

CHAPTER

3

Angles

What You'll Learn

Key Ideas

- Name and identify parts of an angle. (*Lesson 3-1*)
- Measure, draw, classify angles, and find the bisector of an angle. (*Lessons 3-2 and 3-3*)
- Identify and use adjacent angles, linear pairs of angles, complementary and supplementary angles, and congruent and vertical angles. (*Lessons 3-4 to 3-6*)
- Identify, use properties of, and construct perpendicular lines and segments. (*Lesson 3-7*)

Key Vocabulary

acute angle (*p. 98*)

angle (*p. 90*)

obtuse angle (*p. 98*)

right angle (*p. 98*)

straight angle (*p. 90*)

Why It's Important

Hobbies Kites first appeared in China around 500 B.C. Silk and lightweight bamboo were used to make kites in rectangular shapes. After the invention of paper, exotic kites were made in a variety of shapes, such as dragons, birds, insects, and people.

Angles are used to describe relationships between real-life and mathematical objects. You will examine the angles in a flat kite in Lesson 3-1.



Study these lessons to improve your skills.

Lesson 1–2, pp. 12–17

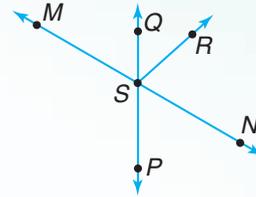
Algebra Review, p. 722

Algebra Review, p. 723

Check Your Readiness

Use the figure to name examples of each term.

- three points
- three rays
- a point that is not on \overline{MS}
- a ray with point N as the endpoint



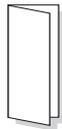
Solve each equation. Check your solution.

- | | |
|----------------------------|-----------------------------|
| 5. $4y = 180$ | 6. $65 + b = 90$ |
| 7. $8v = 168$ | 8. $180 = d + 25$ |
| 9. $90 - k = 12$ | 10. $180 - p = 142$ |
| 11. $90 = 4x + 26$ | 12. $16 + 2t = 84$ |
| 13. $3x + 18 = 180$ | 14. $180 - 2x = 66$ |
| 15. $5n - 45 = 120$ | 16. $6h + 18 = 90$ |
| 17. $52 + (8n - 4) = 160$ | 18. $2x + (4x - 16) = 110$ |
| 19. $75 + (3x + 12) = 180$ | 20. $8y + (10y - 16) = 128$ |

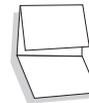
FOLDABLES™ Study Organizer

Make this Foldable to help you organize your Chapter 3 notes. Begin with a sheet of plain $8\frac{1}{2}$ " by 11" paper.

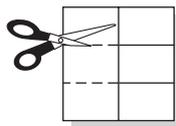
1 Fold in half lengthwise.



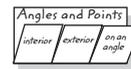
2 Fold again in thirds.



3 Open and cut along the second fold to make three tabs.



4 Label as shown.



Reading and Writing As you read and study the chapter, explain and draw examples of points in the interior, exterior, and on an angle under the tabs. You may want to make another Foldable to record notes about the three types of angles, *right*, *acute*, and *obtuse*.



3-1

Angles

What You'll Learn

You'll learn to name and identify parts of an angle.

Why It's Important

Design Bicycle manufacturers use angles in their bicycle designs. See Exercise 25.

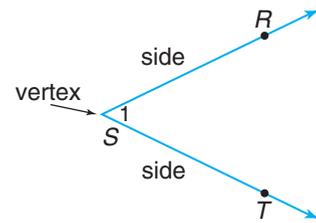
Opposite rays are two rays that are part of the same line and have only their endpoints in common.



\overrightarrow{XY} and \overrightarrow{XZ} are opposite rays.

The figure formed by opposite rays is also referred to as a **straight angle**.

There is another case where two rays can have a common endpoint. This figure is called an **angle**. *Unless otherwise noted, the term "angle" in this book means a nonstraight angle.* Some parts of angles have special names. The common endpoint is called the **vertex**, and the two rays that make up the angle are called the **sides** of the angle.



There are several ways to name the angle shown above.

Reading Geometry

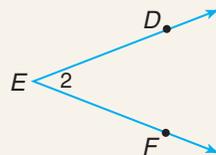
Read the symbol \angle as angle.

Method	Symbol
1. Use the vertex and a point from each side. <i>The vertex letter is always in the middle.</i>	$\angle RST$ or $\angle TSR$
2. Use the vertex only. <i>If there is only one angle at a vertex, then the angle can be named with that vertex.</i>	$\angle S$
3. Use a number.	$\angle 1$

Definition of Angle

Words: An angle is a figure formed by two noncollinear rays that have a common endpoint.

Model:



Symbols: $\angle DEF$
 $\angle FED$
 $\angle E$
 $\angle 2$

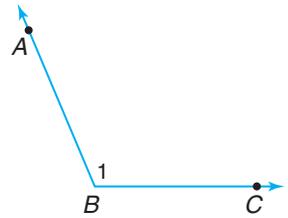
Example

Look Back
Naming Rays:
Lesson 1-2

1

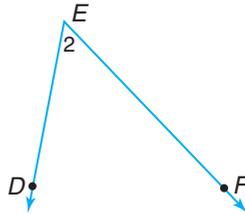
Name the angle in four ways.
Then identify its vertex and its sides.

The angle can be named in four ways:
 $\angle ABC$, $\angle CBA$, $\angle B$, and $\angle 1$.
Its vertex is point B . Its sides are \overrightarrow{BA} and \overrightarrow{BC} .

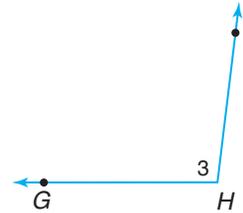


Your Turn

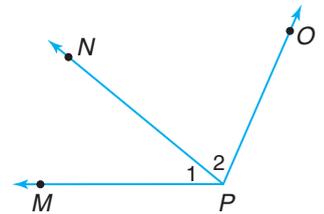
a.



b.



Look at the figure at the right. Three angles have P as their vertex. Whenever there is more than one angle at a given vertex, use three points or a number to name an angle as shown at the right.



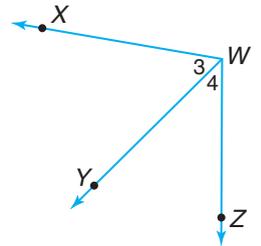
Example

2

Name all angles having W as their vertex.

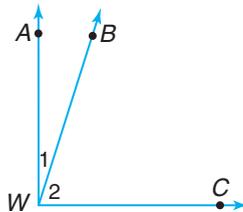
There are three distinct angles with vertex W :
 $\angle 3$, $\angle 4$, and $\angle XWZ$.

What other names are there for $\angle 3$?
What other names are there for $\angle 4$?
What other name is there for $\angle XWZ$?
Is there an angle that can be named $\angle W$?

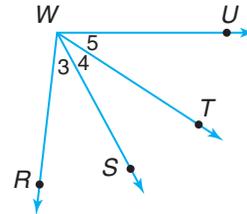


Your Turn

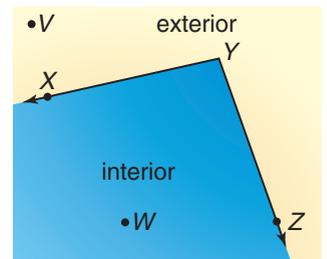
c.



d.



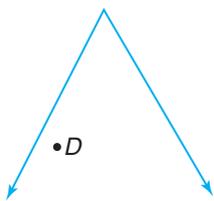
An angle separates a plane into three parts: the **interior** of the angle, the **exterior** of the angle, and the angle itself. In the figure shown, point W and all other points in the blue region are in the interior of the angle. Point V and all other points in the yellow region are in the exterior of the angle. Points X , Y , and Z are on the angle.



Examples

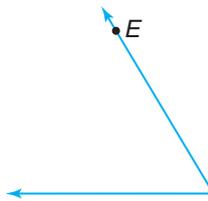
Tell whether each point is in the *interior*, *exterior*, or *on the angle*.

3



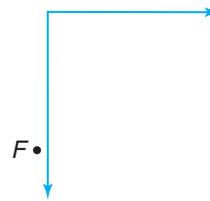
Point D is in the interior of the angle.

4



Point E is on the angle.

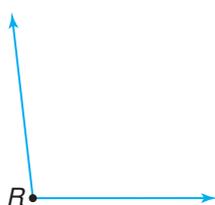
5



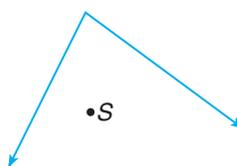
Point F is in the exterior of the angle.

Your Turn

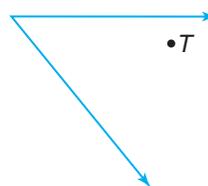
e.



f.



g.



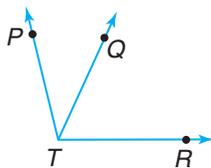
Check for Understanding

Communicating Mathematics

- Sketch and label an angle with sides \overrightarrow{EF} and \overrightarrow{EG} .
- Draw an angle MNP that has a point Q in the interior of the angle.
- Explain why angle PTR cannot be labeled $\angle T$.

Vocabulary

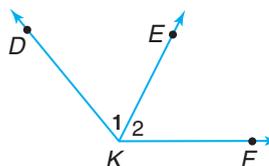
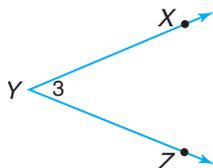
opposite rays
straight angle
angle
vertex
sides
interior
exterior



Guided Practice

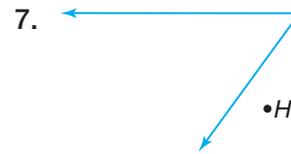
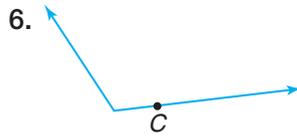
Examples 1 & 2

- Name the angle in four ways. Then identify its vertex and its sides.
- Name all angles having K as their vertex.



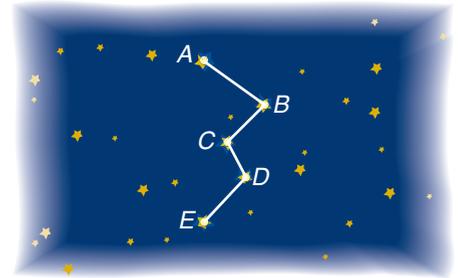
Examples 3–5

Tell whether each point is in the *interior*, *exterior*, or *on the angle*.



Examples 1 & 2

8. **Science** The constellation Cassiopeia is one of the 88 constellations in the sky.
- How many angles are formed by the arrangement of the stars in the constellation?
 - Name each angle in two ways.



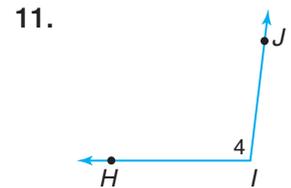
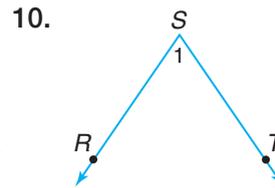
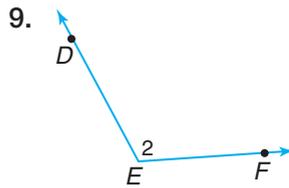
The constellation Cassiopeia

Exercises

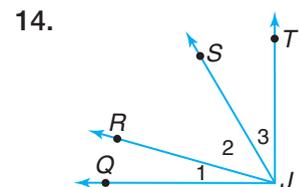
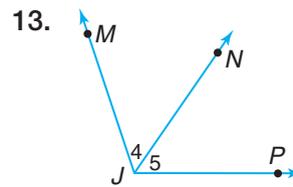
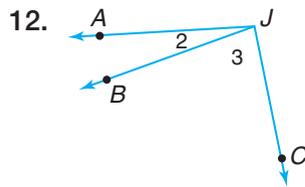
Practice

Homework Help	
For Exercises	See Examples
9–11, 21, 23	1
12–14	2
15–20, 22	3–5
Extra Practice	
See page 729.	

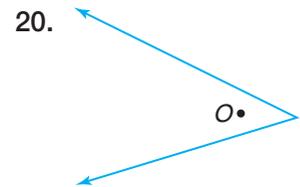
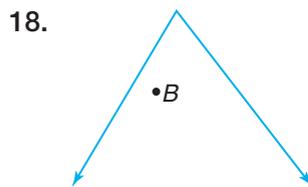
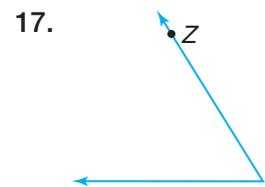
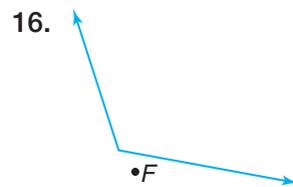
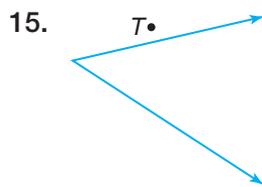
Name each angle in four ways. Then identify its vertex and its sides.



Name all angles having *J* as their vertex.



Tell whether each point is in the *interior*, *exterior*, or *on the angle*.

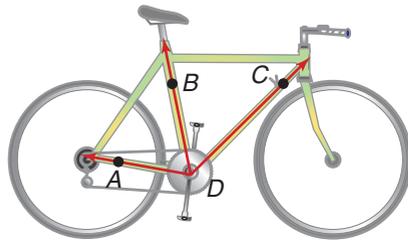
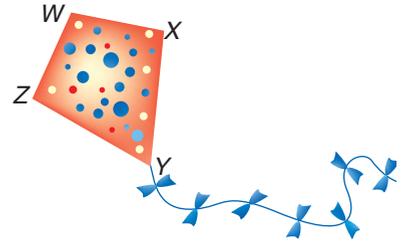


Determine whether each statement is *true* or *false*.

21. Angles may have four different names.
22. The vertex is in the interior of an angle.
23. The sides of $\angle ABC$ are \overrightarrow{AB} and \overrightarrow{BC} .

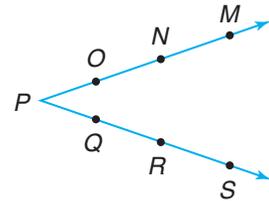
Applications and Problem Solving

24. **Hobbies** The oldest basic type of kite is called the *flat kite*.
 - a. How many angles are formed by the corners of a flat kite?
 - b. Name each angle in two ways.



25. **Design** Bicycle manufacturers use angles when designing bicycles. Name each angle shown. Then identify the sides of each angle.

26. **Critical Thinking** Using three letters, how many different ways can the angle at the right be named? List them.



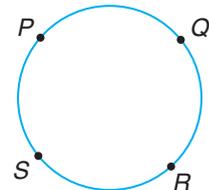
Mixed Review

27. The coordinates of the endpoints of a segment are (2, 3) and (4, 5). Find the coordinates of the midpoint. (*Lesson 2-5*)
28. Draw and label a coordinate plane. Then graph and label point A at (2, -3). (*Lesson 2-4*)
29. **Interior Design** Ke Min is planning to add a wallpaper border to his rectangular bathroom. How much border will he need if the length of the room is 8 feet and the width is 5 feet? (*Lesson 1-6*)
30. Use a compass and a straightedge to construct a five-sided figure. (*Lesson 1-5*)

Standardized Test Practice

- A B C D

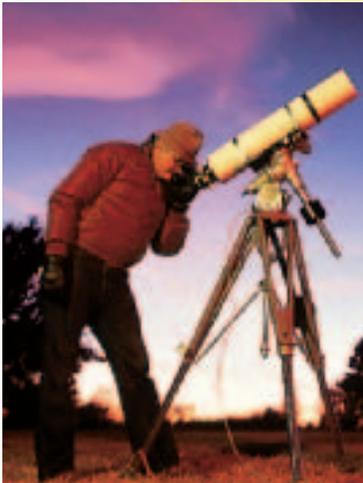
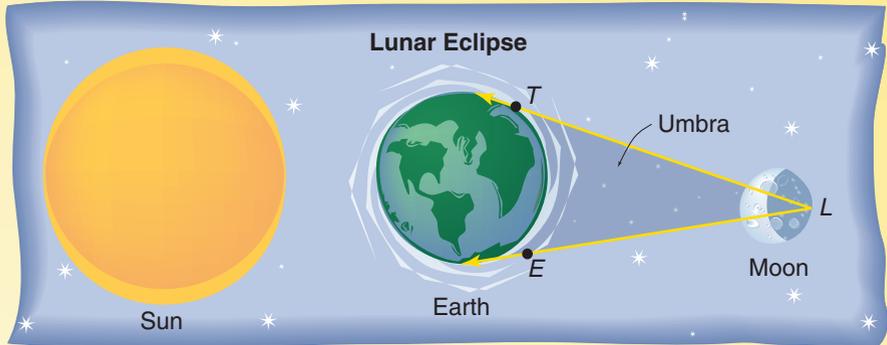
31. **Short Response** Points P, Q, R, and S lie on a circle. List all of the lines that contain exactly two of these four points. (*Lesson 1-2*)



32. **Multiple Choice** Simplify $4y + 3(6 + 2y)$. (*Algebra Review*)
 - A $6y + 18$
 - B $9y + 9$
 - C $10y + 18$
 - D $18y + 18$

Astronomer

Do the stars in the night sky captivate you? If so, you may want to consider a career as an astronomer. In addition to learning about stars, galaxies, the sun, moon, and planets, astronomers study events such as *eclipses*.



