

## CHAPTER

# 5

# Triangles and Congruence

## What You'll Learn

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### Key Ideas

- Identify parts of triangles and classify triangles by their parts. (*Lesson 5-1*)
- Use the Angle Sum Theorem. (*Lesson 5-2*)
- Identify translations, reflections, and rotations and their corresponding parts. (*Lesson 5-3*)
- Name and label corresponding parts of congruent triangles. (*Lesson 5-4*)
- Use the SSS, SAS, ASA, and AAS tests for congruence. (*Lessons 5-5 and 5-6*)

### Key Vocabulary

congruent triangles (*p. 203*)

right triangle (*p. 188*)

triangle (*p. 188*)

## Why It's Important

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**Social Studies** The Hopi people have lived on three isolated mesas in what is now northern Arizona for more than a thousand years. Oraibi, a Hopi village on Third Mesa, is the oldest continuously inhabited village in the United States. The Hopi live in multilevel adobe or stone villages called pueblos.

**Triangles** are the simplest of the polygons. You will determine how triangles are used to create the design on Hopi pottery in Lesson 5-3.



Study these lessons to improve your skills.

# ✓ Check Your Readiness

✓ Lesson 3–2, pp. 96–101

✓ Lesson 3–5, pp. 116–121

✓ Algebra Review, p. 722

✓ Algebra Review, p. 723

Use a protractor to draw an angle having each measurement. Then classify each angle as *acute*, *obtuse*, or *right*.

1.  $52^\circ$
2.  $145^\circ$
3.  $18^\circ$
4.  $90^\circ$
5.  $75^\circ$
6.  $98^\circ$

Determine the measures of the complement and supplement of each angle.

7.  $34^\circ$
8.  $12^\circ$
9.  $44^\circ$
10.  $78^\circ$
11.  $66^\circ$
12.  $5^\circ$

Solve each equation. Check your solution.

13.  $114 + n = 180$
14.  $58 + x = 90$
15.  $5m = 90$
16.  $180 = 12g$
17.  $90 - k = 23$
18.  $180 - q = 121$
19.  $90 = 4b - 18$
20.  $48 + 3g = 90$
21.  $8y - 16 = 180$
22.  $12c + 6 = 90$
23.  $(n - 4) + n = 180$
24.  $2x + 4x + 6x = 180$

## FOLDABLES™ Study Organizer

Make this Foldable to help you organize your Chapter 5 notes. Begin with 3 sheets of plain  $8\frac{1}{2}$ " by 11" paper.

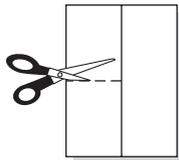
1 Fold in half lengthwise.



2 Fold the top to the bottom.



3 Open and cut along the second fold to make two tabs.



4 Label each tab as shown.



**Reading and Writing** As you read and study the chapter, write what you learn about the two methods of classifying triangles under the tabs.



# 5-1

## Classifying Triangles

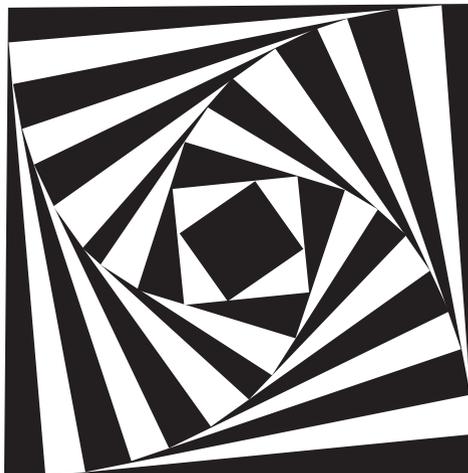
### What You'll Learn

You'll learn to identify the parts of triangles and to classify triangles by their parts.

### Why It's Important

**Art** Abstract artists use geometric shapes in their designs. See Exercise 24.

Optical art is a form of abstract art that creates special effects by using geometric patterns. The design at the right looks like a spiral staircase, but it is made mostly of triangles.

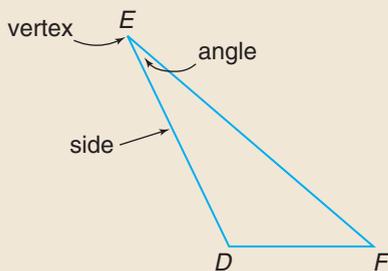


In geometry, a **triangle** is a figure formed when three noncollinear points are connected by segments. Each pair of segments forms an angle of the triangle. The **vertex** of each angle is a vertex of the triangle.

Triangles are named by the letters at their vertices. Triangle  $DEF$ , written  $\triangle DEF$ , is shown below.

### Reading Geometry

Read the symbol  $\triangle$  as *triangle*. Other names for  $\triangle DEF$  are  $\triangle FDE$ ,  $\triangle EDF$ ,  $\triangle FED$ ,  $\triangle DFE$ , and  $\triangle EFD$ .

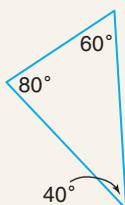


The sides are  $\overline{DE}$ ,  $\overline{EF}$ , and  $\overline{DF}$ .  
The vertices are  $D$ ,  $E$ , and  $F$ .  
The angles are  $\angle D$ ,  $\angle E$ , and  $\angle F$ .

In Chapter 3, you classified angles as acute, obtuse, or right. Triangles can also be classified by their angles. All triangles have at least two acute angles. The third angle is either acute, obtuse, or right.

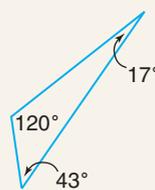
### Triangles Classified by Angles

**acute**



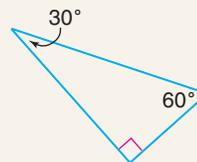
all acute angles

**obtuse**



one obtuse angle

**right**



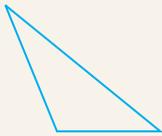
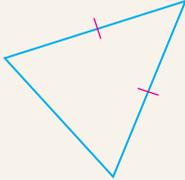
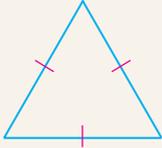
one right angle



## Reading Geometry

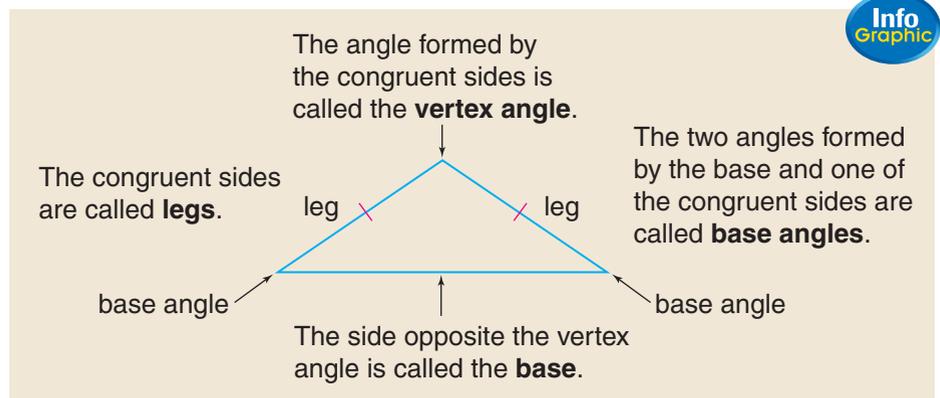
An equal number of slashes on the sides of a triangle indicate that those sides are congruent.

Triangles can also be classified by their sides.

	scalene	isosceles	equilateral
<b>Triangles Classified by Sides</b>			
	no sides congruent	at least two sides congruent	all sides congruent

Since all sides of an equilateral triangle are congruent, then at least two of its sides are congruent. So, *all equilateral triangles are also isosceles triangles.*

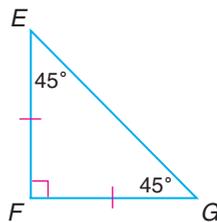
Some parts of isosceles triangles have special names.



## Examples

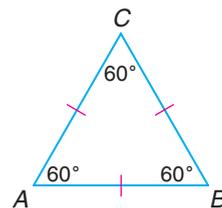
Classify each triangle by its angles and by its sides.

1



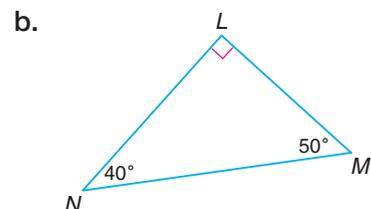
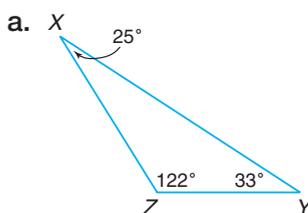
$\triangle EFG$  is a right isosceles triangle.

2



$\triangle ABC$  is an acute equilateral triangle.

## Your Turn



### Example

#### Algebra Link

3

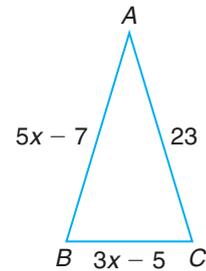
Find the measures of  $\overline{AB}$  and  $\overline{BC}$  of isosceles triangle  $ABC$  if  $\angle A$  is the vertex angle.

**Explore** You know that  $\angle A$  is the vertex angle. Therefore,  $\overline{AB} \cong \overline{AC}$ .

**Plan** Since  $\overline{AB} \cong \overline{AC}$ ,  $AB = AC$ . You can write and solve an equation.

**Solve**

$AB = AC$	<i>Definition of congruent segments</i>
$5x - 7 = 23$	<i>Substitution</i>
$5x - 7 + 7 = 23 + 7$	<i>Add 7 to each side.</i>
$5x = 30$	<i>Simplify.</i>
$\frac{5x}{5} = \frac{30}{5}$	<i>Divide each side by 5.</i>
$x = 6$	<i>Simplify.</i>



To find the measures of  $\overline{AB}$  and  $\overline{BC}$ , replace  $x$  with 6 in the expression for each measure.

<b>AB</b>	<b>BC</b>
$AB = 5x - 7$	$BC = 3x - 5$
$= 5(6) - 7$	$= 3(6) - 5$
$= 30 - 7$ or 23	$= 18 - 5$ or 13

Therefore,  $AB = 23$  and  $BC = 13$ .

**Examine** Since  $AB = 23$  and  $AC = 23$ , the triangle is isosceles.

## Check for Understanding

### Communicating Mathematics

1. Draw a scalene triangle.
2. Sketch and label an isosceles triangle in which the vertex angle is  $\angle X$  and the base is  $\overline{YZ}$ .
3. Is an equilateral triangle also an isosceles triangle? Explain why or why not.

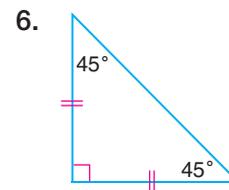
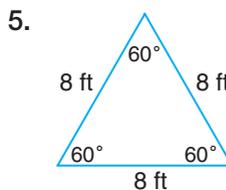
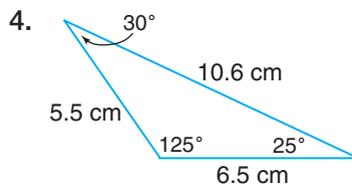
### Vocabulary

triangle  
vertex  
equilateral  
isosceles  
scalene

### Guided Practice

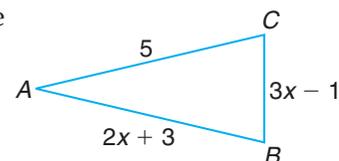
#### Examples 1 & 2

Classify each triangle by its angles and by its sides.



#### Example 3

7. **Algebra**  $\triangle ABC$  is an isosceles triangle with base  $\overline{BC}$ . Find  $AB$  and  $BC$ .

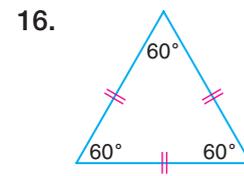
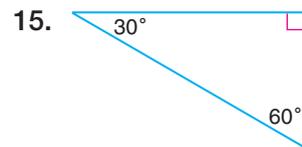
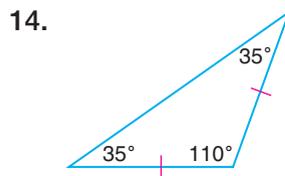
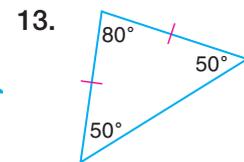
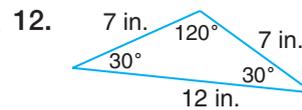
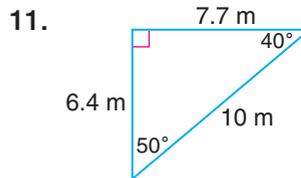
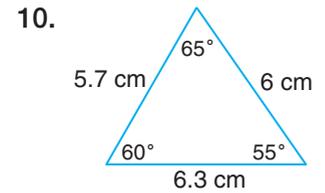
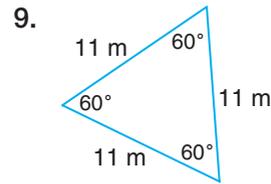
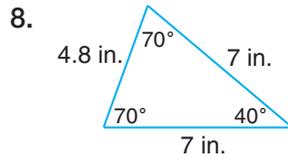


# Exercises

## Practice

Homework Help	
For Exercises	See Examples
8–17	1, 2
18–25	1, 2
26–27	3
Extra Practice	
See page 734.	

Classify each triangle by its angles and by its sides.



17. Triangle XYZ has angles that measure 30°, 60°, and 90°. Classify the triangle by its angles.

Make a sketch of each triangle. If it is not possible to sketch the figure, write *not possible*.

18. acute isosceles
19. right equilateral
20. obtuse and *not* isosceles
21. right and *not* scalene
22. obtuse equilateral

23. **Architecture** Refer to the photo at the right. Classify each triangle by its angles and by its sides.

