CHAPTER 1

Reasoning in Geometry

What You'll Learn

Key Ideas

- Identify patterns and use inductive reasoning. *(Lesson 1–1)*
- Identify, draw models of, and use postulates about points, lines, and planes. *(Lessons 1–2 and 1–3)*
- Write statements in if-then form and write their converses. *(Lesson 1–4)*
- Use geometry tools. *(Lesson 1–5)*
- Use a four-step plan to solve problems. *(Lesson 1–6)*

Key Vocabulary

line *(p. 12)*

line segment *(p. 13)*

plane *(p. 14)*

point *(p. 12)*

ray *(p. 13)*

Why It's Important

Interior Design The goal of an interior designer is to make a room beautiful and functional. Designers listen carefully to their client's needs and preferences and put together a design plan and budget. The plan includes coordinating colors and selecting furniture, floor coverings, and window treatments.

Reasoning in geometry is used to solve real-life problems. You will use the four-step plan for problem solving to find the amount of wallpaper border an interior designer would need for a room in Lesson 1-6.

Study these lessons to improve your skills.

✓ Check Your Readiness

✓ Algebra Review, p. 718

Evaluate each expression.

1. $2 \times 9 + 2 \times 3$	**2.** $2(4) + 2(7)$	**3.** $2(9) + 2(12)$
4. $2(14) + 2(18)$	**5.** 2×11	**6.** $9(10)$
7. 9×3	**8.** 9×8	**9.** $12(7)$

✓ Algebra Review, p. 719

✓ Algebra Review, p. 720

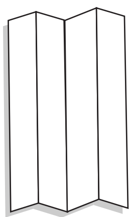
Solve each equation.

10. $10.1 - 0.2 = x$	**11.** $y = 2.6 - 1.4$	**12.** $n = 4.7 - 3.1$
13. $j = 100.4 - 94.9$	**14.** $1.43 + 0.84 = p$	**15.** $4.6 + 2.9 = n$
16. $0.8 + 1.3 = d$	**17.** $11.1 + 0.2 + 0.2 = t$	**18.** $h = 7.4(4.1)$
19. $m = 2.3(8.8)$	**20.** $(10.7)(15.5) = a$	**21.** $0.6(143.5) = g$
22. $q = \frac{5}{12} - \frac{1}{12}$	**23.** $\frac{7}{10} - \frac{1}{10} = t$	**24.** $\frac{2}{3} - \frac{1}{6} = w$
25. $y = \frac{11}{12} - \frac{1}{3}$	**26.** $b = \frac{3}{5} - \frac{1}{4}$	**27.** $\frac{4}{5} - \frac{1}{2} = c$
28. $v = \frac{4}{5} \cdot \frac{1}{3}$	**29.** $\frac{3}{5}\left(\frac{7}{8}\right) = d$	**30.** $z = \frac{5}{9} \cdot \frac{3}{4}$

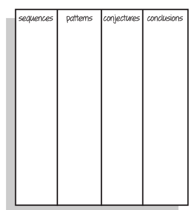
FOLDABLES™ Study Organizer

Make this Foldable to help you organize your Chapter 1 notes. Begin with a sheet of $8\frac{1}{2}$" by 11" paper.

❶ **Fold** lengthwise in fourths.



❷ **Draw** lines along the folds and label each column *sequences, patterns, conjectures,* and *conclusions.*



Reading and Writing As you read and study the chapter, record different sequences and describe their patterns. Also, record conjectures and state whether they are true or false; if false, provide at least one counterexample.

1–1 Patterns and Inductive Reasoning

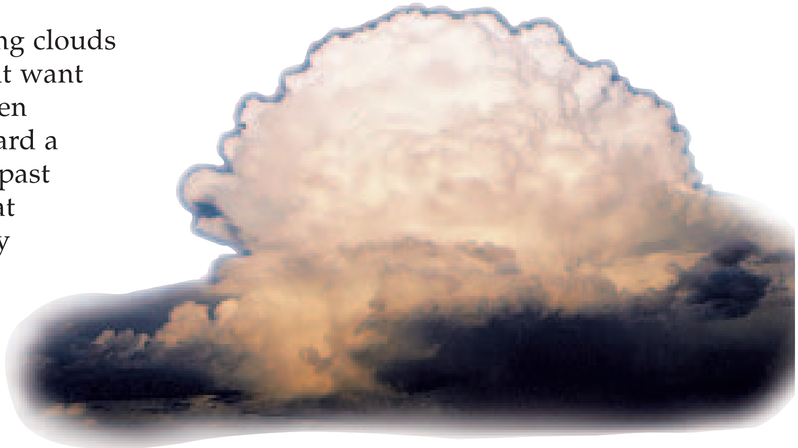
What You'll Learn

You'll learn to identify patterns and use inductive reasoning.

Why It's Important

Business Businesses look for patterns in data. *See Example 5.*

If you see dark, towering clouds approaching, you might want to take cover. Why? Even though you haven't heard a weather forecast, your past experience tells you that a thunderstorm is likely to happen. Every day you make decisions based on past experiences or patterns that you observe.



Rain clouds approaching

When you make a conclusion based on a pattern of examples or past events, you are using **inductive reasoning**. Originally, mathematicians used inductive reasoning to develop geometry and other mathematical systems to solve problems in their everyday lives.

You can use inductive reasoning to find the next terms in a sequence.

Example 1

Find the next three terms of the sequence 33, 39, 45, ...

Study the pattern in the sequence.

33, 39, 45

+6 +6

Each term is 6 more than the term before it. Assume that this pattern continues. Then, find the next three terms using the pattern of adding 6.

33, 39, 45, 51, 57, 63

+6 +6 +6 +6 +6

The next three terms are 51, 57, and 63.

Your Turn

Find the next three terms of each sequence.

a. 1.25, 1.45, 1.65, . . .

b. 13, 8, 3, . . .

c. 1, 3, 9, . . .

d. 32, 16, 8, . . .

Example 2 **Find the next three terms of the sequence 1, 3, 7, 13, 21, ...**

$$1 \xrightarrow{+2} 3 \xrightarrow{+4} 7 \xrightarrow{+6} 13 \xrightarrow{+8} 21$$

Notice the pattern 2, 4, 6, 8, ... To find the next terms in the sequence, add 10, 12, and 14.

$$1 \xrightarrow{+2} 3 \xrightarrow{+4} 7 \xrightarrow{+6} 13 \xrightarrow{+8} 21 \xrightarrow{+10} 31 \xrightarrow{+12} 43 \xrightarrow{+14} 57$$

The next three terms are 31, 43, and 57.

Your Turn

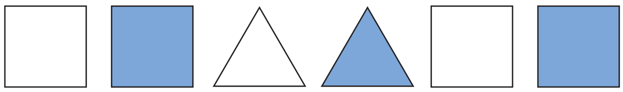
Find the next three terms of each sequence.

e. 10, 12, 15, 19, . . .

f. 1, 2, 6, 24, . . .

Some patterns involve geometric figures.

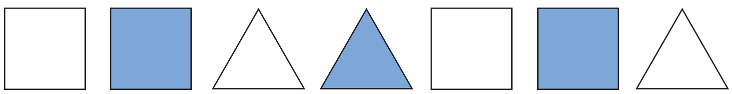
Example 3 **Draw the next figure in the pattern.**



There are two patterns to study.

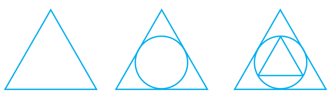
- First, the pattern with the squares (S) and triangles (T) is SSTTSS. The next figure should be a triangle (T).
- Next, the pattern with the colors white (W) and blue (B) is WBWBWB. The next figure should be white.

Therefore, the next figure should be a white triangle.



Your Turn

g.



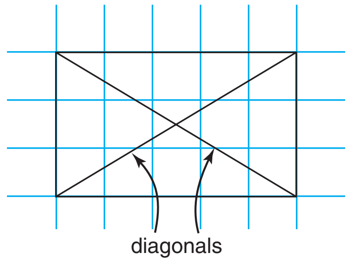
Throughout this text, you will study many patterns and make conjectures. A **conjecture** is a conclusion that you reach based on inductive reasoning. In the following activity, you will make a conjecture about rectangles.

Hands-On Geometry

Materials: grid paper ruler

Step 1 Draw several rectangles on the grid paper. Then draw the diagonals by connecting each corner with its opposite corner.

Step 2 Measure the diagonals of each rectangle. Record your data in a table.



Try These

1. **Make a conjecture** about the diagonals of a rectangle.
2. **Verify** your conjecture by drawing another rectangle and measuring its diagonals.

A conjecture is an educated guess. Sometimes it may be true, and other times it may be false. How do you know whether a conjecture is true or false? Try out different examples to test the conjecture. If you find one example that does not follow the conjecture, then the conjecture is false. Such a false example is called a **counterexample**.

Example 4
Number Theory Link

Akira studied the data in the table at the right and made the following conjecture.

The product of two positive numbers is always greater than either factor.

Factors		Product
2	8	16
5	15	75
20	38	760
54	62	3348

Find a counterexample for his conjecture.

The numbers $\frac{1}{2}$ and 10 are positive numbers.

However, the product of $\frac{1}{2}$ and 10 is 5, which is less than 10.

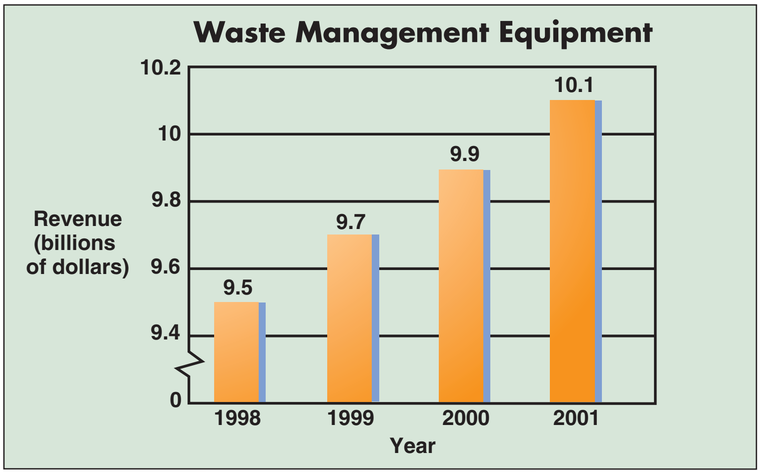
Therefore, the conjecture is false.

Businesses often look for patterns in data to find trends.

Real World **Example** 5

Business Link

The following graph shows the revenue from the sale of waste management equipment in billions of dollars. Find a pattern in the graph and then make a conjecture about the revenue for 2005.



Source: Statistical Abstract of the United States

The graph shows an increase of 200 million dollars ($0.2 billion) each year from the sale of waste management equipment. In 2001, the revenue was 10.1 billion dollars.

2001		*2002*		*2003*		*2004*		*2005*
10.1	+0.2	10.3	+0.2	10.5	+0.2	10.7	+0.2	10.9

One possible conjecture is that the revenue in 2005 will grow to 10.9 billion dollars.

Check for Understanding

Communicating Mathematics

1. **Write** a definition of *conjecture*.
2. **Explain** how you can show that a conjecture is false.
3. Writing Math Write your own sequence of numbers. Then write a sentence that describes the pattern in the numbers.

Vocabulary
inductive reasoning
conjecture
counterexample

Guided Practice

Getting Ready **Tell how to find the next term in each pattern.**

Sample: 15, 18, 21, 24, . . . **Solution:** Add 3.

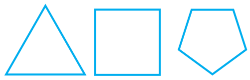
4. 20, 26, 32, 38, . . .
5. 10, 7, 4, 1, . . .
6. 3, 6, 12, 24, . . .
7. 30, 31, 33, 36, . . .

