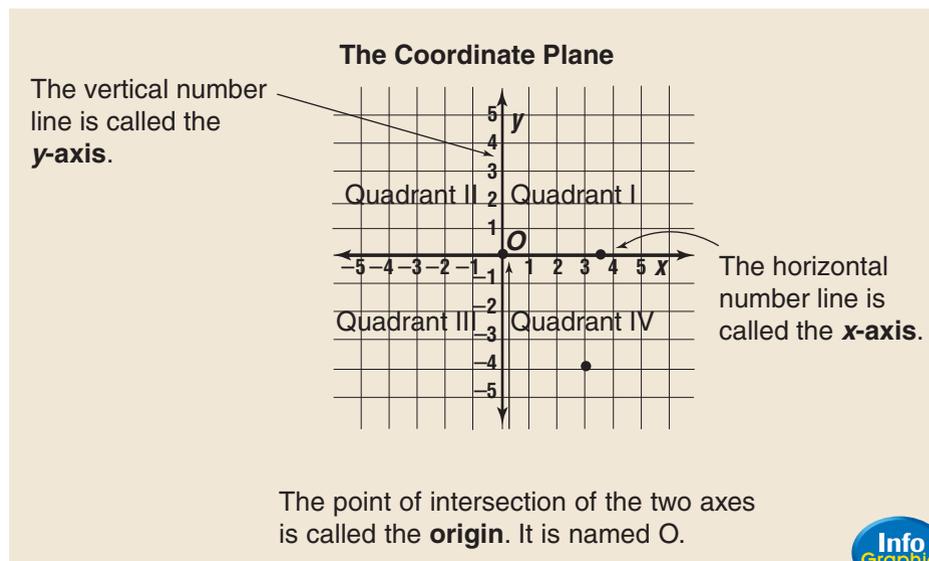
What You'll Learn

You'll learn to name and graph ordered pairs on a coordinate plane.

Why It's Important

Art Artists can use grids to locate points in the same manner as points are located on a coordinate plane. See Exercise 1.

In coordinate geometry, grid paper is used to locate points. The plane of the grid is called the **coordinate plane**.



The two axes separate the plane into four regions called **quadrants**. Points can lie in one of the four quadrants or on an axis. The points on the x -axis to the right of the origin correspond to positive numbers. To the left of the origin, the points correspond to negative numbers. The points on the y -axis above the origin correspond to positive numbers. Below the origin, the points correspond to negative numbers.

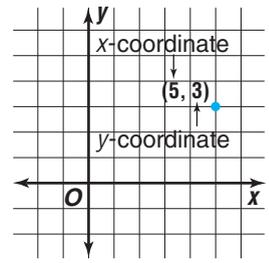
An **ordered pair** of real numbers, called the **coordinates** of a point, locates a point in the coordinate plane. Each ordered pair corresponds to exactly one point in the coordinate plane.

The point in the coordinate plane is called the **graph** of the ordered pair. Locating a point on the coordinate plane is called *graphing* the ordered pair.

Postulate 2-4 Completeness Property for Points in the Plane

Each point in a coordinate plane corresponds to exactly one ordered pair of real numbers. Each ordered pair corresponds to exactly one point in a coordinate plane.

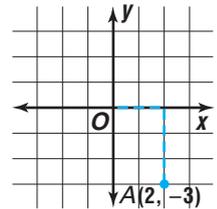
The figure at the right shows the graph of the ordered pair $(5, 3)$. The first number, 5, is called the **x -coordinate**. It tells the number of units the point lies to the left or right of the origin. The second number, 3, is called the **y -coordinate**. It tells the number of units the point lies above or below the origin. What are the coordinates of the origin?



Examples

1 Graph point A at $(2, -3)$.

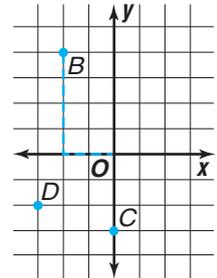
Start at the origin. Move 2 units to the right. Then, move 3 units down. Label this point A .
The location of A at $(2, -3)$ is also written as $A(2, -3)$.



2 Name the coordinates of points B and C .

Point B is 2 units to the left of the origin and 4 units above the origin. Its coordinates are $(-2, 4)$.

Point C is zero units to the left or right of the origin and 3 units below the origin. Its coordinates are $(0, -3)$.



Your Turn

- Graph point E at $(-3, 4)$.
- Name the coordinates of point D .

Hands-On Geometry

Materials:  grid paper

- Draw lines representing the x -axis and y -axis on a piece of grid paper. Label the x -axis, y -axis, and the origin.
- Graph the points $P(3, 4)$, $Q(3, 0)$, $R(3, -1)$, and $S(3, -3)$.

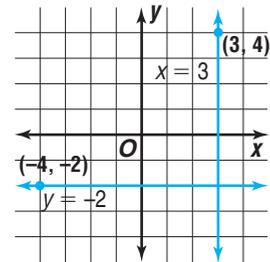
(continued on the next page)



Try These

1. What do you notice about the graphs of these points?
2. What do you notice about the x -coordinates of these points?
3. Name and graph three other points with an x -coordinate of 3. What do you notice about these points?
4. Write a general statement about ordered pairs that have the same x -coordinate.
5. Now graph $T(-4, -2)$, $U(0, -2)$, $V(1, -2)$, and $W(2, -2)$.
6. What do you notice about the graphs of these points?
7. What do you notice about the y -coordinates of these points?
8. Write a general statement about ordered pairs that have the same y -coordinate.