- a.  $\triangle ABC$
- b.  $\triangle ACD$
- c.  $\triangle BCD$

24. **Art** Refer to the optical art design on page 188. Classify the triangles by their angles and by their sides.



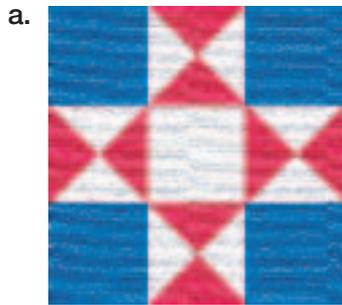
Alcoa Office Building, San Francisco, CA

## Applications and Problem Solving

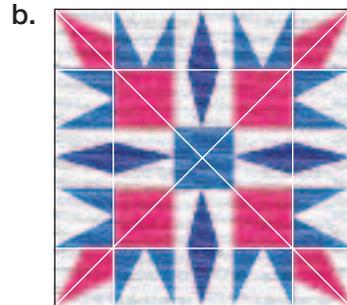
**InterNET CONNECTED**

**Data Update** For the latest information about optical art, visit: [www.geomconcepts.com](http://www.geomconcepts.com)

25. **Quilting** Classify the triangles that are used in the quilt blocks.



Ohio Star

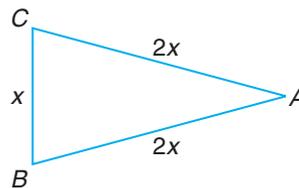


Duck's Foot in the Mud

26. **Algebra**  $\triangle DEF$  is an equilateral triangle in which  $ED = x + 5$ ,  $DF = 3x - 3$ , and  $EF = 2x + 1$ .

- a. Draw and label  $\triangle DEF$ .  
b. Find the measure of each side.

27. **Algebra** Find the measure of each side of isosceles triangle  $ABC$  if  $\angle A$  is the vertex angle and the perimeter of the triangle is 20 meters.



28. **Critical Thinking** Numbers that can be represented by a triangular arrangement of dots are called *triangular numbers*. The first four triangular numbers are 1, 3, 6, and 10.



Find the next two triangular numbers.

**Mixed Review**

Write an equation in slope-intercept form of the line with the given slope that passes through the given point. (Lesson 4-6)

29.  $m = -3$ ,  $(0, 4)$       30.  $m = 0$ ,  $(0, -2)$       31.  $m = -2$ ,  $(-2, 1)$

Find the slope of the lines passing through each pair of points. (Lesson 4-5)

32.  $(5, 7)$ ,  $(4, 5)$       33.  $(8, 4)$ ,  $(-2, 4)$       34.  $(5, -2)$ ,  $(5, 1)$

Photo Graphic

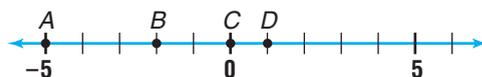


35. **Sports** In the Olympic ski-jumping competition, the skier tries to make the angle between his body and the front of his skis as small as possible. If a skier is aligned so that the front of his skis makes a  $20^\circ$  angle with his body, what angle is formed by the tail of the skis and his body? (Lesson 3-5)

**Standardized Test Practice**

(A) (B) (C) (D)

36. **Multiple Choice** Use the number line to find  $DA$ . (Lesson 2-1)



- (A) -10      (B) -6      (C) 6      (D) 10

# 5-2

# Angles of a Triangle

### What You'll Learn

You'll learn to use the Angle Sum Theorem.

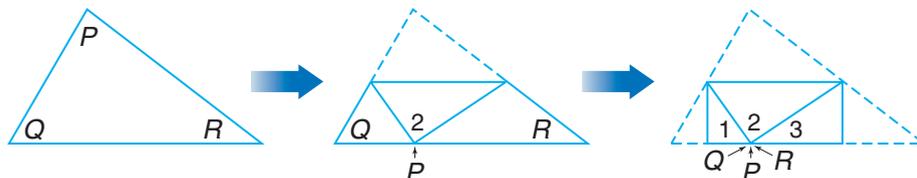
### Why It's Important

#### Construction

Builders use the measure of the vertex angle of an isosceles triangle to frame buildings.

See Exercise 21.

If you measure and add the angles in any triangle, you will find that the sum of the angles have a special relationship. Cut and fold a triangle as shown below. Make a conjecture about the sum of the angle measures of a triangle.

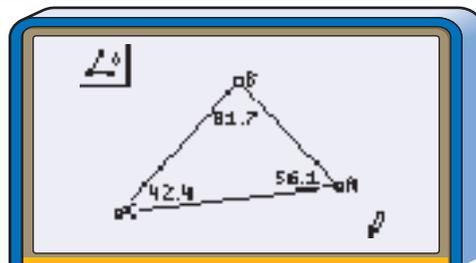


You can use a graphing calculator to verify your conjecture.



### Graphing Calculator Exploration

**Step 1** Use the Triangle tool on the **F2** menu. Move the pencil cursor to each location where you want a vertex and press **ENTER**. The calculator automatically draws the sides. Label the vertices  $A$ ,  $B$ , and  $C$ .



**Step 2** Use the Angle tool under Measure on the **F5** menu to measure each angle.

### Try These

- Determine the sum of the measures of the angles of your triangle.
- Drag any vertex to a different location, measure each angle, and find the sum of the measures.
- Repeat Exercise 2 several times.
- Make a conjecture** about the sum of the angle measures of any triangle.

The results of the activities above can be stated in the Angle Sum Theorem.

### Theorem 5-1 Angle Sum Theorem

**Words:** The sum of the measures of the angles of a triangle is 180.

**Model:**



**Symbols:**  $x + y + z = 180$

### Graphing Calculator Tutorial

See pp. 782–785.



You can use the Angle Sum Theorem to find missing measures in triangles.

## Examples

**Algebra Review**  
Solving One-Step Equations, p. 722

1 Find  $m\angle T$  in  $\triangle RST$ .

$$m\angle R + m\angle S + m\angle T = 180$$

$$54 + 67 + m\angle T = 180$$

$$121 + m\angle T = 180$$

$$121 - 121 + m\angle T = 180 - 121$$

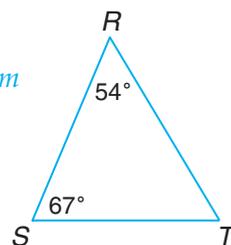
$$m\angle T = 59$$

Angle Sum Theorem

Substitution

Add.

Subtract 121 from each side.



2 Find the value of each variable in  $\triangle DCE$ .

$\angle ACB$  and  $\angle DCE$  are vertical angles.

Vertical angles are congruent, so

$m\angle ACB = m\angle DCE$ . Therefore,  $x = 85$ .

Now find the value of  $y$ .

$$m\angle D + m\angle DCE + m\angle E = 180$$

$$55 + 85 + y = 180$$

$$140 + y = 180$$

$$140 - 140 + y = 180 - 140$$

$$y = 40$$

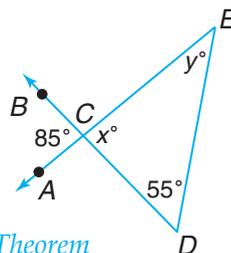
Angle Sum Theorem

Substitution

Add.

Subtract 140 from each side.

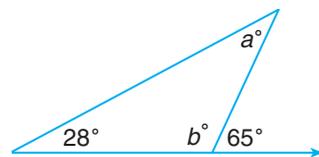
Simplify.



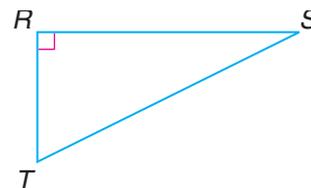
Therefore,  $x = 85$  and  $y = 40$ .

## Your Turn

- Find  $m\angle L$  in  $\triangle MNL$  if  $m\angle M = 25$  and  $m\angle N = 25$ .
- Find the value of each variable in the figure at the right.



You can use the Angle Sum Theorem to discover a relationship between the acute angles of a right triangle. In  $\triangle RST$ ,  $\angle R$  is a right angle.



$$m\angle R + m\angle T + m\angle S = 180$$

$$90 + m\angle T + m\angle S = 180$$

$$90 - 90 + m\angle T + m\angle S = 180 - 90$$

$$m\angle T + m\angle S = 90$$

Angle Sum Theorem

Substitution

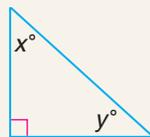
Subtract 90 from each side.

Simplify.

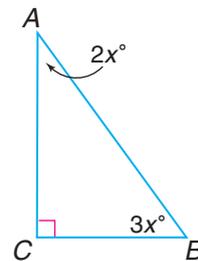
## Look Back

Complementary Angles: Lesson 3-5

By the definition of complementary angles,  $\angle T$  and  $\angle S$  are complementary. This relationship is stated in the following theorem.

**Theorem 5–2****Words:** The acute angles of a right triangle are complementary.**Model:****Symbols:**  $x + y = 90$ **Example****Algebra Link****3** Find  $m\angle A$  and  $m\angle B$  in right triangle  $ABC$ .

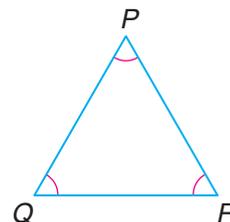
$$\begin{aligned} m\angle A + m\angle B &= 90 && \text{Theorem 5–2} \\ 2x + 3x &= 90 && \text{Substitution} \\ 5x &= 90 && \text{Combine like terms.} \\ \frac{5x}{5} &= \frac{90}{5} && \text{Divide each side by 5.} \\ x &= 18 && \text{Simplify.} \end{aligned}$$

Now replace  $x$  with 18 in the expression for each angle.

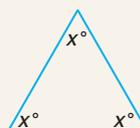
$\angle A$	$\angle B$
$m\angle A = 2x$	$m\angle B = 3x$
$= 2(18)$ or 36	$= 3(18)$ or 54

An **equiangular triangle** is a triangle in which all three angles are congruent. You can use the Angle Sum Theorem to find the measure of each angle in an equiangular triangle.

Triangle  $PQR$  is an equiangular triangle. Since  $m\angle P = m\angle Q = m\angle R$ , the measure of each angle of  $\triangle PQR$  is  $180 \div 3$  or 60.



This relationship is stated in Theorem 5–3.

**Theorem 5–3****Words:** The measure of each angle of an equiangular triangle is 60.**Model:****Symbols:**  $x = 60$

# Check for Understanding

## Communicating Mathematics

- Choose the numbers that are not measures of the three angles of a triangle.
  - 10, 20, 150
  - 30, 60, 90
  - 40, 70, 80
  - 45, 55, 80
- Explain how to find the measure of the third angle of a triangle if you know the measures of the other two angles.
- Writing Math** Is it possible to have two obtuse angles in a triangle? Write a few sentences explaining why or why not.

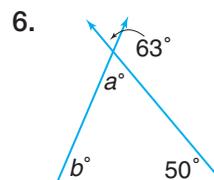
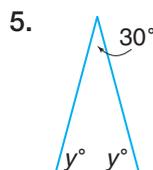
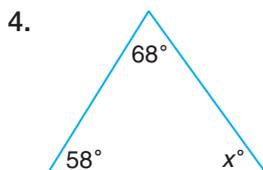
## Vocabulary

equiangular triangle

## Guided Practice

### Examples 1 & 2

Find the value of each variable.



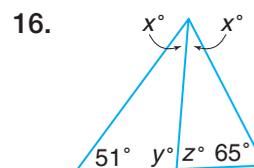
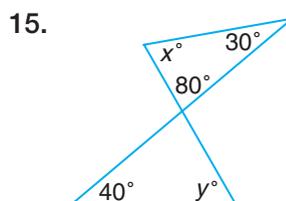
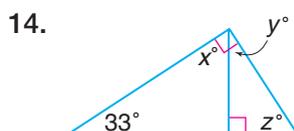
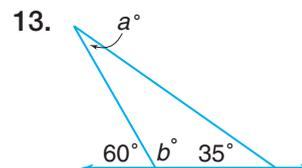
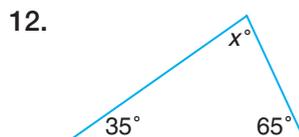
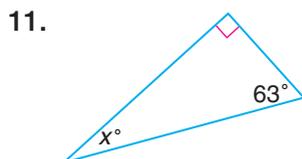
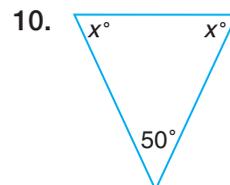
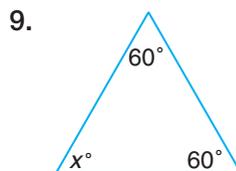
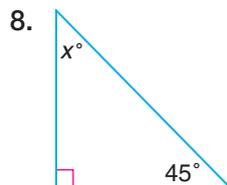
### Example 3

- Algebra** The measures of the angles of a triangle are  $2x$ ,  $3x$ , and  $4x$ . Find the measure of each angle.

# Exercises

## Practice

Find the value of each variable.



### Homework Help

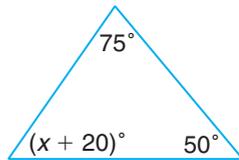
For Exercises	See Examples
8–12, 20, 21	1
13–16	2
17–19, 22	3

### Extra Practice

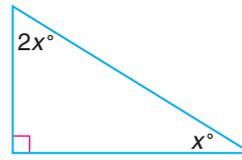
See page 734.

Find the measure of each angle in each triangle.

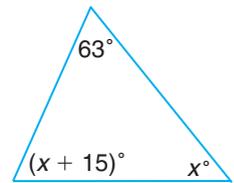
17.



18.



19.