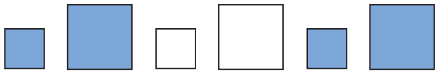
Examples 1 & 2

Find the next three terms of each sequence.

8. 1, 3, 5, 7, . . .
9. 9, 6, 3, 0, . . .
10. 96, 48, 24, 12, . . .
11. 7, 8, 11, 16, . . .

Example 3 **Draw the next figure in the pattern.**

12. 

13. 

Example 4 14. **Number Theory** Jacqui made the following conjecture about the information in the table. *If the first number is negative and the second number is positive, the sum is always negative.* Find a counterexample for her conjecture.

Addends		Sum
−5	3	−2
−3	2	−1
−8	4	−4
−10	6	−4

Exercises

Practice

Homework Help

For Exercises	See Examples
15–26, 33, 34	1, 2
27–32, 38	3
35, 37	4
36	5

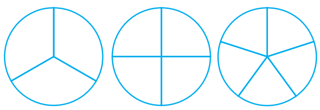
Extra Practice

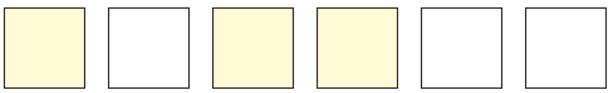
See page 726.

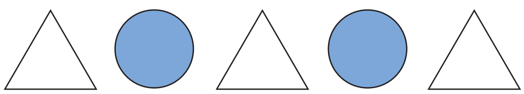
Find the next three terms of each sequence.

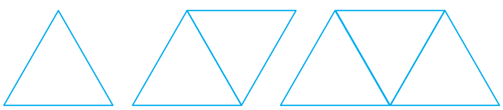
15. 5, 9, 13, 17, . . .
16. 12, 8, 4, 0, . . .
17. 12, 21, 30, 39, . . .
18. 1, 2, 4, 8, . . .
19. 3, 15, 75, 375, . . .
20. 2, −3, −8, −13, . . .
21. −1.4, 2.6, 6.6, 10.6, . . .
22. 6, 7, 9, 12, . . .
23. 13, 14, 16, 19, . . .
24. 20, 22, 26, 32, . . .
25. 10, 13, 19, 28, . . .
26. 10, 17, 31, 52, . . .

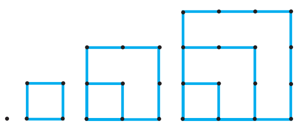
Draw the next figure in each pattern.

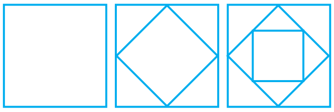
27. 

28. 

29. 

30. 

31. 

32. 

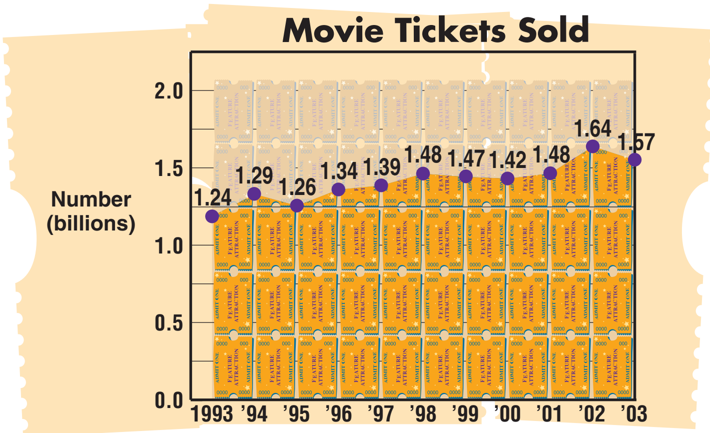
33. Find the next term in the sequence $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$.

34. What operation would you use to find the next term in the sequence 3, 6, 12, 24, . . . ?

Applications and Problem Solving

35. **Pets** Find a counterexample for this statement: All dogs have spots.

36. **Entertainment** The graph shows the number of movie tickets sold yearly in the United States from 1993 to 2003. Predict the number of movie tickets that will be sold in 2006.

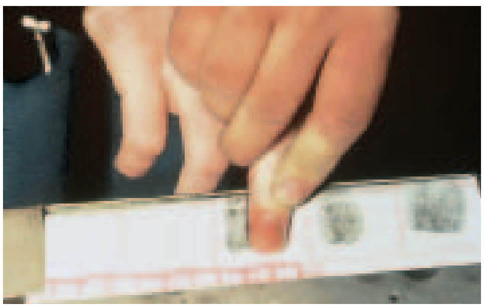


Source: The Motion Picture Association of America

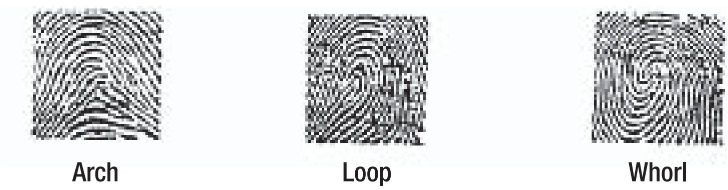
interNET CONNECTION

Data Update For the latest information about movie attendance, visit: www.geomconcepts.com

37. **Law Enforcement** All fingerprint patterns can be divided into three main groups: arches, loops, and whorls.

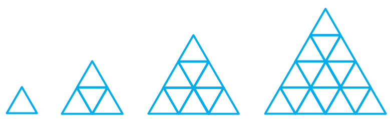


Fingerprinting procedure



Name a situation that would provide a counterexample to this statement.

38. **Critical Thinking** Find the total number of small triangles in the eighth figure of the pattern.



Standardized Test Practice

39. **Multiple Choice** Choose the expression that represents the value, in cents, of n nickels and d dimes. *(Algebra Review)*

(A) $n + d$ (B) $10n + 5d$

(C) $5n + 10d$ (D) $15nd$

Chapter 1 Investigation

To Grandmother's House We Go!

Materials

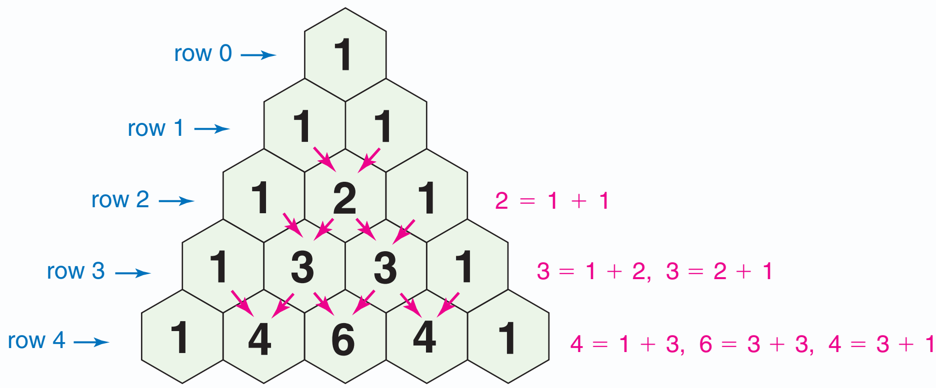
calculator

colored pencils

Number Patterns in Pascal's Triangle

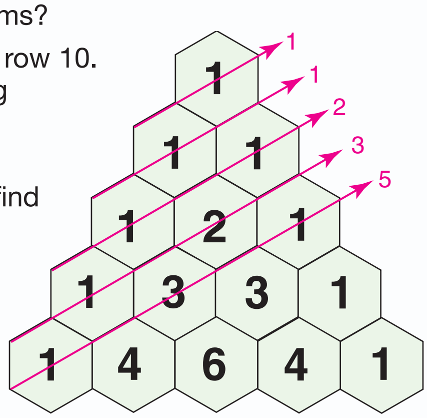
The triangular-shaped pattern of numbers below is called **Pascal's triangle**. Each row begins and ends with the number 1. Each other number is the sum of the two numbers above it.

Pascal's triangle is named for French mathematician Blaise Pascal (1623–1662).



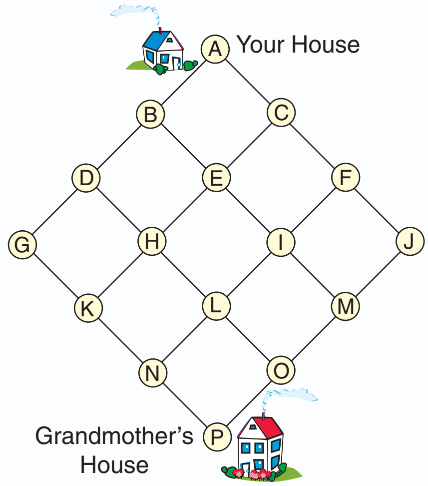
Investigate

1. Copy row 0 through row 4 of Pascal's triangle on your paper.
   a. Complete row 5 through row 9 following the pattern.
   b. Find the sum of the numbers in each row. Examine the sums. What pattern do you see in the sums?
   c. Predict the sum of the numbers in row 10. Then check your answer by finding row 10 of Pascal's triangle and finding its sum.
2. The figure at the right shows how to find the sum of the diagonals of Pascal's triangle.
   a. Describe the pattern in the sums of the diagonals.
   b. Predict the sum of the next two diagonals.



The pattern in the sum of the diagonals is called the Fibonacci sequence.

Suppose the grid at the right represents all of the streets between your house and your grandmother's house. You will start at your house and move down the grid to get to your grandmother's house.



3. Copy the grid. How many different routes are there between each pair of points? Write your answers on the grid.

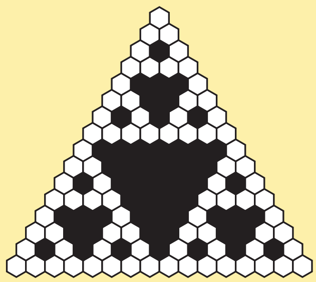
a. A and B	b. A and C
c. A and D	d. A and E
e. A and F	f. A and G
g. A and H	h. A and I

4. Explain how Pascal's triangle is related to the numbers on the grid.

5. Extend the pattern to find the number of routes between your house and your grandmother's house.

Extending the Investigation

In this extension, you will find more patterns in Pascal's triangle. Here are some suggestions.



- The figure at the right shows the pattern when multiples of 2 are shaded. Show the patterns when multiples of 3, 4, 5, and 6 are highlighted. (*Hint:* You will need to use at least 16 rows of Pascal's triangle.)
- Square numbers are 1, 4, 9, 16, ... Find two places where square numbers appear. (*Hint:* Look for the sum of adjacent numbers.)
- Find the powers of 11 in Pascal's triangle.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Write a paragraph about some of the patterns found in Pascal's triangle.
- Make a bulletin board that shows some of the visual patterns found in Pascal's triangle.

Investigation For more information on Pascal's triangle, visit: www.geomconcepts.com

1–2 Points, Lines, and Planes

What You'll Learn

You'll learn to identify and draw models of points, lines, and planes, and determine their characteristics.

Why It's Important

Art Artists use points and lines to add shading to their drawings.
See Exercise 31.

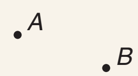
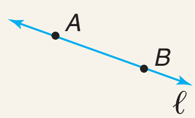
Geometry is the study of points, lines, and planes and their relationships. Everything we see contains elements of geometry. Even the painting below is made entirely of small, carefully placed dots of color.

Georges Seurat, *Sunday Afternoon on the Island of La Grande Jatte*, 1884–1886

Reading Geometry

Points, lines, and planes are referred to as **undefined terms**.

Each dot in the painting represents a point. A **point** is the basic unit of geometry. The shoreline in the painting represents part of a line. A **line** is a series of points that extends without end in two directions.

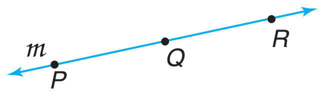
Term	Description	Model
Point	• A point has no size. • Points are named using capital letters. • The points at the right are named *point A* and *point B*.	
Line	• A line is made up of an infinite number of points. • The arrows show that the line extends without end in both directions. • A line can be named with a single lowercase script letter or by two points on the line. • The line at the right is named *line AB*, *line BA*, or line ℓ. • The symbol for line AB is $\overleftrightarrow{AB}$.	

