# More Coordinate Graphing and Transformations

## What You'll Learn

### Key Ideas

- Solve systems of equations by graphing, substitution, or elimination. *(Lessons 16–1 and 16–2)*

- Investigate and draw translations, reflections, rotations, and dilations on a coordinate plane. *(Lessons 16–3 to 16–6)*

### Key Vocabulary

dilation *(p. 703)*

reflection *(p. 692)*

rotation *(p. 697)*

translation *(p. 687)*

system of equations *(p. 676)*

## Why It's Important

**Architecture** The Parthenon in Athens, Greece is one of the greatest monuments of antiquity. Made completely from marble, it took ten years to build and was finished in 438 B.C. The famous frieze that decorated the inside walls was 524 feet long and contained 115 plaques.

**Coordinate graphing and transformations** allow for exact study of geometric figures. You will use transformations to create a repeating frieze in Lesson 16–3.

# ✓ Check Your Readiness

✓ **Lesson 4–6,**
**pp. 174–179**

**Graph each equation using the slope and *y*-intercept.**

**1.** $y = 2x - 3$      **2.** $y = -x + 2$      **3.** $y = -1$

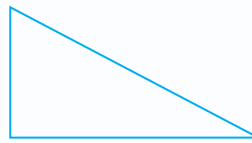**4.** $2x + 3y = 6$      **5.** $3x + 4y = 12$      **6.** $5x - y = 2$

✓ **Lesson 10–6,**
**pp. 434–439**

**Determine whether each figure has line symmetry. If it does, copy the figure, and draw all lines of symmetry. If not, write *no.***

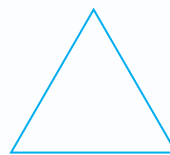**7.**            **8.**            **9.**

✓ **Lesson 10–6,**
**pp. 434–439**
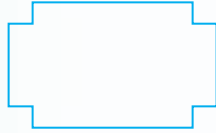
**Determine whether each figure has rotational symmetry. Write *yes* or *no.***

**10.**            **11.**            **12.**

✓ **Lesson 9–2,**
**pp. 356–361**

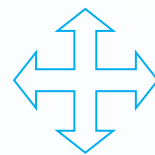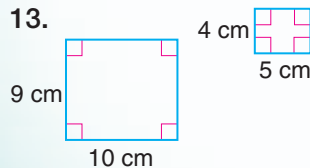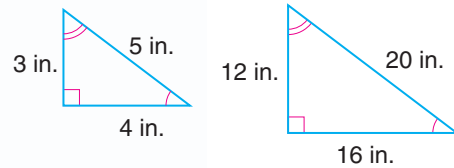**Determine whether each pair of polygons is similar. Justify your answer.**

**13.**    4 cm      **14.**    5 in.      20 in.

9 cm    5 cm    3 in.    12 in.

10 cm    4 in.    16 in.

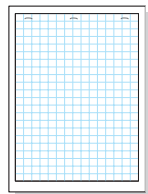## FOLDABLES™
### Study Organizer

Make this Foldable to help you organize your Chapter 16 notes. Begin with six sheets of grid paper and an $8\frac{1}{2}$" by 11" poster board.

❶ **Staple** the six sheets of grid paper onto the poster board.

❷ **Label** each sheet with a lesson title.

*Lesson 16-1*
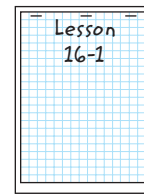
**Reading and Writing** As you read and study the chapter, use the pages to write the main ideas and graph examples of each lesson.

## What You'll Learn

You'll learn to solve systems of equations by graphing.

## Why It's Important

**Business** Business analysts can find the break-even point in producing and selling a product by solving systems of equations. *See Exercise 25*.

A set of two or more equations is called a **system of equations**. The solution of a system of equations is the intersection point of the graphs of these equations. The ordered pair for this point satisfies all equations in the system.

To solve a system of equations, first graph each equation in the system. To graph the equations, you can find ordered pairs, use the slope and $y$-intercept, or use a graphing calculator. *Choose the method that is easiest for you to use. The results will be the same for all methods.*

When solving a system of two linear equations in two variables, there are three possibilities.

- The lines intersect, so the point of intersection is the solution.

- The lines are parallel, so there is no point of intersection and, therefore, no solution.

- The lines coincide (both graphs are the same), so there is an infinite number of solutions.

### Example 1

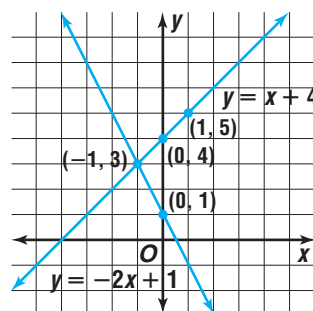**Solve the system of equations by graphing.**

$$y = x + 4$$
$$y = -2x + 1$$

In this example, we find ordered pairs by choosing values for $x$ and finding the corresponding values of $y$.

| $y = x + 4$ |
|---|---|---|---|
| $x$ | $x + 4$ | $y$ | $(x, y)$ |
| 0 | 4 | 4 | (0, 4) |
| 1 | 5 | 5 | (1, 5) |

| $y = -2x + 1$ |
|---|---|---|---|
| $x$ | $-2x + 1$ | $y$ | $(x, y)$ |
| −1 | 3 | 3 | (−1, 3) |
| 0 | 1 | 1 | (0, 1) |

Graph the ordered pairs and draw the graphs of the equations. The graphs intersect at the point whose coordinates are $(-1, 3)$. Therefore, the solution of the system of equations is $(-1, 3)$.