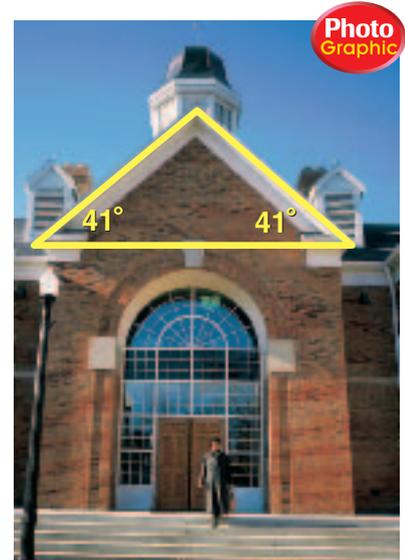
20. The measure of one acute angle of a right triangle is 25. Find the measure of the other acute angle.

**Applications and Problem Solving**

21. **Construction** The roof lines of many buildings are shaped like the legs of an isosceles triangle. Find the measure of the vertex angle of the isosceles triangle shown at the right.

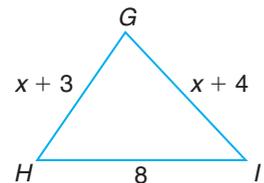
22. **Algebra** The measures of the angles of a triangle are  $x + 5$ ,  $3x + 14$ , and  $x + 11$ . Find the measure of each angle.

23. **Critical Thinking** If two angles of one triangle are congruent to two angles of another triangle, what is the relationship between the third angles of the triangles? Explain your reasoning.



**Mixed Review**

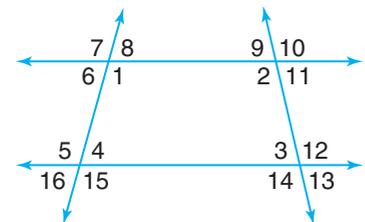
24. The perimeter of  $\triangle GHI$  is 21 units. Find  $GH$  and  $GI$ . (Lesson 5-1)



25. State the slope of the lines perpendicular to the graph of  $y = 3x - 2$ . (Lesson 4-6)

Exercise 24

Identify each pair of angles as *alternate interior*, *alternate exterior*, *consecutive interior*, or *vertical*. (Lesson 4-2)



26.  $\angle 1, \angle 5$

27.  $\angle 9, \angle 11$

28.  $\angle 2, \angle 3$

29.  $\angle 7, \angle 15$

**Standardized Test Practice**

(A) (B) (C) (D)

30. **Short Response** Points  $X, Y,$  and  $Z$  are collinear, and  $XY = 45, YZ = 23,$  and  $XZ = 22.$  Locate the points on a number line. (Lesson 2-2)



### What You'll Learn

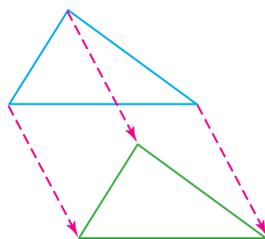
You'll learn to identify translations, reflections, and rotations and their corresponding parts.

### Why It's Important

**Art** Artists use motion geometry to make designs. See Example 6.

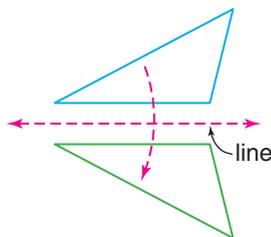
We live in a world of motion. Geometry helps us define and describe that motion. In geometry, there are three fundamental types of motion: **translation, reflection, and rotation.**

#### Translation



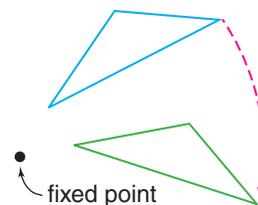
In a **translation**, you slide a figure from one position to another without turning it. Translations are sometimes called *slides*.

#### Reflection



In a **reflection**, you flip a figure over a line. The new figure is a mirror image. Reflections are sometimes called *flips*.

#### Rotation



In a **rotation**, you turn the figure around a fixed point. Rotations are sometimes called *turns*.

When a figure is translated, reflected, or rotated, the lengths of the sides of the figure do not change.

### Examples

Identify each motion as a *translation, reflection, or rotation.*

**1**

rotation

**2**

reflection

**3**

translation

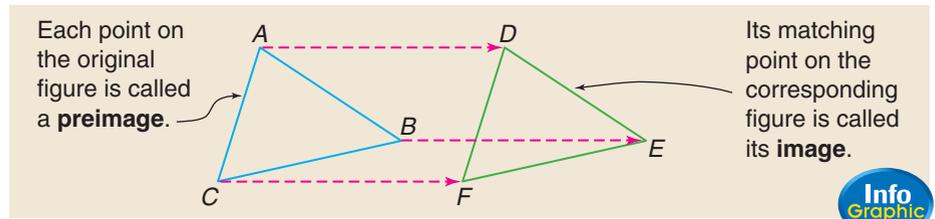
### Your Turn

a.

b.

c.

The figure below shows a translation.



**Reading Geometry**

Read  $\triangle ABC \rightarrow \triangle DEF$  as triangle ABC maps to triangle DEF.

Each point on the preimage can be paired with exactly one point on its image, and each point on the image can be paired with exactly one point on the preimage. This one-to-one correspondence is an example of a **mapping**.

The symbol  $\rightarrow$  is used to indicate a mapping. In the figure,  $\triangle ABC \rightarrow \triangle DEF$ . In naming the triangles, the order of the vertices indicates the corresponding points.

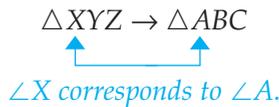
Preimage	Image	Preimage	Image
$A$	$\rightarrow$ $D$	$\overline{AB}$	$\rightarrow$ $\overline{DE}$
$B$	$\rightarrow$ $E$	$\overline{BC}$	$\rightarrow$ $\overline{EF}$
$C$	$\rightarrow$ $F$	$\overline{AC}$	$\rightarrow$ $\overline{DF}$

This mapping is called a **transformation**.

**Examples**

In the figure,  $\triangle XYZ \rightarrow \triangle ABC$  by a reflection.

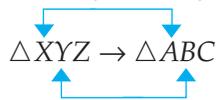
4 Name the image of  $\angle X$ .



So,  $\angle A$  is the image of  $\angle X$ .

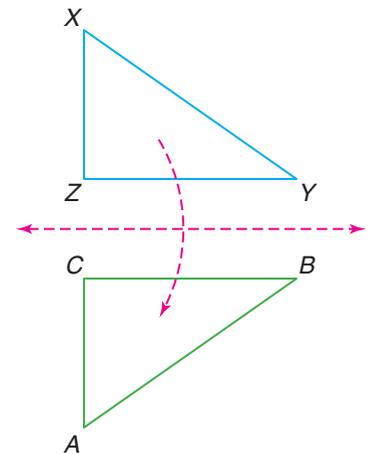
5 Name the side that corresponds to  $\overline{AB}$ .

Point A corresponds to point X.



Point B corresponds to point Y.

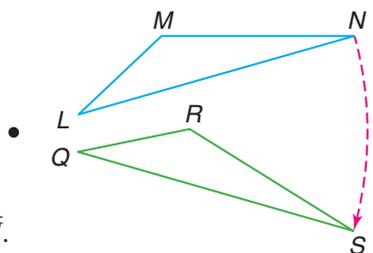
So,  $\overline{AB}$  corresponds to  $\overline{XY}$ .



**Your Turn**

In the figure,  $\triangle LMN \rightarrow \triangle QRS$  by a rotation.

- d. Name the image of  $\angle M$ .
- e. Name the angle that corresponds to  $\angle S$ .
- f. Name the image of  $\overline{LM}$ .
- g. Name the side that corresponds to  $\overline{LN}$ .



Translations, reflections, and rotations are all **isometries**. An isometry is a movement that does not change the size or shape of the figure being moved. Artists often use isometries in designs. One of the most famous artists to use this technique was M. C. Escher.



**Example**  
**Art Link**

**6** Identify the type of transformation in the artwork at the right.

Each figure can be moved to match another without turning or flipping. Therefore, the motion is a translation.

M. C. Escher, *Pegasus*

**Check for Understanding**

**Communicating Mathematics**

1. Explain the difference between a translation and a rotation.
2. **You Decide?** Suppose  $\triangle ABC \rightarrow \triangle RST$ . Antonio says that  $\angle C$  corresponds to  $\angle T$ . Keisha says she needs to see the drawing to know which angles correspond. Who is correct? Explain your reasoning.

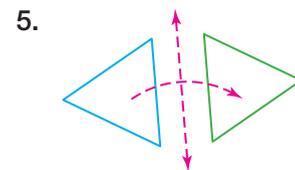
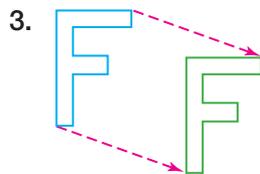
**Vocabulary**

- translation
- reflection
- rotation
- transformation
- preimage
- isometry
- image
- mapping

**Guided Practice**

**Examples 1–3**

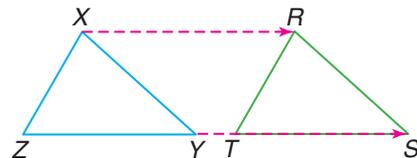
Identify each motion as a *translation, reflection, or rotation*.



**Examples 4 & 5**

In the figure at the right,  $\triangle XYZ \rightarrow \triangle RST$ .

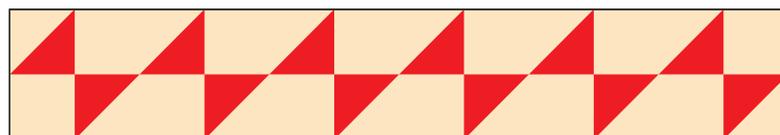
6. Name the image of  $\overline{XY}$ .
7. Name the angle that corresponds to  $\angle R$ .



**Example 6**



8. **Native American Designs** The design below was found on food bowls that were discovered in the ruins of an ancient Hopi pueblo. Identify the transformations in the design.

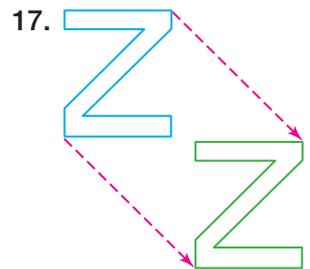
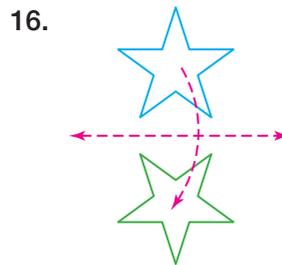
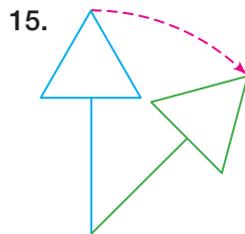
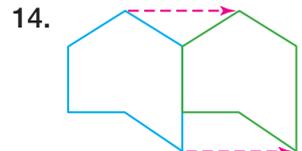
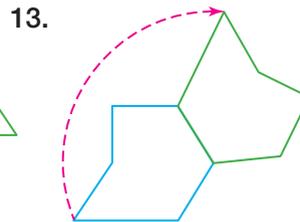
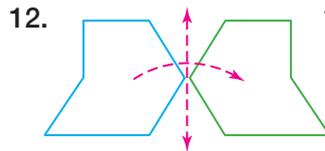
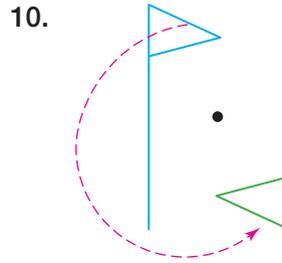
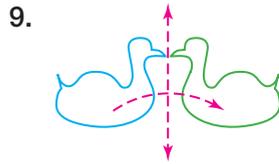


# Exercises

## Practice

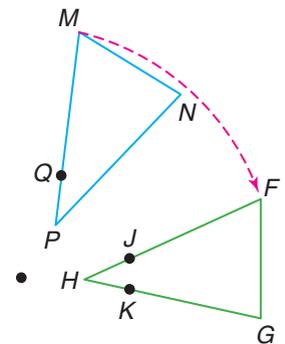
Homework Help	
For Exercises	See Examples
9–17	1–3
18–24	4, 5
25–27	6
Extra Practice	
See page 734.	

Identify each motion as a *translation*, *reflection*, or *rotation*.



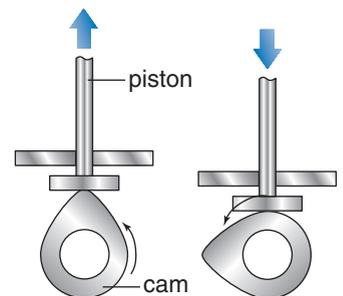
In the figure at the right,  $\triangle MNP \rightarrow \triangle FGH$ .

- Which angle corresponds to  $\angle N$ ?
- Which side corresponds to  $\overline{MN}$ ?
- Name the angle that corresponds to  $\angle H$ .
- Name the image of point  $Q$ .
- Name the side that corresponds to  $\overline{GH}$ .
- Name the image of  $\overline{PQ}$ .
- If  $\triangle ABC \rightarrow \triangle PQR$ , which angle corresponds to  $\angle R$ ?



## Applications and Problem Solving

25. **Engines** *Cams* are important parts of engines because they change motion from one direction to another. As the cam turns around, the pistons move up and down. Identify the transformation that occurs in the cams.



- 26. **Art** The figure at the left shows an untitled work by M. C. Escher. Identify the type of transformation used to complete the work.
- 27. **Critical Thinking** The transformation below is called a *glide reflection*. How is this transformation different from a translation, reflection, and rotation?