When we show the figure of a line, this is only a small part of a line.

Examples

1 **Name two points on line *m*.**

Two points are point P and point Q.



2 **Give three names for the line.**

Any two points on the line or the script letter can be used to name it. Three names are $\overleftrightarrow{PQ}$, $\overleftrightarrow{QR}$, and line m.

Your Turn

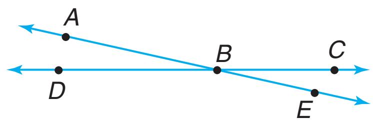
a. Name another point on line m.

b. Give two other names for line m.

Three points may lie on the same line, as in Example 1. These points are **collinear**. Points that do *not* lie on the same line are **noncollinear**.

Example

3 **Name three points that are collinear and three points that are noncollinear.**



D, B, and C are collinear.
A, B, and C are noncollinear.

Your Turn

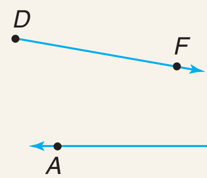
c. Name three other points that are collinear.

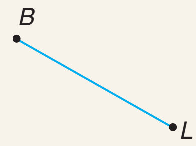
d. Name three other points that are noncollinear.

Reading Geometry

The order of the letters can be switched with lines, but *not* with rays. Ray *DF* and ray *FD* are *not* the same ray.

Rays and line segments are parts of lines. A **ray** has a definite starting point and extends without end in one direction. The sun's rays represent a ray. A **line segment** has a definite beginning and end.

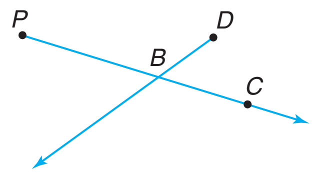
Term	Description	Model
Ray	• The starting point of a ray is called the **endpoint**. • A ray is named using the endpoint first, then another point on the ray. • The rays at the right are named *ray DF* and *ray CA*. • The symbol for ray *CA* is $\overrightarrow{CA}$.	

Term	Description	Model
Line Segment	• A line segment is part of a line containing two endpoints and all points between them. • A line segment is named using its endpoints. • The line segment at the right is named *segment BL* or *segment LB*. • The symbol for segment *BL* is $\overline{BL}$.	

In this text, we will refer to line segments as segments.

Example 4

Name two segments and one ray.

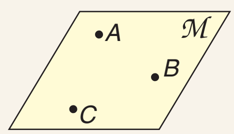


Two segments are $\overline{PB}$ and $\overline{BC}$.
One ray is $\overrightarrow{PC}$.
$\overrightarrow{PB}$ is another name for $\overrightarrow{PC}$.

Your Turn

e. Name another segment and another ray.

The painting on page 12 was painted on a flat surface called a canvas. A canvas represents a plane. A **plane** is a flat surface that extends without end in all directions.

Term	Description	Model
Plane	• For any three noncollinear points, there is only one plane that contains all three points. • A plane can be named with a single uppercase script letter or by three noncollinear points. • The plane at the right is named plane *ABC* or plane $\mathcal{M}$.	

Whenever we draw a plane in this text, it is only a part of the whole plane. The whole plane continues in all directions.

Points that lie in the same plane are **coplanar**. Points that do *not* lie in the same plane are **noncoplanar**.

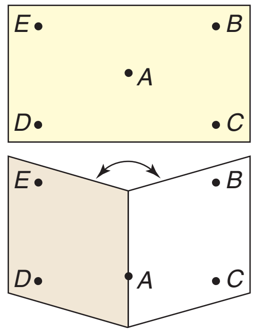
Hands-On Geometry
Paper Folding

Materials: unlined paper

Step 1 Place points *A*, *B*, *C*, *D*, and *E* on a piece of paper as shown in the drawing.

Step 2 Fold the paper so that point *A* is on the crease.

Step 3 Open the paper slightly. The two sections of the paper represent different planes.



Try These

1. Name three points that are coplanar.
2. Name three points that are noncoplanar.
3. Name a point that is in both planes.

Check for Understanding

Communicating Mathematics

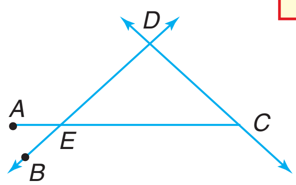
1. **Explain** the difference between a line and a segment.
2. **You Decide?** Joel says that $\overrightarrow{JL}$ and $\overrightarrow{LJ}$ name the same ray. Pat says they name different rays. Who is correct? Explain your reasoning.

Vocabulary
point
line
collinear
noncollinear
ray
line segment
plane
coplanar
noncoplanar

Guided Practice

Use the figure below to name an example of each term.

Example 1 — 3. point
Example 2 — 4. line
Example 4 — 5. ray
Example 4 — 6. segment



Example 3 — 7. **Maps** The map shows the state of Colorado. Name three cities that appear to be collinear. Name three cities that are *not* collinear.

Exercises

Practice

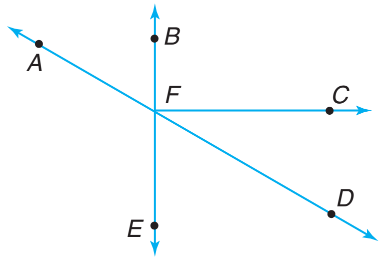
Homework Help

For Exercises	See Examples
8, 18, 20	1
9, 24	2
10, 11, 14, 15, 21, 22, 25, 29, 31	4
12, 13, 16, 17, 27, 28	3

Extra Practice

See page 726.

Use the figure at the right to name examples of each term.



8. three points
9. two lines
10. three rays
11. three segments
12. point that is *not* on $\overline{AD}$
13. line that does *not* contain point E
14. ray with point A as the endpoint
15. segment with points E and F as its endpoints
16. three collinear points
17. three noncollinear points

Determine whether each model suggests a point, a line, a ray, a segment, or a plane.

18. the tip of a needle
19. a wall
20. a star in the sky
21. rules on notebook paper
22. a beam from a flashlight
23. a skating rink

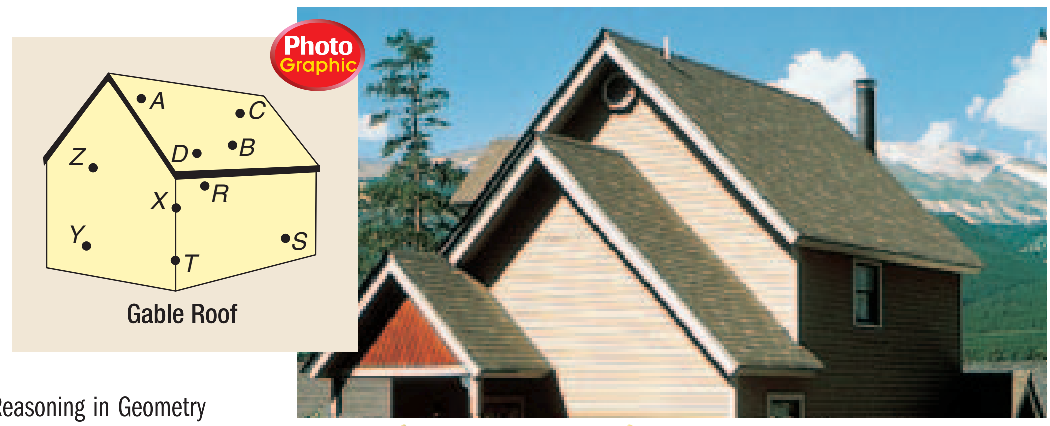
Draw and label a figure for each situation described.

24. line ℓ
25. $\overline{CD}$
26. plane XYZ
27. collinear points A, B, and C
28. lines ℓ and m intersecting at point T
29. $\overrightarrow{BD}$ and $\overrightarrow{BE}$ so that point B is the only point common to both rays

Applications and Problem Solving

30. Construction The gable roof is the most common type of roof. It has two surfaces that meet at the top. The roof and the walls of the building are models of planes.

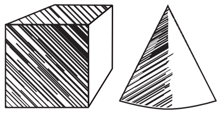
a. Name a point that is coplanar with points C and D.
b. Name a point that is noncoplanar with points R and S.
c. Name one point that is in two different planes.



Gable Roof

CONTENTS

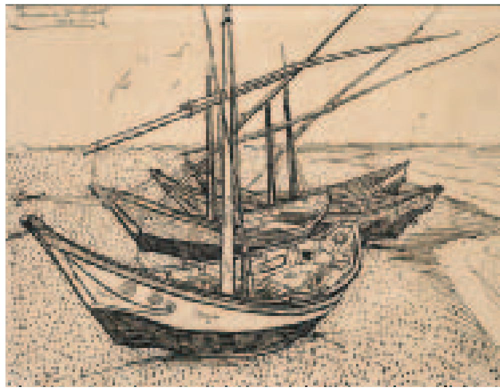
31. **Art** Artists use segments to add shading to their drawings.



Hatching

a. How are the segments placed to create dark images?

b. How are the segments placed to create light images?



Fishing Boats on the Beach at Saintes-Maries-de-la-Mer, 1888
Vincent van Gogh

32. **Critical Thinking** Is the following statement *true* or *false*? Explain. *Two rays can have at most one point in common.*

Mixed Review

Find the next three terms of each sequence. *(Lesson 1–1)*

33. 5, 10, 20, 40, . . .

34. 112, 115, 118, 121, . . .

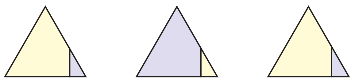
35. 1, −1, −3, −5, . . .

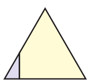
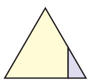
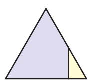
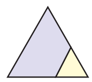
36. 1, 2, 4, 7, . . .

Standardized Test Practice

37. **Short Response** Draw a figure that is a counterexample for the following conjecture: *All figures with four sides are squares.* *(Lesson 1–1)*

38. **Multiple Choice** Choose the figure that will continue the pattern. *(Lesson 1–1)*



A  B  C  D 

Quiz 1 Lessons 1–1 and 1–2

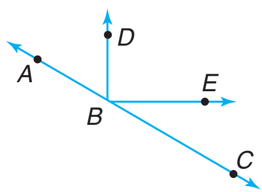
Find the next three terms of each sequence. *(Lesson 1–1)*

1. 15, 11, 7, 3, . . .

2. 8, 10, 14, 20, . . .

Use the figure to name an example of each term. *(Lesson 1–2)*

3. line
4. point
5. ray

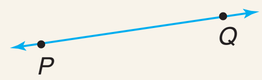
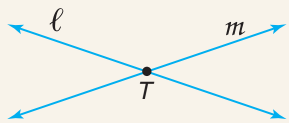
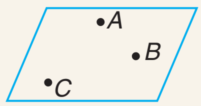


1–3 Postulates

What You'll Learn
You'll learn to identify and use basic postulates about points, lines, and planes.

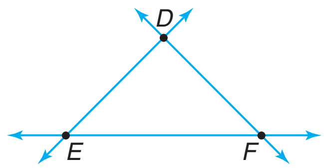
Why It's Important
Architecture
Architects use points and lines in perspective drawings.
See page 23.

Geometry is built on statements called postulates. **Postulates** are statements in geometry that are accepted as true. The postulates in this lesson describe how points, lines, and planes are related.
*Another term for postulate is **axiom**.*

Postulate	Words	Models
1–1	Two points determine a unique line. *There is only one line that contains points P and Q.*	
1–2	If two distinct lines intersect, then their intersection is a point. *Lines ℓ and m intersect at point T.*	
1–3	Three noncollinear points determine a unique plane. *There is only one plane that contains points A, B, and C.*	

Examples

Points D, E, and F are noncollinear.