Horizontal and vertical lines in the coordinate plane have special characteristics. All lines can be described, or named, by equations. If a vertical line passes through $(3, 4)$, then the x -coordinate of all points on the line is 3. If a horizontal line passes through $(-4, -2)$, then the y -coordinate of all points on the line is -2 .



Theorem 2-4 summarizes this relationship for any vertical or horizontal line.

Theorem 2-4

If a and b are real numbers, a vertical line contains all points (x, y) such that $x = a$, and a horizontal line contains all points (x, y) such that $y = b$.

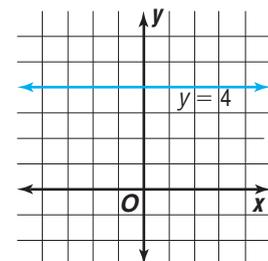
The equation of a vertical line is $x = a$, and the equation of a horizontal line is $y = b$.

Example

Algebra Link

3 Graph $y = 4$.

The graph of $y = 4$ is a horizontal line that intersects the y -axis at 4.



Your Turn

c. Graph $x = -3$.

Check for Understanding

Communicating Mathematics

- Describe how an artist can use a grid to create a larger or smaller drawing.
- Graph several points that form a horizontal line. Describe the common coordinate for each of these points.
 - Graph several points that form a vertical line. Describe the common coordinate for each of these points.
- Writing Math** List at least five words that start with *quad*. Recall that the x - and y -axes divide the coordinate plane into four regions called *quadrants*. Consult a dictionary to see if all the words in your list relate to the number four.

Vocabulary

coordinate plane
 y -axis
 x -axis
 quadrant
 origin
 ordered pair
 coordinates
 graph
 x -coordinate
 y -coordinate

Guided Practice

Getting Ready

Name the x -coordinate and y -coordinate of each ordered pair.

Sample: $(-7, 2)$

Solution: $x = -7, y = 2$

4. $(0, -2)$ 5. $(-3, -6)$ 6. $(5, 8)$ 7. $(11, 0)$

Example 1

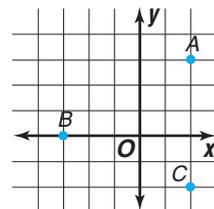
Draw and label a coordinate plane on a piece of grid paper. Then graph and label each point.

8. $M(-6, -2)$ 9. $J(2, 0)$ 10. $P(-5, 3)$

Example 2

Refer to the coordinate plane at the right. Name the ordered pair for each point.

11. A
 12. B
 13. C



Exercises 11–13

Example 3

14. **Algebra** Graph $x = -5$.

Exercises

Practice

Draw and label a coordinate plane on a piece of grid paper. Then graph and label each point.

15. $T(0, -1)$ 16. $R(-2, -4)$ 17. $Q(5, 5)$
 18. $C(0, 5)$ 19. $N(1, -5)$ 20. $S(3, 6)$
 21. $G(-4, 0)$ 22. $L(-1, 4)$ 23. $F(6, -2)$



Homework Help

For Exercises

See Examples

24–31, 35

2

34

3

36, 37

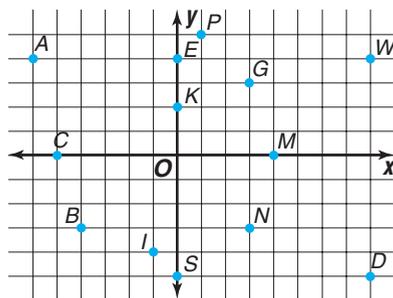
1

Extra Practice

See page 729.

Applications and Problem Solving

Name the ordered pair for each point on the coordinate plane at the right.



24. A 25. I
26. W 27. P
28. D 29. S
30. B 31. C
32. What point is located at $(4, 0)$?
33. Name the point at $(3, -3)$.
34. **Algebra** Graph $y = 6$.
35. **Geography** In geography, places are located using latitude (horizontal) and longitude (vertical) lines in much the same way as points are located in a coordinate plane.
- Name the city that is located at 30°N and 90°W .
 - State the latitude and longitude of St. Petersburg. Round to the nearest ten.
 - Suppose you are standing at 30°S and 20°E . Name the country you are visiting.
 - State the latitude and longitude of the city or town where you live.
36. **Science** The average weight and top speeds of various animals are given below.

Animal	Avg. Weight (pounds)	Top Speed (miles per hour)
Cheetah	128	70
Chicken	7	9
Coyote	75	43
Fox	14	42
Horse	950	43
Polar Bear	715	35
Rabbit (domestic)	8	35

Sources: *Comparisons and The World Almanac*

- If the x -coordinate of an ordered pair represents the average weight and the y -coordinate represents the top speed, then $(128, 70)$ would represent the cheetah. Write an ordered pair for each animal.
- Graph the ordered pairs.
- Look for patterns in the graph. Are larger animals usually faster or slower than smaller animals?



37. **Critical Thinking** Graph $A(-3, -2)$ and $B(2, -2)$. Draw \overline{AB} . Find the coordinates of two other points that when connected with A and B would form a 5-by-3 rectangle.

Mixed Review

Use the number line to determine whether each statement is *true* or *false*. Explain your reasoning. (Lesson 2-3)



38. D is the midpoint of \overline{CE} . 39. $\overline{AC} \cong \overline{CE}$

Refer to the line below for Exercises 40–42. (Lesson 2-2)



40. If $XY = 14$ and $YZ = 27$, find XZ .
 41. If $WX = 15$ and $WZ = 54$, find XZ .
 42. If $WY = 21$ and $YZ = 21$, find WZ .