M. C. Escher, *Flying Fish*

### Mixed Review

- 28. The measure of one acute angle of a right triangle is 30. Find the measure of the other acute angle. (Lesson 5-2)
- 29. **Algebra**  $\triangle XYZ$  is an equilateral triangle in which  $XY = 2x + 2$ ,  $YZ = x + 7$ , and  $XZ = 4x - 8$ . Find the measure of each side. (Lesson 5-1)

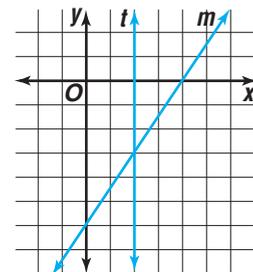
Draw a figure for each pair of planes or segments. (Lesson 4-1)

- 30. parallel planes
- 31. skew segments
- 32. intersecting planes

### Standardized Test Practice

A B C D

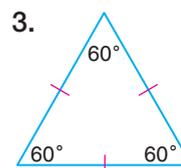
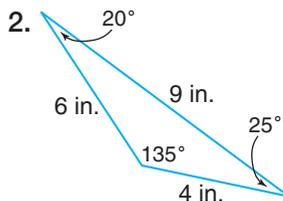
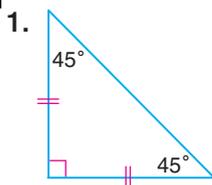
- 33. **Multiple Choice** Which ordered pair represents the intersection of line  $t$  and line  $m$ ? (Lesson 2-4)
  - (A) (2, 3)
  - (B) (-2, -3)
  - (C) (2, -3)
  - (D) (-2, 3)



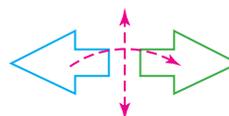
## Quiz 1

## Lessons 5-1 through 5-3

Classify each triangle by its angles and by its sides. (Lesson 5-1)



- 4. **Algebra** The measures of the angles of a triangle are  $2x$ ,  $5x$ , and  $5x$ . Find the measure of each angle. (Lesson 5-2)
- 5. Identify the motion as a *translation*, *reflection*, or *rotation*. (Lesson 5-3)



**What You'll Learn**

You'll learn to identify corresponding parts of congruent triangles.

**Why It's Important**

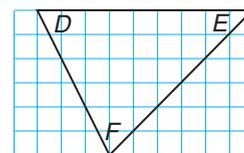
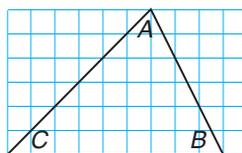
**Crafts** The pieces of fabric used to make a quilt are congruent to a template. See Exercise 27.

You've learned that congruent segments have the same length and congruent angles have the same degree measure. In the following activity, you will learn about congruent triangles.

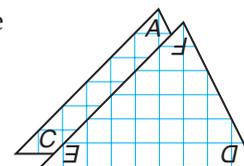
**Hands-On Geometry**

**Materials:**  grid paper  scissors  straightedge

**Step 1** On a piece of grid paper, draw two triangles like the ones below. Label the vertices as shown.



**Step 2** Cut out the triangles. Put one triangle over the other so that the parts with the same measures match up.

**Try These**

1. Identify all of the pairs of angles and sides that match or correspond.
2. Triangle  $ABC$  is congruent to  $\triangle FDE$ . What is true about their corresponding sides and angles?

**Reading Geometry**

Arcs are used to show which angles are congruent. Slash marks are used to show which sides are congruent.

If a triangle can be translated, rotated, or reflected onto another triangle so that all of the vertices correspond, the triangles are **congruent triangles**. The parts of congruent triangles that “match” are called **corresponding parts**.

In the figure,  $\triangle ABC \cong \triangle FDE$ . As in a mapping, the order of the vertices indicates the corresponding parts.

**Congruent Angles**      **Congruent Sides**

$$\angle A \cong \angle F$$

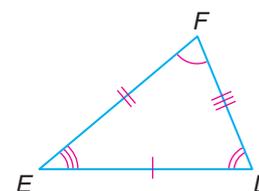
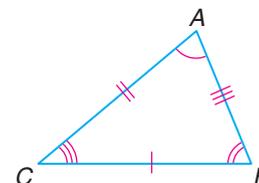
$$\angle B \cong \angle D$$

$$\angle C \cong \angle E$$

$$\overline{AB} \cong \overline{FD}$$

$$\overline{BC} \cong \overline{DE}$$

$$\overline{AC} \cong \overline{FE}$$



These relationships help to define congruent triangles.

**Definition of Congruent Triangles (CPCTC)**

If the corresponding parts of two triangles are congruent, then the two triangles are congruent.

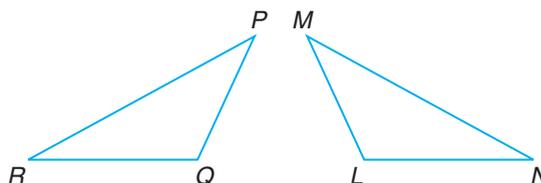
If two triangles are congruent, then the corresponding parts of the two triangles are congruent.

*CPCTC is an abbreviation for Corresponding Parts of Congruent Triangles are Congruent.*

**Examples**

**1** If  $\triangle PQR \cong \triangle MLN$ , name the congruent angles and sides. Then draw the triangles, using arcs and slash marks to show the congruent angles and sides.

First, name the three pairs of congruent angles by looking at the order of the vertices in the statement  $\triangle PQR \cong \triangle MLN$ .

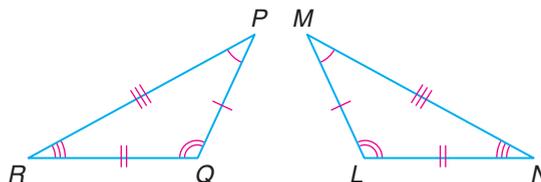


So,  $\angle P \cong \angle M$ ,  $\angle Q \cong \angle L$ , and  $\angle R \cong \angle N$ .

Since  $P$  corresponds to  $M$ , and  $Q$  corresponds to  $L$ ,  $\overline{PQ} \cong \overline{ML}$ .

Since  $Q$  corresponds to  $L$ , and  $R$  corresponds to  $N$ ,  $\overline{QR} \cong \overline{LN}$ .

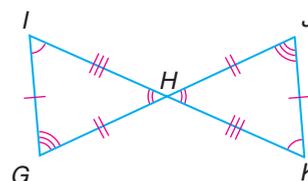
Since  $P$  corresponds to  $M$ , and  $R$  corresponds to  $N$ ,  $\overline{PR} \cong \overline{MN}$ .



**2** The corresponding parts of two congruent triangles are marked on the figure. Write a congruence statement for the two triangles.

List the congruent angles and sides.

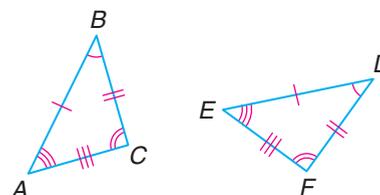
$$\begin{aligned} \angle I &\cong \angle K & \overline{IH} &\cong \overline{KH} \\ \angle G &\cong \angle J & \overline{GH} &\cong \overline{JH} \\ \angle GHI &\cong \angle JHK & \overline{GI} &\cong \overline{JK} \end{aligned}$$

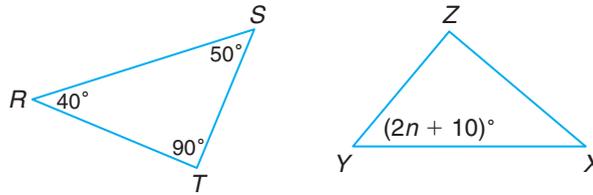


The congruence statement can be written by matching the vertices of the congruent angles. Therefore,  $\triangle IGH \cong \triangle KJH$ .

**Your Turn**

The corresponding parts of two congruent triangles are marked on the figure. Write a congruence statement for the two triangles.



**Example****Algebra Link****3** $\triangle RST$  is congruent to  $\triangle XYZ$ . Find the value of  $n$ .Since  $\triangle RST \cong \triangle XYZ$ , the corresponding parts are congruent.

$$m\angle S = m\angle Y$$

$$50 = 2n + 10 \quad \text{Substitution}$$

$$50 - 10 = 2n + 10 - 10 \quad \text{Subtract 10 from each side.}$$

$$40 = 2n \quad \text{Simplify.}$$

$$\frac{40}{2} = \frac{2n}{2} \quad \text{Divide each side by 2.}$$

$$20 = n \quad \text{Simplify.}$$

**Algebra Review**

Solving Multi-Step Equations, p. 723

**Check for Understanding****Communicating Mathematics**

1. Explain what it means when one triangle is congruent to another.
2. Describe how transformations are used to determine whether triangles are congruent.

**Vocabulary**congruent triangles  
corresponding parts**Guided Practice****Getting Ready**If  $\triangle ABC \cong \triangle DEF$ , name the corresponding side or angle.**Sample:**  $\angle B$ **Solution:**  $\angle B$  corresponds to  $\angle E$ .

3.  $\angle F$

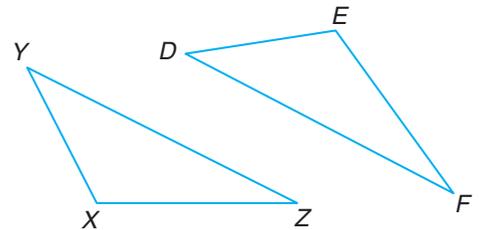
4.  $\angle A$

5.  $\overline{AC}$

6.  $\overline{EF}$

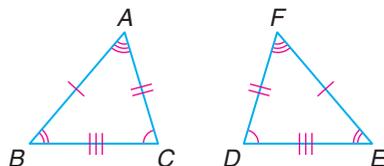
**Example 1**

7. If  $\triangle XYZ \cong \triangle EDF$ , name the congruent angles and sides. Then draw the triangles, using arcs and slash marks to show the congruent angles and sides.

**Example 2**

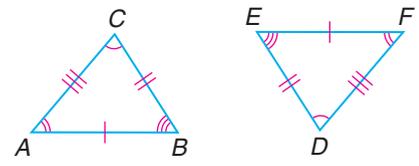
Complete each congruence statement.

8.



$\triangle ABC \cong \triangle \underline{\quad} \underline{\quad} ?$

9.

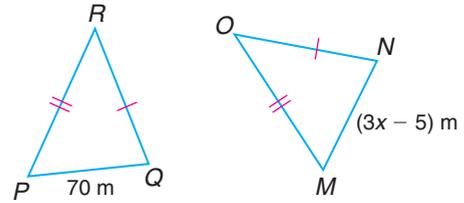


$\triangle CBA \cong \triangle \underline{\quad} \underline{\quad} ?$



**Example 3**

10. **Algebra**  $\triangle RQP$  is congruent to  $\triangle ONM$ . Find the value of  $x$ .

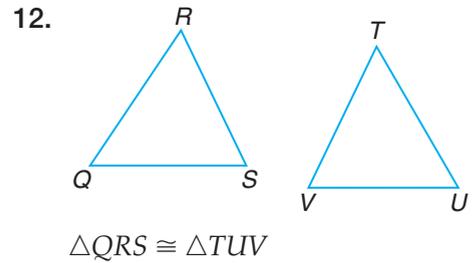
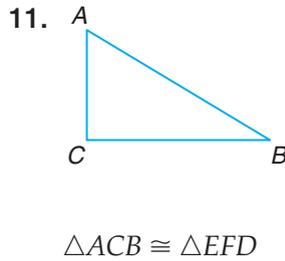


## Exercises

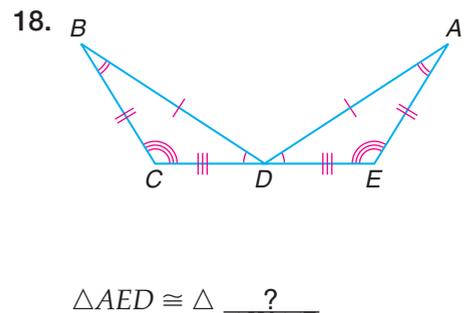
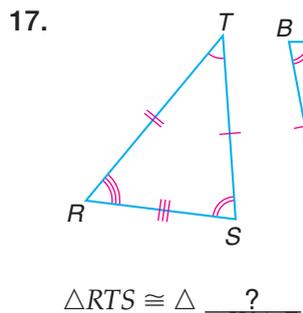
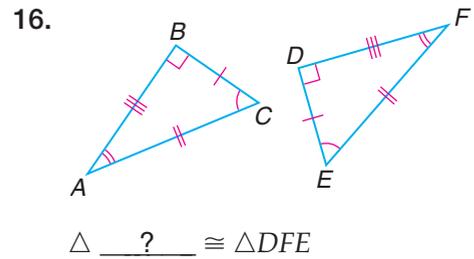
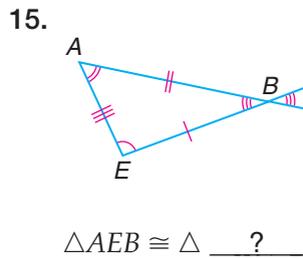
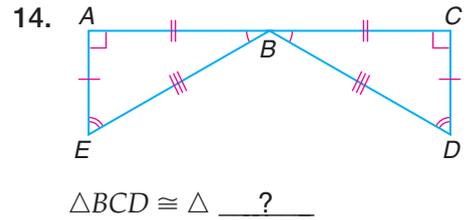
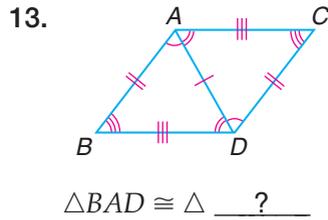
**Practice**

For each pair of congruent triangles, name the congruent angles and sides. Then draw the triangles, using arcs and slash marks to show the congruent angles and sides.

Homework Help	
For Exercises	See Examples
11, 12, 19–23, 26, 27	1
13–18	2
24, 25	3
Extra Practice	
See page 735.	



Complete each congruence statement.