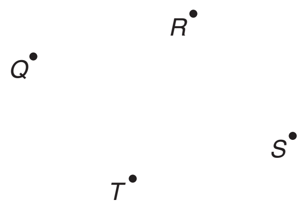
1 **Name all of the different lines that can be drawn through these points.**

There is only one line through each pair of points. Therefore, the lines that contain D, E, and F, taken two at a time, are $\overleftrightarrow{EF}$, $\overleftrightarrow{DF}$, and $\overleftrightarrow{DE}$.

2 **Name the intersection of $\overleftrightarrow{DE}$ and $\overleftrightarrow{EF}$.**

The intersection of $\overleftrightarrow{DE}$ and $\overleftrightarrow{EF}$ is point E.

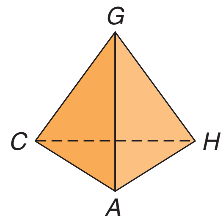
Your Turn



a. Points Q, R, S, and T are noncollinear. Name all of the different lines that can be drawn through these points.
b. Name the intersection of $\overleftrightarrow{QR}$ and $\overleftrightarrow{RS}$.

Example 3 **Name all of the planes that are represented in the figure.**

There are four points, *C*, *G*, *A*, and *H*. There is only one plane that contains three noncollinear points. Therefore, the planes that can contain the points, taken three at a time, are plane *ACG*, plane *GCH*, plane *GHA*, and plane *AHC*.

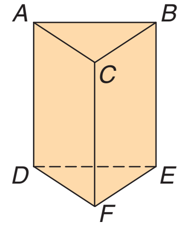


Reading Geometry

When four points are coplanar, you can name the plane by choosing any three points.

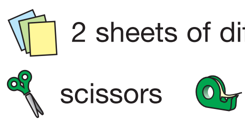
Your Turn

c. Name all of the planes that are represented in the figure.



When two distinct lines intersect, they have only one point in common. In the following activity, you will investigate what happens when two planes intersect.

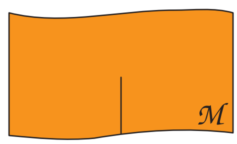
Hands-On Geometry
Paper Folding



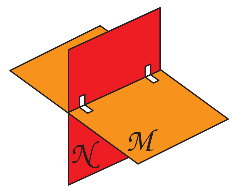
Materials: 2 sheets of different-colored paper

scissors   tape

Step 1 Label one sheet of paper *M* and the other *N*. Hold the two sheets of paper together and cut a slit halfway through both sheets.

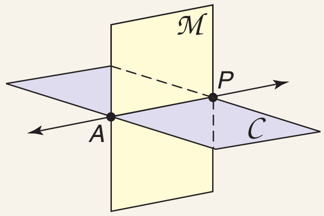


Step 2 Turn the papers so that the two slits meet and insert one sheet into the slit of the other sheet. Use tape to hold the two sheets together.



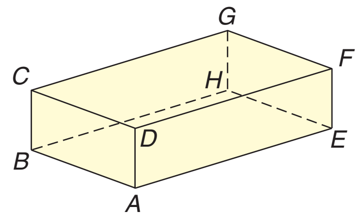
Try These

1. Draw two points, *D* and *E*, so they lie in both planes.
2. Draw the line determined by points *D* and *E*.
3. Describe the intersection of planes *M* and *N*.

Postulate 1–4	
	Words: If two distinct planes intersect, then their intersection is a line.
	Model:  *Plane $\mathcal{M}$ and plane $\mathcal{C}$ intersect in line AP.*

Example 4

The figure shows the intersection of six planes. Name the intersection of plane *CDG* and plane *BCD*.



The intersection is $\overleftrightarrow{CD}$.

Your Turn

d. Name two planes that intersect in $\overleftrightarrow{BA}$.

Check for Understanding

Communicating Mathematics

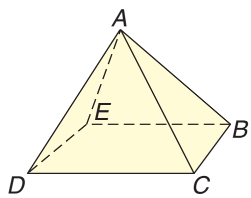
1. **Draw** plane *ABC* and plane *DEF* that intersect in $\overleftrightarrow{GH}$.
2. **State** the number of lines that are determined by two points.
3. Writing Math Write in your own words a sentence describing each postulate in this lesson. Include a diagram with each postulate.

Vocabulary
postulate

Guided Practice

Example 1
4. Points *X*, *Y*, and *Z* are noncollinear. Name all of the different lines that can be drawn through these points.

Refer to the figure at the right.



Example 2
5. Name the intersection of $\overleftrightarrow{DC}$ and $\overleftrightarrow{CB}$.

Example 3
6. Name all of the planes that are represented.

Example 4
7. Name the intersection of plane *ABC* and plane *ACD*.

Example 3

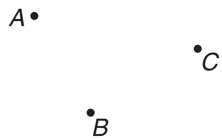
8. **Photography** Cameras are often mounted on tripods to stabilize them. A tripod has three legs. Which postulate in the lesson guarantees that the tripod is stable?

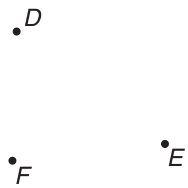
Exercises

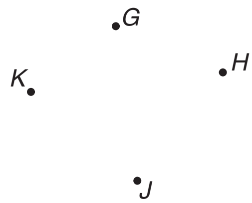
Practice

Homework Help	
For Exercises	See Examples
9–11, 23, 25	1
12–14, 22	2
15–17, 26, 29, 30	3
18–21, 24, 27, 28, 32	4
Extra Practice	
See page 726.	

Name all of the different lines that can be drawn through each set of points.

9. 

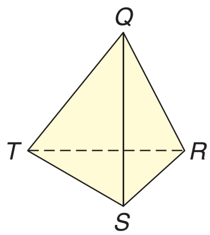
10. 

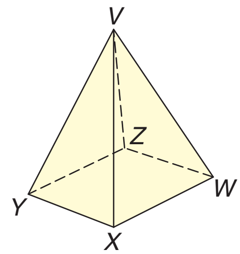
11. 

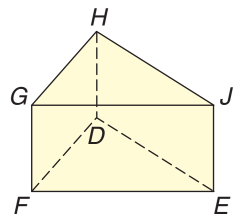
Refer to Exercises 9–11. Name the intersection of each pair of lines.

12. $\overleftrightarrow{AC}$ and $\overleftrightarrow{AB}$
13. $\overleftrightarrow{FE}$ and $\overleftrightarrow{ED}$
14. $\overleftrightarrow{KJ}$ and $\overleftrightarrow{GK}$

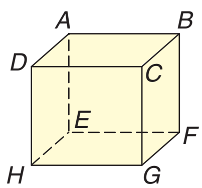
Name all of the planes that are represented in each figure.

15. 

16. 

17. 

Refer to the figure at the right.



18. Name the intersection of plane *ABC* and plane *BCG*.
19. Name the intersection of plane *DCG* and plane *HGF*.
20. Name two planes that intersect in $\overleftrightarrow{AD}$.
21. Name two planes that intersect in $\overleftrightarrow{EF}$.

Determine whether each statement is *true* or *false*.

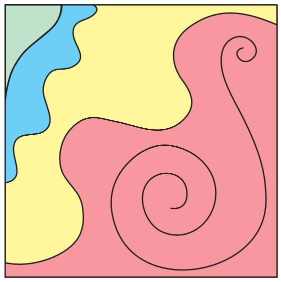
22. If two lines intersect in a point, then the point is in both lines.
23. More than one line can be drawn through two points.
24. Two planes can intersect in a line.
25. Two points determine two lines.

Determine whether each statement is *true* or *false*.

26. Three noncollinear points determine a plane.
27. If two planes intersect in a line, then the line is in both planes.
28. Two planes can intersect in a point.
29. It is possible for two lines to lie in the same plane.
30. Three planes can intersect in a point.

Applications and Problem Solving

31. **Art** In art, a line is *the path of a dot through space.* Using this definition, the figures at the right are *lines*. Explain why these curves are not called *lines* in geometry.



Irene Rice Pereira, *Untitled*, 1951

32. **Buildings** You can think of your classroom as a model of six planes: the ceiling, the floor, and the four walls.
    a. Find two planes that intersect.
    b. Find two planes that do *not* intersect.
    c. Is it possible for three planes to intersect? If so, find the intersection.

33. **Critical Thinking** Three noncollinear points determine a plane. How many planes can contain three collinear points?

Mixed Review

Use the painting at the right to describe examples of each term. *(Lesson 1–2)*

34. point 35. line 36. plane

37. Explain how a ray is different from a line. *(Lesson 1–2)*

Standardized Test Practice

38. **Grid In** Dyani shoots baskets every day to increase her free throw percentage. On Sunday, she shoots 20 free throws and plans to increase the number by 5 each day until Saturday. How many free throws will she shoot on Saturday? *(Lesson 1–1)*

39. **Short Response** Add two numbers to the data below so that the median does not change. *(Statistics Review)*

    11, 13, 16, 12, 25, 8, 25, 33, 51

Math In the Workplace

Architect

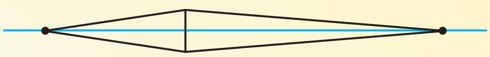
Did you ever spend time building castles out of blocks or designing a dream home for your dolls? Then you might enjoy a career as an architect.

In addition to preparing blueprints and technical drawings, architects often prepare *perspective drawings* for their clients. The following steps show how to make a two-point perspective drawing of an office building.

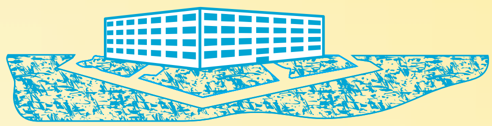
Step 1: Draw a horizon line. Mark two *vanishing points*.

Step 2: Draw a vertical line, called the *key edge*. This will be a corner of the building.

Step 3: Connect the end of the key edge to each vanishing point. Draw the edges of the two visible sides of the building.



Step 4: Add details to the building.



1. Make a two-point perspective drawing of a building.
2. Do research about other kinds of perspective drawings.

FAST FACTS About Architects

Working Conditions

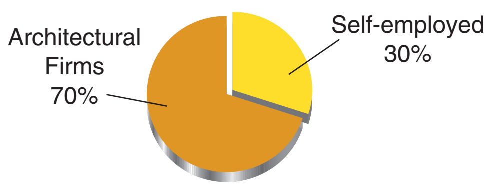
- generally work in a comfortable environment
- may be under great stress, working nights and weekends to meet deadlines

Education

- training period required after college degree
- knowledge of computer-aided design and drafting
- artistic ability is helpful, but not essential

Employment

Where Architects Are Employed



Source: Bureau of Labor Statistics

interNET CONNECTION **Career Data** For the latest information about a career as an architect, visit: www.geomconcepts.com

1–4 Conditional Statements and Their Converses

What You'll Learn
You'll learn to write statements in if-then form and write the converses of the statements.

Why It's Important
Advertising Many advertisements are written in if-then form. *See Example 4.*

In mathematics, you will come across many **if-then statements**. For example, if a number is even, then the number is divisible by 2.

If-then statements join two statements based on a condition: a number is divisible by 2 only if the number is even. Therefore, if-then statements are also called **conditional statements.**

Conditional statements have two parts. The part following *if* is the **hypothesis.** The part following *then* is the **conclusion.**

Hypothesis: a number is even.
Conclusion: the number is divisible by 2.

How do you determine whether a conditional statement is true or false?

Conditional Statement	True or False?	Why?
If it is the fourth of July, then it is a holiday.	True	The statement is true because the conclusion follows from the hypothesis.
If an animal lives in the water, then it is a fish.	False	You can show that this statement is false by giving one counterexample. Whales live in water, but whales are mammals, not fish.

In geometry, postulates are often written as if-then or conditional statements. You can easily identify the hypothesis and conclusion in a conditional statement.