A *total lunar eclipse* occurs when the moon passes totally into Earth's dark shadow, or *umbra*. Notice the angle that is formed by Earth's umbra.

1. Name the angle in three ways.
2. Identify the vertex and its sides.
3. Is the umbra in the *exterior*, in the *interior*, or *on the angle*?
4. Research lunar eclipses. Explain the difference between a lunar eclipse and a total lunar eclipse.

FAST FACTS About Astronomers

Working Conditions

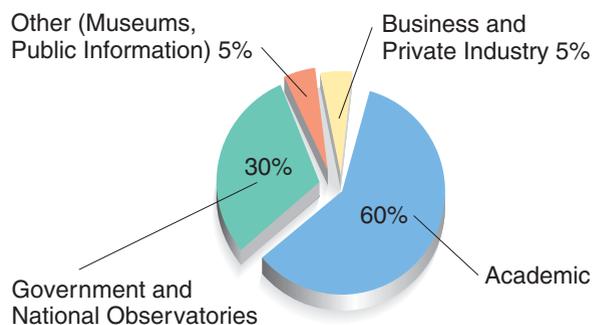
- usually work in observatories
- may have to travel to remote locations
- may work long hours and nights

Education/Skills

- high school math and physical science courses
- college degree in astronomy or physics
- mathematical ability, computer skills, and the ability to work independently are essential

Employment

Where Astronomers Are Employed



Career Data For the latest information on careers in astronomy, visit:

www.geomconcepts.com

3-2

Angle Measure

What You'll Learn

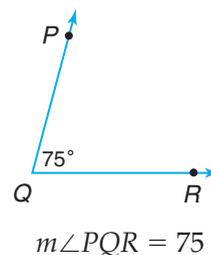
You'll learn to measure, draw, and classify angles.

Why It's Important

Sports Golfers use angles when hitting a golf ball. See Exercise 30.

In geometry, angles are measured in units called **degrees**. The symbol for degree is $^\circ$.

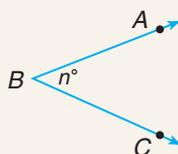
The angle shown measures 75 degrees. In the notation, there is no degree symbol with 75 because the measure of an angle is a real number with no unit of measure. This is summarized in the following postulate.



Postulate 3-1 Angles Measure Postulate

Words: For every angle, there is a unique positive number between 0 and 180 called the *degree measure* of the angle.

Model:



Symbols: $m\angle ABC = n$
and $0 < n < 180$



Read $m\angle PQR = 75$ as the *degree measure* of angle PQR is 75.

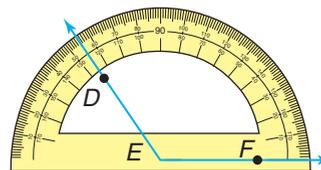
In this text, the term degree measure will be used in all appropriate theorems and postulates. Elsewhere we will refer to the degree measure of an angle as just measure.

You can use a **protractor** to measure angles and sketch angles of given measure.

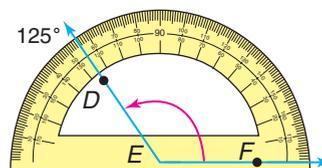
Examples

1 Use a protractor to measure $\angle DEF$.

Step 1 Place the center point of the protractor on vertex E . Align the straightedge with side \overline{EF} .



Step 2 Use the scale that begins with 0 at \overline{EF} . Read where the other side of the angle, \overline{ED} , crosses this scale.



Angle DEF measures 125° .

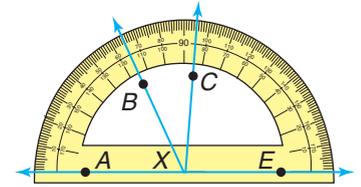
$m\angle DEF = 125$

- 2** Find the measures of $\angle BXE$, $\angle CXE$, and $\angle AXB$.

$m\angle BXE = 115$ \overrightarrow{XE} is at 0° on the right.

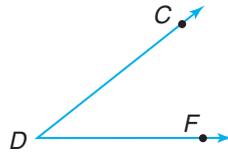
$m\angle CXE = 85$ \overrightarrow{XE} is at 0° on the right.

$m\angle AXB = 65$ \overrightarrow{XA} is at 0° on the left.

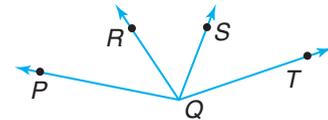


Your Turn

- a. Use a protractor to measure $\angle CDF$.



- b. Find the measure of $\angle PQR$, $\angle PQS$, and $\angle PQT$.

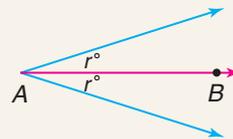


Just as Postulate 3–1 provides a way to measure angles, Postulate 3–2 describes the relationship between angle measures and numbers.

**Postulate 3–2
Protractor
Postulate**

Words: On a plane, given \overrightarrow{AB} and a number r between 0 and 180, there is exactly one ray with endpoint A , extending on each side of \overrightarrow{AB} such that the degree measure of the angle formed is r .

Model:



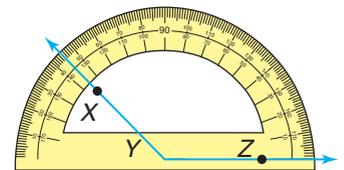
Example

- 3** Use a protractor to draw an angle having a measure of 135.

Step 1 Draw \overrightarrow{YZ} .

Step 2 Place the center point of the protractor on Y . Align the mark labeled 0 with the ray.

Step 3 Locate and draw point X at the mark labeled 135. Draw \overrightarrow{YX} .



Your Turn

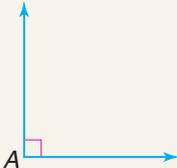
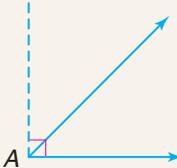
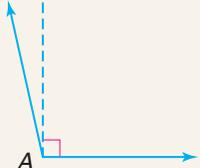
- c. Use a protractor to draw an angle having a measure of 65.



Reading Geometry

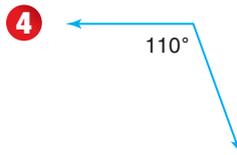
The symbol \square is used to indicate a right angle.

Once the measure of an angle is known, the angle can be classified as one of three types of angles. These types are defined in relation to a right angle.

Types of Angles		
		
<p>right angle $m\angle A = 90$</p>	<p>acute angle $0 < m\angle A < 90$</p>	<p>obtuse angle $90 < , m\angle A < 180$</p>

Examples

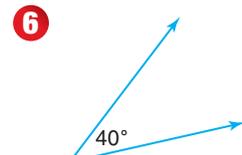
Classify each angle as *acute*, *obtuse*, or *right*.



obtuse

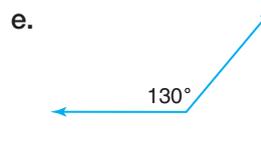
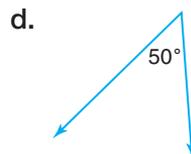


right



acute

Your Turn



Example

Algebra Link

7 The measure of $\angle B$ is 138. Solve for x .

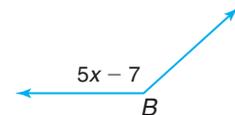
Explore You know that $m\angle B = 138$
and $m\angle B = 5x - 7$.

Plan Write and solve an equation.

Solve

$138 = 5x - 7$	<i>Substitution</i>
$138 + 7 = 5x - 7 + 7$	<i>Add 7 to each side.</i>
$145 = 5x$	<i>Simplify.</i>
$\frac{145}{5} = \frac{5x}{5}$	<i>Divide each side by 5.</i>
$29 = x$	<i>Simplify.</i>

Examine Since $m\angle B = 5x - 7$, replace x with 29.
 $5(29) - 7 = 138$ and $m\angle B = 138$.



Algebra Review

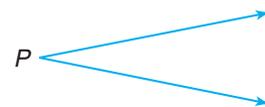
Solving Multi-Step Equations, p. 723

To construct two angles of the same measure requires a compass and straightedge.

Hands-On Geometry Construction

Materials:  compass  straightedge

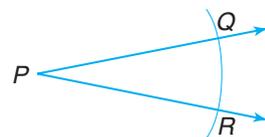
Step 1 Draw an angle like $\angle P$ on your paper.



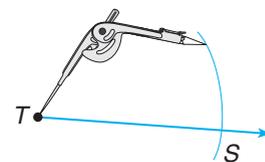
Step 2 Use a straightedge to draw a ray on your paper. Label its endpoint T .



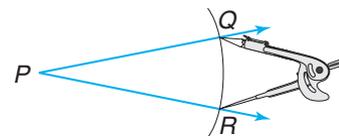
Step 3 With P as the center, draw a large arc that intersects both sides of $\angle P$. Label the points of intersection Q and R .



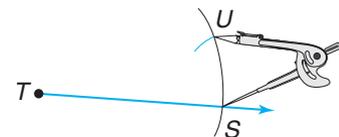
Step 4 Using the same compass setting, put the compass at point T and draw a large arc that starts above the ray and intersects the ray. Label the point of intersection S .



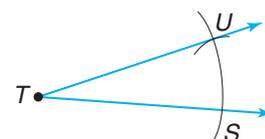
Step 5 Place the point of the compass on R and adjust so that the pencil tip is on Q .



Step 6 Without changing the setting, place the compass at point S and draw an arc to intersect the larger arc you drew in Step 4. Label the point of intersection U .

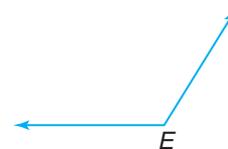


Step 7 Use a straightedge to draw \overline{TU} .



Try These

1. Cut out $\angle QPR$ and $\angle UTS$ and then compare them.
2. Do the two angles have the same measure? If so, write an equation.
3. Construct an angle whose measure is equal to the measure of $\angle E$.



Check for Understanding

Communicating Mathematics

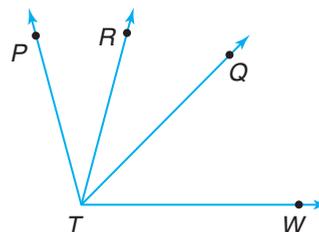
1. Draw an angle having a measure of 70 using a protractor.
2. Draw any angle. Then construct an angle whose measure is equal to the measure of the angle drawn.
3. **Writing Math** Write a few sentences describing how rulers and protractors are used in geometry.

Vocabulary

degrees
protractor
right angle
acute angle
obtuse angle

Guided Practice

Use a protractor to find the measure of each angle. Then classify each angle as *acute*, *obtuse*, or *right*.



Examples 1, 2, 4–6

4. $m\angle PTR$
5. $m\angle PTW$
6. $m\angle RTW$
7. $m\angle PTQ$

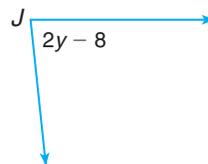
Examples 3–6

Use a protractor to draw an angle having each measurement. Then classify each angle as *acute*, *obtuse*, or *right*.

8. 45°
9. 115°

Example 7

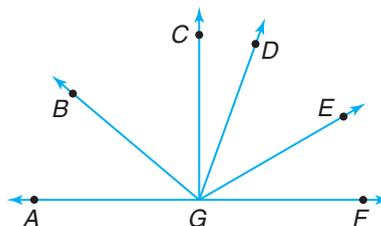
10. **Algebra** The measure of $\angle J$ is 84. Solve for y .



Exercises

Use a protractor to find the measure of each angle. Then classify each angle as *acute*, *obtuse*, or *right*.

11. $\angle AGD$
12. $\angle CGD$
13. $\angle EGF$
14. $\angle BGE$
15. $\angle CGF$
16. $\angle EGC$
17. $\angle AGB$
18. $\angle FGD$
19. $\angle BGF$
20. $\angle BGC$
21. $\angle AGC$
22. $\angle BGD$



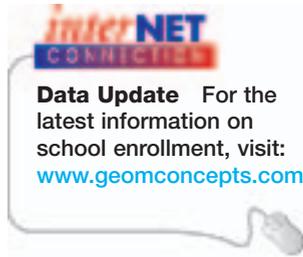
Use a protractor to draw an angle having each measurement. Then classify each angle as *acute*, *obtuse*, or *right*.

23. 42°
24. 155°
25. 26°
26. 95°
27. 75°
28. 138°

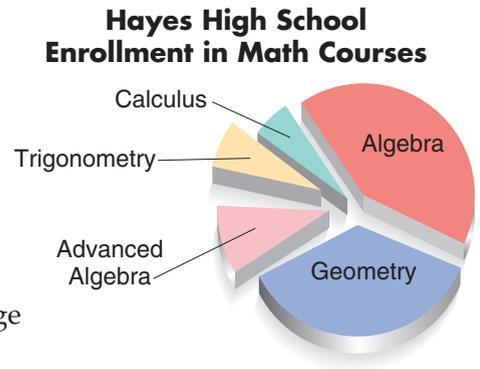
Homework Help

For Exercises	See Examples
11–22	1–2, 4–6
23–28	3, 4–6
29	1–6
Extra Practice	
See page 730.	

Applications and Problem Solving



29. **Statistics** The circle graph shows the enrollment in math courses at Hayes High School.



- Use a protractor to find the measure of each angle of the circle graph.
- Classify each angle as *acute*, *obtuse*, or *right*.
- What is the greatest percentage that an acute angle could represent on a circle graph? Explain your reasoning.

30. **Sports** In golf, the launch angle is the angle of a ball's initial flight path relative to horizontal. While most amateur golfers hit the ball at a 7° angle, professional golfers hit the ball at a 10° angle. A launch angle of 13° is optimal.

- Draw a diagram that shows these launch angles.
- Explain why an angle of 13° is optimal.
- Explain why an angle of 30° is not optimal.

31. **Algebra** The measure of $\angle ABC$ is 6 more than twice the measure of $\angle EFG$. The sum of the measures of the two angles is 90. Find the measure of each angle.

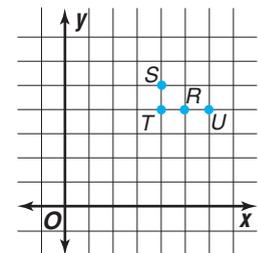
32. **Critical Thinking** Tell how a corner of a sheet of notebook paper could be used to classify an angle.