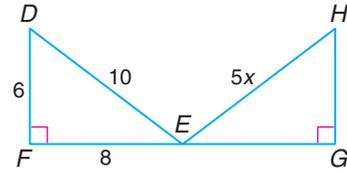


If  $\triangle BCA \cong \triangle GFH$ , name the part that is congruent to each angle or segment.

19.  $\angle F$       20.  $\overline{BA}$       21.  $\angle A$       22.  $\overline{FG}$       23.  $\angle G$

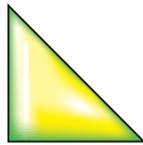
24. If  $\triangle PRQ \cong \triangle YXZ$ ,  $m\angle P = 63$ , and  $m\angle Q = 57$ , find  $m\angle X$ .

25. **Algebra** If  $\triangle DEF \cong \triangle HEG$ , what is the value of  $x$ ?

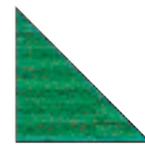
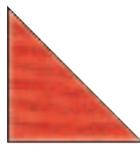


26. **Landscaping** Two triangular gardens have the same size and shape. The landscaper needed 24 feet of fencing for one garden. How much fencing is needed for the second garden? Explain your reasoning.

27. **Crafts** Many quilts are designed using triangles. Quilters start with a template and trace around the template, outlining the triangles to be cut out. Explain why the triangles are congruent.



template

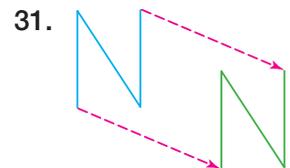
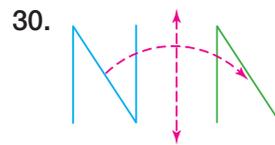
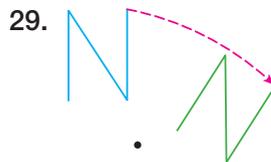


fabric triangles

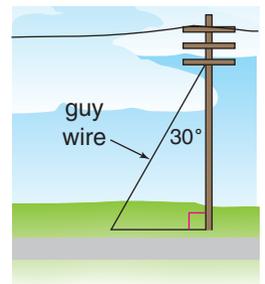
28. **Critical Thinking** Determine whether each statement is *true* or *false*. If *true*, explain your reasoning. If *false*, show a counterexample.
- If two triangles are congruent, their perimeters are equal.
  - If two triangles have the same perimeter, they are congruent.

Identify each motion as a *translation*, *reflection*, or *rotation*.

(Lesson 5-3)



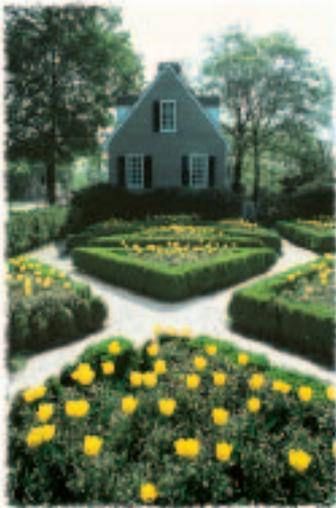
32. **Communication** A support cable called a guy wire is attached to a utility pole to give it stability. Safety regulations require a minimum angle of  $30^\circ$  between the pole and the guy wire. Determine the measure of the angle between the guy wire and the ground. (Lesson 5-2)



33. **Short Response** If  $m\angle R = 45$ , classify  $\angle R$  as *acute*, *right*, or *obtuse*. (Lesson 3-2)
34. **Multiple Choice** Choose the *false* statement. (Lesson 1-3)

- Two points determine two lines.
- A line contains at least two points.
- Three points that are not on the same line determine a plane.
- If two planes intersect, then their intersection is a line.

### Applications and Problem Solving



### Mixed Review

### Standardized Test Practice

- (A) (B) (C) (D)



# Take a Shortcut

## Materials



patty paper



scissors



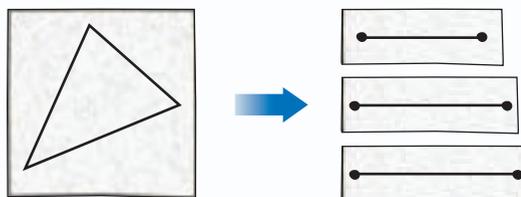
straightedge

## Introducing the Congruence Postulates

Is it possible to show that two triangles are congruent without showing that all six pairs of corresponding parts are congruent? Let's look for a shortcut.

### Investigate

1. Use patty paper to investigate three pairs of congruent sides.
  - a. Draw a triangle on a piece of patty paper.
  - b. Copy the sides of the triangle onto another piece of patty paper and cut them out.

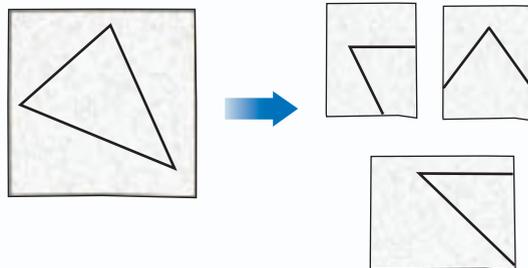


- c. Arrange the pieces so that they form a triangle.
- d. Is this triangle congruent to the original triangle? Explain your reasoning.
- e. Try to form another triangle. Is it congruent to the original triangle?
- f. Can three pairs of congruent sides be used to show that two triangles are congruent?



2. Use patty paper to investigate three pairs of congruent angles.
- Draw a triangle on a piece of patty paper.

- Copy each angle of the triangle onto a separate piece of patty paper and cut them out. Extend each ray of each angle to the edge of the patty paper.



- Arrange the pieces so that they form a triangle.
- Is this triangle congruent to the original triangle? Explain your reasoning.
- Try to form another triangle. Is this triangle congruent to the original triangle?
- Can three pairs of congruent angles be used to show that two triangles are congruent?

### Extending the Investigation

In this investigation, you will determine which three pairs of corresponding parts can be used to show that two triangles are congruent.

Use patty paper or graphing software to investigate these six cases. (You have already investigated the first two.)

- three pairs of congruent sides
- three pairs of congruent angles
- two pairs of congruent sides and the pair of congruent angles between them
- two pairs of congruent sides and one pair of congruent angles *not* between them
- two pairs of congruent angles and the pair of congruent sides between them
- two pairs of congruent angles and one pair of congruent sides *not* between them

### Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a poster that summarizes your results.
- Make a model with straws that illustrates why certain pairs of corresponding parts cannot be used to show that two triangles are congruent. Be sure to show counterexamples.



**Investigation** For more information on the congruence postulates, visit: [www.geomconcepts.com](http://www.geomconcepts.com)



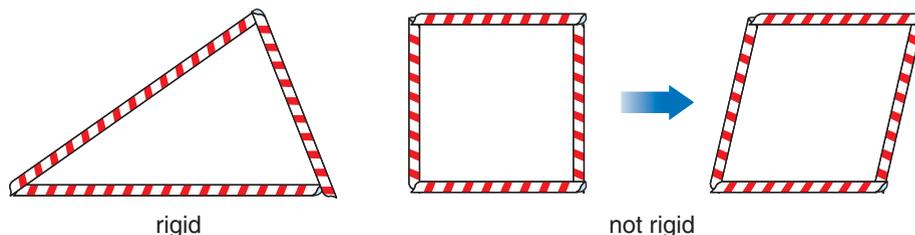
**What You'll Learn**

You'll learn to use the SSS and SAS tests for congruence.

**Why It's Important**  
**Construction**

Architects add strength to their buildings by using triangles for support. See Exercise 7.

Triangles are common in construction, because triangles, unlike squares, maintain their shape under stress. You can see this yourself if you use straws and a string to make a triangle and a four-sided figure.

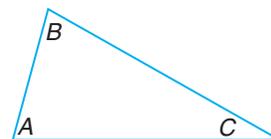


This rigidity hints at an underlying geometric concept: a triangle with three sides of a set length has exactly one shape.

**Hands-On Geometry**  
**Construction**

**Materials:** compass straightedge scissors

**Step 1** Draw an acute scalene triangle on a piece of paper. Label its vertices  $A$ ,  $B$ , and  $C$  on the interior of each angle.



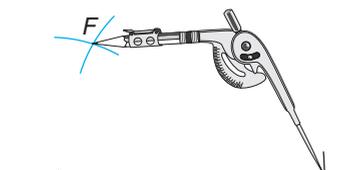
Step 1

**Step 2** Construct a segment congruent to  $\overline{AC}$ . Label the endpoints of the segment  $D$  and  $E$ .



Step 2

**Step 3** Adjust the compass setting to the length of  $\overline{AB}$ . Place the compass at point  $D$  and draw a large arc above  $\overline{DE}$ .



Step 4

**Step 4** Adjust the compass setting to the length of  $\overline{CB}$ . Place the compass at point  $E$  and draw an arc to intersect the one drawn from point  $D$ . Label the intersection  $F$ .

**Step 5** Draw  $\overline{DF}$  and  $\overline{EF}$ .

**Try These**

1. Label the vertices of  $\triangle DEF$  on the interior of each angle. Then cut out the two triangles. **Make a conjecture.** Are the triangles congruent?
2. If the triangles are congruent, write a congruence statement.
3. **Verify** your conjecture with another triangle.

In the previous activity, you constructed a congruent triangle by using only the measures of its sides. This activity suggests the following postulate.

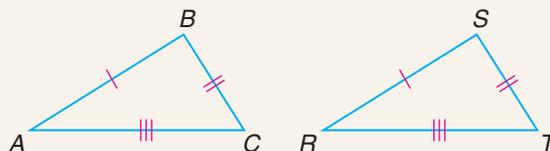
**Reading Geometry**

The abbreviation **SSS** is read as **Side-Side-Side**.

**Postulate 5-1  
SSS Postulate**

**Words:** If three sides of one triangle are congruent to three corresponding sides of another triangle, then the triangles are congruent.

**Model:**

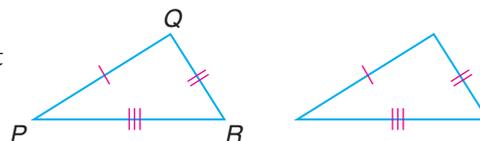


**Symbols:** If  $\overline{AB} \cong \overline{RS}$ ,  $\overline{BC} \cong \overline{ST}$ , and  $\overline{CA} \cong \overline{TR}$ , then  $\triangle ABC \cong \triangle RST$ .

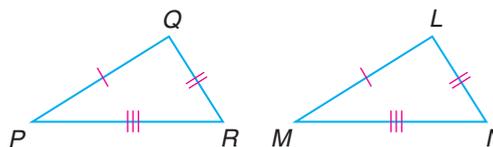
**Example**

1 In two triangles,  $\overline{PQ} \cong \overline{ML}$ ,  $\overline{PR} \cong \overline{MN}$ , and  $\overline{RQ} \cong \overline{NL}$ . Write a congruence statement for the two triangles.

Draw a pair of congruent triangles. Identify the congruent parts with slashes. Label the vertices of one triangle.



Use the given information to label the vertices of the second triangle.



By SSS,  $\triangle PQR \cong \triangle MLN$ .

**Your Turn**

a. In two triangles,  $\overline{ZY} \cong \overline{FE}$ ,  $\overline{XY} \cong \overline{DE}$ , and  $\overline{XZ} \cong \overline{DF}$ . Write a congruence statement for the two triangles.

In a triangle, the angle formed by two given sides is called the **included angle** of the sides.

**Info Graphic**

$\angle C$  is the included angle of  $\overline{AC}$  and  $\overline{BC}$ .

$\angle A$  is the included angle of  $\overline{AB}$  and  $\overline{AC}$ .

$\angle B$  is the included angle of  $\overline{AB}$  and  $\overline{CB}$ .

Using the SSS Postulate, you can show that two triangles are congruent if their corresponding sides are congruent. You can also show their congruence by using two sides and the included angle.

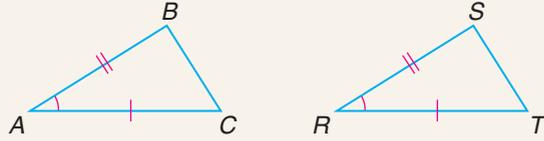
## Reading Geometry

The abbreviation **SAS** is read as *Side-Angle-Side*.

### Postulate 5–2 SAS Postulate

**Words:** If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the triangles are congruent.

**Model:**



**Symbols:** If  $\overline{AB} \cong \overline{RS}$ ,  $\angle A \cong \angle R$ , and  $\overline{AC} \cong \overline{RT}$ , then  $\triangle ABC \cong \triangle RST$ .

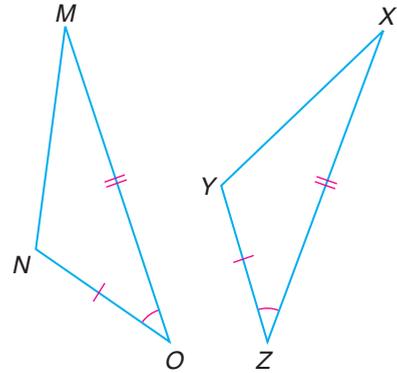
### Example

2

Determine whether the triangles shown at the right are congruent. If so, write a congruence statement and explain why the triangles are congruent. If not, explain why not.

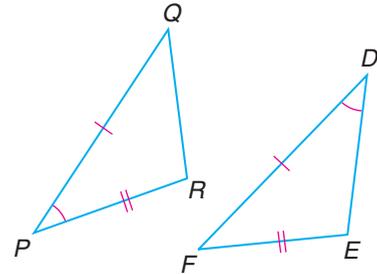
There are two pairs of congruent sides,  $\overline{NO} \cong \overline{YZ}$  and  $\overline{MO} \cong \overline{XZ}$ . There is one pair of congruent angles,  $\angle O \cong \angle Z$ , which is included between the sides.

Therefore,  $\triangle MNO \cong \triangle XYZ$  by SAS.