Example 1

Identify the hypothesis and conclusion in this statement.
If it is Saturday, then Elisa plays soccer.

Hypothesis: it is Saturday
Conclusion: Elisa plays soccer

Your Turn

a. If two lines intersect, then their intersection is a point.

There are different ways to express a conditional statement. The following statements all have the same meaning.

- If you are a member of Congress, then you are a U.S. citizen.
- All members of Congress are U.S. citizens.
- You are a U.S. citizen if you are a member of Congress.


U.S. Capitol, Washington, D.C.

Example 2

Write two other forms of this statement.
If points are collinear, then they lie on the same line.

All collinear points lie on the same line.
Points lie on the same line if they are collinear.

Your Turn

b. If three points are noncollinear, then they determine a plane.

The **converse** of a conditional statement is formed by exchanging the hypothesis and the conclusion.

Example 3

Write the converse of this statement.
If a figure is a triangle, then it has three angles.

To write the converse, exchange the hypothesis and conclusion.

Conditional: If a figure is a triangle, then *it has three angles*.

Converse: If *a figure has three angles*, then it is a triangle.

Your Turn

c. If you are at least 16 years old, then you can get a driver's license.

If a conditional statement is true, is its converse always true?

Conditional: If a figure is a square, then it has four sides.
Converse: If a figure has four sides, then it is a square.

Look Back
Counterexample, Lesson 1–1

The conditional statement is true. But there are many four-sided figures that are not squares. One counterexample is a rectangle. Therefore, the converse of this conditional is false.

Real World **Example** 4

Advertising Link

Write the statement from the advertisement at the right in if-then form. Then write the converse of the statement.

Statement:	It's OK to buy the wrong lipstick if you buy it in the right place.
If-then form:	If you buy lipstick in the right place, then it's OK to buy the wrong lipstick.
Converse:	If it's OK to buy the wrong lipstick, then you should buy it in the right place.

Check for Understanding

Communicating Mathematics

1. **Write** a conditional in which *there are clouds in the sky* is the hypothesis and *it may rain* is the conclusion.
2. **Explain** how to form the converse of a conditional.

Vocabulary
if-then statement
conditional statement
hypothesis
conclusion
converse

Guided Practice

Example 1 — **Identify the hypothesis and the conclusion of each statement.**

3. If a figure is a quadrilateral, then it has four sides.
4. If a player misses three practices, he is off the team.

Example 2 — **Write two other forms of each statement.**

5. You can vote if you are at least 18 years old.
6. All students who fight in school will be suspended.

Examples 3 & 4 — **Write the converse of each statement.**

7. If it is raining, then the ground is wet.
8. If you cut class, you will be assigned a detention.

Example 2

9. **Biology** Write the if-then form of this statement. *All cats are mammals.*

Exercises

Practice

For Exercises	See Examples
10–15	1
16–21	2
22–27, 29, 30	3
28	4

Homework Help

Extra Practice
See page 727.

Identify the hypothesis and the conclusion of each statement.

10. If the dog barks, it will wake the neighbors.
11. If a set of points has two endpoints, it is a line segment.
12. School will be cancelled if it snows more than six inches.
13. I will call my friend if I finish my homework.
14. All butterflies are arthropods.
15. All students should report to the gymnasium.

Write two other forms of each statement.

16. If the probability of an event is close to 1, it is very likely to happen.
17. If you eat fruits and vegetables, you will be healthy.
18. Your teeth will be whiter if you use a certain brand of toothpaste.
19. You'll win the race if you run the fastest.
20. All whole numbers are integers.
21. All people over age 18 can serve in the armed forces.

Write the converse of each statement.

22. If $2x = 20$, then $x = 10$.
23. If you play a musical instrument, you will do well in school.
24. The football team will play for the championship if it wins tonight.
25. You'll play softball if it stops raining.
26. All even numbers have a factor of 2.
27. All lines extend without end in two directions.

Applications and Problem Solving

28. **Comics** Write two conditional statements from the comic below in if-then form.

THE MIDDLETONS

29. **Advertising** Find an advertisement in a magazine or newspaper that contains an if-then statement. Write the converse of the statement.

30. **Number Theory** Consider this statement.
If two numbers are negative, then their product is positive.

a. Write the converse of the statement.

b. Determine whether the converse is *true* or *false*. If *false*, give a counterexample.

31. **Critical Thinking** The **inverse** of a conditional is formed by negating both the hypothesis and conclusion of the conditional.

Conditional: If it is raining, then it is cloudy.

Inverse: If it is *not* raining, then it is *not* cloudy.

The **contrapositive** of a conditional is formed by negating both the hypothesis and conclusion of the *converse* of the conditional.

Converse: If it is cloudy, then it is raining.

Contrapositive: If it is *not* cloudy, then it is *not* raining.

a. Write the inverse and contrapositive of this statement.
If a figure has five sides, then it is a pentagon.

b. Write a conditional. Then write its converse, inverse, and contrapositive.

Mixed Review

32. Determine whether the following statement is *true* or *false*.
If two planes intersect, then their intersection is a point. *(Lesson 1–3)*

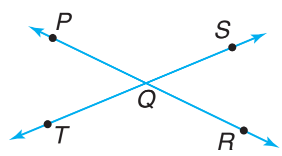
Refer to the figure at the right. *(Lesson 1–2)*

33. Name a ray.

34. Name a segment.

35. Name three collinear points.

36. Name three noncollinear points.



Exercises 33–36

Standardized Test Practice

37. **Multiple Choice** Find the next term of the sequence 0, 3, 9, 18, ... *(Lesson 1–1)*

(A) 21 (B) 30 (C) 54 (D) 162

Quiz 2 Lessons 1–3 and 1–4

1. Name the intersection of plane $\mathcal{X}$ and plane $\mathcal{W}$. *(Lesson 1–3)*

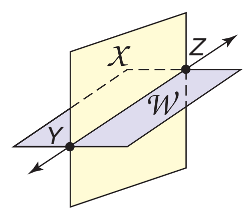
2. Points R, S, and T are noncollinear. Name all the different lines that can be drawn through these points. *(Lesson 1–3)*

Exercises 3–5 refer to this statement.
If today is Monday, then I have band practice. *(Lesson 1–4)*

3. Identify the hypothesis.

4. Identify the conclusion.

5. Write the converse of the statement.



Exercise 1

1–5 Tools of the Trade

What You'll Learn
You'll learn to use geometry tools.

Why It's Important
Landscaping
Landscapers use a compass-like device to draw large circles.
See Exercise 13.

Industrial designers usually make rough sketches to begin new designs. Then they use drafting tools to make technical drawings of their plans. Some drafting tools are shown at the right.

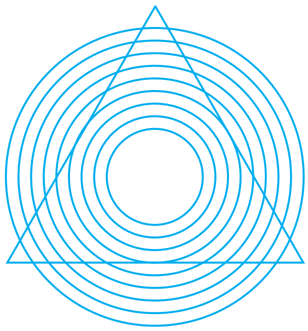
As you study geometry, you will use some of these basic tools. The first tool is a straightedge. A **straightedge** is an object used to draw a straight line. A credit card, a piece of cardboard, or a ruler can serve as a straightedge. A straightedge is also used to check if a line is straight.

An *optical illusion* is a misleading image. Points, lines, and planes in geometry can be arranged to create such illusions.

Example 1

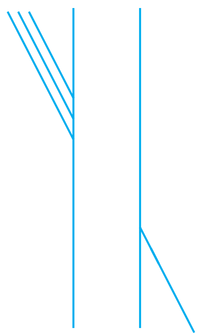
Determine whether the sides of the triangle are straight.

Place a straightedge along each side of the triangle. You can see that the sides are straight.



Your Turn

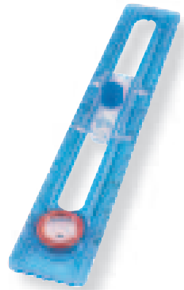
a. Use a straightedge to determine which of the three segments at the upper left forms a straight line with the segment at the lower right.



A **compass** is another useful tool. A common use for a compass is drawing arcs and circles. *An arc is part of a circle.*

The figures below show two kinds of compasses.

A-shaped compass

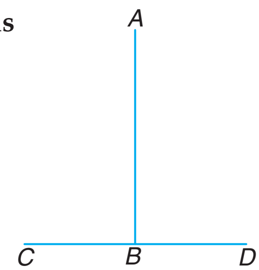


Safety compass

The A-shaped compass usually has a point and a pencil or lead point. The two legs of the compass are on a hinge so they can be adjusted for different settings. This kind of compass is often used by engineers and cartographers, people who draw maps. One use of a compass is to compare lengths of segments.

Example 2

Use a compass to determine which segment is longer, $\overline{AB}$ or $\overline{CD}$.

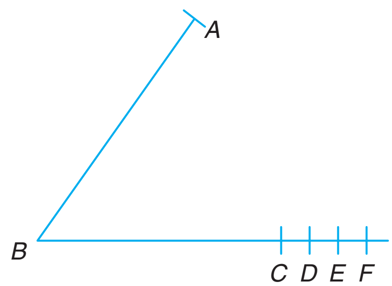


Place the point of the compass on B and adjust the compass so that the pencil is on A.

Without changing the setting of the compass, place the point of the compass on C. The pencil point does not reach point D. Therefore, $\overline{CD}$ is longer.

Your Turn

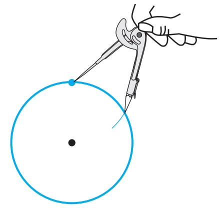
b. Which is longer, the segment from A to B or the segment from B to D?



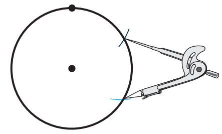
In geometry, you will draw figures using only a compass and a straightedge. These drawings are called **constructions**. No standard measurements are used in constructions. A construction is shown in Example 3.

Example 3 **Use a compass and straightedge to construct a six-sided figure.**

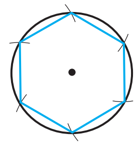
First, use the compass to draw a circle. Then using the same compass setting, put the point on the circle and draw a small arc on the circle.



Move the compass point to the arc and then draw another arc along the circle. Continue doing this until there are six arcs.



Use a straightedge to connect the points in order.



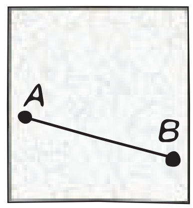
Another tool of the trade is patty paper. It can be used to do constructions such as finding a point in the middle of a segment, which is called a **midpoint**.

Hands-On Geometry
Paper Folding

Materials: patty paper, straightedge

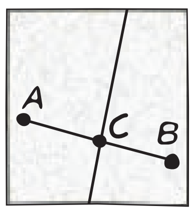
Step 1 Draw points A and B anywhere on a sheet of patty paper. Connect the points to form $\overline{AB}$.

Step 2 Fold the paper so that the endpoints lie on top of each other. Pinch the paper to make a crease on the segment.



Step 1

Step 3 Open the paper and label the point where the crease intersects $\overline{AB}$ at C. C is the midpoint of $\overline{AB}$.



Step 3

Try This

Draw a circle on patty paper and cut it out. Fold it in half and then in half again. What do you call the point where the fold lines meet?

You can use graphing calculators and computers as tools in geometry. The following activity shows how to use a TI-83 Plus or TI-84 Plus to construct a triangle with three sides of equal length.

Graphing Calculator Exploration

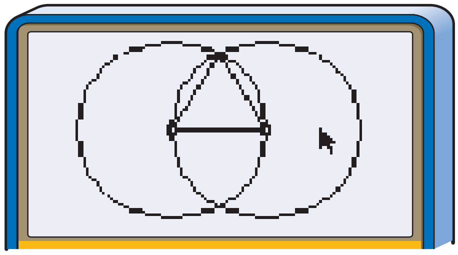
To open a geometry session, press APPS, select CabriJr and press ENTER.