Mixed Review

33. Draw $\angle XYZ$ that has a point W on the angle. (Lesson 3-1)

34. Find the midpoint of a segment that has endpoints at $(3, -5)$ and $(-1, 1)$. (Lesson 2-5)

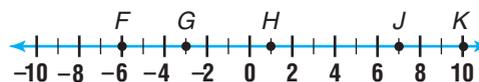
35. What is the ordered pair for point R ? (Lesson 2-4)



Standardized Test Practice

(A) (B) (C) (D)

36. **Extended Response** Use the number line to determine whether H is the midpoint of \overline{FJ} . Explain your reasoning. (Lesson 2-3)



37. **Short Response** Write a sequence in which each term is 6 more than the previous term. (Lesson 1-1)

Those Magical Midpoints

Materials

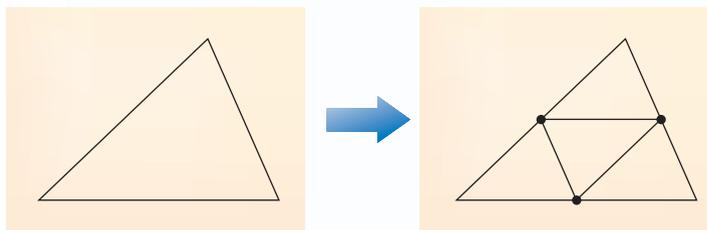
-  straightedge
-  compass
-  scissors

Triangles, Quadrilaterals, and Midpoints

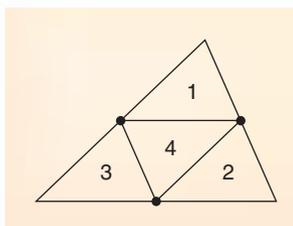
What happens when you find the midpoints of the sides of a three-sided figure and connect them to form a new figure? What if you connect the midpoints of the sides of a four-sided figure? Let's find out.

Investigate

1. A three-sided closed figure is called a **triangle**. Use paper and scissors to investigate the midpoints of the sides of a triangle.
 - a. On a piece of paper, draw a triangle with all angles acute and all sides of different lengths.
 - b. Use a compass to construct the midpoints of the three sides of your triangle. Connect the three midpoints as shown.

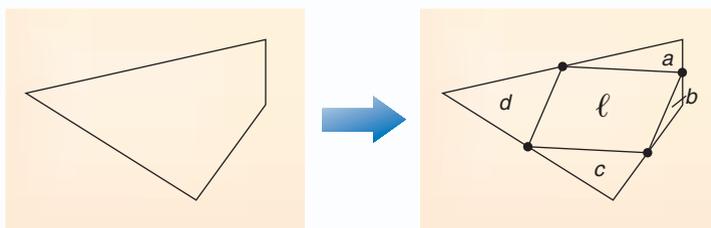


- c. Label the inner triangle 4. Label the outer triangles 1, 2, 3. Cut out each triangle. Compare the shape and size of the triangles.



- d. What appears to be true about the four triangles?

2. A four-sided closed figure is called a **quadrilateral**. Use paper and scissors to investigate the midpoints of the sides of a quadrilateral.
- On a piece of paper, draw a large quadrilateral with all sides of different lengths.
 - Use a compass to construct the midpoints of the four sides of your quadrilateral. Connect the four midpoints with line segments as shown.



- Label the inner quadrilateral ℓ and the outer triangles a , b , c , and d . Cut out each triangle. Compare the shape and size of the triangles.
- Assemble all four triangles to cover quadrilateral ℓ completely. Sketch the arrangement on quadrilateral ℓ .

Extending the Investigation

In this extension, you will investigate other triangles and quadrilaterals and their midpoints.

Use paper and scissors or geometry software to complete these investigations.

- Make a conjecture about the triangles formed when the midpoints of a triangle are connected. Test your conjecture on at least four triangles of different shapes and sizes. Include one triangle with a right angle and one with an obtuse angle.
- Make a conjecture about the inner quadrilateral and the four triangles formed by connecting the midpoints of a quadrilateral. Test your conjecture on at least four quadrilaterals of different shapes and sizes. Include one quadrilateral with at least one right angle and one quadrilateral with at least one obtuse angle.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a poster that summarizes your results.
- Design an experiment using geometry software to test your conjectures about triangles, quadrilaterals, and the midpoints of their sides.



Investigation For more information on midpoints and fractals, visit: www.geomconcepts.com

What You'll Learn

You'll learn to find the measure of an angle and the bisector of an angle.

Why It's Important

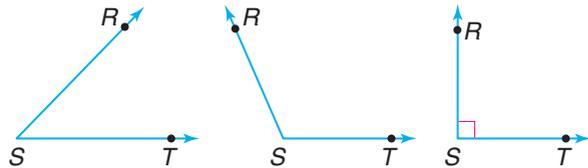
Sailing Angle measures can be used to determine sailing positions. See Exercise 24.

In the following activity, you will learn about the Angle Addition Postulate.

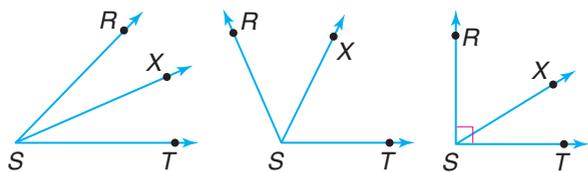
Hands-On Geometry

Materials:  straightedge  protractor

Step 1 Draw an acute, an obtuse, and a right angle. Label each angle RST .



Step 2 Draw and label a point X in the interior of each angle. Then draw \overrightarrow{SX} .



Step 3 For each angle, find $m\angle RSX$, $m\angle XST$, and $m\angle RST$.

Try These

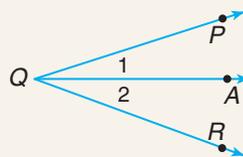
- For each angle, how does the sum of $m\angle RSX$ and $m\angle XST$ compare to $m\angle RST$?
- Make a conjecture** about the relationship between the two smaller angles and the larger angle.

The activity above leads to the following postulate.

Postulate 3-3 Angle Addition Postulate (A-A Postulate)

Words: For any angle PQR , if A is in the interior of $\angle PQR$, then $m\angle PQA + m\angle AQR = m\angle PQR$.

Model:



Symbols:

$$m\angle 1 + m\angle 2 = m\angle PQR$$

There are two equations that can be derived using Postulate 3-3.

$$m\angle 1 = m\angle PQR - m\angle 2 \quad \text{These equations are true no matter where } A \text{ is located in the interior of } \angle PQR.$$

$$m\angle 2 = m\angle PQR - m\angle 1$$

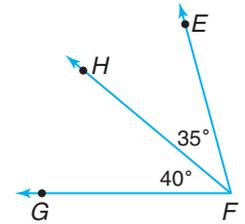
Examples

1

If $m\angle EFH = 35$ and $m\angle HFG = 40$, find $m\angle EFG$.

$$\begin{aligned} m\angle EFG &= m\angle EFH + m\angle HFG \\ &= 35 + 40 && \text{Substitution} \\ &= 75 && \text{Add.} \end{aligned}$$

So, $m\angle EFG = 75$.

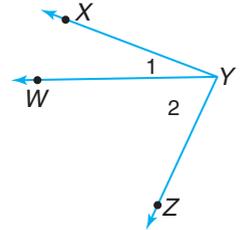


2

Find $m\angle 2$ if $m\angle XYZ = 86$ and $m\angle 1 = 22$.

$$\begin{aligned} m\angle 2 &= m\angle XYZ - m\angle 1 \\ &= 86 - 22 && \text{Substitution} \\ &= 64 && \text{Subtract.} \end{aligned}$$

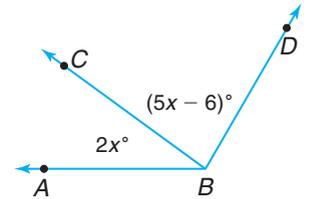
So, $m\angle 2 = 64$.



Algebra Link

3

Find $m\angle ABC$ and $m\angle CBD$ if $m\angle ABD = 120$.



$$\begin{aligned} m\angle ABC + m\angle CBD &= m\angle ABD && \text{Postulate 3-3} \\ 2x + (5x - 6) &= 120 && \text{Substitution} \\ 7x - 6 &= 120 && \text{Combine like terms.} \\ 7x - 6 + 6 &= 120 + 6 && \text{Add 6 to each side.} \\ 7x &= 126 && \text{Simplify.} \\ \frac{7x}{7} &= \frac{126}{7} && \text{Divide each side by 7.} \\ x &= 18 && \text{Simplify.} \end{aligned}$$

To find $m\angle ABC$ and $m\angle CBD$, replace x with 18 in each expression.

$$\begin{aligned} m\angle ABC &= 2x && m\angle CBD = 5x - 6 \\ &= 2(18) && = 5(18) - 6 && x = 18 \\ &= 36 && = 90 - 6 \text{ or } 84 \end{aligned}$$

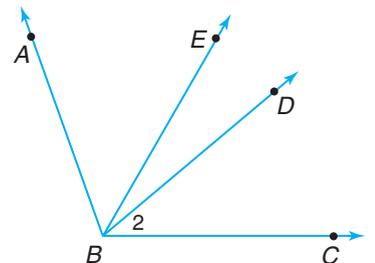
So, $m\angle ABC = 36$ and $m\angle CBD = 84$.

Check: Is the sum of the measures 120?

Algebra Review
Solving Multi-Step Equations, p. 723

Your Turn

- Find $m\angle ABC$ if $m\angle ABD = 70$ and $m\angle DBC = 43$.
- If $m\angle EBC = 55$ and $m\angle EBD = 20$, find $m\angle 2$.
- Find $m\angle ABD$ if $m\angle ABC = 110$ and $m\angle 2 = 36$.



Look Back

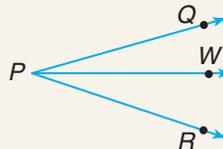
Bisector of a Segment:
Lesson 2-3

Just as every segment has a midpoint that bisects the segment, every angle has a ray that bisects the angle. This ray is called an **angle bisector**.

Definition of an Angle Bisector

Words: The bisector of an angle is the ray with its endpoint at the vertex of the angle, extending into the interior of the angle. The bisector separates the angle into two angles of equal measure.

Model:

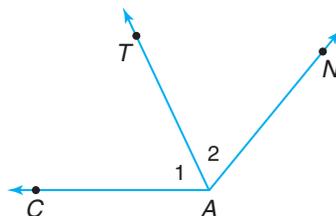


Symbols:

\overrightarrow{PW} is the bisector of $\angle P$.
 $m\angle QPW = m\angle WPR$

Example

4 If \overrightarrow{AT} bisects $\angle CAN$ and $m\angle CAN = 130$, find $m\angle 1$ and $m\angle 2$.



Since \overrightarrow{AT} bisects $\angle CAN$, $m\angle 1 = m\angle 2$.

$$m\angle 1 + m\angle 2 = m\angle CAN \quad \text{Postulate 3-3}$$

$$m\angle 1 + m\angle 2 = 130 \quad \text{Replace } m\angle CAN \text{ with } 130.$$

$$m\angle 1 + m\angle 1 = 130 \quad \text{Replace } m\angle 2 \text{ with } m\angle 1.$$

$$2(m\angle 1) = 130 \quad \text{Combine like terms.}$$

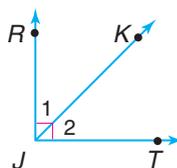
$$\frac{2(m\angle 1)}{2} = \frac{130}{2} \quad \text{Divide each side by 2.}$$

$$m\angle 1 = 65 \quad \text{Simplify.}$$

Since $m\angle 1 = m\angle 2$, $m\angle 2 = 65$.

Your Turn

d. If \overrightarrow{JK} bisects $\angle RJT$ and $\angle RJT$ is a right angle, find $m\angle 1$ and $m\angle 2$.

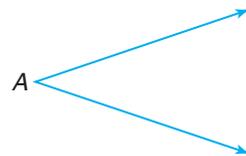


The angle bisector of a given angle can be constructed using the following procedure.

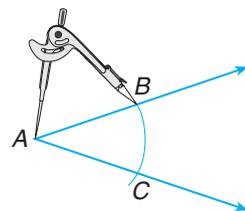
Hands-On Geometry Construction

Materials:  compass  straightedge

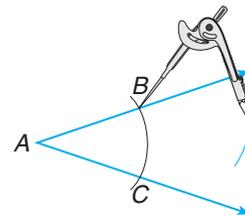
Step 1 Draw an angle like $\angle A$ on your paper.



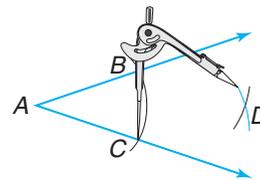
Step 2 Place a compass at point A and draw a large arc that intersects both sides of $\angle A$. Label the points of intersection B and C .



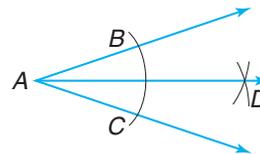
Step 3 With the compass at point B , draw an arc in the interior of $\angle A$.



Step 4 Keeping the same compass setting, place the compass at point C . Draw an arc that intersects the arc drawn in Step 3. Label the point of intersection D .

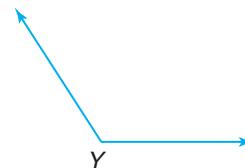


Step 5 Draw \overrightarrow{AD} .



Try These

1. How does $m\angle BAD$ compare to $m\angle DAC$?
2. Name the bisector of $\angle BAC$.
3. Draw an angle like $\angle Y$ on your paper. Then construct the angle bisector of $\angle Y$.



Check for Understanding

Communicating Mathematics

1. State the Angle Addition Postulate in your own words.
2. Draw an acute angle and label it $\angle D$. Then construct the angle bisector and label it \overline{DM} .
3. **You Decide?** Josh says that you get two obtuse angles after bisecting an angle. Brandon disagrees. Who is correct, and why?

Guided Practice

Getting Ready

Use the Angle Addition Postulate to solve each of the following.

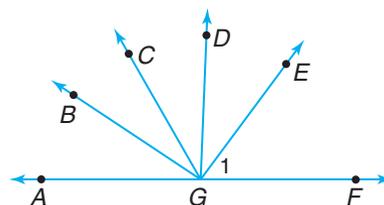
Sample: If $m\angle 1 = 36$ and $m\angle 2 = 73$, find $m\angle 1 + m\angle 2$.

Solution: $m\angle 1 + m\angle 2 = 36 + 73$ or 109

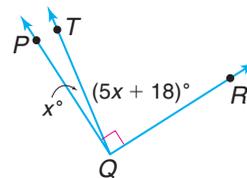
4. If $m\angle 1 + m\angle 2 = 134$ and $m\angle 2 = 90$, find $m\angle 1$.
5. If $m\angle 1 + m\angle 2 = 158$ and $m\angle 1 = m\angle 2$, find $m\angle 1$.
6. If $m\angle 1 + m\angle 2 = 5x$ and $m\angle 1 = 2x + 1$, find $m\angle 2$.

Refer to the figure at the right.

7. If $m\angle AGB = 40$ and $m\angle BGC = 24$, find $m\angle AGC$.
8. If $m\angle BGD = 52$ and $m\angle BGC = 24$, find $m\angle CGD$.
9. If \overline{GE} bisects $\angle CGF$ and $m\angle CGF = 116$, find $m\angle 1$.



10. **Algebra** Find $m\angle PQT$ and $m\angle TQR$ if $m\angle PQT = x$, $m\angle TQR = 5x + 18$, and $m\angle PQR = 90$.



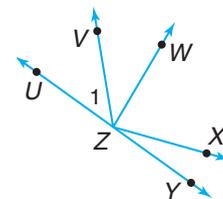
Exercises

Practice

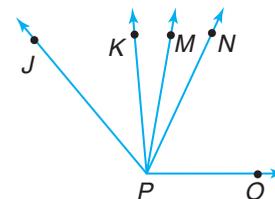
Homework Help	
For Exercises	See Examples
11–20, 23, 24	1–3
22–24	4
Extra Practice	
See page 730.	

Refer to the figures at the right.