### Your Turn

b. Determine whether the triangles shown at the right are congruent by SAS. If so, write a congruence statement and tell why the triangles are congruent. If not, explain why not.



## Check for Understanding

### Communicating Mathematics

- Sketch and label a triangle in which  $\angle X$  is the included angle of  $\overline{YX}$  and  $\overline{ZX}$ .
- You Decide?** Karen says that there is only one triangle with sides of 3 inches, 4 inches, and 5 inches. Mika says that there can be many different triangles with those measures. Who is correct? Explain your reasoning.

### Vocabulary

included angle

### Guided Practice

#### Example 1

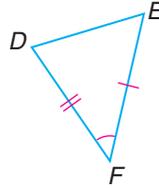
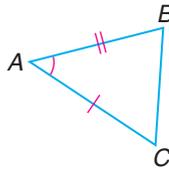
Write a congruence statement for each pair of triangles represented.

- $\overline{RT} \cong \overline{UW}$ ,  $\overline{RS} \cong \overline{UV}$ ,  $\overline{TS} \cong \overline{WV}$
- $\overline{AB} \cong \overline{GH}$ ,  $\overline{BC} \cong \overline{HI}$ ,  $\angle B \cong \angle H$

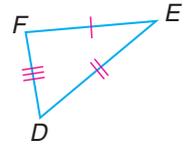
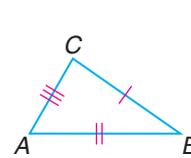
**Example 2**

Determine whether each pair of triangles is congruent. If so, write a congruence statement and explain why the triangles are congruent.

5.



6.



7. **Construction** Most roofs on residential buildings are made of triangular roof trusses. Explain how the SSS postulate guarantees that the triangles in the roof truss will remain rigid. **Example 1**

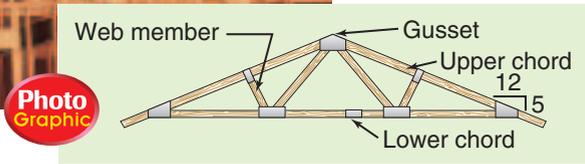


Photo Graphic

**Exercises**

**Practice**

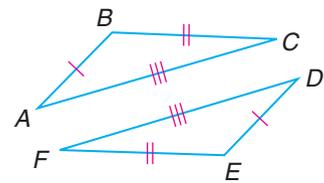
Write a congruence statement for each pair of triangles represented.

8.  $\overline{JK} \cong \overline{MN}$ ,  $\overline{LK} \cong \overline{ON}$ ,  $\angle K \cong \angle N$     9.  $\overline{CB} \cong \overline{EF}$ ,  $\overline{CA} \cong \overline{ED}$ ,  $\overline{BA} \cong \overline{FD}$   
 10.  $\overline{XY} \cong \overline{CA}$ ,  $\overline{XZ} \cong \overline{CB}$ ,  $\angle X \cong \angle C$     11.  $\overline{GH} \cong \overline{RT}$ ,  $\overline{GI} \cong \overline{RS}$ ,  $\overline{HI} \cong \overline{TS}$

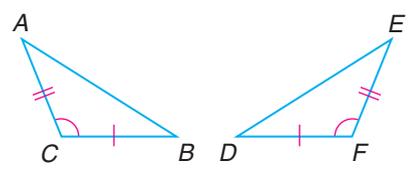
Homework Help	
For Exercises	See Examples
8–11, 20	1
12–15, 16–19, 22	2
Extra Practice	
See page 735.	

Determine whether each pair of triangles is congruent. If so, write a congruence statement and explain why the triangles are congruent.

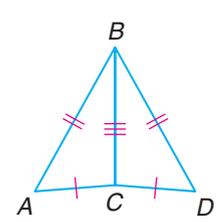
12.



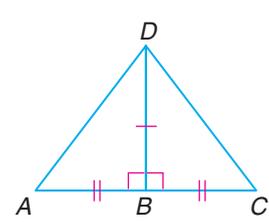
13.



14.

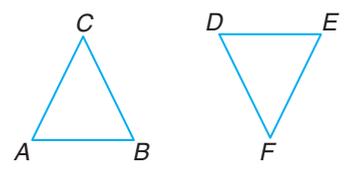


15.



Use the given information to determine whether the two triangles are congruent by SAS. Write yes or no.

16.  $\angle A \cong \angle D$ ,  $\overline{AB} \cong \overline{DE}$ ,  $\overline{BC} \cong \overline{EF}$   
 17.  $\overline{EF} \cong \overline{CA}$ ,  $\overline{BC} \cong \overline{ED}$ ,  $\angle C \cong \angle E$   
 18.  $\overline{BC} \cong \overline{DF}$ ,  $\overline{BA} \cong \overline{EF}$ ,  $\angle B \cong \angle F$   
 19.  $\overline{AB} \cong \overline{DF}$ ,  $\overline{CA} \cong \overline{DE}$ ,  $\angle C \cong \angle F$

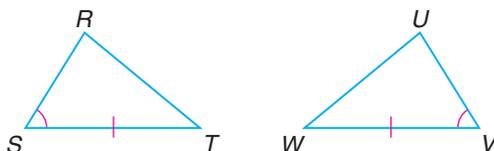


## Applications and Problem Solving

20. **Carpentry** Suppose you are building a rectangular bookcase. How could you provide additional support so that the back of the bookcase won't shift?
21. **Landscaping** When small trees are planted, they are usually supported with a wooden stake as shown at the right. Explain how the stake provides support against the wind.
22. **Critical Thinking** Name the additional corresponding part needed to prove that the triangles below are congruent by SAS.

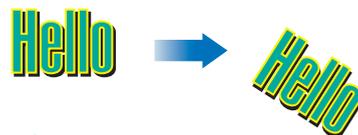


Exercise 21



## Mixed Review

23. If  $\triangle PQR \cong \triangle CAB$ ,  $m\angle P = 45$ , and  $m\angle R = 38$ , find  $m\angle A$ . (Lesson 5-4)
24. **Word Processing** The button in some computer programs makes the indicated change in the position of the word "Hello." Identify the change as a rotation, reflection, or translation. (Lesson 5-3)



The coordinates of the endpoints of a segment are given. Find the coordinates of the midpoint of each segment. (Lesson 2-5)

25.  $(-1, -2)$ ,  $(-3, -8)$     26.  $(4, 8)$ ,  $(-3, -4)$     27.  $(0, 0)$ ,  $(x, y)$

## Standardized Test Practice

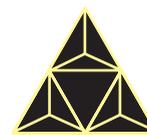
A B C D

28. **Multiple Choice** Express 0.0025 in scientific notation. (Algebra Review)
- (A)  $2.5 \times 10^3$     (B)  $2.5 \times 10^4$     (C)  $2.5 \times 10^{-3}$     (D)  $2.5 \times 10^{-4}$

## Quiz 2

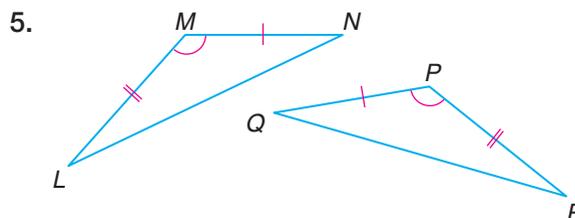
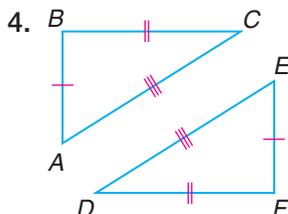
## Lessons 5-4 and 5-5

1. **Design** Which triangles in the figure appear to be congruent? (Lesson 5-4)
2. If  $\triangle XYZ \cong \triangle RST$ , which angle is congruent to  $\angle S$ ? (Lesson 5-4)
3. In two triangles,  $\overline{XZ} \cong \overline{BC}$ ,  $\overline{YZ} \cong \overline{AC}$ , and  $\overline{YX} \cong \overline{AB}$ . Write a congruence statement for the two triangles. (Lesson 5-5)



Exercise 1

Determine whether each pair of triangles is congruent. If so, write a congruence statement and explain why the triangles are congruent. (Lesson 5-5)



# 5-6

# ASA and AAS

### What You'll Learn

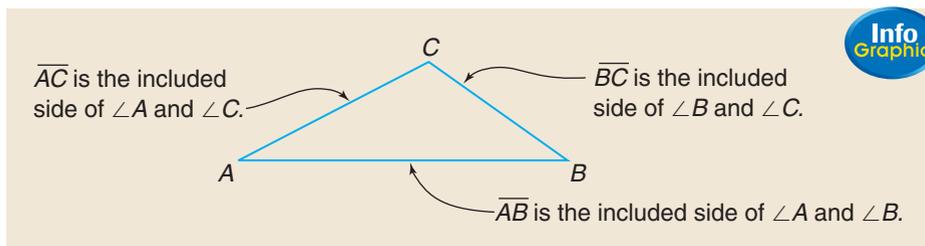
You'll learn to use the ASA and AAS tests for congruence.

### Why It's Important

**Surveying** Surveyors use the ASA Postulate when setting up sight markers.

See Exercise 10.

The side of a triangle that falls between two given angles is called the **included side** of the angles. It is the one side common to both angles.



You can show that two triangles are congruent by using two angles and the included side of the triangles.

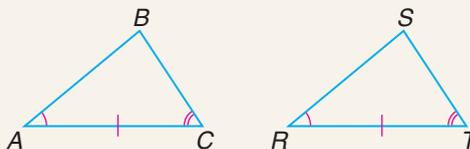
### Reading Geometry

The abbreviation **ASA** is read as **Angle-Side-Angle**.

### Postulate 5-3 ASA Postulate

**Words:** If two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the triangles are congruent.

**Model:**



**Symbols:** If  $\angle A \cong \angle R$ ,  $\overline{AC} \cong \overline{RT}$ , and  $\angle C \cong \angle T$ , then  $\triangle ABC \cong \triangle RST$ .

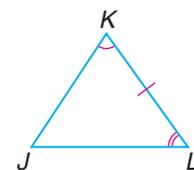
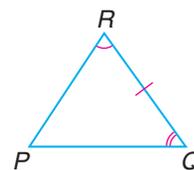
### Example

- 1 In  $\triangle PQR \setminus \triangle KJL$ ,  $\angle R \cong \angle K$ ,  $\overline{RQ} \cong \overline{KL}$ , and  $\angle Q \cong \angle L$ . Write a congruence statement for the two triangles.

Begin by drawing a pair of congruent triangles. Mark the congruent parts with arcs and slashes. Label the vertices of one triangle  $P$ ,  $Q$ , and  $R$ .

Locate  $K$  and  $L$  on the unlabeled triangle in the same positions as  $R$  and  $Q$ . The unassigned vertex must be  $J$ .

Therefore,  $\triangle PQR \cong \triangle JLK$  by ASA.



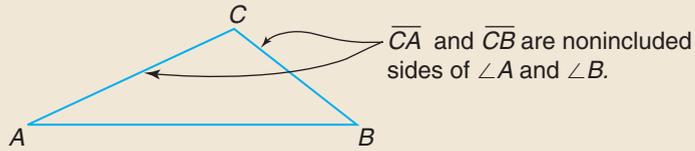
### Your Turn

- a. In  $\triangle DEF$  and  $\triangle LMN$ ,  $\angle D \cong \angle N$ ,  $\overline{DE} \cong \overline{NL}$ , and  $\angle E \cong \angle L$ . Write a congruence statement for the two triangles.

## Reading Geometry

The abbreviation **AAS** is read as **Angle-Angle-Side**.

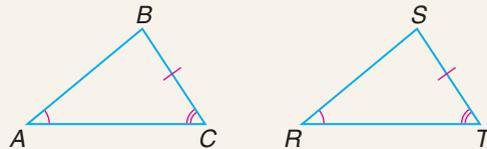
The Angle-Angle-Side Theorem is called a theorem because it can be derived from the ASA Postulate. In AAS, the S is *not* between the two given angles. Therefore, the S indicates a side that is not included between the two angles.



### Theorem 5-4 AAS Theorem

**Words:** If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and nonincluded side of another triangle, then the triangles are congruent.

**Model:**

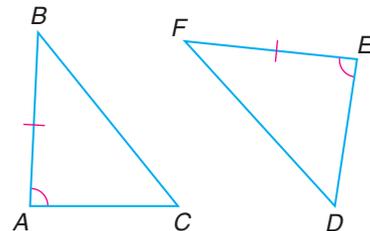


**Symbols:** If  $\angle A \cong \angle R$ ,  $\angle C \cong \angle T$ , and  $\overline{BC} \cong \overline{ST}$ , then  $\triangle ABC \cong \triangle RST$ .

### Example

2

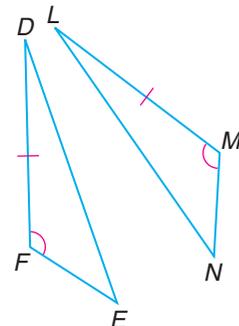
$\triangle ABC$  and  $\triangle EDF$  each have one pair of sides and one pair of angles marked to show congruence. What other pair of angles must be marked so that the two triangles are congruent by AAS?



If  $\angle B$  and  $\angle F$  are marked congruent, then  $\overline{AB}$  and  $\overline{EF}$  would be included sides. However, AAS requires the nonincluded sides. Therefore,  $\angle C$  and  $\angle D$  must be marked congruent.

### Your Turn

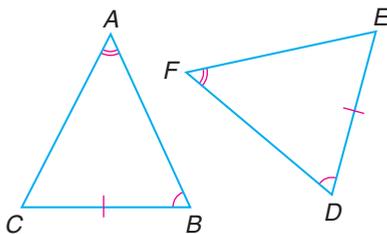
- $\triangle DEF$  and  $\triangle LMN$  each have one pair of sides and one pair of angles marked to show congruence. What other pair of angles must be marked so that the two triangles are congruent by AAS?
- What other pair of angles must be marked so that the two triangles are congruent by ASA?