Standardized Test Practice

(A) (B) (C) (D)

43. **Short Response** Write the following statement in if-then form. (Lesson 1-4)
Students who do their homework will pass the course.
44. **Multiple Choice** Charo walks 15 minutes the first day, 22 minutes the second day, and 29 minutes the third day. If she continues this pattern, how many minutes will Charo walk the fifth day? (Lesson 1-1)
- (A) 33 min (B) 36 min
 (C) 39 min (D) 43 min



Quiz 2

Lessons 2-3 and 2-4

Use the number line to determine whether each statement is *true* or *false*. Explain your reasoning. (Lesson 2-3)



1. $\overline{AC} \cong \overline{EF}$ 2. $\overline{AB} \cong \overline{CE}$ 3. D is the midpoint of \overline{AF} .

Draw and label a coordinate plane on a piece of grid paper. Then graph and label each point. (Lesson 2-4)

4. $G(-2, 4)$ 5. $H(0, -3)$



“V”s for Vector

Materials

 centimeter grid paper

 uncooked spaghetti

 red and blue markers

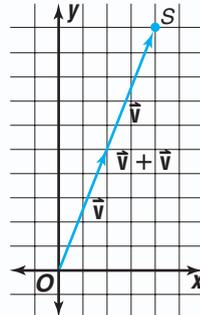
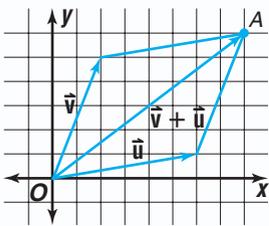
Vectors

What looks like a ray and is used in navigation, animation, and meteorology? The answer is a **vector**. A vector is a directed line segment. The length of a vector is called its *magnitude*, and the arrowhead of the vector shows its direction. Vectors are used to show movement in a certain direction.



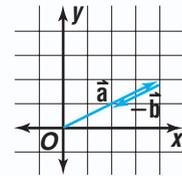
The magnitude of vector \mathbf{a} is 1 inch.

Investigate

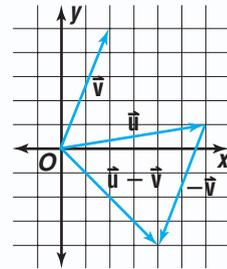
- Use a sheet of centimeter grid paper and some uncooked spaghetti to model addition of vectors.
 - Draw and label a coordinate plane on centimeter grid paper. Let each centimeter represent one unit. Place $(0, 0)$ at the center of the grid.
 - Break two pieces of spaghetti so that each is the length of a segment that goes from $(0, 0)$ to $(2, 5)$. Mark one end of each piece with a red marker. Each of these represents vector \vec{v} or $(2, 5)$.
 - Repeat this process to make two pieces of spaghetti that are the length of a segment that goes from $(0, 0)$ to $(6, 1)$. Mark one end of each piece with a blue marker. Each of these represents vector \vec{u} or $(6, 1)$. The colors at the ends of the spaghetti represent the arrowheads of the vectors.
 - To add two vectors with the same direction, lay them arrowhead (marked end) to tail (unmarked end) on the coordinate plane. Place your two \vec{v} vectors as shown. What are the coordinates of point S ? This is the vector representing the sum of the two red vectors.
 
 - To add vectors with different lengths and directions, form a parallelogram on your grid paper as shown at the left. The sum is represented by the diagonal of the parallelogram. The coordinates of point A are the vector sum. What is $\vec{v} + \vec{u}$?
 

2. Use a sheet of centimeter grid paper and some uncooked spaghetti to model subtraction of vectors.

a. Break two more pieces of spaghetti. One should be the length of a segment that goes from $(0, 0)$ to $(4, 2)$ and the other the length of a segment that goes from $(0, 0)$ to $(2, 1)$. To subtract $\vec{b} = (2, 1)$ from $\vec{a} = (4, 2)$, think of adding the opposite of \vec{b} to \vec{a} . The opposite of \vec{b} is a vector that points in the opposite direction as \vec{b} with the same length. Lay the spaghetti as shown. What is $\vec{a} - \vec{b}$?



b. Use your \vec{u} and \vec{v} vectors to model $\vec{u} - \vec{v}$ as shown. What is $-\vec{v}$? What is $\vec{u} - \vec{v}$?



Extending the Investigation

In this extension, you will determine how to add and subtract vectors and to multiply vectors by an integer.

Use grid paper and spaghetti vectors to find shortcuts for operations with vectors.

- Describe a way to add two vectors without using spaghetti or grid paper. Give at least three examples that verify your answer.
- Describe a way to subtract vectors without using spaghetti or grid paper. Give at least three examples that verify your answer.
- Describe a way to find the product of an integer and a vector \vec{v} .
 - First experiment using spaghetti and grid paper. Remember that multiplication is the same as repeated addition. Write 2 times the vector \vec{v} as $2\vec{v}$.
 - Now describe a way to multiply an integer and a vector without using spaghetti or grid paper. Give at least three examples of an integer times a vector. One example should have a negative integer as a factor, such as $-3\vec{v}$.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a poster that explains how to add and subtract vectors and to multiply vectors by an integer.
- Research the use of vectors in science. Write a report about your findings. Include at least three specific ways in which they are used and a real-life example of each.



Investigation For more information on vectors, visit: www.geomconcepts.com



What You'll Learn

You'll learn to find the coordinates of the midpoint of a segment.

Why It's Important**Interior Design**

Interior designers can determine where to place things by finding a midpoint. See Exercise 34.

The midpoint of a line segment, \overline{AB} , is the point C that bisects the segment.



$$\overline{AC} \cong \overline{CB} \text{ (or } AC = CB\text{)}$$

You can use a number line to find the coordinates of the midpoint of a line segment.