Step 1 Open the F2 menu and select Segment. Draw a line segment for the first side of the triangle.

Step 2 Open the F3 menu and the Compass tool. The length of the segment you drew is the setting for the compass. Move your cursor to the segment and select it. The calculator will show a circle. Move the cursor so that the center of the circle is at one endpoint of the line segment and press ENTER.

Graphing Calculator Tutorial
See pp. 782–785.

Step 3 Repeat Step 2 and place the circle at the other endpoint of the line segment.



Step 4 Use the Segment tool on the F2 menu. Draw the line segments from each endpoint to the point of intersection of the two circles.

Try These

1. Open menu F5 and select Measure and then D. & Length. Then select each side of the triangle to find the lengths of the sides. Are the sides of the triangle all the same length?
2. Change the length of the original line segment. How is the triangle affected?

Check for Understanding

Communicating Mathematics

1. **Explain** the difference between a construction and other kinds of drawings.
2. **Name** four tools that are used in geometry.
3. You Decide? Mario says that a straightedge and a ruler are the same. Curtis says they are different. Who is correct? Explain your reasoning.

Vocabulary
straightedge
compass
construction
midpoint

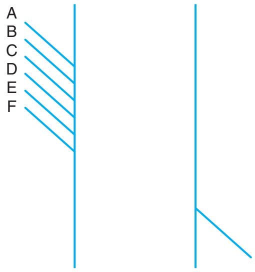
CONTENTS

Guided Practice

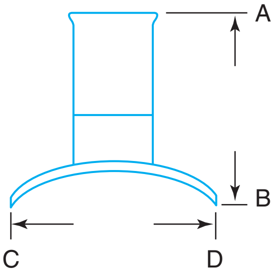
Examples 1 & 2

Use a straightedge or compass to answer each question.

4. Which segment on the upper left forms a straight line with the segment on the lower right?

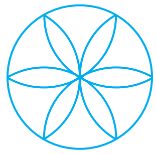


5. Which is greater, the height of the hat (from *A* to *B*) or the width of the hat (from *C* to *D*)?



Example 3

6. Design Use a compass to make a design like the one shown at the right. (*Hint:* Draw large arcs from one "side" of the circle to the other.)

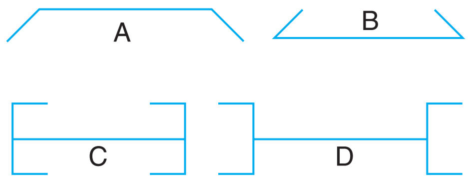


Exercises

Practice

Use a straightedge or compass to answer each question.

7. Which segment is longest?

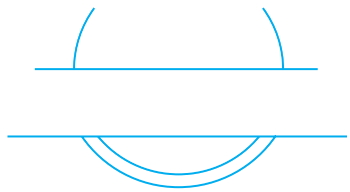


Homework Help	
For Exercises	See Examples
7	2
8, 11–13	3
9, 10	1

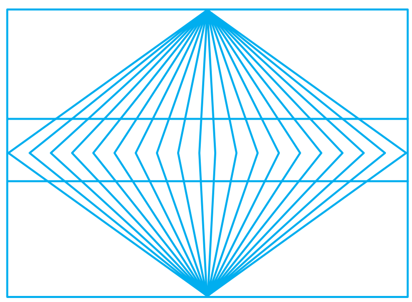
Extra Practice

See page 727.

8. Which arc in the lower part of the figure goes with the upper arcs to form part of a circle?

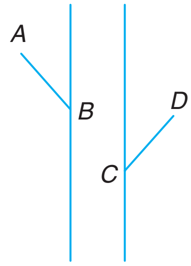


9. Are the two horizontal segments straight or do they bend?



10. If extended, will $\overline{AB}$ intersect $\overline{CD}$ at C?

11. Use a compass to draw three different-sized circles that all have the same center.

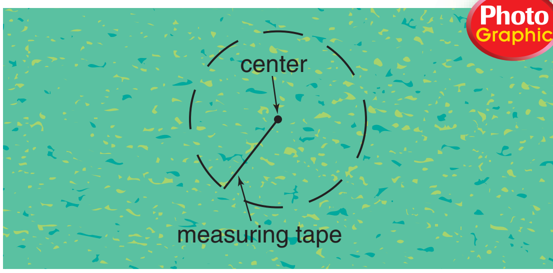


Exercise 10

Applications and Problem Solving

12. **Sewing** Explain how you could use a pencil and a long piece of string to outline a circular cloth for a round table.

13. **Landscaping** One way to mark off a circle for a bed of flowers is by using a measuring tape. From the center of the bed, extend the measuring tape a given distance and walk around the center, making marks on the ground. Explain how this method is similar to drawing a circle with a compass.



14. **Critical Thinking** In this text, you will be asked to make conjectures about geometric figures. Explain why you should not make conclusions about figures based only on their appearance. (*Hint:* Think of optical illusions.)

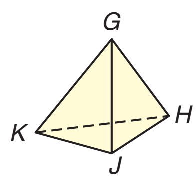
Mixed Review

Write the converse of each statement. *(Lesson 1–4)*

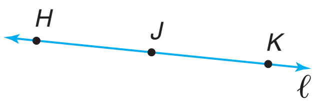
15. If a figure is a triangle, then it has three sides.
16. All whole numbers can be written as decimals.
17. You like the ocean if you are a surfer.

Standardized Test Practice

18. **Short Response** Name all of the planes that are represented in the figure at the right. *(Lesson 1–3)*



19. **Multiple Choice** Which is *not* a name for this figure? *(Lesson 1–2)*



(A) $\overleftrightarrow{JK}$ (B) $\overleftrightarrow{\ell H}$ (C) $\overleftrightarrow{KH}$ (D) line ℓ

1–6 A Plan for Problem Solving

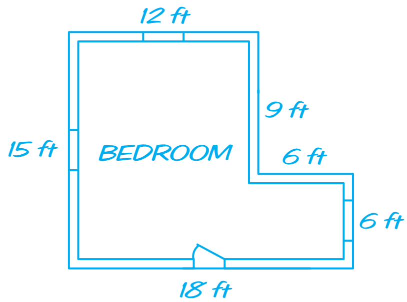
What You'll Learn

You'll learn to use a four-step plan to solve problems that involve the perimeters and areas of rectangles and parallelograms.

Why It's Important

Interior Design Designers use formulas for perimeter and area to order materials. *See Exercise 11.*

A useful measurement in geometry is perimeter. **Perimeter** is the distance around a figure. It is the sum of the lengths of the sides of the figure. The perimeter of the room shown at the right is found by adding.



$P = 12 + 9 + 6 + 6 + 18 + 15$ or 66

The perimeter of the room is 66 feet.

Some figures have special characteristics. For example, the opposite sides of a rectangle have the same length. This allows us to use a formula to find the perimeter of a rectangle. A **formula** is an equation that shows how certain quantities are related.

Perimeter of a Rectangle

Words: The perimeter P of a rectangle is the sum of the measures of its sides. It can also be expressed as two times the length ℓ plus two times the width w.

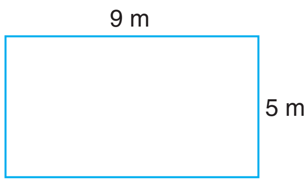
Symbols: $P = \ell + w + \ell + w$
$P = 2\ell + 2w$

Model:

Example 1

Algebra Review Evaluating Expressions, p. 718

Find the perimeter of the rectangle.



$P = 2\ell + 2w$ *Perimeter formula*

$P = 2(9) + 2(5)$ *Replace ℓ with 9 and w with 5.*

$P = 18 + 10$ or 28 *Simplify.*

The perimeter is 28 meters.

Your Turn

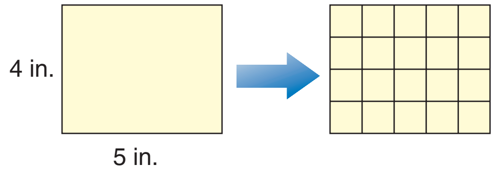
a. Find the perimeter of a rectangle with a length of 17 feet and a width of 8 feet.

Reading Geometry

Abbreviations for units of area use the exponent 2.
square inch → in^2
square centimeter → cm^2

Another important measure is area. The **area** of a figure is the number of square units needed to cover its surface. Two common units of area are the *square centimeter* and the *square inch*.

The area of the rectangle below can be found by dividing it into 20 unit squares.



The area of a rectangle is also found by multiplying the length and the width.

Area of a Rectangle

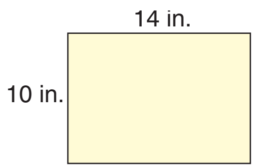
Words: The area A of a rectangle is the product of the length ℓ and the width w.

Symbols: $A = \ell w$

Model:

Example 2

Find the area of the rectangle.



$A = \ell w$ *Area formula*

$A = (14)(10)$ *Replace ℓ with 14 and w with 10.*

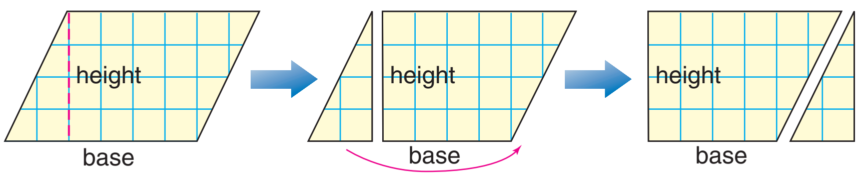
$A = 140$

The area of the rectangle is 140 square inches.

Your Turn

b. Find the area of a rectangle with a length of 8.2 meters and a width of 2.4 meters.

The opposite sides of a parallelogram also have the same length. The area of a parallelogram is closely related to the area of a rectangle.



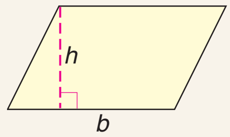
The area of a parallelogram is found by multiplying the base and height.

Area of a Parallelogram

Words: The area A of a parallelogram is the product of the base b and the height h.

Symbols: $A = bh$

Model:



Example 3

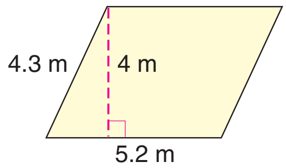
Algebra Review
Operations with Decimals, p. 720

Find the area of the parallelogram.

$A = bh$ *Area formula*

$A = (5.2)(4)$ *Replace b with 5.2 and h with 4.*

$A = 20.8$



The area of the parallelogram is 20.8 square meters.

Your Turn

c. Find the area of a parallelogram with a height of 6 feet and a base of 8 feet.

Some mathematics problems can be solved by using a formula. Others can be solved by using a problem-solving strategy like finding a pattern or making a model. No matter what type of problem you need to solve, you can always use a **four-step plan**.

Problem-Solving Plan

1. **Explore** the problem.
2. **Plan** the solution.
3. **Solve** the problem.
4. **Examine** the solution.

Real World

Example 4

Interior Design Link

Julia wants to paint two rectangular walls of her bedroom. One bedroom wall is 15 feet long and 8 feet high. The other wall is 12 feet long and 8 feet high. She wants to put two coats of paint on the walls. She knows that 1 gallon of paint will cover about 350 square feet of surface. Will one gallon of paint be enough?

Explore You know the dimensions of each wall. You also know that one gallon of paint covers about 350 square feet. You also know that she wants to use two coats of paint. You need to determine whether one gallon of paint is enough.

(continued on the next page)

Plan Since you need to find the total area that will be covered with paint, you can use the formula for the area of a rectangle. Find the total area of the two walls with two coats of paint. Then compare to 350 square feet.

Solve

Area of first wall	Area of second wall
$A = \ell w$	$A = \ell w$
$A = (15)(8)$	$A = (12)(8)$
$A = 120$	$A = 96$

The total area of the two walls is $120 + 96$ or 216 square feet.