11. If $m\angle UZW = 77$ and $m\angle VZW = 35$, find $m\angle 1$.
12. Find $m\angle VZX$ if $m\angle VZW = 35$ and $m\angle WZX = 78$.
13. If $m\angle WZX = 78$ and $m\angle XZY = 25$, find $m\angle WZY$.
14. If $m\angle UZW = 76$ and \overline{ZV} bisects $\angle UZW$, find $m\angle UZV$.
15. Find $m\angle KPM$ if \overline{PM} bisects $\angle KPN$ and $m\angle KPN = 30$.
16. If $m\angle JPM = 48$ and $m\angle KPM = 15$, find $m\angle JPK$.
17. If $m\angle JPO = 126$ and \overline{PN} bisects $\angle JPO$, find $m\angle NPO$.



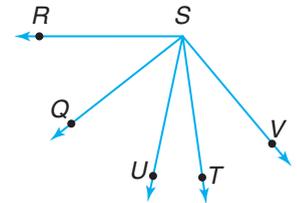
Exercises 11–14



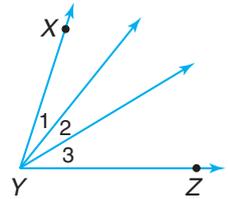
Exercises 15–17

Refer to the figure at the right.

18. If $m\angle QSU = 38$ and $m\angle UST = 18$, find $m\angle QST$.
19. If RST is a right angle and $m\angle UST = 18$, find $m\angle RSU$.
20. Find $m\angle QSV$ if $m\angle TSU = 18$, $m\angle TSV = 24$, and $m\angle QSU = 38$.
21. If an acute angle is bisected, what type of angles are formed?
22. What type of angles are formed when an obtuse angle is bisected?
23. **Algebra** If $m\angle 1 = 21$, $m\angle 2 = 5x$, $m\angle 3 = 7x + 3$, and $m\angle XYZ = 18x$, find x .



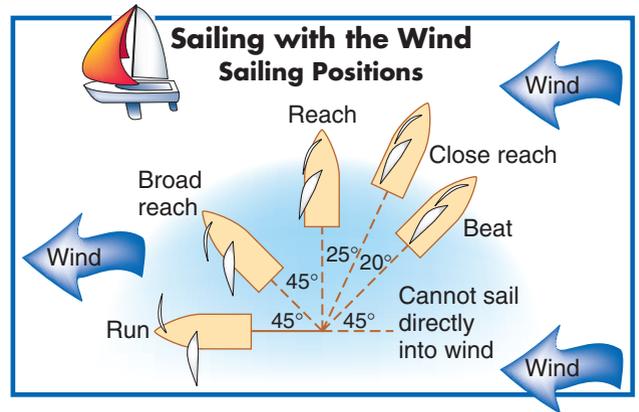
Exercises 18–20



Exercise 23

Applications and Problem Solving

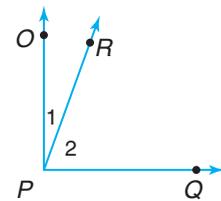
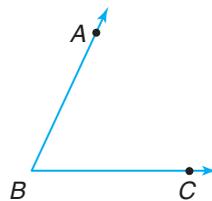
24. **Sailing** The graph shows sailing positions. Suppose a sailboat is in the run position. How many degrees must the sailboat be turned so that it is in the close reach position?



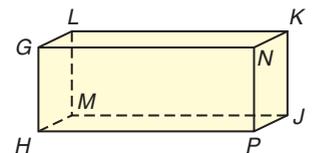
Source: Coast Guard

25. **Critical Thinking** What definition involving segments and points is similar to the Angle Addition Postulate?
26. Use a protractor to measure $\angle ABC$. (Lesson 3–2)
27. Name all angles having P as their vertex. (Lesson 3–1)

Mixed Review



28. Points A , B , and C are collinear. If $AB = 12$, $BC = 37$, and $AC = 25$, determine which point is between the other two. (Lesson 2–2)



Exercise 29

Standardized Test Practice

- (A) (B) (C) (D)

29. **Short Response** Name the intersection of plane GNK and plane PJK . (Lesson 1–3)
30. **Multiple Choice** A stock rose in price from \$2.50 to \$2.75 a share. Find the percent of increase in the price of the stock. (Percent Review)

- (A) 10% (B) 9% (C) 0.1% (D) 0.09%



Adjacent Angles and Linear Pairs of Angles

What You'll Learn

You'll learn to identify and use adjacent angles and linear pairs of angles.

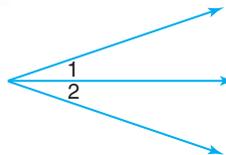
Why It's Important

Architecture

Adjacent angles and linear pairs are used in architecture. See Example 6.

When you bisect an angle, you create two angles of equal measure. The two angles are called **adjacent angles**.

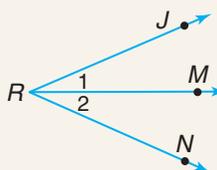
Angles 1 and 2 are examples of adjacent angles. They share a common ray.



Definition of Adjacent Angles

Words: Adjacent angles are angles that share a common side and have the same vertex, but have no interior points in common.

Model:

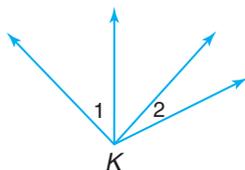


$\angle 1$ and $\angle 2$ are adjacent with the same vertex R and common side \overrightarrow{RM} .

Examples

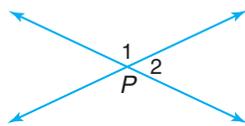
Determine whether $\angle 1$ and $\angle 2$ are adjacent angles.

1



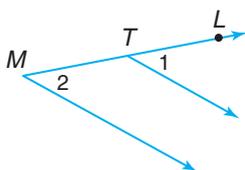
No. They have the same vertex K , but no common side.

2



Yes. They have the same vertex P and a common side with no interior points in common.

3

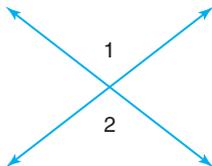


No. They do not have a common side or a common vertex.

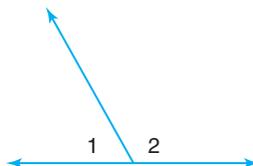
The side of $\angle 1$ is \overline{TL} .
The side of $\angle 2$ is \overline{ML} .

Your Turn

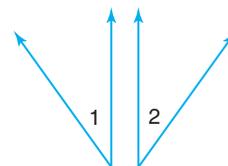
a.



b.



c.

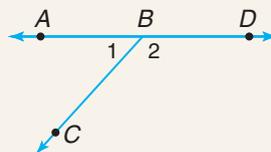


In Example 2, the noncommon sides of the adjacent angles form a straight line. These angles are called a **linear pair**.

Definition of Linear Pair

Words: Two angles form a linear pair if and only if they are adjacent and their noncommon sides are opposite rays.

Model:



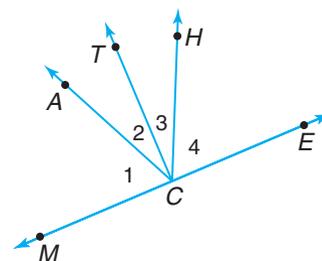
$\angle 1$ and $\angle 2$ are a linear pair.

Examples

In the figure, \overrightarrow{CM} and \overrightarrow{CE} are opposite rays.

- 4** Name the angle that forms a linear pair with $\angle 1$.

$\angle ACE$ and $\angle 1$ have a common side \overrightarrow{CA} , the same vertex C , and opposite rays \overrightarrow{CM} and \overrightarrow{CE} . So, $\angle ACE$ forms a linear pair with $\angle 1$.



- 5** Do $\angle 3$ and $\angle TCM$ form a linear pair? Justify your answer.

No, their noncommon sides are not opposite rays.

Your Turn

- d. Name the angle that forms a linear pair with $\angle MCH$.
e. Tell whether $\angle TCE$ and $\angle TCM$ form a linear pair. Justify your answer.



Example

Architecture Link

- 6** The John Hancock Center in Chicago, Illinois, contains many types of angles. Describe the highlighted angles.

The angles are adjacent, and they form a linear pair.



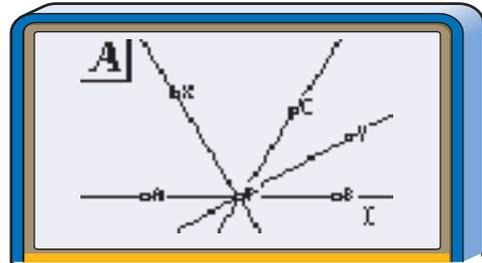
You can use a TI-83/84 Plus graphing calculator to investigate how the angle bisectors for a linear pair are related.



Graphing Calculator Exploration

**Graphing
Calculator Tutorial**
See pp. 782–785.

Step 1 Construct a line that passes through a point P . Use the Point on Object tool on the **F2** menu to mark points A and B on opposite sides of point P . Use the Line tool on **F2** to construct line PC .



Step 2 Use the Angle Bisector tool on the **F3** menu to construct the lines that bisect $\angle APC$ and $\angle BPC$. Label points X and Y on these lines.

Step 3 Use the Measure Angle tool on the **F5** menu to display the measure of $\angle XPY$.

Try These

1. What value does the calculator display for $\angle XPY$?
2. Use the Angle tool to display the measures of $\angle XPC$ and $\angle CPY$. What is the sum of these measures?
3. Drag point C . Describe what happens to the angle measures.
4. **Make a conjecture** about the relationship between bisectors of a linear pair.

Check for Understanding

Communicating Mathematics

1. Draw and label two adjacent angles for which the sum of their measures is 90.
2. **Writing Math** Write a sentence explaining why you think the term *linear pair* is used to describe angles such as $\angle 1$ and $\angle ACE$ in Example 4.

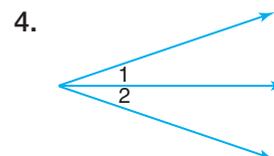
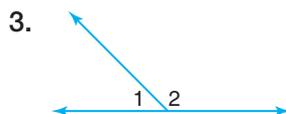
Vocabulary

adjacent angles
linear pair

Guided Practice

Examples 1–5

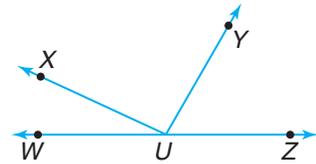
Use the terms *adjacent angles*, *linear pair*, or *neither* to describe angles 1 and 2 in as many ways as possible.



Examples 4 & 5

In the figure at the right, \overrightarrow{UZ} and \overrightarrow{UW} are opposite rays.

- Name two angles that are adjacent to $\angle WUX$.
- Which angle forms a linear pair with $\angle YUZ$?



Exercises 5–6

Example 6

- Science** Describe the illustrated angles in the spider web.

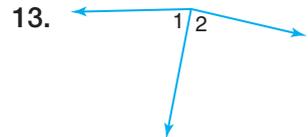
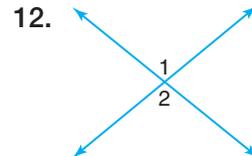
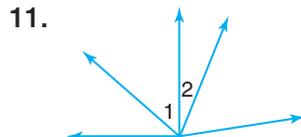
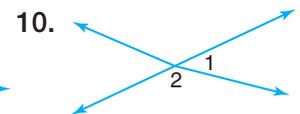
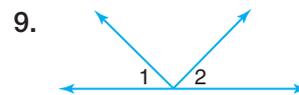
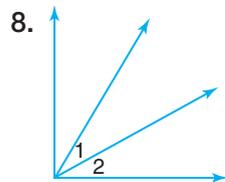


Exercises

Practice

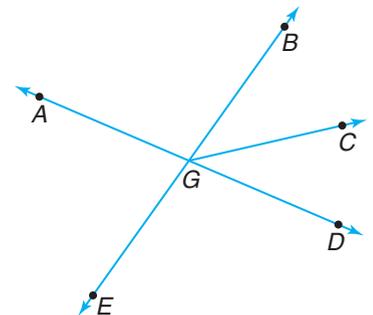
Use the terms *adjacent angles*, *linear pair*, or *neither* to describe angles 1 and 2 in as many ways as possible.

Homework Help	
For Exercises	See Examples
8–13	1–5
14–15	4–5
16, 18	1–2
17, 19, 21	4–5
20	1–2, 4–5
Extra Practice	
See page 730.	



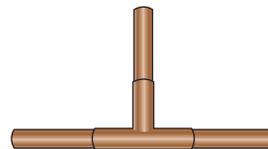
In the figure, \overrightarrow{GA} and \overrightarrow{GD} , and \overrightarrow{GB} and \overrightarrow{GE} are opposite rays.

- Which angle forms a linear pair with $\angle DGC$?
- Do $\angle BGC$ and $\angle EGD$ form a linear pair? Justify your answer.
- Name two angles that are adjacent to $\angle CGD$.
- Name two angles that form a linear pair with $\angle BGD$.
- Name three angles that are adjacent to $\angle AGB$.
- Do $\angle CGE$ and $\angle CGB$ form a linear pair? Justify your answer.



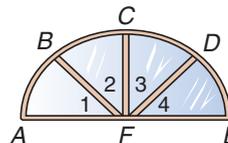
Applications and Problem Solving

20. **Plumbing** A plumber uses a T-fitting to join three pieces of copper piping as shown. Describe the type of angles formed by the three pieces of pipe and the fitting.



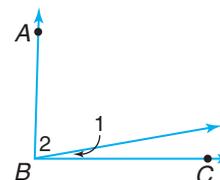
21. **Flags** Sailors use international code flags to communicate at sea. The flag shown represents the letter z. How many linear pairs are in the design of the flag?

22. **Critical Thinking** How many pairs of adjacent angles are in the design of the window shown at the right? Name them.



Mixed Review

23. $\angle ABC$ is shown at the right. Find $m\angle 2$ if $m\angle ABC = 87$ and $m\angle 1 = 19$. (Lesson 3-3)
24. Use a protractor to draw an 85° angle. Then classify the angle. (Lesson 3-2)
25. Draw $\angle ABC$ that has point T in the exterior of the angle. (Lesson 3-1)



Exercise 23

Standardized Test Practice



26. **Grid In** Find the measure of the distance between B and C . (Lesson 2-1)
27. **Multiple Choice** Find the area of a rectangle with length 16 feet and width 9 feet. (Lesson 1-6)
- (A) 50 ft^2 (B) 71 ft^2 (C) 86 ft^2 (D) 144 ft^2



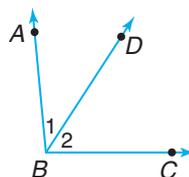
Quiz 1

Lessons 3-1 through 3-4

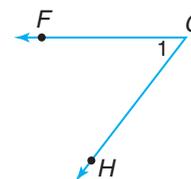
1. Name the angle in four ways. Then identify its vertex and its sides. (Lesson 3-1)

Use a protractor to draw an angle for each measurement. Then classify each angle as *acute*, *obtuse*, or *right*. (Lesson 3-2)

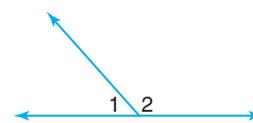
2. 97° 3. 35°
4. **Algebra** If $m\angle 1 = 3x$, $m\angle 2 = 5x$, and $m\angle ABC = 96$, find x . (Lesson 3-3)
5. Use the terms *adjacent angles*, *linear pair*, or *neither* to describe the pair of angles in as many ways as possible. (Lesson 3-4)



Exercise 4



Exercise 1



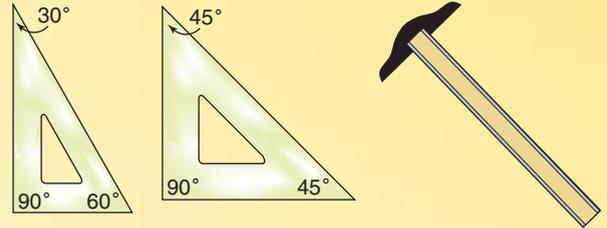
Exercise 5



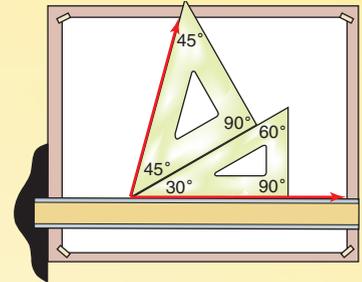
Drafter

Do you like to draw? Does a career that involves drawing interest you? If so, you may enjoy a career as a drafter. Drafters prepare drawings and plans that are used to build everything from ordinary houses to space stations.