Hands-On Geometry

Materials:  grid paper  scissors  straightedge

- Step 1** Draw a number line and mark the coordinates of the points from -10 to 10 . Locate point A at -7 and point B at 5 .
- Step 2** Cut out \overline{AB} . Fold the segment so that points A and B are together. What is the coordinate of the midpoint of \overline{AB} ?
- Step 3** Find the sum of the coordinates of A and B . Divide the sum by 2 .

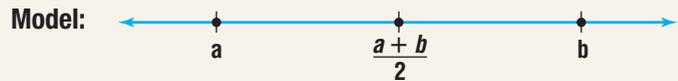
Try These

- How do the results of Steps 2 and 3 compare?
- On a number line, locate point C with coordinate 2 and point D with coordinate 10 . What is the coordinate of the midpoint of \overline{CD} ?
 - Find $(2 + 10) \div 2$.
 - Compare your answers.
- Repeat Exercise 2 with point E with coordinate -9 and point F with coordinate 1 . What are the results?
- Make a conjecture** about the coordinate of the midpoint of a line segment on a number line.

In the activity above, you discovered that the coordinate of the midpoint of a segment on the number line equals the sum of the coordinates of the endpoints divided by 2 .

Theorem 2-5
Midpoint
Formula for a
Number Line

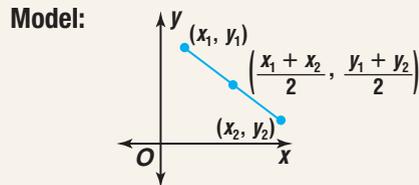
Words: On a number line, the coordinate of the midpoint of a segment whose endpoints have coordinates a and b is $\frac{a+b}{2}$.



A similar relationship is true for the midpoint of a segment on a coordinate plane.

Theorem 2-6
Midpoint
Formula for a
Coordinate
Plane

Words: On a coordinate plane, the coordinates of the midpoint of a segment whose endpoints have coordinates (x_1, y_1) and (x_2, y_2) are $(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$.



Example

1 Find the coordinate of the midpoint of \overline{RS} .



Use the Midpoint Formula to find the coordinate of the midpoint of \overline{RS} .

$$\begin{aligned} \frac{a+b}{2} &= \frac{-10+(-3)}{2} && \text{The coordinate of } R \text{ is } -10. \text{ So, } a = -10. \\ &= \frac{-13}{2} \text{ or } -6\frac{1}{2} && \text{The coordinate of } S \text{ is } -3. \text{ So, } b = -3. \\ &&& \text{Simplify.} \end{aligned}$$

The coordinate of the midpoint is $-6\frac{1}{2}$.

Your Turn

a. Refer to the number line above. Find the coordinate of the midpoint of \overline{RT} .



Examples

- 2 Find the coordinates of M , the midpoint of \overline{JK} , given endpoints $J(2, -9)$ and $K(8, 3)$.

Use the Midpoint Formula to find the coordinates of M .

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) &= \left(\frac{2 + 8}{2}, \frac{-9 + 3}{2} \right) & (x_1, y_1) &= (2, -9) \\ & & (x_2, y_2) &= (8, 3) \\ &= \left(\frac{10}{2}, \frac{-6}{2} \right) \text{ or } (5, -3) & \text{Simplify.} \end{aligned}$$

The coordinates of M are $(5, -3)$.

Your Turn

- b. Find the coordinates of N , the midpoint of \overline{VW} , given the endpoints $V(-4, -3)$ and $W(6, 11)$.
- c. Find the coordinates of Q , the midpoint of \overline{PR} , given the endpoints $P(-5, 1)$ and $R(2, -8)$.

Algebra Link

- 3 Suppose $G(8, -9)$ is the midpoint of \overline{FE} and the coordinates of E are $(18, -21)$. Find the coordinates of F .

Let (x_1, y_1) be the coordinates of F and let (x_2, y_2) or $(18, -21)$ be the coordinates of E . So, $x_2 = 18$ and $y_2 = -21$. Use the Midpoint Formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = (8, -9)$$

x-coordinate of F

$$\frac{x_1 + x_2}{2} = 8$$

$$\frac{x_1 + 18}{2} = 8$$

$$\frac{x_1 + 18}{2}(2) = 8(2)$$

$$x_1 + 18 = 16$$

$$x_1 + 18 - 18 = 16 - 18$$

$$x_1 = -2$$

Midpoint formula

Replace x_2 with 18 and y_2 with -21.

Multiply each side by 2.

Simplify.

Add or subtract to isolate the variable.

Simplify.

y-coordinate of F

$$\frac{y_1 + y_2}{2} = -9$$

$$\frac{y_1 + (-21)}{2} = -9$$

$$\frac{y_1 - 21}{2}(2) = -9(2)$$

$$y_1 - 21 = -18$$

$$y_1 - 21 + 21 = -18 + 21$$

$$y_1 = 3$$

The coordinates of F are $(-2, 3)$.

Your Turn

- d. Suppose $K(-10, 17)$ is the midpoint of \overline{IJ} and the coordinates of J are $(4, 12)$. Find the coordinates of I .
- e. Suppose $S\left(3, -\frac{3}{4}\right)$ is the midpoint of \overline{RT} and the coordinates of T are $(-2, 6)$. Find the coordinates of R .

Algebra Review

Solving Multi-Step Equations, p. 723



You can use a TI-83/84 Plus calculator to draw figures on a coordinate plane.

**Graphing
Calculator Tutorial**
See pp. 782–785.

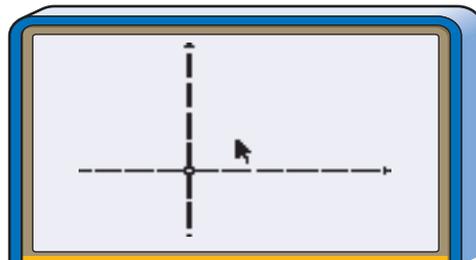


Graphing Calculator Exploration

Step 1 To display a coordinate plane, open the **F5** menu.