## Examples

Determine whether each pair of triangles is congruent by SSS, SAS, ASA, or AAS. If it is not possible to prove that they are congruent, write *not possible*.

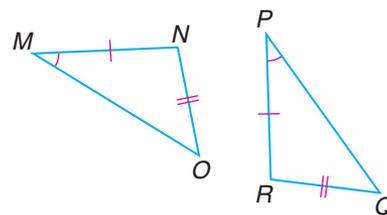
3



There are two pairs of congruent angles,  $\angle A \cong \angle F$  and  $\angle B \cong \angle D$ . There is one pair of corresponding congruent sides,  $\overline{CB} \cong \overline{ED}$ , which is *not included* between the angles.

Therefore,  $\triangle ABC \cong \triangle FDE$  by AAS.

4

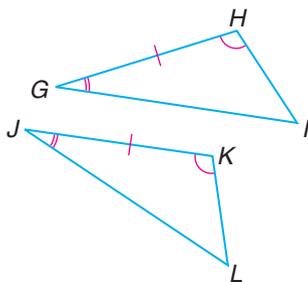


There are two pairs of congruent sides,  $\overline{MN} \cong \overline{RP}$  and  $\overline{NO} \cong \overline{RQ}$ . There is one pair of congruent angles,  $\angle M \cong \angle P$ , which is *not included* between the sides.

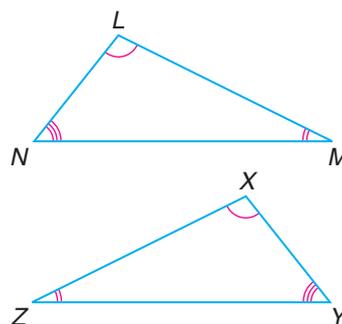
Since SSA is *not* a test for congruence, it is *not possible* to show the triangles are congruent from this information.

### Your Turn

d.



e.



## Check for Understanding

### Communicating Mathematics

1. Sketch and label triangle  $XYZ$  in which  $\overline{XZ}$  is an included side. Then name the two angles  $\overline{XZ}$  is between.
2. Explain how you could construct a triangle congruent to a given triangle using ASA.
3. **Writing Math** Write a few sentences explaining the SSS, SAS, ASA, and AAS tests for congruence. Give an example of each.

### Vocabulary

included side

### Guided Practice

#### Example 1

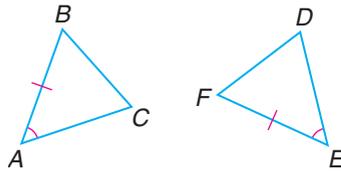
Write a congruence statement for each pair of triangles represented.

4. In  $\triangle DEF$  and  $\triangle RST$ ,  $\angle D \cong \angle R$ ,  $\angle E \cong \angle T$ , and  $\overline{DE} \cong \overline{RT}$ .
5. In  $\triangle ABC$  and  $\triangle XYZ$ ,  $\angle A \cong \angle X$ ,  $\angle B \cong \angle Y$ , and  $\overline{BC} \cong \overline{YZ}$ .

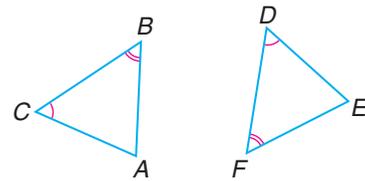
**Example 2**

Name the additional congruent parts needed so that the triangles are congruent by the postulate or theorem indicated.

6. ASA



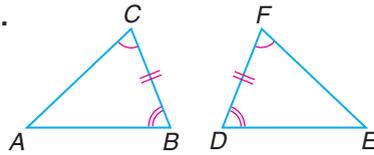
7. AAS



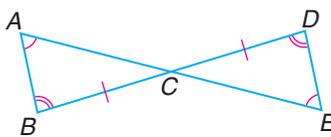
**Examples 3 & 4**

Determine whether each pair of triangles is congruent by SSS, SAS, ASA, or AAS. If it is not possible to prove that they are congruent, write *not possible*.

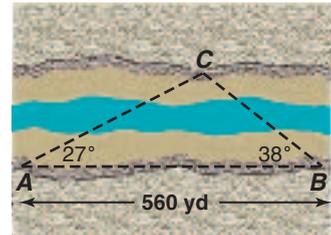
8.



9.



10. **Surveying** Two surveyors 560 yards apart sight a marker C on the other side of a canyon at angles of  $27^\circ$  and  $38^\circ$ . What will happen if they repeat their measurements from the same positions on another day? Explain your reasoning.



Surveying land

**Example 1**

**Exercises**

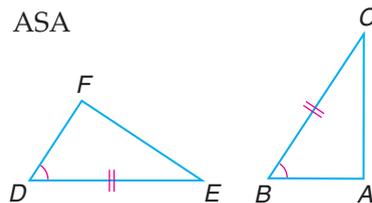
**Practice**

Write a congruence statement for each pair of triangles represented.

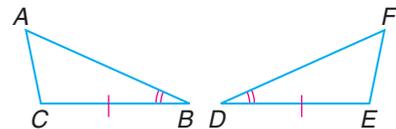
11. In  $\triangle QRS$  and  $\triangle TUV$ ,  $\angle Q \cong \angle T$ ,  $\angle S \cong \angle U$ , and  $\overline{QS} \cong \overline{TU}$ .
12. In  $\triangle ABC$  and  $\triangle DEF$ ,  $\overline{AC} \cong \overline{ED}$ ,  $\angle C \cong \angle D$ , and  $\angle B \cong \angle F$ .
13. In  $\triangle RST$  and  $\triangle XYZ$ ,  $\angle S \cong \angle X$ ,  $\overline{ST} \cong \overline{XZ}$ , and  $\angle T \cong \angle Z$ .
14. In  $\triangle MNO$  and  $\triangle PQR$ ,  $\angle M \cong \angle P$ ,  $\angle N \cong \angle R$ , and  $\overline{NO} \cong \overline{RQ}$ .

Name the additional congruent parts needed so that the triangles are congruent by the postulate or theorem indicated.

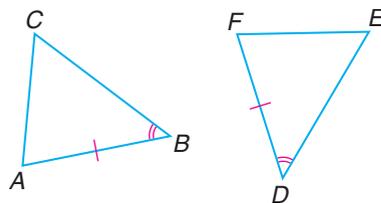
15. ASA



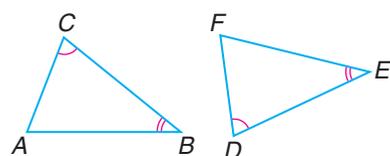
16. AAS



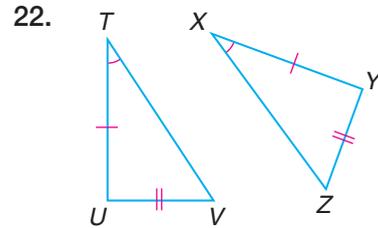
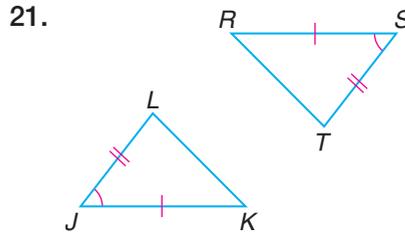
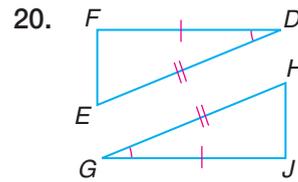
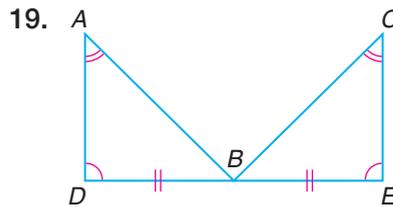
17. AAS



18. ASA

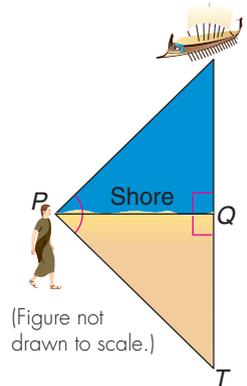


Determine whether each pair of triangles is congruent by SSS, SAS, ASA, or AAS. If it is not possible to prove that they are congruent, write *not possible*.



### Applications and Problem Solving

23. **Math History** The figure shows how the Greek mathematician Thales (624 B.C.–547 B.C.) determined the distance from the shore to enemy ships during a war. He sighted the ship from point  $P$  and then duplicated the angle at  $\angle QPT$ . The angles at point  $Q$  are right angles. Explain why  $QT$  represents the distance from the shore to the ship.



24. **Critical Thinking** In  $\triangle RST$  and  $\triangle UVW$ ,  $\angle R \cong \angle U$ ,  $\angle S \cong \angle V$ , and  $\overline{RT} \cong \overline{UW}$ . So,  $\triangle RST \cong \triangle UVW$  by AAS. Prove  $\triangle RST \cong \triangle UVW$  by ASA.

### Mixed Review

25. In two triangles,  $\overline{MN} \cong \overline{PQ}$ ,  $\overline{MO} \cong \overline{PR}$ , and  $\overline{NO} \cong \overline{QR}$ . Write a congruence statement for the two triangles and explain why the triangles are congruent. (Lesson 5–5)

If  $\triangle HRT \cong \triangle MNP$ , complete each statement. (Lesson 5–4)

26.  $\angle R \cong$  ?  
 27.  $\overline{HT} \cong$  ?  
 28.  $\angle P \cong$  ?

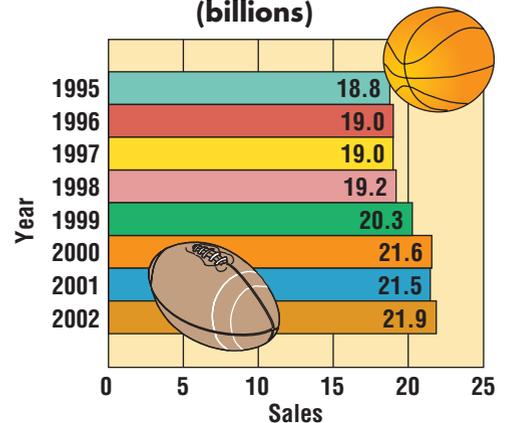
### Standardized Test Practice

- (A) (B) (C) (D)

29. **Multiple Choice** The graph shows the sales of athletic and sports equipment from 1995 to 2002. Between which two years was the percent of increase the greatest? (Statistics Review)

- (A) 1996 to 1997  
 (B) 1998 to 1999  
 (C) 1999 to 2000  
 (D) 2000 to 2001

**Athletic and Sports Equipment Sales (billions)**



Source: National Sporting Goods Association

## Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

acute triangle (p. 188)

base (p. 189)

base angles (p. 189)

congruent triangle (p. 203)

corresponding parts (p. 203)

equiangular triangle (p. 195)

equilateral triangle (p. 189)

image (p. 199)

included angle (p. 211)

included side (p. 215)

isometry (p. 200)

isosceles triangle (p. 189)

legs (p. 189)

mapping (p. 199)

obtuse triangle (p. 188)

preimage (p. 199)

reflection (p. 198)

right triangle (p. 188)

**InterNET**  
CONNECTIONS **Review Activities**

For more review activities, visit:  
[www.geomconcepts.com](http://www.geomconcepts.com)

rotation (p. 198)

scalene triangle (p. 189)

transformation (p. 199)

translation (p. 198)

triangle (p. 188)

vertex (p. 188)

vertex angle (p. 189)

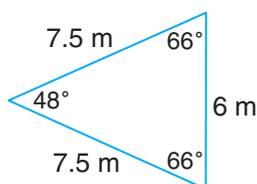
State whether each sentence is *true* or *false*. If false, replace the underlined word(s) to make a true statement.

1. Triangles can be classified by their angles and sides.
2. An isosceles triangle has two vertex angles.
3. The sum of the measures of the angles of a triangle is 360°.
4. An equiangular triangle is defined as a triangle with three congruent sides.
5. The acute angles of a right triangle are supplementary.
6. SSS, SAS, ASA, and AAS are ways to show that two triangles are congruent.
7. A translation is an example of a transformation.
8. An equilateral triangle is also an isosceles triangle.
9. AAS refers to two angles and their included side.
10. Reflections are sometimes called *turns*.

## Skills and Concepts

### Objectives and Examples

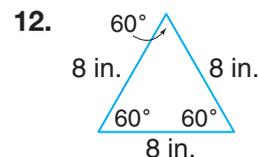
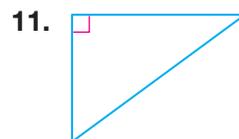
- **Lesson 5–1** Identify the parts of triangles and classify triangles by their parts.



The triangle is acute and isosceles.

### Review Exercises

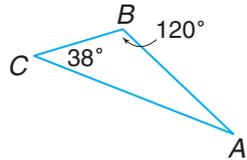
Classify each triangle by its angles and by its sides.



**Objectives and Examples**

- **Lesson 5–2** Use the Angle Sum Theorem.

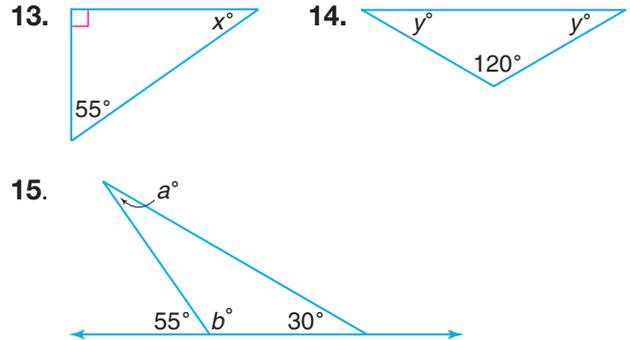
Find  $m\angle A$  in  $\triangle ABC$ .