Since Julia wants to use two coats of paint, she needs to cover 2×216 or 432 square feet of area. One gallon covers only 350 square feet, so one gallon will not be enough.

Examine Is the answer reasonable? The area of the first wall with two coats of paint is 2×120 or 240 square feet, which is more than one-half of 350 square feet. The answer seems reasonable.

Check for Understanding

Communicating Mathematics

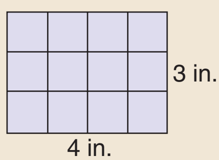
1. **Draw and label** two rectangles and one parallelogram, each having an area of 12 square feet.
2. **Explain** the difference between *perimeter* and *area*.
3. **Name** the four steps of the four-step plan for problem solving.

Vocabulary
perimeter
formula
area
four-step plan

Guided Practice

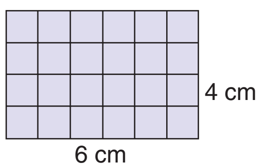
Getting Ready **Find the area of each figure.**

Sample:

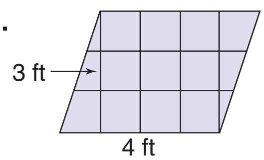


Solution: The surface can be covered by 12 unit squares. The area is 4×3 or 12 square inches.

4.

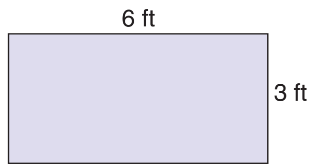


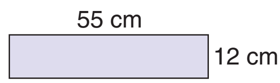
5.



Examples 1 & 2

Find the perimeter and area of each rectangle.

6. 

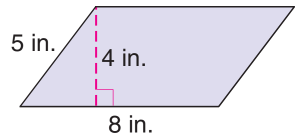
7. 

8. $\ell = 18$ cm, $w = 12$ cm

9. $\ell = 25$ ft, $w = 5$ ft

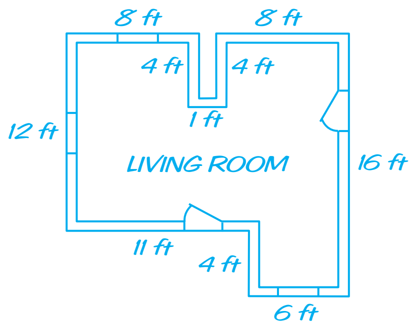
Example 3

10. Find the area of the parallelogram.



Example 4

11. **Interior Design** An interior designer wants to order wallpaper border to place at the top of the walls in the room shown at the right. If one roll of border is 5 yards long, how many rolls of border should the designer order?

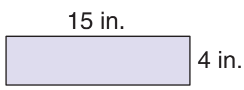


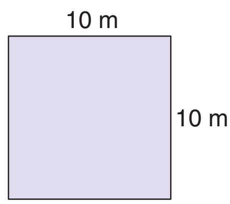
Exercises

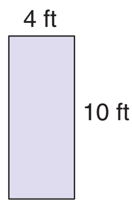
Practice

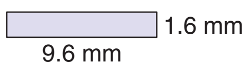
Find the perimeter and area of each rectangle.

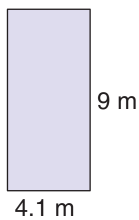
Homework Help	
For Exercises	See Examples
12–23, 30	1, 2
24–26, 29	3
27, 28, 31	2
Extra Practice	
See page 727.	

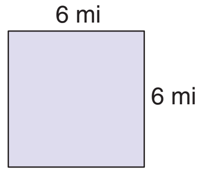
12. 

13. 

14. 

15. 

16. 

17. 

Find the perimeter and area of each rectangle described.

18. $\ell = 12$ in., $w = 6$ in.

19. $\ell = 15$ ft, $w = 10$ ft

20. $\ell = 14$ m, $w = 4$ m

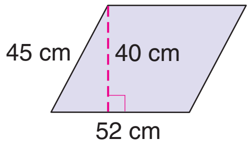
21. $\ell = 18$ cm, $w = 18$ cm

22. $\ell = 8.4$ mm, $w = 5$ mm

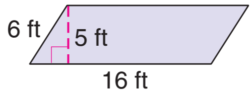
23. $\ell = 10$ mi, $w = 6.5$ mi

Find the area of each parallelogram.

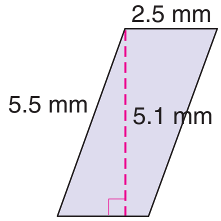
24.



25.



26.



27. Find the area of a rectangle with length 15 meters and width 2.3 meters.

28. The length of a rectangle is 24 inches, and the width of the rectangle is 18 inches. What is the area?

Applications and Problem Solving

29. Algebra What is the base of a parallelogram with area 45 square yards and height 9 yards?

30. Algebra What is the width of a rectangle with perimeter 18 centimeters and length 5 centimeters?

31. Remodeling A remodeler charges \$3.25 per square foot to refinish a wood floor. How much would it cost to refinish a wood floor that measures 30 feet by 18 feet?

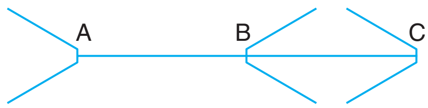
32. Critical Thinking A square is a rectangle in which all four sides have the same measure. Suppose s represents the measure of one side of a square.

a. Write a formula for the perimeter of a square.

b. Write a formula for the area of a square.

Mixed Review

33. Use a compass to determine which segment is longer, $\overline{AB}$ or $\overline{BC}$. *(Lesson 1–5)*



34. Advertising A billboard reads *If you want an exciting vacation, come to Las Vegas.* Identify the hypothesis and conclusion of this statement. *(Lesson 1–4)*

Find the next three terms of each sequence. *(Lesson 1–1)*

35. 13, 9, 5, 1, . . .

36. 50, 51, 53, 56, . . .

Standardized Test Practice

A B C D

37. Multiple Choice Which expression can be used to find the total cost of b bats and g gloves if a bat costs \$50 and a glove costs \$75? *(Algebra Review)*

(A) $(50 + 75) \times (b + g)$

(B) $(50 \times 75) + (b \times g)$

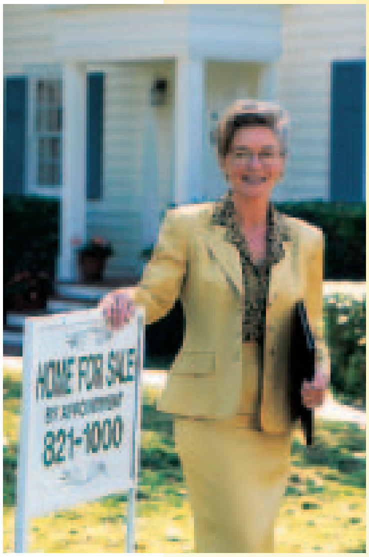
(C) $50b + 75g$

(D) $75b + 50g$

Math In the Workplace

Real Estate Agent

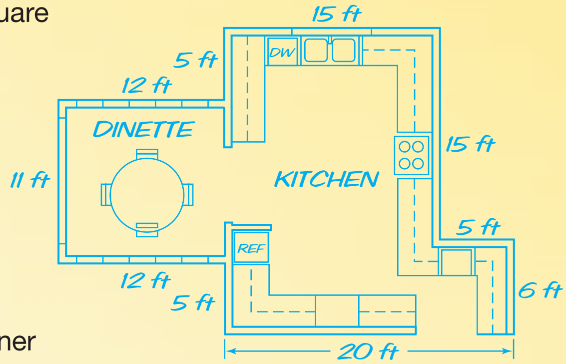
Do you like to work with people? Are you enthusiastic, well organized, and detail oriented? Then you may enjoy a career as a real estate agent. Real estate agents help people with an important event—buying and selling a home.



But before you can be a real estate agent, you must obtain a license. All states require prospective agents to pass a written test, which usually contains a section on real estate mathematics. Here are some typical questions.

1. Determine the total square footage of the kitchen and dinette in the blueprint.

2. How many square feet of concrete would be needed to construct a walk 7 feet wide around the outside corner of a corner lot measuring 50 feet by 120 feet?



FAST FACTS About Real Estate Agents

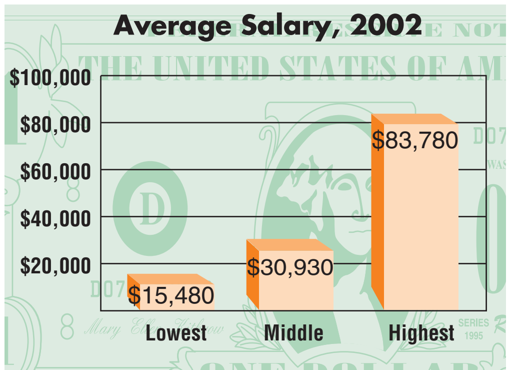
Working Conditions

- growing number work from their homes because of advances in telecommunications
- work evenings and weekends to meet the needs of their clients

Education

- high school graduate
- 30 to 90 hours of classroom instruction about real estate mathematics and laws
- continuing education for license renewal

Earnings



Source: Bureau of Labor Statistics

Career Data For up-to-date information about a career as a real estate agent, visit: www.geomconcepts.com

CHAPTER 1 Study Guide and Assessment

Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

Review Activities
For more review activities, visit:
www.geomconcepts.com

Geometry

area *(p. 36)*
axiom *(p. 18)*
collinear *(p. 13)*
compass *(p. 30)*
construction *(p. 30)*
coplanar *(p. 14)*
endpoint *(p. 13)*
formula *(p. 35)*
four-step plan *(p. 37)*
line *(p. 12)*
line segment *(p. 13)*
midpoint *(p. 31)*
noncollinear *(p. 13)*
noncoplanar *(p. 14)*
Pascal's triangle *(p. 10)*
perimeter *(p. 35)*
plane *(p. 14)*
point *(p. 12)*
postulate *(p. 18)*
ray *(p. 13)*
straightedge *(p. 29)*
undefined terms *(p. 12)*

Logic

conclusion *(p. 24)*
conditional statement *(p. 24)*
conjecture *(p. 6)*
contrapositive *(p. 28)*
converse *(p. 25)*
counterexample *(p. 6)*
hypothesis *(p. 24)*
if-then statement *(p. 24)*
inductive reasoning *(p. 4)*
inverse *(p. 28)*

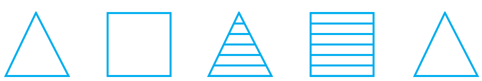
Choose the correct term to complete each sentence.

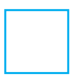
1. A (line, plane) is named using three noncollinear points.
2. The intersection of two planes is a (point, line).
3. The part following *if* in an if-then statement is called the (hypothesis, conclusion).
4. (Conjectures, Constructions) are special drawings created using only a compass and a straightedge.
5. The distance around a figure is called its (perimeter, area).
6. A conclusion reached using inductive reasoning is called a (hypothesis, conjecture).
7. A (line segment, ray) has a definite beginning and end.
8. (Hypotheses, Postulates) are facts about geometry that are accepted to be true.
9. A credit card or a piece of cardboard can serve as a (straightedge, compass).
10. It takes only one (converse, counterexample) to show that a conjecture is not true.

Skills and Concepts

Objectives and Examples

- **Lesson 1–1** Identify patterns and use inductive reasoning.



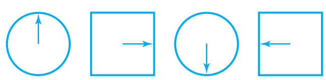
The next figure in this pattern is .

Review Exercises

Find the next three terms of each sequence.

11. 2, 3, 6, 11, . . .
12. 27, 21, 15, 9, . . .

Draw the next figure in the pattern.

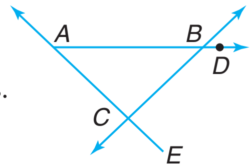
13. 