Step 2 Select Hide/Show and then select Axes.

The calculator will display a coordinate plane on which you can construct geometric figures.



Try These

1. Use the Segment tool on the **F2** menu to construct a segment in Quadrant I. Select the Midpoint tool on the **F3** menu and construct the midpoint of the segment. Use the Coordinates & Equation tool on the **F5** menu to display the coordinates of the endpoints and midpoint of the segment. What do you notice about the coordinates of the midpoint?
2. Drag one endpoint of the segment into Quadrant III. How do the coordinates of the midpoint change as you do this?
3. Open the **F5** menu. Select Measure, then select Distance & Length to display the distance from the midpoint to each endpoint of the segment. How are the two distances related? What happens to these distances if you drag an endpoint of the segment?

Check for Understanding

Communicating Mathematics

1. **Graph** $A(1, 3)$ and $B(5, 1)$. Draw \overline{AB} . Use your graph to estimate the midpoint of \overline{AB} . Check your answer by using the Midpoint Formula.
2. **Explain** why it is correct to say that the coordinates of the midpoint of a segment are the means of the coordinates of the endpoints of the segment.
3. **You Decide?** Fina wants to find the midpoint of a segment on a number line. She finds the length of the segment and divides by 2. She adds this number to the coordinate of the left endpoint to find the midpoint. Kenji says she should subtract the number from the coordinate of the right endpoint to find the midpoint. Who is correct? Explain your reasoning.



Guided Practice



Getting Ready Find the mean of each pair of numbers.

Sample: -4 and 10

Solution: $\frac{-4 + 10}{2} = \frac{6}{2}$ or 3

4. 4 and 8 5. -2 and 6 6. 5 and -6 7. -4 and -10

Example 1

Use the number line to find the coordinate of the midpoint of each segment.



8. \overline{AB}

9. \overline{AC}

Example 2

The coordinates of the endpoints of a segment are given. Find the coordinates of the midpoint of each segment.

10. $(-3, 6)$, $(-5, -2)$ 11. $(-8, 6)$, $(1, -3)$ 12. $(-3, 2)$, $(6, 5)$

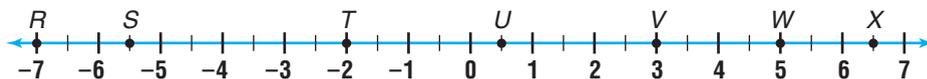
Example 3

13. **Algebra** Suppose $R(3, -5)$ is the midpoint of \overline{PQ} and the coordinates of P are $(7, -2)$. Find the coordinates of Q .

Exercises

Practice

Use the number line to find the coordinate of the midpoint of each segment.



14. \overline{RV}

15. \overline{TW}

16. \overline{UX}

17. \overline{SU}

18. \overline{TX}

19. \overline{ST}

The coordinates of the endpoints of a segment are given. Find the coordinates of the midpoint of each segment.

20. $(0, 4)$, $(0, 0)$ 21. $(-1, -2)$, $(-3, -6)$
 22. $(6, 0)$, $(13, 0)$ 23. $(4, 6)$, $(-2, -3)$
 24. $(-3, 2)$, $(-5, 6)$ 25. $(-1, -7)$, $(6, 1)$
 26. $(-8, 3)$, $(6, -6)$ 27. $(-18, 5)$, $(-3, -16)$
 28. (a, b) , $(0, 0)$ 29. (a, b) , (c, d)

30. Find the midpoint of the segment that has endpoints at $(-1, 6)$ and $(-5, -18)$.

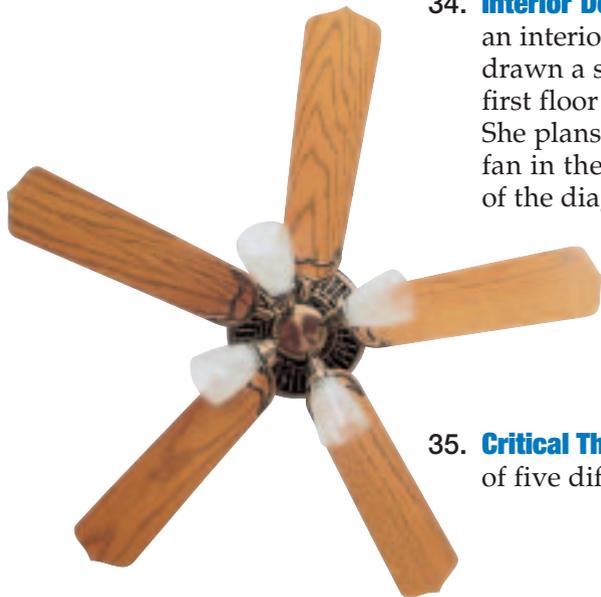
31. What is the midpoint of \overline{ST} if the endpoints are $S(2a, 2b)$ and $T(0, 0)$?

Homework Help	
For Exercises	See Examples
14–19	1
20–31, 33, 34	2
32	3
Extra Practice	
See page 729.	