$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180 \\ m\angle A + 120 + 38 &= 180 \\ m\angle A + 158 &= 180 \\ m\angle A + 158 - 158 &= 180 - 158 \\ m\angle A &= 22 \end{aligned}$$

**Review Exercises**

Find the value of each variable.

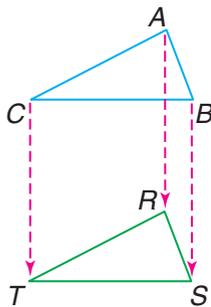


- **Lesson 5–3** Identify translations, reflections, and rotations and their corresponding parts.

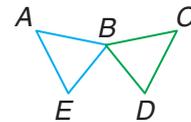
$\triangle ABC \rightarrow \triangle RST$  by a translation.

$\angle R$  is the image of  $\angle A$ .

$\overline{BC}$  corresponds to  $\overline{ST}$ .



Suppose  $\triangle ABE \rightarrow \triangle CBD$ .

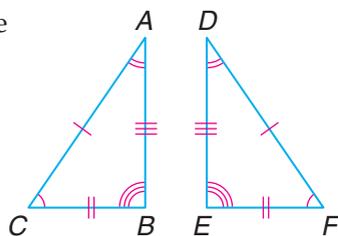


16. Name the angle that corresponds to  $\angle D$ .
17. Name the image of  $\angle ABE$ .
18. Name the image of  $\overline{AE}$ .
19. Identify the transformation that occurred in the mapping.

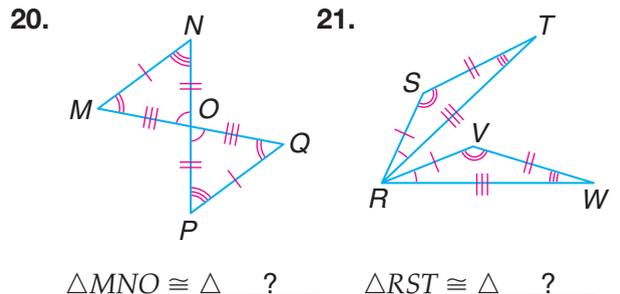
- **Lesson 5–4** Name and label corresponding parts of congruent triangles.

Write a congruence statement for the two triangles.

$\triangle ABC \cong \triangle DEF$



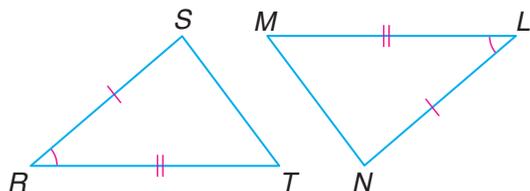
Complete each congruence statement.



$\triangle MNO \cong \triangle \underline{\hspace{1cm}} ? \hspace{1cm}$        $\triangle RST \cong \triangle \underline{\hspace{1cm}} ? \hspace{1cm}$

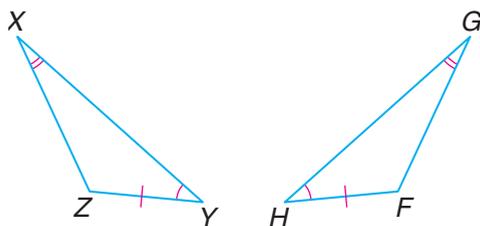
Objectives and Examples

- **Lesson 5-5** Use the SSS and SAS tests for congruence.



$\triangle RST \cong \triangle LNM$  by SAS.

- **Lesson 5-6** Use the ASA and AAS tests for congruence.

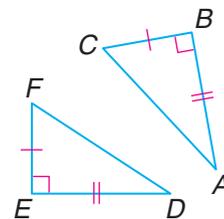
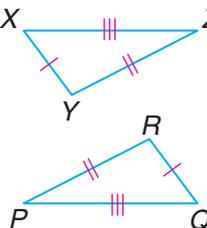


$\triangle XYZ \cong \triangle GHF$  by AAS.

Review Exercises

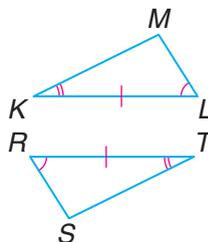
Determine whether each pair of triangles is congruent. If so, write a congruence statement and explain why the triangles are congruent.

- 22.

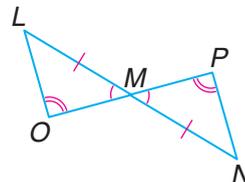


Determine whether each pair of triangles is congruent by SSS, SAS, ASA, or AAS. If it is not possible to prove that they are congruent, write *not possible*.

- 24.



- 25.

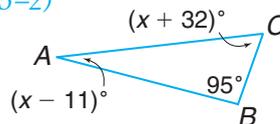


Applications and Problem Solving

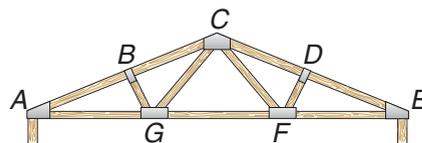
26. **Maps** Classify the triangle by its sides. (Lesson 5-1)



27. **Algebra** Find the measure of  $\angle A$  in  $\triangle ABC$ . (Lesson 5-2)



28. **Construction** The W-truss is the most widely used of light wood trusses. Identify two pairs of triangles in the truss below that appear to be congruent. (Lesson 5-4)

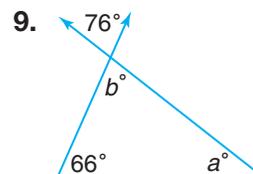
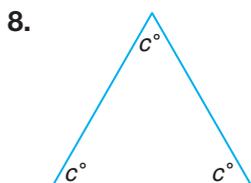
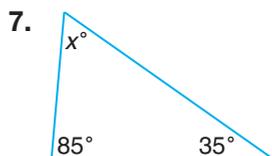


Choose the letter of the description that best matches each term.

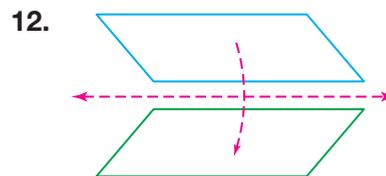
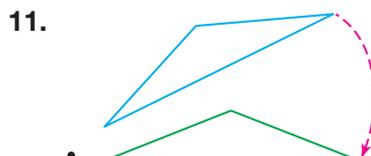
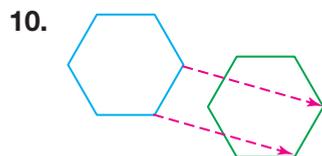
1. scalene triangle
2. right triangle
3. isosceles triangle
4. acute triangle
5. equilateral triangle
6. equiangular triangle

- a. has a right angle
- b. all sides are congruent
- c. no sides are congruent
- d. has a vertex angle
- e. all angles are acute
- f. all angles are congruent

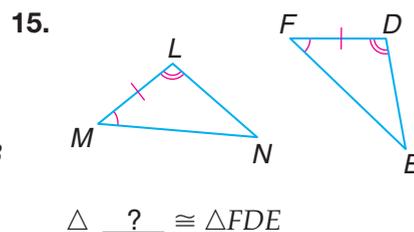
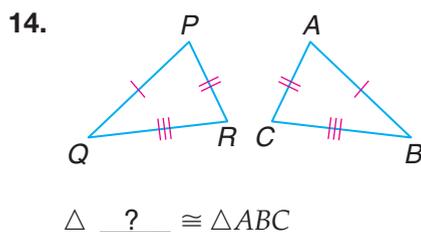
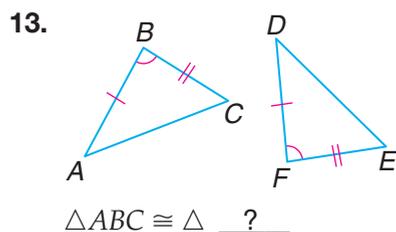
Find the value of each variable.



Identify each motion as a translation, reflection, or rotation.

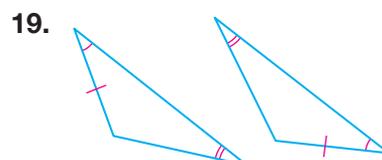
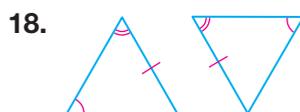
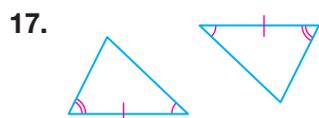


Complete each congruence statement.



16. In  $\triangle CDE$ , identify the included angle for sides  $\overline{CD}$  and  $\overline{EC}$ .

Determine whether each pair of triangles is congruent by SSS, SAS, ASA, or AAS. If it is not possible to prove that they are congruent, write *not possible*.



20. **Sports** The sail for a sailboat looks like a right triangle. If the angle at the top of the sail measures  $54^\circ$ , what is the measure of the acute angle at the bottom?



## Statistics Problems

On some standardized tests, you will calculate the mean, median, and mode of a data set. You will also choose the most appropriate measure for a data set. On the SAT and ACT, you will apply the concept of the mean to solve problems.

$$\text{mean} = \frac{\text{sum of the numbers}}{\text{number of numbers}}$$

median = middle number of a set arranged in numerical order

mode = the number(s) that occurs most often

### Test-Taking Tip

**Memory Tip** A highway *median* is in the middle of the road. So a *median* is the middle number of an ordered data set.

### Example 1

The heights of ten National Champion Trees are listed in the table below. What is the median, in feet, of the heights?

Tree	Height (ft)	Tree	Height (ft)
American Beech	115	Loblolly Pine	148
Black Willow	76	Pinyon Pine	69
Coast Douglas Fir	329	Sugar Maple	87
Coast Redwood	313	Sugar Pine	232
Giant Sequoia	275	White Oak	79

**Hint** If there is no single middle number, find the median by calculating the mean of the two middle values.

**Solution** To find the median, first list the heights in numerical order.

69 76 79 87 115 148 232 275 313 329

Since there are ten numbers, there is no middle number. The two numbers in the middle are 115 and 148. Calculate the mean of these two numbers.

$$\frac{115 + 148}{2} = \frac{263}{2} \text{ or } 131\frac{1}{2}$$

The median is  $131\frac{1}{2}$  feet.

### Example 2

If the average of five numbers is 32 and the average of two of the numbers is 20, then what is the *sum* of the remaining three numbers?

- (A) 12      (B) 40      (C)  $46\frac{2}{3}$   
 (D) 120      (E) 140

**Hint** Use the formula for mean to calculate the sum of the numbers.

**Solution** On the SAT, *average* is the same as *mean*. First find the sum of the five numbers. Then use the formula for the mean. You know the average (32) and the number of numbers (5).

$$32 = \frac{\text{sum of the five numbers}}{5}$$

$$5 \cdot 32 = 5 \cdot \frac{\text{sum of the five numbers}}{5}$$

$$160 = \text{sum of the five numbers}$$

Use the same method to find the sum of the two numbers.

$$20 = \frac{\text{sum of the two numbers}}{2}$$

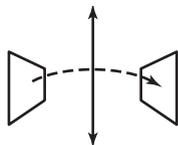
$$40 = \text{sum of the two numbers}$$

You can find the sum of the other three numbers by subtracting: (sum of the five numbers) – (sum of the two numbers) =

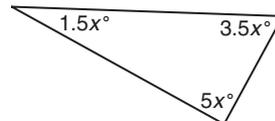
160 – 40 or 120. The answer is D.

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

**Multiple Choice**

- Mr. Mendosa obtained estimates for painting from five companies. The estimates were \$950, \$850, \$995, \$1000, and \$950. What is the mode of these estimates? *(Statistics Review)*  
 A \$150    B \$949    C \$950    D \$995
- $\sqrt{64 + 36} = ?$  *(Algebra Review)*  
 A 10    B 14    C 28  
 D 48    E 100
- Jared’s study group recorded the time they spent on math homework one day. Here are the results (in minutes): 30, 29, 32, 25, 36, 20, 30, 26, 56, 45, 33, and 34. What was the median time spent? *(Statistics Review)*  
 A 20 min    B 25 min  
 C 30 min    D 31 min
- The figure below shows an example of a— *(Lesson 5–3)*  
  
 A dilation.    B reflection.  
 C rotation.    D translation.
- Yoshi wants to buy a sweater priced at \$59.95. If the sales tax rate is 6%, which is the best estimate of the tax paid on the sweater? *(Percent Review)*  
 A \$3.00    B \$3.60  
 C \$4.00    D \$4.20
- How many even integers are there between 2 and 100, not including 2 and 100? *(Algebra Review)*  
 A 98    B 97    C 50  
 D 49    E 48

- Jenny recorded high temperatures every day for a week. The temperatures, in degrees Fahrenheit, were 48, 55, 60, 55, 52, 47, and 40. What was the mean temperature? *(Statistics Review)*  
 A 51    B 52    C 55    D 60
- What is the value of  $x$  in the figure? *(Lesson 5–2)*  
 A 10    B 18  
 C 27    D 63



**Grid In**

- There are 24 fish in an aquarium. If  $\frac{1}{8}$  of them are tetras and  $\frac{2}{3}$  of the remaining fish are guppies, how many guppies are there? *(Algebra Review)*

**Extended Response**

- The table shows the percent of new passenger cars imported into the United States by country of origin in 2003. *(Statistics Review)*

Percent of New Passenger Cars Imported into U.S. by Country of Origin	
Country	New Cars (percent)
Canada	27
Germany	17
Japan	28
Mexico	10
Korea	7
Other	11

**Source:** Bureau of Census, Foreign Trade Division

**Part A** Make a circle graph to show the data. Label each section of the graph with the percent of imported cars.

**Part B** The total value of cars imported was about \$114 billion. Use this information to determine the value of cars imported from outside North America.