When preparing a drawing, drafters may use *drafting triangles* along with a *T-square* to draw various angles.



The diagram at the right shows how a drafter would use these tools to draw a 75° angle.



Draw a diagram that shows how a drafter would use drafting triangles and a T-square to draw each angle measure.

1. 105°
2. 150°
3. 135°

FAST FACTS About Drafters

Working Conditions

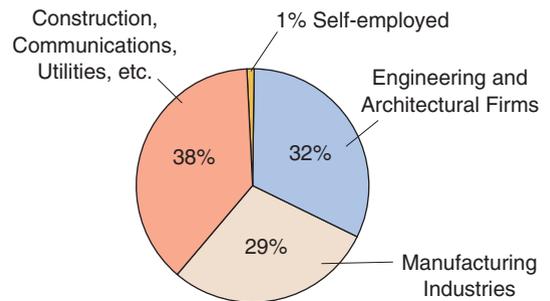
- usually work in a comfortable office
- sit at drafting tables or computer terminals
- may be susceptible to eyestrain, hand and wrist problems, and back discomfort

Education

- high school math, science, computer, design, and drafting courses
- postsecondary training in drafting at a technical school or community college

Employment

Where Drafters Are Employed



Career Data For the latest information on careers in drafting, visit:

www.geomconcepts.com

3-5

Complementary and Supplementary Angles

What You'll Learn

You'll learn to identify and use complementary and supplementary angles.

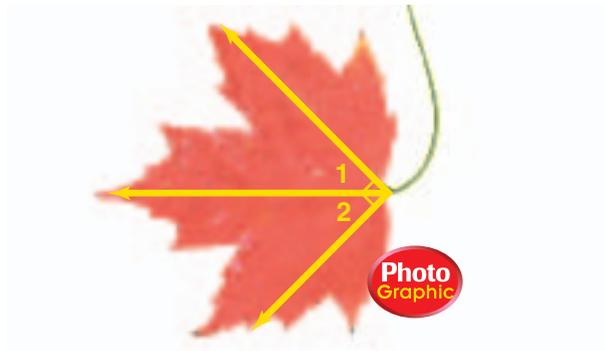
Why It's Important

Carpentry

Carpenters use angles when cutting lumber. See Exercise 30.

Angles are all around us, even in nature. The veins of a maple leaf show a pair of **complementary angles**.

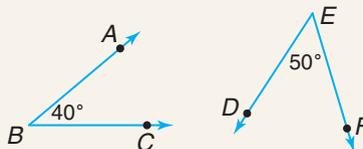
$$m\angle 1 + m\angle 2 = 90$$



Definition of Complementary Angles

Words: Two angles are complementary if and only if the sum of their degree measures is 90.

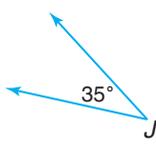
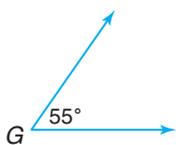
Model:



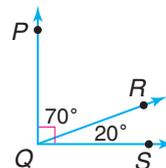
Symbols: $m\angle ABC + m\angle DEF = 90$

If two angles are complementary, each angle is a *complement* of the other. For example, $\angle ABC$ is the complement of $\angle DEF$ and $\angle DEF$ is the complement of $\angle ABC$.

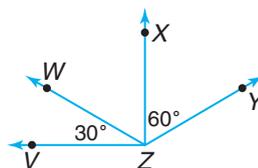
Complementary angles do not need to have a common side or even the same vertex. Some examples of complementary angles are shown.



$$m\angle G + m\angle J = 90$$



$$m\angle PQR + m\angle RQS = 90$$

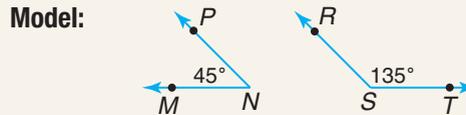


$$m\angle VZW + m\angle XZY = 90$$

If the sum of the measures of two angles is 180, they form a special pair of angles called **supplementary angles**.

Definition of Supplementary Angles

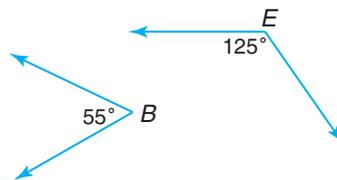
Words: Two angles are supplementary if and only if the sum of their degree measures is 180.



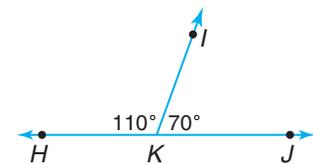
Symbols: $m\angle MNP + m\angle RST = 180$

If two angles are supplementary, each angle is a *supplement* of the other. For example, $\angle MNP$ is the supplement of $\angle RST$ and $\angle RST$ is the supplement of $\angle MNP$.

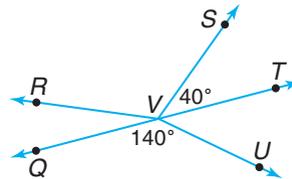
Like complementary angles, supplementary angles do not need to have a common side or the same vertex. The figures below are examples of supplementary angles.



$$m\angle B + m\angle E = 180$$



$$m\angle HKI + m\angle IKJ = 180$$



$$m\angle QVU + m\angle SVT = 180$$

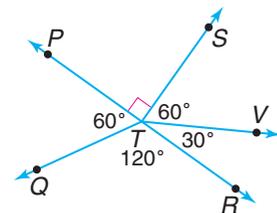
Example

1

Name a pair of adjacent complementary angles.

$m\angle STV + m\angle VTR = 90$, and they have the same vertex T and common side \overline{TV} with no overlapping interiors.

So, $\angle STV$ and $\angle VTR$ are adjacent complementary angles.



Your Turn

a. Name a pair of nonadjacent complementary angles.



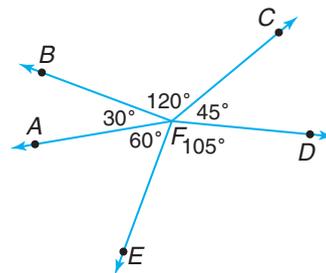
Examples

2

Name a pair of nonadjacent supplementary angles.

$m\angle BFC + m\angle AFE = 180$, and they have the same vertex F , but no common side.

So, $\angle BFC$ and $\angle AFE$ are nonadjacent supplementary angles.



3

Find the measure of an angle that is supplementary to $\angle CFD$.

Let x = the measure of the angle that is supplementary to $\angle CFD$.

$$m\angle CFD + x = 180 \quad \text{Supplementary angles have a sum of 180.}$$

$$45 + x = 180 \quad m\angle CFD = 45$$

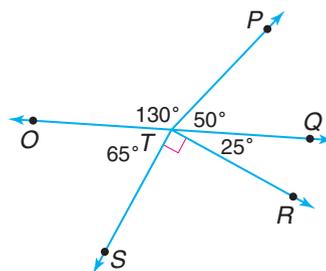
$$45 + x - 45 = 180 - 45 \quad \text{Subtract 45 from each side.}$$

$$x = 135 \quad \text{Simplify.}$$

The measure of an angle that is supplementary to $\angle CFD$ is 135.

Your Turn

- Name a pair of adjacent supplementary angles.
- Find the measure of the angle that is complementary to $\angle QTR$.



Example

Algebra Link

4

Angles A and B are complementary. If $m\angle A = x$ and $m\angle B = 5x$, find x . Then find $m\angle A$ and $m\angle B$.

$$m\angle A + m\angle B = 90 \quad \text{Definition of Complementary Angles}$$

$$x + 5x = 90 \quad \text{Substitution}$$

$$6x = 90 \quad \text{Combine like terms.}$$

$$\frac{6x}{6} = \frac{90}{6} \quad \text{Divide each side by 6.}$$

$$x = 15 \quad \text{Simplify.}$$

Substitute the value of x into each expression.

$$\begin{aligned} m\angle A &= x & x &= 15 \\ &= 15 \end{aligned}$$

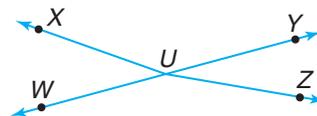
$$\begin{aligned} m\angle B &= 5x & x &= 15 \\ &= 5(15) \text{ or } 75 \end{aligned}$$

So, $x = 15$, $m\angle A = 15$, and $m\angle B = 75$.

Algebra Review

Solving One-Step Equations, p. 722

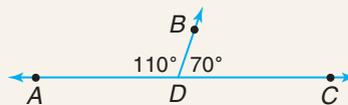
In the figure, $\angle WUX$ and $\angle XUY$ form a linear pair. Postulate 3–4 states that if two angles form a linear pair, the angles are supplementary.



Postulate 3–4 Supplement Postulate

Words: If two angles form a linear pair, then they are supplementary.

Model:



Symbols: $m\angle ADB + m\angle BDC = 180$

Example

5 If $m\angle 1 = 57$ and $\angle 1$ and $\angle 2$ form a linear pair, find $m\angle 2$.

If $\angle 1$ and $\angle 2$ form a linear pair, then they are supplementary.

$$m\angle 1 + m\angle 2 = 180$$

$$57 + m\angle 2 = 180$$

$$57 + m\angle 2 - 57 = 180 - 57$$

$$m\angle 2 = 123$$

So, $m\angle 2 = 123$.

*Supplement Postulate and
Definition of Supplementary Angles*

Replace $m\angle 1$ with 57.

Subtract 57 from each side.

Simplify.

Your Turn

d. If $m\angle 2 = 39$ and $\angle 1$ and $\angle 2$ form a linear pair, find $m\angle 1$.

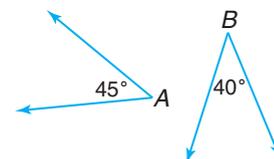
Check for Understanding

Communicating Mathematics

1. Draw a pair of adjacent angles that are complementary and have the same measure. What is the measure of each angle?
2. Explain why an obtuse angle cannot have a complement.
3. Tell whether the angles shown are complementary, supplementary, or neither.

Vocabulary

complementary angles
supplementary angles



Exercise 3

Guided Practice

Getting Ready

Determine the measures of the complement and supplement of each angle.

Sample: 62

Solution: $90 - 62 = 28$; $180 - 62 = 118$

4. 38

5. 42

6. 79

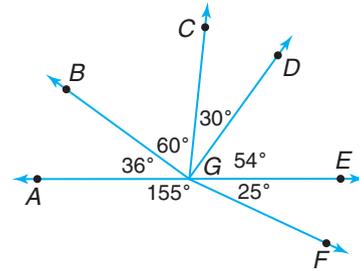
7. 55



Examples 1–3

Refer to the figure at the right.

8. Name a pair of adjacent supplementary angles.
9. Name a pair of nonadjacent complementary angles.
10. Find the measure of an angle that is supplementary to $\angle DGE$.



Example 4

11. **Algebra** Angles G and H are supplementary. If $m\angle G = x + 3$ and $m\angle H = 2x$, find the measure of each angle.

Example 5

12. Angles XYZ and WYX form a linear pair. If $m\angle WYX = 56$, what is $m\angle XYZ$?

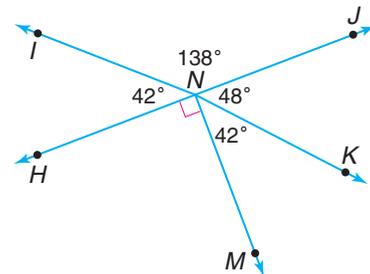
Exercises

Practice

Homework Help	
For Exercises	See Examples
13, 20	1
14, 19, 25–27, 30	3
15, 17	1, 2
16	1, 3
18, 21	2
22–24, 29	5
28	4
Extra Practice	
See page 731.	

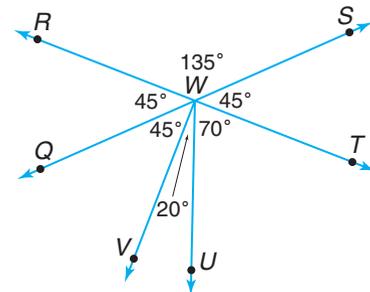
Refer to the figures at the right.

13. Name two pairs of complementary angles.
14. Find the measure of an angle that is supplementary to $\angle HNM$.
15. Name a pair of adjacent supplementary angles.
16. Find the measure of an angle that is complementary to $\angle VWU$.
17. Name a pair of nonadjacent complementary angles.
18. Name two pairs of supplementary angles.

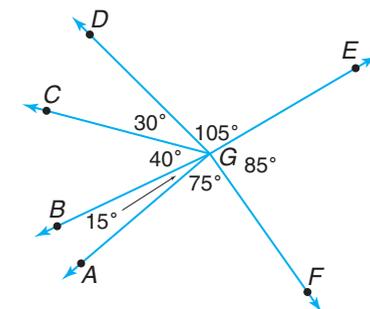


Exercises 13–15

19. Find the measure of an angle that is supplementary to $\angle EGF$.
20. Name a pair of adjacent complementary angles.
21. Name a pair of nonadjacent supplementary angles.



Exercises 16–18



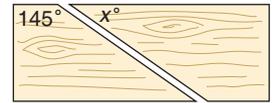
Exercises 19–21

22. If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 96$, find $m\angle 1$.
23. Find $m\angle 2$ if $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 1 = 127$.
24. Angles ABC and DEF form a linear pair. If $m\angle DEF = 49$, what is $m\angle ABC$?
25. Can two acute angles be supplementary? Explain.
26. What kind of angle is the supplement of an acute angle?
27. What kind of angle is the supplement of a right angle?

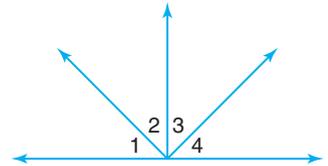
Applications and Problem Solving

28. **Algebra** Angles 1 and 2 are complementary. If $m\angle 1 = 3x + 2$ and $m\angle 2 = 2x + 3$, find the measure of each angle.
29. **Algebra** Angles J and K are supplementary. Find the measures of the two angles if $m\angle J = x$ and $m\angle K = x - 60$.

30. **Carpentry** A carpenter uses a circular saw to cut a piece of lumber at a 145° angle. What is the measure of the other angle formed by the cut?



31. **Critical Thinking** Angles 1 and 2 are complementary, and $\angle 1$ and $\angle 3$ are also complementary. Describe the relationship that exists between $\angle 2$ and $\angle 3$.

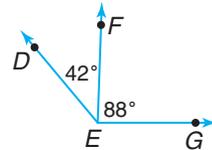


Mixed Review

32. Use the terms *adjacent angles*, *linear pair*, or *neither* to describe the pair of angles in as many ways as possible. (Lesson 3–4)



33. If $m\angle DEF = 42$ and $m\angle FEG = 88$, find $m\angle DEG$. (Lesson 3–3)



34. **Technology** A videotape cartridge has a length of 18.7 centimeters and a width of 10.3 centimeters. What is the perimeter of the cartridge? (Lesson 1–6)

Write the converse of each statement. (Lesson 1–4)

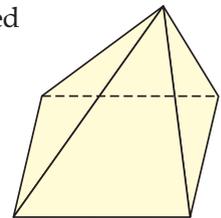
35. If it snows, then he will go skiing.
36. If she has 10 dollars, then she will go to the movies.

Standardized Test Practice

A B C D

37. **Multiple Choice** How many planes are represented in the figure? (Lesson 1–3)

- A 4
B 5
C 6
D 7



What You'll Learn

You'll learn to identify and use congruent and vertical angles.

Why It's Important

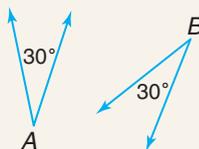
Quilting Congruent and vertical angles are often found in quilt patterns. See Exercise 22.

Recall that congruent segments have the same measure. **Congruent angles** also have the same measure.

Definition of Congruent Angles

Words: Two angles are congruent if and only if they have the same degree measure.