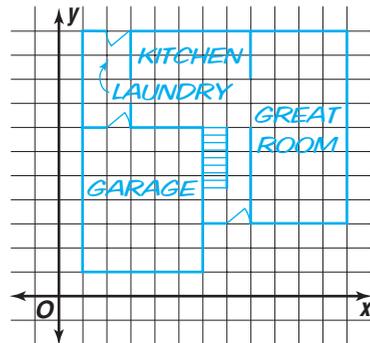


Applications and Problem Solving

32. **Algebra** Suppose $C(-4, 5)$ is the midpoint of \overline{AB} and the coordinates of A are $(2, 17)$. Find the coordinates of B .
33. **Travel** Donte is traveling on I-70 in Kansas. He gets on the interstate at the 128-mile marker and gets off the interstate at the 184-mile marker to go to Russell. Which mile marker is the midpoint of his drive on I-70?



34. **Interior Design** Chapa is an interior designer. She has drawn a scale model of the first floor of a client's house. She plans to install a paddle fan in the ceiling at the midpoint of the diagonals of the great room. Name the coordinates of the location for the fan.

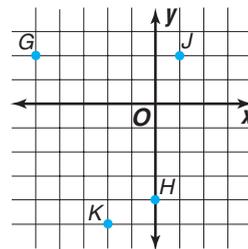


35. **Critical Thinking** Name the coordinates of the endpoints of five different segments with $M(6, 8)$ as the midpoint.

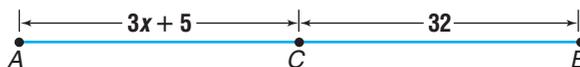
Mixed Review

Refer to the coordinate plane at the right. Name the ordered pair for each point. (Lesson 2-4)

36. G
 37. H
 38. J
 39. K



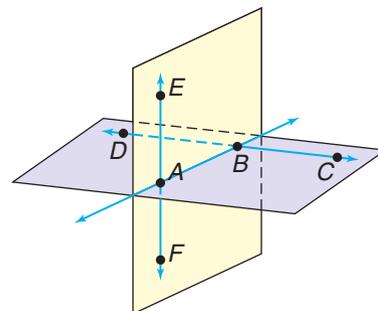
40. **Algebra** In the figure, C is the midpoint of \overline{AB} . Find the value of x . (Lesson 2-3)



Standardized Test Practice

- (A) (B) (C) (D)

41. **Short Response** Name the intersection of plane DAC and plane EBF . (Lesson 1-3)
42. **Short Response** How would you describe any three points that lie in the same plane? (Lesson 1-2)



Exercise 41



Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

Internet
CONTENTS Review Activities

For more review activities, visit:
www.geomconcepts.com

Geometry

betweenness (p. 56)
bisect (p. 64)
congruent segments (p. 62)
greatest possible error (p. 58)
measure (p. 52)
measurements (p. 58)
midpoint (p. 63)
percent of error (p. 58)
precision (p. 58)
theorems (p. 62)
unit of measure (p. 58)
vector (p. 74)

Algebra

absolute value (p. 52)
coordinate (p. 52)
coordinate plane (p. 68)
coordinates (p. 68)
equation (p. 57)
graph (p. 68)
integers (p. 50)
irrational numbers (p. 51)
natural numbers (p. 50)
nonterminating (p. 50)
ordered pair (p. 68)

origin (pp. 52, 68)
quadrants (p. 68)
rational numbers (p. 50)
real numbers (p. 51)
terminating (p. 50)
whole numbers (p. 50)
 x -axis (p. 68)
 x -coordinate (p. 69)
 y -axis (p. 68)
 y -coordinate (p. 69)

Choose the term or terms from the list above that best complete each statement.

- The ___?___ numbers include 0 and the natural numbers.
- A ___?___ is any number of the form $\frac{a}{b}$, where a and b are integers and b cannot equal zero.
- Decimals that are nonterminating and do not repeat are called ___?___ numbers.
- The number that corresponds to a point on a number line is called the ___?___ of the point.
- The number of units from zero to a number on the number line is called its ___?___.
- The second component of an ordered pair is called the ___?___.
- Two segments are ___?___ if and only if they have the same length.
- ___?___ are statements that can be justified using logical reasoning.
- To ___?___ a segment means to separate it into two congruent segments.
- The two axes separate a coordinate plane into four regions called ___?___.

Skills and Concepts

Objectives and Examples

- Lesson 2-1** Find the distance between two points on a number line.

Use the number line at the right to find BE .

$$BE = |-3 - 1| \quad \text{The coordinate of B is } -3.$$

$$= |-4| \text{ or } 4 \quad \text{The coordinate of E is } 1.$$

Review Exercises

Use the number line to find each measure.



- AD
- FH
- CG

Objectives and Examples

- **Lesson 2–2** Apply the properties of real numbers to the measure of segments.

If $XY = 39$ and $XZ = 62$, find YZ .



$$\begin{aligned} XY + YZ &= XZ && \text{Def. of Betweenness} \\ 39 + YZ &= 62 && \text{Substitution} \\ 39 + YZ - 39 &= 62 - 39 && \text{Subtraction} \\ YZ &= 23 && \text{Simplify.} \end{aligned}$$

- **Lesson 2–3** Identify congruent segments, and find the midpoints of segments.

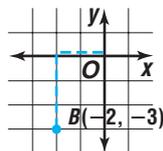
Determine whether B is the midpoint of \overline{AC} .



Because $AB = 3$ and $BC = 2$, $AB \neq BC$. So, B is not the midpoint of AC .

- **Lesson 2–4** Name and graph ordered pairs on a coordinate plane.

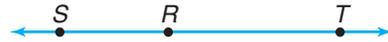
Graph point B at $(-2, -3)$.



Start at the origin. Move 2 units to the left. Then, move 3 units down. Label this point B .

Review Exercises

Refer to the line for Exercises 14–15.



- If $ST = 15$ and $SR = 6$, find RT .
- If $SR = 6$ and $RT = 4.5$, find ST .
- Find the length of the segment below in centimeters and in inches.