Objectives and Examples

Review Exercises

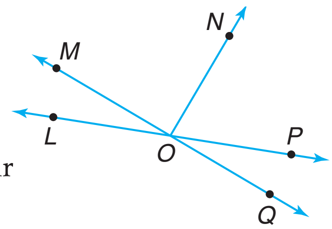
- **Lesson 1–2** Identify and draw models of points, lines, and planes and determine their characteristics.

$\overleftrightarrow{CB}$ is a line.
$\overrightarrow{EA}$ and $\overrightarrow{AD}$ are rays.
$\overline{CE}$ and $\overline{BA}$ are segments.
Points A, B, and D are collinear.



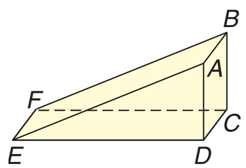
Use the figure to name examples of each term.

14. three segments
15. two rays
16. a line containing point P
17. three noncollinear points



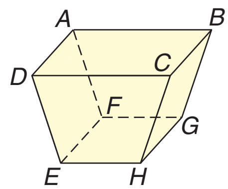
- **Lesson 1–3** Identify and use basic postulates about points, lines, and planes.

Two of the planes represented in this figure are planes ABF and ADE.



Planes ABC and BCF intersect at $\overleftrightarrow{BC}$.

18. Name the intersection of planes ADE and CDE.
19. Name three other planes represented in the figure.



- **Lesson 1–4** Write statements in if-then form and write the converses of the statements.

Statement: All integers are rational numbers.

Write the statement in if-then form and then write its converse.

If-then: If a number is an integer, then it is a rational number.

Converse: If a number is a rational number, then it is an integer.

Identify the hypothesis and the conclusion of each statement.

20. If an animal has wings, then it is a bird.
21. All school buses are yellow.

Write two other forms of each statement.

22. Every cloud has a silver lining.
23. People who own pets live long lives.

Write the converse of each statement.

24. If the month is December, then it has 31 days.
25. All students like to play baseball.

Mixed Problem Solving
See pages 758–765.

Objectives and Examples

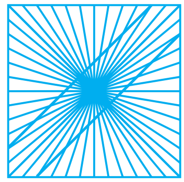
• **Lesson 1–5** Use geometry tools.

In which figure is the middle segment longer?

The middle segment in figure a is longer.

Review Exercises

26. Use a straightedge to determine whether the two heavy segments are straight.



27. Use a compass to draw two circles that have two points of intersection.

• **Lesson 1–6** Use a four-step plan to solve problems that involve the perimeters and areas of rectangles and parallelograms.

Find the perimeter and area of a rectangle with length 17 inches and width 5.5 inches.

Perimeter

$$P = 2\ell + 2w = 2(17) + 2(5.5) = 34 + 11 \text{ or } 45$$

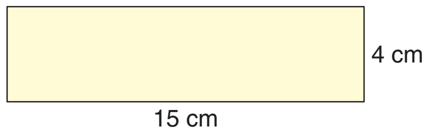
Area

$$A = \ell w = (17)(5.5) = 93.5$$

The perimeter is 45 inches, and the area is 93.5 square inches.

Find the perimeter and area of each rectangle.

28.



29. $\ell = 18$ ft, $w = 23$ ft

30. $\ell = 4.2$ m, $w = 1.5$ m

31. Find the base of a parallelogram with height 9 centimeters and area 108 square centimeters.

Application and Problem Solving

32. Number Theory Consider this statement. *If a number is divisible by 6, it is also divisible by 3.*

a. Write the converse of the statement.

b. Determine whether the converse is *true* or *false*. If false, give a counterexample. *(Lesson 1–4)*

33. Construction A rectangular patio measures 15 feet by 18 feet. The patio will be covered with square tiles measuring 1 foot on each side. If the tiles are $4.50 each, find the total cost of the tiles. *(Lesson 1–6)*

34. Retail Sales A display of cereal boxes is stacked in the shape of a pyramid. There are 4 boxes in the top row, 6 boxes in the next row, 8 boxes in the next row, and so on. The display contains 7 rows of boxes. How many boxes are in the seventh row? *(Lesson 1–1)*

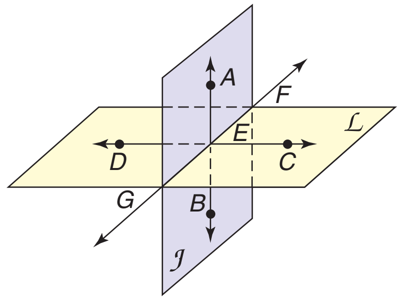
CHAPTER 1 Test

1. **Explain** the difference between a drawing and a construction.
2. **Draw** a ray with endpoint A that also contains point B.
3. **Draw and label** a parallelogram that has an area of 24 square inches.
4. **Compare and contrast** lines and rays.

Find the next three terms of each sequence.

5. 1, 2, 4, 7, . . .
6. -800, 400, -200, 100, . . .
7. 11, 15, 19, 23, . . .

For Exercises 8–11, refer to the figure at the right.



8. Name the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.
9. Name the intersection of plane $\mathcal{J}$ and plane $\mathcal{L}$.
10. Name a point that is coplanar with points A and E.
11. Name three collinear points.

Determine whether each statement is *true* or *false*. If false, replace the underlined word(s) to make a true statement.

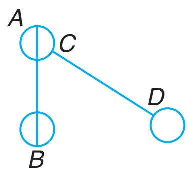
12. The intersection of two planes is a point.
13. Two points determine a line.
14. A line segment has two endpoints.
15. Three collinear points determine a plane.

Write the converse of each statement. Then determine whether the converse is *true* or *false*. If false, give a counterexample.

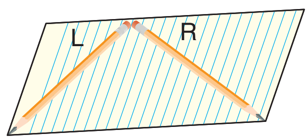
16. If $x = 3$, then $x + 10 = 13$.
17. The sum of two odd numbers is an even number.
18. If you live in Vermont, then you live in the United States.

Use a straightedge or compass to answer each question.

19. Is the segment from A to B as long as the segment from C to D?



20. Which is longer, pencil L or R?



Find the perimeter and area of each rectangle described.

21. $\ell = 5$ mm, $w = 12$ mm
22. $\ell = 22$ ft, $w = 3$ ft
23. $\ell = 16$ m, $w = 14.25$ m

24. Find the area of a parallelogram with base 15 feet and height 10 feet.

25. **Agriculture** A sod farmer wants to fertilize and seed a rectangular plot of land 150 feet by 240 feet. A bag of fertilizer covers 5000 square feet, and a bag of grass seed covers 3000 square feet. How many bags of each does the farmer need to buy for this plot of land?

Number Concept Problems

All standardized tests contain numerical problems. You'll need to understand and apply these mathematical terms.

absolute value	decimals	divisibility
exponents	factors	fractions
integers	odd and even	positive and negative
prime numbers	roots	scientific notation

Problems on standardized tests often use these terms. Be sure you understand each term and read the problem carefully!

Test-Taking Tip

Know the properties of 0 and 1. For example, 0 is even, neither positive nor negative, and not prime. 1 is the only integer with only one divisor. 1 is not prime.

Example 1

Ricky earned about 3.6×10^4 dollars last year. If he worked 50 weeks during the year, how much did he earn per week?

(A) $72 (B) $180
(C) $720 (D) $7200

Hint Look for key terms, like *per week*. "Per" tells you to use division.

Solution You know the total amount earned in a year. You need to find the amount earned in one week. So divide the total amount by the number of weeks, 50. The total amount is written in scientific notation. Express this amount in standard notation. Then divide.

$$\frac{3.6 \times 10^4}{50} = \frac{3.6 \times 10{,}000}{50} \qquad 10^4 = 10{,}000$$

$$= \frac{3.6 \times \cancel{10{,}000}^{200}}{\cancel{50}_{1}} \qquad \textit{Divide the numerator and denominator by 50.}$$

$$= 3.6 \times 200 \qquad \textit{Simplify.}$$

$$= 720 \qquad \textit{Multiply.}$$

The answer is C.

Example 2

What is the sum of the positive even factors of 12?

Hint Look for terms like *positive, even,* and *factor*.

Solution First, find all the factors of 12. To be sure you don't miss any factors, write all the *integers* from 1 to 12. Then cross out the numbers that are *not* factors of 12.

1 2 3 4 ~~5~~ 6 ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ 12

Reread the question. It asks for the sum of *even* factors. Circle the factors that are even numbers.

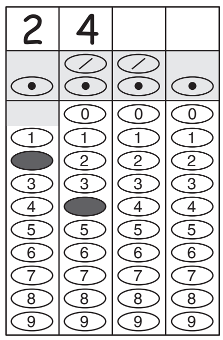
1 2 3 4 ~~5~~ 6 ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ 12

Now add these factors to find the sum.

$$2 + 4 + 6 + 12 = 24$$

The answer is 24. Record it on the grid.

- Start with the *left* column.
- Write the answer in the boxes at the top. Write one digit in each column.
- Mark the correct oval in each column.
- *Never* grid a mixed number; change it to a fraction or a decimal.



Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 766–781.

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

1. The daily cost of renting a car is \$25.00 plus \$0.30 per mile driven. What is the cost of renting the car for one day and driving it 75 miles? *(Algebra Review)*

Ⓐ \$22.50 Ⓑ \$27.50
Ⓒ \$47.50 Ⓓ \$55.00

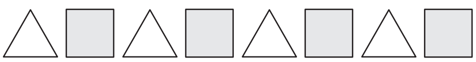
2. The product of a number and 1.85 is less than 1.85. Which of the following is the number? *(Algebra Review)*

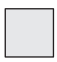
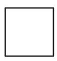
Ⓐ 1.5 Ⓑ 1
Ⓒ 185 Ⓓ 0.75

3. Four students were asked to find the distance between their homes and school. Their responses were: 3.5 miles, $3\frac{3}{8}$ miles, $3\frac{3}{5}$ miles, and $3\frac{1}{3}$ miles. Which is the *greatest* distance? *(Algebra Review)*

Ⓐ 3.5 miles Ⓑ $3\frac{3}{8}$ miles
Ⓒ $3\frac{3}{5}$ miles Ⓓ $3\frac{1}{3}$ miles

4. If the pattern below continues, what will the 18th figure look like? *(Lesson 1–1)*



Ⓐ  Ⓑ 
Ⓒ  Ⓓ 

5. In 2003, about 746,000,000 CDs were shipped in the United States. What is another way of expressing the number 746,000,000? *(Algebra Review)*

Ⓐ 7.46×10^8 Ⓑ 74.6×10^8
Ⓒ 74.6 million Ⓓ 74.6 billion

6. Which of the following expresses the prime factorization of 54? *(Algebra Review)*

Ⓐ 9×6 Ⓑ $3 \times 3 \times 6$
Ⓒ $3 \times 3 \times 2$ Ⓓ $3 \times 3 \times 3 \times 2$
Ⓔ 5.4×10

7. If 8 and 12 are each factors of K, what is the value of K? *(Algebra Review)*

Ⓐ 6 Ⓑ 24
Ⓒ 8 Ⓓ 96
Ⓔ It cannot be determined from the information given.

8. A rectangle has a perimeter of 38 feet and an area of 48 square feet. What are the dimensions of the rectangle? *(Lesson 1–6)*

Ⓐ 4 ft $\times$ 12 ft Ⓑ 6 ft $\times$ 8 ft
Ⓒ 16 ft $\times$ 3 ft Ⓓ 24 ft $\times$ 2 ft

Grid In

9. Dr. Cronheim has 379 milliliters of solution to use for a class experiment. She divides the solution evenly among the 24 students. If she has 19 milliliters of the solution left after the experiment, how many milliliters of the solution did she give to each student? *(Algebra Review)*

Extended Response

10. The altitude in Galveston, Texas, is about 10 feet. There are 5280 feet in a mile. *(Algebra Review)*

Part A Explain how you can find this altitude in miles.

Part B Give your answer both as a fraction and as a decimal to the nearest ten-thousandth.