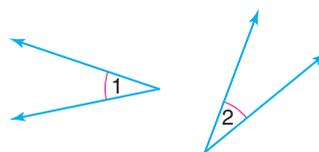
Model:



Symbols:

$\angle A \cong \angle B$ if and only if $m\angle A = m\angle B$.

If and only if means that if $m\angle 1 = m\angle 2$, then $\angle 1 \cong \angle 2$ and if $\angle 1 \cong \angle 2$, then $m\angle 1 = m\angle 2$.

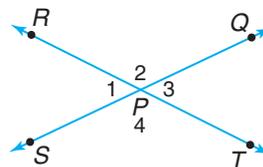


The arcs are used to show congruent angles.

Reading Geometry

The notation $\angle A \cong \angle B$ is read as *angle A is congruent to angle B*.

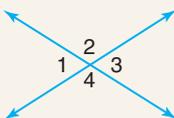
In the figure at the right, \overleftrightarrow{SQ} and \overleftrightarrow{RT} intersect. When two lines intersect, four angles are formed. There are two pairs of nonadjacent angles. These pairs are called **vertical angles**.



Definition of Vertical Angles

Words: Two angles are vertical if and only if they are two nonadjacent angles formed by a pair of intersecting lines.

Model:

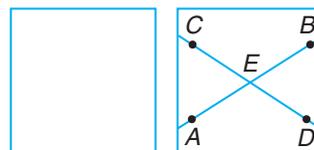


Vertical angles:

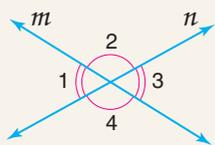
$\angle 1$ and $\angle 3$

$\angle 2$ and $\angle 4$

Vertical angles are related in a special way. Suppose you cut out and fold a piece of patty paper twice as shown. Compare the angles formed. What can you say about the measures of the vertical angles?

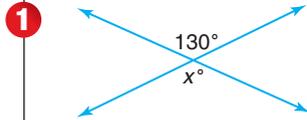


These results are stated in the Vertical Angle Theorem.

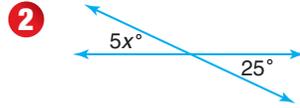
Theorem 3-1 Vertical Angle Theorem	Words: Vertical angles are congruent.	Symbols: $\angle 1 \cong \angle 3$ $\angle 2 \cong \angle 4$
	Model: 	

Examples

Find the value of x in each figure.



The angles are vertical angles.
So, the value of x is 130.

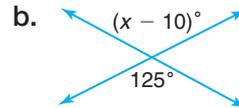
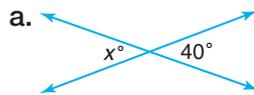


Since the angles are vertical angles, they are congruent.

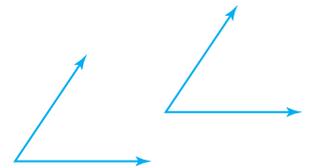
$$5x = 25 \quad \div 5 \quad x = 5$$

So, the value of x is 5.

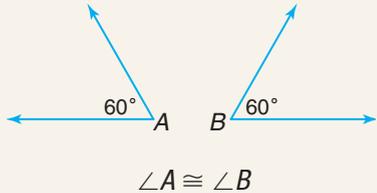
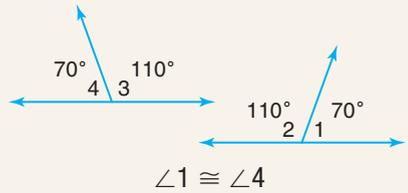
Your Turn



Suppose two angles are congruent. What do you think is true about their complements? What is true about their supplements? Draw several examples and make a conjecture.



These results are stated in the following theorems.

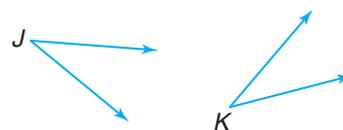
Theorem	Words	Models
3-2	If two angles are congruent, then their complements are congruent. <i>The measure of angles complementary to $\angle A$ and $\angle B$ is 30.</i>	
3-3	If two angles are congruent, then their supplements are congruent. <i>The measure of angles supplementary to $\angle 1$ and $\angle 4$ is 110.</i>	



Theorem	Words	Models
3-4	<p>If two angles are complementary to the same angle, then they are congruent.</p> <p>$\angle 3$ is complementary to $\angle 4$. $\angle 5$ is complementary to $\angle 4$. $\angle 3 \cong \angle 5$</p>	
3-5	<p>If two angles are supplementary to the same angle, then they are congruent.</p> <p>$\angle 1$ is supplementary to $\angle 2$. $\angle 3$ is supplementary to $\angle 2$. $\angle 1 \cong \angle 3$</p>	

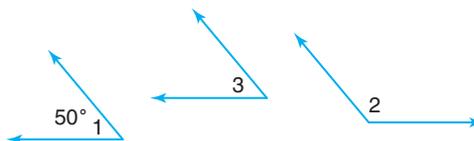
Examples

- 3 Suppose $\angle J \cong \angle K$ and $m\angle K = 35$. Find the measure of an angle that is complementary to $\angle J$.



Since $\angle J \cong \angle K$, their complements are congruent. The complement of $\angle K$ is $90 - 35$ or 55 . So, the measure of an angle that is complementary to $\angle J$ is 55 .

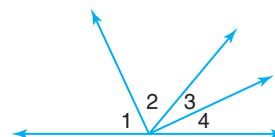
- 4 In the figure, $\angle 1$ is supplementary to $\angle 2$, $\angle 3$ is supplementary to $\angle 2$, and $m\angle 1 = 50$. Find $m\angle 2$ and $m\angle 3$.



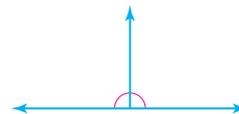
$\angle 1$ and $\angle 2$ are supplementary. So, $m\angle 2 = 180 - 50$ or 130 .
 $\angle 2$ and $\angle 3$ are supplementary. So, $m\angle 3 = 180 - 130$ or 50 .

Your Turn

- c. Suppose $\angle A \cong \angle B$ and $m\angle A = 52$. Find the measure of an angle that is supplementary to $\angle B$.
- d. If $\angle 1$ is complementary to $\angle 3$, $\angle 2$ is complementary to $\angle 3$, and $m\angle 3 = 25$, what are $m\angle 1$ and $m\angle 2$?



Suppose you draw two angles that are congruent and supplementary as shown at the right. What is true about the angles?



Theorem	Words	Models
3-6	If two angles are congruent and supplementary, then each is a right angle. <i>$\angle 1$ is supplementary to $\angle 2$.</i> <i>$m\angle 1$ and $m\angle 2 = 90$.</i>	
3-7	All right angles are congruent.	

Check for Understanding

Communicating Mathematics

- Construct a pair of congruent angles.
- Explain the difference between $m\angle F = m\angle G$ and $\angle F \cong \angle G$.
- You Decide?** Keisha says that if $m\angle A = 45$ and $m\angle B = 45$, then it is correct to write $m\angle A \cong m\angle B$. Roberta disagrees. She says that it is correct to write $m\angle A = m\angle B$. Who is correct? Explain your reasoning.

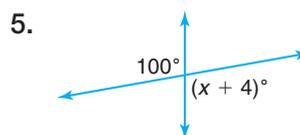
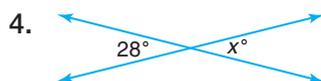
Vocabulary

congruent angles
vertical angles

Guided Practice

Examples 1 & 2

Find the value of x in each figure.

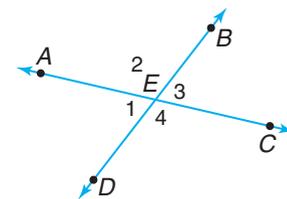


Example 3

6. If $m\angle BEC = 68$, what is the measure of an angle that is complementary to $\angle AED$?

Example 4

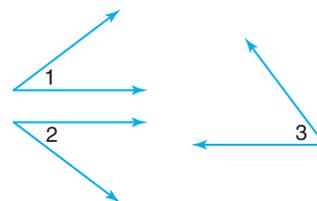
7. If $\angle 1$ is supplementary to $\angle 4$, $\angle 3$ is supplementary to $\angle 4$, and $m\angle 1 = 64$, what are $m\angle 3$ and $m\angle 4$?



Exercises 6-7

Example 4

8. **Algebra** $\angle 1$ is complementary to $\angle 3$, and $\angle 2$ is complementary to $\angle 3$. If $m\angle 2 = 2x + 9$ and $m\angle 3 = 4x - 3$, find $m\angle 1$ and $m\angle 3$.

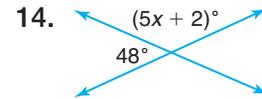
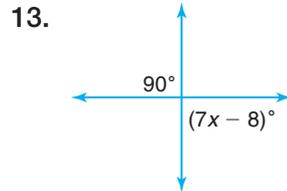
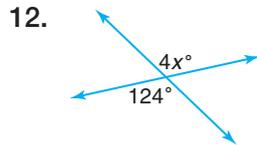
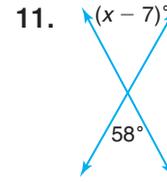
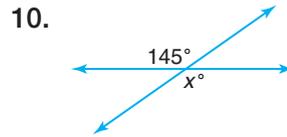
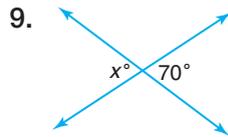


Exercises

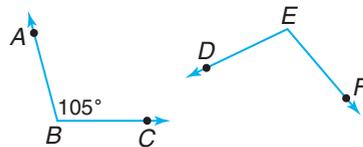
Practice

Homework Help	
For Exercises	See Examples
9–14, 21	1–2
16, 17, 19, 20	3–4
18	4
Extra Practice	
See page 731.	

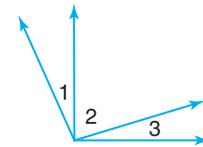
Find the value of x in each figure.



15. What is the measure of an angle that is supplementary to $\angle DEF$ if $\angle ABC \cong \angle DEF$?

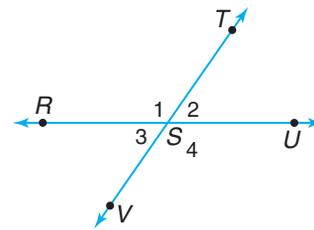


16. If $\angle 1$ is complementary to $\angle 2$, $\angle 3$ is complementary to $\angle 2$, and $m\angle 1 = 28$, what are $m\angle 2$ and $m\angle 3$?



17. If $\angle 2 \cong \angle 3$ and $m\angle 2 = 55$, find the measure of an angle that is supplementary to $\angle 3$.

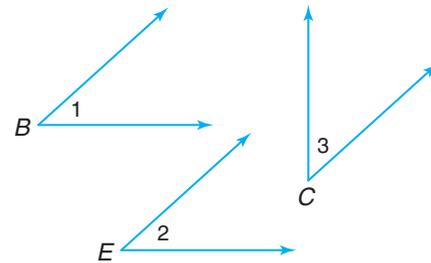
18. If $\angle RST$ is supplementary to $\angle TSU$, $\angle VSU$ is supplementary to $\angle TSU$, and $m\angle TSU = 62$, find $m\angle RST$ and $m\angle VSU$.



Exercises 17–18

19. Find the measure of an angle that is complementary to $\angle B$ if $\angle B \cong \angle E$ and $m\angle E = 43$.

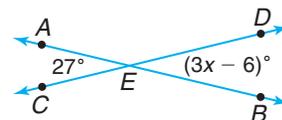
20. If $\angle 1$ is complementary to $\angle 3$, $\angle 2$ is complementary to $\angle 3$, and $m\angle 1 = 42$, what are $m\angle 2$ and $m\angle 3$?



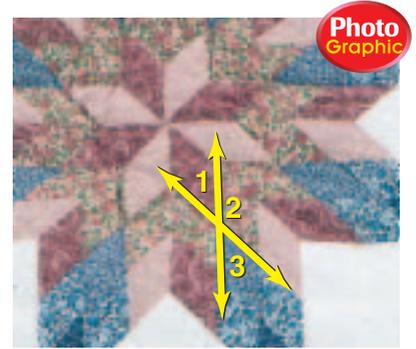
Exercises 19–20

Applications and Problem Solving

21. **Algebra** What is the value of x if $\angle AEC$ and $\angle DEB$ are vertical angles and $m\angle AEC = 27$ and $m\angle DEB = 3x - 6$?



22. **Quilting** The quilt pattern shown is called the *Lone Star*. If $\angle 1$ is supplementary to $\angle 2$, $\angle 3$ is supplementary to $\angle 2$, and $m\angle 1 = 45$, what are $m\angle 2$ and $m\angle 3$?

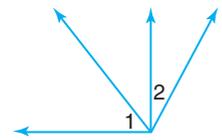


23. **Critical Thinking** Show that Theorem 3–6 is true.

Mixed Review

24. **Algebra** Angles G and H are supplementary. If $m\angle G = x$ and $m\angle H = 4x$, what are $m\angle G$ and $m\angle H$? (Lesson 3–5)

25. Use the terms *adjacent angles*, *linear pair*, or *neither* to describe the relationship between $\angle 1$ and $\angle 2$. (Lesson 3–4)



26. Draw an angle having a measure of 35° . (Lesson 3–2)

Standardized Test Practice

A B C D

27. **Short Response** Write an irrational number between 2 and 3 that has ten digits to the right of the decimal point. (Lesson 2–1)

28. **Multiple Choice** Tamika is planning to install vinyl floor tiles in her basement. Her basement measures 20 feet by 16 feet. How many boxes of vinyl floor tile should she buy if one box covers an area of 20 square feet? (Lesson 1–6)

A 4 B 12 C 16 D 20

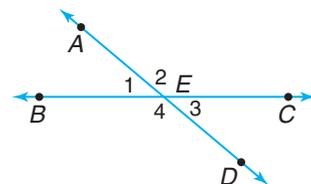
Quiz 2

Lessons 3–5 and 3–6

1. Draw a pair of adjacent complementary angles. (Lesson 3–5)
2. If $m\angle 1 = 62$ and $\angle 1$ and $\angle 2$ form a linear pair, find $m\angle 2$. (Lesson 3–5)
3. Angles J and K are vertical angles. If $m\angle J = 37$, what is $m\angle K$? (Lesson 3–6)

Refer to the figure at the right. (Lesson 3–6)

4. If $m\angle AEB = 35$, what is the measure of an angle complementary to $\angle CED$?
5. If $m\angle 2 = 135$, find $m\angle 3$ and $m\angle 4$.



What You'll Learn

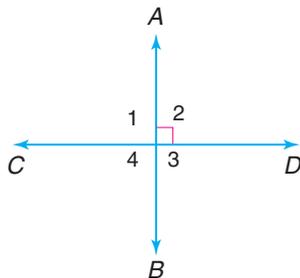
You'll learn to identify, use properties of, and construct perpendicular lines and segments.

Why It's Important Engineering

Site planners use perpendicular lines when planning a construction site. See Exercise 26.

Lines that intersect at an angle of 90 degrees are **perpendicular lines**.

In the figure below, lines \overleftrightarrow{AB} and \overleftrightarrow{CD} are perpendicular.



The square symbol where the two lines intersect indicates that the two lines are perpendicular. In the figure, four right angles are formed at the point of intersection.

Also, notice that the four pairs of adjacent angles $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, and $\angle 4$ and $\angle 1$ are supplementary. These adjacent angles also form linear pairs because the nonadjacent sides in each pair are opposite rays.

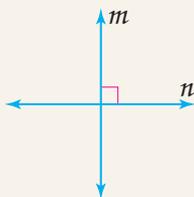
Reading Geometry

Read the symbol \perp as *is perpendicular to*. For example, read $m \perp n$ as *line m is perpendicular to line n*.

Definition of Perpendicular Lines

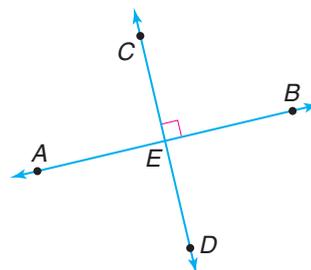
Words: Perpendicular lines are lines that intersect to form a right angle.