Use the number line at the left to determine whether each statement is *true* or *false*. Explain your reasoning.

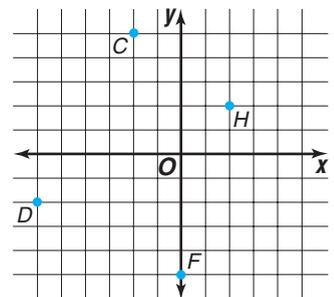
- $\overline{BD} \cong \overline{EG}$
- $\overline{AB} \cong \overline{DE}$
- The midpoint of \overline{AE} is C .

Determine whether each statement is *true* or *false*. Explain your reasoning.

- If $\overline{RQ} \cong \overline{TP}$ and $\overline{RQ} \cong \overline{FG}$, then $\overline{TP} \cong \overline{FG}$.
- \overline{LM} is not congruent to \overline{ML} .
- If points K , L , and M are collinear, then L is the midpoint of \overline{KM} .

Name the ordered pair for each point.

- F
- C
- H
- D



Draw and label a coordinate plane on a piece of grid paper. Then graph and label each point.

- $A(5, 5)$
- $B(0, 4)$
- $E(-4, 0)$
- $G(2, -2)$



Objectives and Examples

- **Lesson 2–5** Find the coordinates of the midpoint of a segment.

Find the coordinates of M , the midpoint of \overline{CD} , given the endpoints $C(3, 1)$ and $D(9, 9)$.

Let $x_1 = 3$, $x_2 = 9$, $y_1 = 1$, and $y_2 = 9$.

$$\begin{aligned} \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) &= \left(\frac{3 + 9}{2}, \frac{1 + 9}{2} \right) \\ &= \left(\frac{12}{2}, \frac{10}{2} \right) \text{ or } (6, 5) \end{aligned}$$

The coordinates of M are $(6, 5)$.

Review Exercises

Use the number line to find the coordinate of the midpoint of each segment.



31. \overline{UW}

32. \overline{VX}

The coordinates of the endpoints of a segment are given. Find the coordinates of the midpoint of each segment.

33. $(-1, -5), (3, -3)$ 34. $(4, 7), (-1, 2)$

Applications and Problem Solving

- 35. **Temperatures** Temperatures on the planet Mars range from -122°C to 31°C . What is the difference between these two temperatures? (*Lesson 2–1*)
- 36. **Geography** The highest point in Asia is Mount Everest at 29,028 feet above sea level. The lowest point in Asia is the Dead Sea at 1312 feet below sea level. What is the vertical distance between these two points? (*Lesson 2–2*)

- 37. **Environment** The table at the right shows the mid-1990s Gross National Product (GNP) per person and municipal waste production for six countries. (*Lesson 2–4*)

Country	GNP (\$ per person)	Waste (kg per person)
United States	27,550	720
France	26,290	560
Japan	41,160	400
Mexico	2521	330
United Kingdom	19,020	490
Spain	14,160	370

Source: *Statistical Abstract of U.S.*

- a. Graph the data. Let the x -coordinate of an ordered pair represent the GNP per person, and let the y -coordinate represent the number of kilograms of waste per person.
- b. Does the graph show that countries with a higher GNP per person generate more or less waste per person? Explain.

- 38. **Algebra** Suppose $K(3, -4)$ is the midpoint of \overline{JL} . The coordinates of J are $(-3, -2)$. Find the coordinates of L . (*Lesson 2–5*)



For each situation, write a real number with ten digits to the right of the decimal point.

- a rational number less than -2 with a 3-digit repeating pattern
- an irrational number between 3.5 and 4

Refer to the number line at the right.



- True or false: $\overline{AD} \cong \overline{CE}$
- What is the measure of \overline{AF} ?
- What is the midpoint of \overline{CG} ?

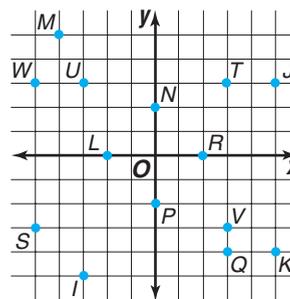
Refer to the line at the right.



- Find the length of \overline{EI} in centimeters and in inches.
- If $GH = 17$ and $FH = 23$, find FG .
- If $FG = 28$ and $GH = 12$, find FH .

Name the ordered pair for each point in the coordinate plane at the right.

- M
- P
- V



What point is located at each of the coordinates in the coordinate plane at the right?

- $(-2, 0)$
- $(3, -4)$
- $(-5, -3)$

Exercises 9–14

The coordinates of the endpoints of a segment are given. Find the coordinates of the midpoint of each segment.

- $(3, -8), (7, 2)$
- $(-4, 2), (-3, 1)$
- $(-11, 9), (3, -5)$

- Algebra** In the figure at right, M is the midpoint of \overline{LN} . Find the value of x .



- Hardware** Naomi purchased an extension ladder consisting of two 8-foot sections. When fully extended, the ladder measures 13 feet 7 inches. By how much do the two ladder sections overlap?

- Algebra** Plot the points for the ordered pairs on grid paper. Connect the points in the given order with straight line segments. What shape is formed? (Lesson 2–4)
 $(0, 2), (1, 3), (2, 3), (3, 2), (3, 0), (2, -2), (1, -3), (0, -4), (-1, -3), (-2, -2), (-3, 0), (-3, 2), (-2, 3), (-1, 3), (0, 2)$



More Number Concept Problems

Numerical problems on standardized tests can involve integers, fractions, decimals, percents, square roots, or exponents.