Model:

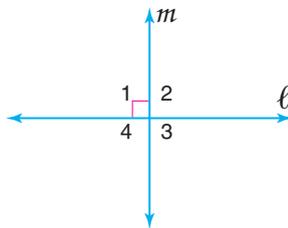


Symbols: $m \perp n$

Because rays and segments are parts of lines, these too can be perpendicular. For rays or segments to be perpendicular, they must be part of perpendicular lines and they must intersect. In the figure at the right, $\overrightarrow{EC} \perp \overrightarrow{EA}$ and $\overline{CD} \perp \overline{AB}$.

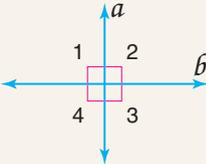
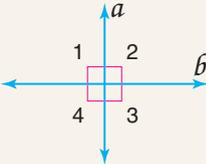
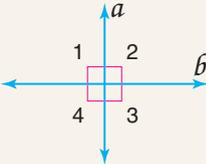


In the figure below, $\ell \perp m$. The following statements are true.



- | | |
|--|--|
| 1. $\angle 1$ is a right angle. | <i>Definition of Perpendicular Lines</i> |
| 2. $\angle 1 \cong \angle 3$ | <i>Vertical angles are congruent.</i> |
| 3. $\angle 1$ and $\angle 4$ form a linear pair. | <i>Definition of Linear Pair</i> |
| 4. $\angle 1$ and $\angle 4$ are supplementary. | <i>Linear pairs are supplementary.</i> |
| 5. $\angle 4$ is a right angle. | $m\angle 4 + 90 = 180, m\angle 4 = 90$ |
| 6. $\angle 4 \cong \angle 2$ | <i>Vertical angles are congruent.</i> |

These statements lead to Theorem 3–8.

Theorem 3–8	Words: If two lines are perpendicular, then they form four right angles.			
	<table border="0"> <tr> <td style="vertical-align: top;">Model:</td> <td style="text-align: center;"></td> <td style="vertical-align: top;">Symbols:</td> <td> $a \perp b$ $m\angle 1 = 90$ $m\angle 2 = 90$ $m\angle 3 = 90$ $m\angle 4 = 90$ </td> </tr> </table>	Model:		Symbols:
Model:		Symbols:	$a \perp b$ $m\angle 1 = 90$ $m\angle 2 = 90$ $m\angle 3 = 90$ $m\angle 4 = 90$	

Examples

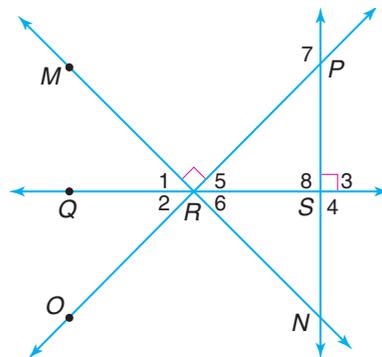
In the figure, $\overline{OP} \perp \overline{MN}$ and $\overline{NP} \perp \overline{QS}$. Determine whether each of the following is true or false.

- 1** $\angle PRN$ is an acute angle.

False. Since $\overline{OP} \perp \overline{MN}$, $\angle PRN$ is a right angle.

- 2** $\angle 4 \cong \angle 8$

True. $\angle 4$ and $\angle 8$ are vertical angles, and vertical angles are congruent.



Your Turn

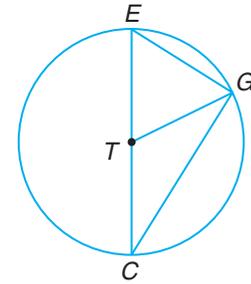
- a. $m\angle 5 + m\angle 6 = 90$ b. $\overline{QR} \perp \overline{PR}$



Example**Algebra Link**

3 Find $m\angle EGT$ and $m\angle TGC$ if $\overline{EG} \perp \overline{CG}$, $m\angle EGT = 7x + 2$, and $m\angle TGC = 4x$.

Since $\overline{EG} \perp \overline{CG}$, $\angle EGC$ is a right angle. So, $m\angle EGT + m\angle TGC = 90$.

**Algebra Review**

Solving Multi-Step Equations, p. 723

$$m\angle EGT + m\angle TGC = 90$$

$$(7x + 2) + 4x = 90$$

$$11x + 2 = 90$$

$$11x + 2 - 2 = 90 - 2$$

$$11x = 88$$

$$\frac{11x}{11} = \frac{88}{11}$$

$$x = 8$$

Definition of Perpendicular Lines

Substitution

Combine like terms.

Subtract 2 from each side.

Simplify.

Divide each side by 11.

Simplify.

To find $m\angle EGT$ and $m\angle TGC$, replace x with 8 in each expression.

$$\begin{aligned} m\angle EGT &= 7x + 2 \\ &= 7(8) + 2 \text{ or } 58 \end{aligned}$$

$$\begin{aligned} m\angle TGC &= 4x \\ &= 4(8) \text{ or } 32 \end{aligned}$$

The following activity demonstrates how to construct a line perpendicular to a line through a point on the line.

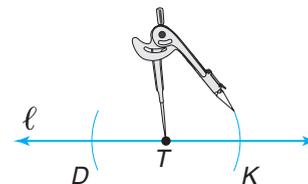
Hands-On Geometry Construction

Materials:  compass  straightedge

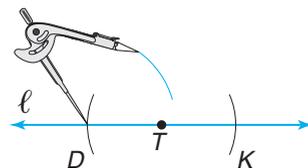
Step 1 Draw a line ℓ that contains a point T .



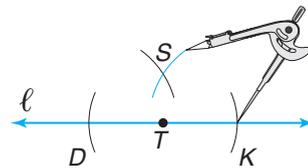
Step 2 Place the compass at point T . Using the same compass setting, draw arcs to the left and right of T , intersecting line ℓ . Label these points D and K .



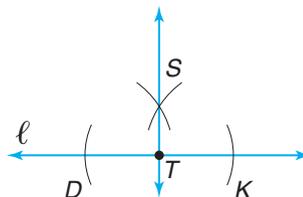
Step 3 Open the compass to a setting greater than \overline{DT} . Put the compass at point D and draw an arc above line ℓ .



Step 4 Using the same compass setting, put the compass at point K and draw an arc to intersect the one previously drawn. Label the point of intersection S .



Step 5 Use a straightedge to draw \overleftrightarrow{ST} .



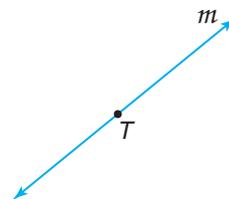
Try These

1. Find $m\angle DTS$ and $m\angle STK$.
2. Describe the relationship between \overleftrightarrow{ST} and line ℓ .
3. Construct a line perpendicular to line n through point B .



In the activity, you constructed a line through point T and perpendicular to line ℓ . Could you have constructed a different line through T that is perpendicular to line ℓ ?

Think of a point T on line m . How many lines can be drawn through that given point? How many lines can be drawn that are perpendicular to line m ? How many lines in a plane can be drawn that are perpendicular to line m and go through point T ? The next theorem answers this question.



Theorem 3–9

If a line m is in a plane and T is a point on m , then there exists exactly one line in that plane that is perpendicular to m at T .

Check for Understanding

Communicating Mathematics

1. Choose the types of angles that are *not* formed by two perpendicular lines.
 - a. vertical
 - b. linear pair
 - c. complementary
2. **Writing Math** Write a few sentences explaining why it is impossible for two perpendicular lines to form exactly one right angle.

Vocabulary

perpendicular

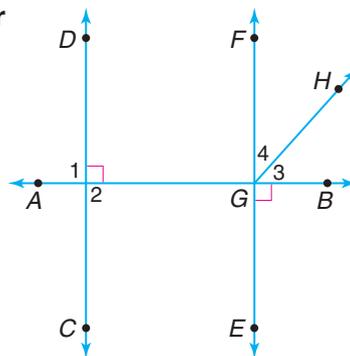
Guided Practice

Examples 1 & 2

Example 3

$\overline{AB} \perp \overline{CD}$ and $\overline{AB} \perp \overline{EF}$. Determine whether each of the following is *true* or *false*.

- $m\angle 1 + m\angle 4 = 180$
 - $m\angle 1 = 90$
 - $\overline{EF} \perp \overline{BG}$
 - $m\angle AGE < m\angle 3$
7. **Algebra** If $m\angle 3 = 2x + 6$ and $m\angle 4 = 2x$, find $m\angle 3$ and $m\angle 4$.



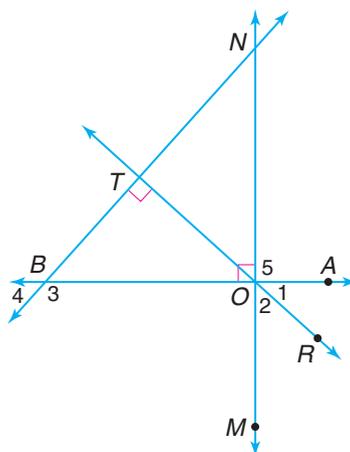
Exercises 3–7

Exercises

Practice

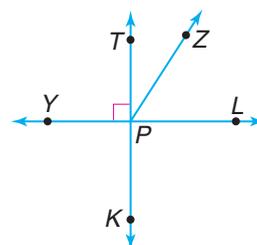
$\overline{BN} \perp \overline{RT}$, $\overline{MN} \perp \overline{AB}$, and point T is the midpoint of \overline{NB} . Determine whether each of the following is *true* or *false*.

- $\angle 5$ is a right angle.
- $\overline{MO} \perp \overline{OR}$
- $\angle 2 \cong \angle TON$
- $\angle NOB \cong \angle MOA$
- $\angle 1$ and $\angle 2$ are complementary.
- $\angle AON$ and $\angle 3$ are supplementary.
- $\overline{BT} \perp \overline{OT}$
- $m\angle BOM + m\angle AOR = 180$
- $\overline{NT} \cong \overline{BT}$
- $m\angle BOM + m\angle 5 = 90$
- $m\angle BTR = m\angle 5$
- $m\angle 1 + m\angle TON \geq 90$
- \overline{AB} is the only line \perp to \overline{MN} at O .
- If $m\angle 1 = 48$, what is $m\angle ROM$?



Exercises 8–21

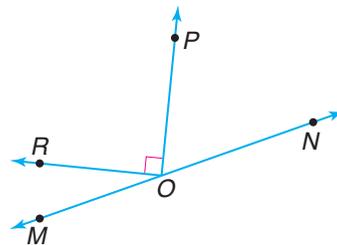
- Name four right angles if $\overline{TK} \perp \overline{LY}$.
- Name a pair of supplementary angles.
- Name a pair of angles whose sum is 90 .



Exercises 22–24

Applications and Problem Solving

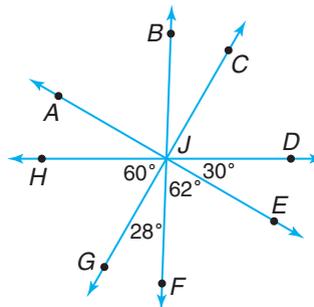
- Algebra** If $\overline{OP} \perp \overline{OR}$, \overline{OM} and \overline{ON} are opposite rays, $m\angle NOP = 5x$, and $m\angle MOR = 2x - 1$, find $m\angle NOP$ and $m\angle MOR$.



26. **Engineering** A site planner is preparing the layout for a new construction site.
- Which street appears to be perpendicular to Fair Avenue?
 - Which streets appear to be perpendicular to Main Street?



27. **Modeling** Two planes are *perpendicular planes* if they form a right angle. Give a real-world example of two perpendicular planes.
28. **Critical Thinking** Refer to the figure below. Explain in writing, which lines, if any, are perpendicular.



Mixed Review

29. Angles P and Q are vertical angles. If $m\angle P = 47$, what is $m\angle Q$? (Lesson 3–6)
30. **Algebra** Angles M and N are complementary. If $m\angle M = 3x$ and $m\angle N = 2x - 5$, find x . Then find $m\angle M$ and $m\angle N$. (Lesson 3–5)
31. Draw and label a coordinate plane. Then graph and label point C at $(-5, 3)$. (Lesson 2–4)

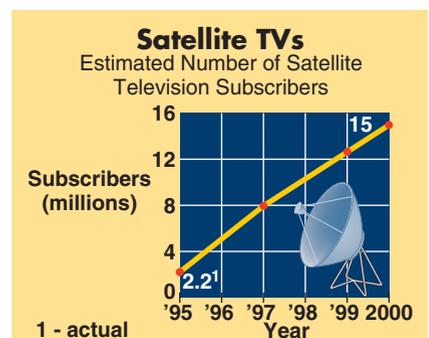
Standardized Test Practice

(A) (B) (C) (D)

32. **Short Response** Find the length of \overline{RS} in centimeters and in inches. (Lesson 2–2)



33. **Multiple Choice** The graph shows the estimated number of satellite television subscribers in the United States over five years. Use the pattern in the graph to predict the number of satellite subscribers in 2005. (Lesson 1–1)
- 20 million
 - 28 million
 - 24 million
 - 32 million



Source: Donaldson, Lufkin & Jenrette



Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

acute angle (p. 98)
 adjacent angles (p. 110)
 angle (p. 90)
 angle bisector (p. 106)
 complementary angles (p. 116)
 congruent angles (p. 122)
 degrees (p. 96)
 exterior (p. 92)

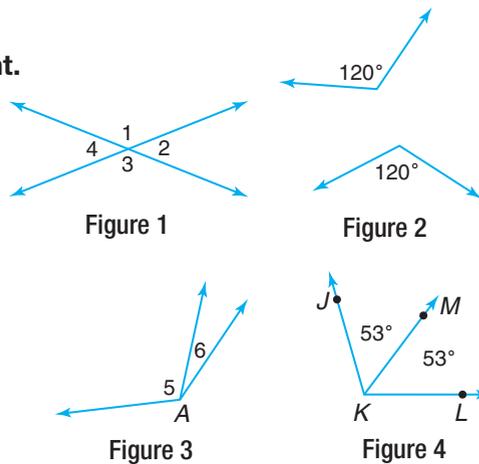
interior (p. 92)
 linear pair (p. 111)
 obtuse angle (p. 98)
 opposite rays (p. 90)
 perpendicular (p. 128)
 protractor (p. 96)
 quadrilateral (p. 103)

InterNET
CONTENTS **Review Activities**
 For more review activities, visit:
www.geomconcepts.com

right angle (p. 98)
 sides (p. 90)
 straight angle (p. 90)
 supplementary angles (p. 116)
 triangle (p. 102)
 vertex (p. 90)
 vertical angles (p. 122)

State whether each sentence is *true* or *false*. If false, replace the underlined word(s) to make a true statement.

- Angles are measured in units called degrees.
- In Figure 1, $\angle 2$ and $\angle 3$ are complementary angles.
- A compass is used to find the measure of an angle.
- In Figure 1, $\angle 3$ is an acute angle.
- In Figure 2, the two angles shown are supplementary.
- In Figure 3, $\angle 5$ and $\angle 6$ are vertical angles.
- Perpendicular lines intersect to form obtuse angles.
- In Figure 3, A is called a side of $\angle 6$.
- In Figure 1, $\angle 1$ and $\angle 4$ form a linear pair.
- In Figure 4, \overline{KM} is the vertex of $\angle JKL$.



Skills and Concepts

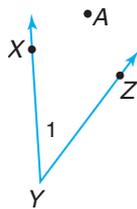
Objectives and Examples

- Lesson 3-1** Name and identify parts of an angle.

This angle can be named in four ways:
 $\angle XYZ$, $\angle ZYX$, $\angle Y$, or $\angle 1$.

The vertex is Y , and the sides are \overline{YX} and \overline{YZ} .

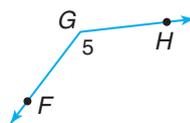
Point A is in the interior of $\angle XYZ$.



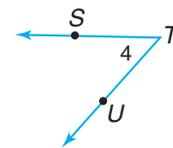
Review Exercises

Name each angle in four ways. Then identify its vertex and its sides.

11.

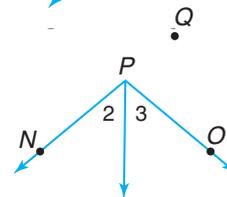


12.



13. Name all angles having P as their vertex.

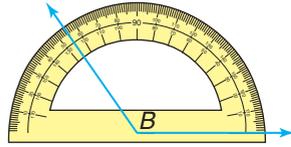
14. Is Q in the *interior*, *exterior*, or *on* $\angle 3$?



Objectives and Examples

- **Lesson 3–2** Measure, draw, and classify angles.

To find the measure of an angle, use a protractor.



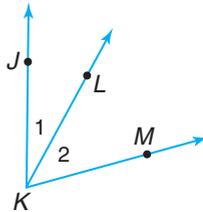
The measure of $\angle B$ is 125° .

Since $90 < m\angle B < 180$, $\angle B$ is obtuse.

- **Lesson 3–3** Find the measure of an angle and the bisector of an angle.

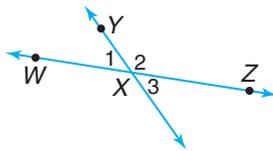
Find $m\angle 2$ if $m\angle JKM = 74$ and $m\angle 1 = 28$.

$$m\angle 2 = m\angle JKM - m\angle 1 = 74 - 28 \text{ or } 46$$

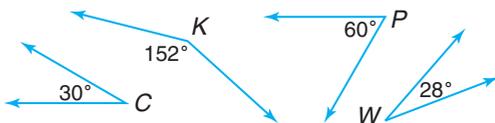


- **Lesson 3–4** Identify and use adjacent angles and linear pairs of angles.

$\angle 1$ and $\angle 2$ are adjacent angles. Since \overrightarrow{XW} and \overrightarrow{XZ} are opposite rays, $\angle 1$ and $\angle 2$ also form a linear pair. $\angle 1$ and $\angle 3$ are nonadjacent angles.



- **Lesson 3–5** Identify and use complementary and supplementary angles.



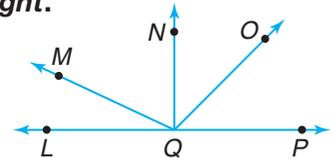
Since $m\angle C + m\angle P = 90$, $\angle C$ and $\angle P$ are complementary angles.

Since $m\angle K + m\angle W = 180$, $\angle K$ and $\angle W$ are supplementary angles.

Review Exercises

Use a protractor to find the measure of each angle. Then classify each angle as acute, obtuse, or right.

15. $\angle MQP$
16. $\angle PQO$
17. $\angle LQN$

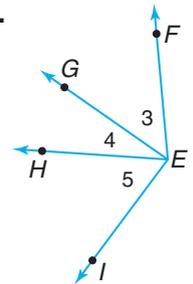


18. Use a protractor to draw a 65° angle.

Exercises 15–17

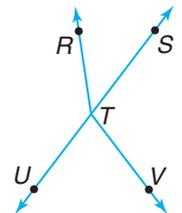
Refer to the figure at the right.

19. Find $m\angle FEH$ if $m\angle 3 = 52$ and $m\angle 4 = 31$.
20. If \overrightarrow{EH} bisects $\angle IEF$ and $m\angle HEF = 57$, find $m\angle 5$.
21. If $m\angle GEI = 90$ and $m\angle 5 = 42$, find $m\angle 4$.



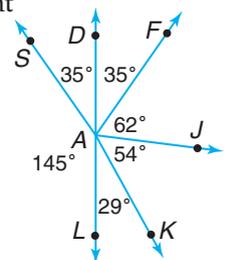
In the figure at the right, \overrightarrow{TU} and \overrightarrow{TS} are opposite rays.

22. Do $\angle VTR$ and $\angle UTV$ form a linear pair? Justify your answer.
23. Name two angles that are adjacent to $\angle VTU$.
24. Which angle forms a linear pair with $\angle STR$?



Refer to the figure.

25. Name a pair of nonadjacent supplementary angles.
26. Name a pair of supplementary angles.
27. Find the measure of an angle that is supplementary to $\angle KAJ$.
28. Find the measure of an angle that is complementary to $\angle DAS$.

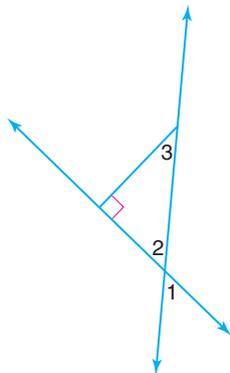


Objectives and Examples

- **Lesson 3–6** Identify and use congruent and vertical angles.

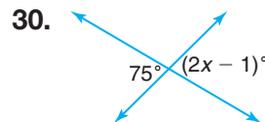
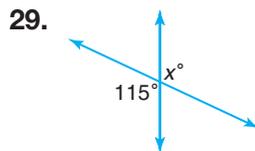
If $m\angle 1 = 51$ and $\angle 2$ and $\angle 3$ are complementary, find $m\angle 3$.

$\angle 1$ and $\angle 2$ are vertical angles. So, $\angle 1 \cong \angle 2$.
 $m\angle 1 = 51$. So, $m\angle 2 = 51$.
 $\angle 2$ and $\angle 3$ are complementary. So, $m\angle 3 = 90 - 51$ or 39.



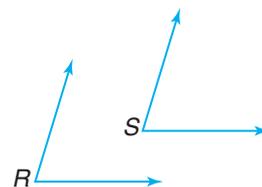
Review Exercises

Find the value of x in each figure.

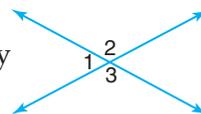


Refer to the figures.

31. Find the measure of an angle that is complementary to $\angle R$ if $\angle R \cong \angle S$ and $m\angle S = 73$.



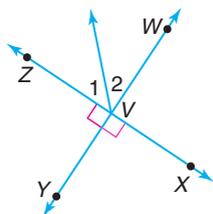
32. If $\angle 1$ is supplementary to $\angle 2$, $\angle 3$ is supplementary to $\angle 1$ and $m\angle 1 = 56$, what are $m\angle 2$ and $m\angle 3$?



- **Lesson 3–7** Identify, use properties of, and construct perpendicular lines and segments.

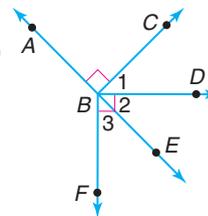
If $\overline{WY} \perp \overline{ZX}$, then the following are true.

1. $\angle WVZ$ is a right angle.
2. $\angle YVZ \cong \angle WVX$
3. $m\angle 1 + m\angle 2 = 90$



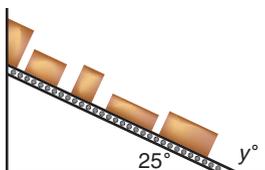
$\overline{BC} \perp \overline{AE}$ and $\overline{BF} \perp \overline{BD}$. Determine whether each of the following is *true* or *false*.

33. $\angle ABC$ is obtuse.
34. $m\angle FBD + m\angle ABC = 180$
35. $\angle DBF \cong \angle CBE$
36. $\overline{BD} \perp \overline{AE}$
37. $\angle 1 \cong \angle 3$

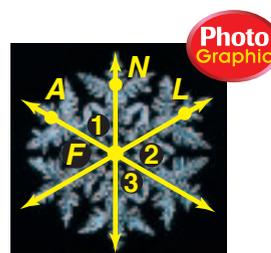


Applications and Problem Solving

38. **Manufacturing** A conveyor belt is set at a 25° angle to the floor of a factory. If this angle is increased, will the value of y increase or decrease? (Lesson 3–2)

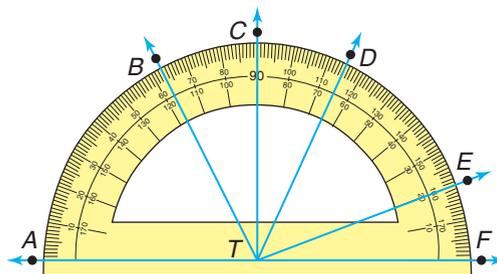


39. **Nature** In the picture of the snowflake, \overline{FN} bisects $\angle AFL$ and $m\angle AFL = 120$. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$. (Lessons 3–3 & 3–6)



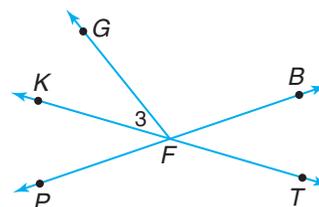
Refer to the figures at the right.

1. Name a pair of opposite rays.
2. *True or false:* $\angle CTE$ is adjacent to $\angle ATC$.
3. Name an angle congruent to $\angle ATC$.
4. Find the measure of an angle that is complementary to $\angle FTE$.
5. Name a pair of supplementary angles.
6. Name an angle that forms a linear pair with $\angle ATB$.
7. Find the measure of $\angle ATE$. Then classify the angle as *acute*, *right*, or *obtuse*.



Exercises 1–7

8. Name $\angle 3$ in two other ways.
9. If \overline{FK} bisects $\angle GFP$ and $m\angle 3 = 38$, find $m\angle KFP$.
10. If $m\angle GFB = 114$ and $m\angle BFT = 34$, find $m\angle GFT$.
11. If $\angle PFK$ is supplementary to $\angle KFB$, $\angle PFK$ is supplementary to $\angle TFP$, and $m\angle PFK = 33$, what is $m\angle KFB$ and $m\angle TFP$?

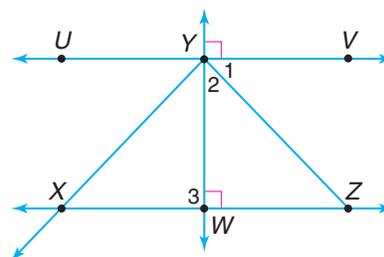


Exercises 8–11

12. Find the measure of an angle that is complementary to $\angle C$ if $\angle C \cong \angle D$ and $m\angle D = 27$.
13. If $\angle JKL$ and $\angle CKD$ are vertical angles and $m\angle JKL = 35$, find $m\angle CKD$.

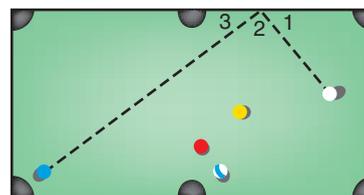
In the figure, $\overline{UV} \perp \overline{YW}$.

14. If $m\angle 2 = 44$, find $m\angle 1$.
15. Find $m\angle VYW + m\angle ZWY$.
16. *True or false:* $\overline{UV} \perp \overline{ZY}$
17. Find $m\angle UYX + m\angle XYW$.
18. Name two pairs of adjacent right angles.



Exercises 14–18

19. **Sports** In pocket billiards, when a ball is hit so that no spin is produced, the angle at which the ball strikes the cushion is equal to the angle at which the ball rebounds off the cushion. That is, $m\angle 1 = m\angle 3$. If $m\angle 1 = 35$, find $m\angle 2$ and $m\angle 3$.
20. **Algebra** $\angle G$ and $\angle H$ are supplementary angles. If $m\angle G = 4x$ and $m\angle H = 7x + 15$, find the measure of each angle.



Exercise 19



Counting and Probability Problems

Standardized tests usually include problems that ask you to count or calculate probabilities. You may need to know these concepts.

combinations permutations tree diagram
 outcomes probability

It's a good idea to memorize the definition of the probability of an event.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{total number of outcomes}}$$

Test-Taking Tip

To solve counting problems, you can use arithmetic, make a list, draw a tree diagram, use permutations, use combinations, or draw a Venn diagram.

Example 1

How many combinations of 5 flowers can you choose from one dozen different flowers?

- (A) 99 (B) 396
 (C) 792 (D) 1024

Solution You need to find the combinations of 5 items out of 12. (These are *combinations*, not permutations, because the order of the flowers does not matter.) Calculate $C(12, 5)$, the number of combinations of 12 things taken 5 at a time.

$$\begin{aligned} C(12, 5) &= \frac{P(12, 5)}{5!} \\ &= \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} \end{aligned}$$

Hint Simplify numeric expressions when possible.

$$\begin{aligned} &= \frac{\overset{1}{12} \times \overset{1}{11} \times \overset{1}{10} \times 9 \times 8}{\underset{1}{5} \times \underset{1}{4} \times \underset{1}{3} \times \underset{1}{2} \times 1} \\ &= 792 \end{aligned}$$

The answer is choice C, 792.

Example 2

A box of donuts contains 3 plain, 5 cream-filled, and 4 chocolate donuts. If one of the donuts is chosen at random from the box, what is the probability that it will NOT be cream-filled?

Hint If the probability of an event is p , then the probability of NOT an event is $1 - p$.

Solution One method for solving this problem is to first find the total number of donuts in the box: $3 + 5 + 4 = 12$.

Then find the number of donuts that are NOT cream-filled. This is the sum of plain plus chocolate: $3 + 4 = 7$.

Calculate the probability of randomly selecting a donut that is NOT cream-filled.

$$\begin{aligned} \text{number of favorable outcomes} &\rightarrow 7 \\ \text{total number of outcomes} &\rightarrow 12 \end{aligned}$$

Another method is to find the probability of selecting a donut that *is* cream-filled, $\frac{5}{12}$. Then subtract this probability from 1.

$$1 - \frac{5}{12} = \frac{7}{12}$$

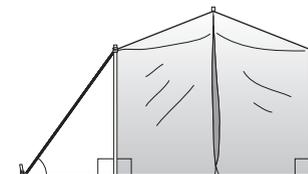
After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

- How many ways can a family of 5 be seated in a theater if the mother sits in the middle?
(Statistics Review)
 (A) 120 (B) 24 (C) 15 (D) 10
- For a class play, student tickets cost \$2 and adult tickets cost \$5. A total of 30 tickets are sold. If the total sales must exceed \$90, then what is the minimum number of adult tickets that must be sold?
(Algebra Review)
 (A) 7 (B) 8 (C) 9
 (D) 10 (E) 11
- Andrew's family wants to fence in a 40-meter by 75-meter rectangular area on their ranch. How much fencing should they buy?
(Lesson 1–6)
 (A) 115 m (B) 230 m
 (C) 1500 m (D) 3000 m
- A coin was flipped 20 times and came up heads 10 times and tails 10 times. If the first and the last flips were both heads, what is the greatest number of consecutive heads that could have occurred?
(Statistics Review)
 (A) 1 (B) 2 (C) 8
 (D) 9 (E) 10
- A suitcase designer determines the longest item that could fit in a particular suitcase to be $\sqrt{360}$ centimeters. Which is equivalent to this value?
(Algebra Review)
 (A) $6\sqrt{10}$ (B) $10\sqrt{6}$
 (C) 36 (D) 180
- $-|-7| - |-5| - 3|-4| =$ *(Algebra Review)*
 (A) -24 (B) -11 (C) 0
 (D) 13 (E) 24

- A rope is used to stake a tent pole as shown. Which could be the measure of an angle that is supplementary to the angle that the rope makes with the ground?
(Lesson 3–5)

- (A) 45°
 (B) 75°
 (C) 90°
 (D) 125°



- Of the 16 people waiting for the subway, 12 have briefcases, 8 have overcoats, and 5 have both briefcases and overcoats. The other people have neither. How many people have just a briefcase?
(Statistics Review)
 (A) 10 (B) 7 (C) 6 (D) 3

Grid In

- Celine made a basket 9 out of 15 times. Based on this, what would be the odds *against* her making a basket the next time she shoots? Write as a fraction.
(Statistics Review)

Extended Response

- Spin the two spinners and add the numbers. If the sum is even, you get one point; if odd, your partner gets one point.
(Statistics Review)



Part A Use a tree diagram to find the probability of getting an even number. Explain why this makes sense.

Part B How could you change the spinners so that the probability of getting an even number equals the probability of getting an odd number?