Many problems ask you to convert between fractions or decimals and percents. It's a good idea to memorize these common decimal-fraction-percent equivalents.

$$0.01 = \frac{1}{100} = 1\%$$

$$0.1 = \frac{1}{10} = 10\%$$

$$0.2 = \frac{2}{10} = 20\%$$

$$0.25 = \frac{1}{4} = 25\%$$

$$0.5 = \frac{1}{2} = 50\%$$

$$0.75 = \frac{3}{4} = 75\%$$

Test-Taking Tip

Remember the order of operations.

1. Parentheses
2. Exponents
3. Multiply, Divide
4. Add, Subtract

Please Excuse My Dear Aunt Sally

Example 1

Evaluate the following expression.

$$[(9 - 5) \times 6] + 4^2 \div 4$$

Hint Begin inside the parentheses.

Solution Use the order of operations. Evaluate the expression inside the parentheses. Then evaluate the resulting expression inside the brackets.

$$\begin{aligned} [(9 - 5) \times 6] + 4^2 \div 4 & \\ = [4 \times 6] + 4^2 \div 4 & \quad 9 - 5 = 4 \\ = 24 + 4^2 \div 4 & \quad 4 \times 6 = 24 \\ = 24 + 16 \div 4 & \quad 4^2 = 16 \\ = 24 + 4 & \quad 16 \div 4 = 4 \\ = 28 & \quad 24 + 4 = 28 \end{aligned}$$

The answer is 28.

Example 2

At a restaurant, diners get an “early bird” discount of 10% off their bill. If a diner orders a meal regularly priced at \$18 and leaves a tip of 15% of the discounted meal, how much does she pay in total?

- (A) \$13.50
- (B) \$16.20
- (C) \$18.63
- (D) \$18.90
- (E) \$20.70

Hint Be sure to read the question carefully.

Solution First, find the amount of the discount.

$$10\% \text{ of } \$18.00 = 0.10(18.00) \text{ or } \$1.8$$

Then subtract to find the cost of the discounted meal.

$$\$18.00 - \$1.80 = \$16.20$$

This is choice B, but it is *not* the answer to the question. You need to find the total cost of the meal plus the tip. Calculate the amount of the tip. 15% of \$16.20 is \$2.43.

The total amount paid is \$16.20 + \$2.43 or \$18.63. The answer is C.

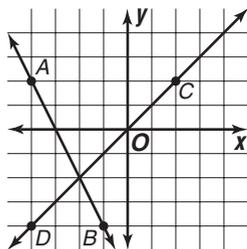
After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

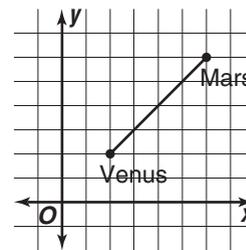
1. Which is the correct order of the set of numbers from least to greatest?
(Algebra Review)

$$-5, 4, 0, -\sqrt{22}, \sqrt{18}, 8$$

- (A) $-\sqrt{22}, \sqrt{18}, 0, 4, -5, 8$
 (B) $-\sqrt{22}, -5, 0, 4, 8, \sqrt{18}$
 (C) $-5, -\sqrt{22}, 0, 4, \sqrt{18}, 8$
 (D) $-5, -\sqrt{22}, 0, 4, 8, \sqrt{18}$
2. What are the coordinates of the intersection of \overleftrightarrow{AB} and \overleftrightarrow{CD} ? (Lesson 2–4)
- (A) $(-2, -2)$
 (B) $(-2, 2)$
 (C) $(-3, 0)$
 (D) $(0, -6)$
3. After $\frac{4\frac{1}{3}}{2\frac{3}{5}}$ has been simplified to a single fraction in lowest terms, what is the denominator? (Algebra Review)
- (A) 2 (B) 3 (C) 5 (D) 9 (E) 13
4. Talia is a travel agent. The agency gives a 7% bonus to any agent who sells at least \$9000 in travel packages each month. If an average travel package is \$855, how many packages must Talia sell to receive a bonus each month? (Percent Review)
- (A) 9 or more (B) 10 or more
 (C) 11 or more (D) less than 9
5. If n is an even integer, which must be an odd integer? (Algebra Review)
- (A) $3n - 2$ (B) $3(n + 1)$ (C) $n - 2$
 (D) $\frac{n}{3}$ (E) n^2



6. Luke is making a model of our solar system. He has placed Venus and Mars in his model on the coordinate grid at the right. He wants to place the model of Earth at the midpoint of the segment connecting Venus and Mars. What will be the coordinates for the model of Earth? (Lesson 2–5)



- (A) $(2, 2)$ (B) $(3, 3)$
 (C) $(4, 4)$ (D) $(5, 5)$
7. The length of the page in a textbook is $10\frac{7}{8}$ inches. The top and bottom margins total $1\frac{1}{16}$ inches. What is the length of the page inside the margins? (Algebra Review)
- (A) $8\frac{3}{16}$ (B) $8\frac{13}{16}$
 (C) $9\frac{13}{16}$ (D) $11\frac{15}{16}$
8. For a positive integer x , 10% of $x\%$ of 1000 equals— (Percent Review)
- (A) x . (B) $10x$. (C) $100x$.
 (D) $1000x$. (E) $10,000x$.

Grid In

9. Set S consists of all multiples of 3 between 11 and 31. Set T consists of all multiples of 5 between 11 and 31. What is one possible number in S but NOT in T? (Algebra Review)

Short Response

10. You must choose between two Internet providers. One charges a flat fee of \$22 per month for unlimited usage, and the other charges a fee of \$10.99 for 10 hours of use per month, plus \$1.95 for each additional hour. Decide which provider would be more economical for you to use. (Algebra Review)