### Your Turn

**a.** $y = -x + 1$
$\quad\ y = x - 5$

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When the graphs of the equations are parallel lines, a system of equations has no solution.

**Example**
—2

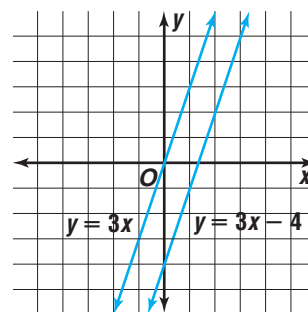**Solve the system of equations by graphing.**

$y = 3x$
$2y = 6x - 8$

In this example, we use the slope and $y$-intercept to graph each equation. Write the second equation in slope-intercept form.

$2y = 6x - 8$  *Original equation*

$\frac{2y}{2} = \frac{6x - 8}{2}$  *Divide each side by 2.*

$y = 3x - 4$  *Simplify.*

| Equation | Slope | y-intercept |
|----------|-------|-------------|
| $y = 3x$ | 3 | 0 |
| $y = 3x - 4$ | 3 | $-4$ |



Note that the slope of each line is 3. So, the graphs are parallel and do not intersect. Therefore, there is no solution to the system of equations.

**Your Turn**

**b.** $y = 2x - 3$
$\quad\, y - 2x = 4$

You can use systems of equations to solve problems.

**Real World**

**Example**
—3

**Gardening Link**

**Toshiro is making a vegetable garden. He wants the length to be twice the width, and he has 24 feet of fencing to put around the garden. If $w$ represents the width of the garden and $\ell$ represents the length, solve the system of equations below to find the dimensions of Toshiro's garden.**

$2w + 2\ell = 24$
$\ell = 2w$
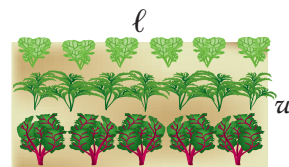


Solve the first equation for $\ell$.

$2w + 2\ell = 24$  *The perimeter is 24 feet.*

$2w + 2\ell - 2w = 24 - 2w$  *Subtract 2w from each side.*

$2\ell = 24 - 2w$  *Simplify.*

$\frac{2\ell}{2} = \frac{24 - 2w}{2}$  *Divide each side by 2.*

$\ell = 12 - w$  *Simplify.*

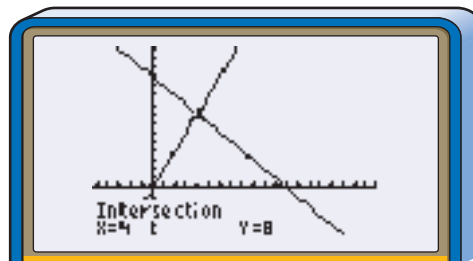Use a TI–83/84 Plus graphing calculator to graph the equations $\ell = 12 - w$ and $\ell = 2w$ and to find the coordinates of the intersection point. *Note that these equations can be written as $y = 12 - x$ and $y = 2x$ and then graphed.*

**Enter:** $\boxed{Y=}$ 12 $\boxed{-}$
$\boxed{X,T,\theta,n}$ $\boxed{ENTER}$ 2
$\boxed{X,T,\theta,n}$ $\boxed{GRAPH}$

Next, use the Intersection tool on the CALC menu to find the coordinates of the point of intersection.

The solution is (4, 8). Since $w = 4$ and $\ell = 8$, the width of the garden will be 4 feet, and the length will be 8 feet.

Check your answer by examining the original problem.

Is the length of the garden twice the width?   √
Does the garden have a perimeter of 24 feet?   √

The solution checks.

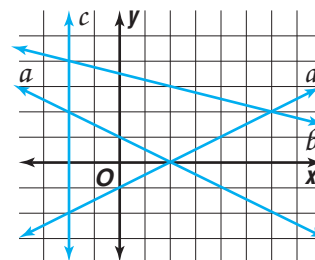# Check for Understanding

**Communicating Mathematics**

1. **Draw** a system of equations whose solution is $P(5, 1)$. Explain why the ordered pair for $P$ is the solution.

**Vocabulary**

system of equations

2. **State** the solution of the system of equations represented by each pair of lines.
   a.  $a$ and $d$
   b.  $b$ and $d$
   c.  $a$ and $c$
   d.  $b$ and $c$

3. a.  **Writing Math**  Graph the system of equations.
   $y = x + 5$
   $2y = 2x + 10$

   b.  List five solutions.

   c.  Make a conjecture about systems of equations in part a.

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⏱ **Getting Ready**  **Write each equation in slope-intercept form.**

**Sample:** $5x - y = 6$         **Solution:** $5x - y = 6$
$-y = 6 - 5x$
$y = 5x - 6$

**4.** $-x + y = -11$     **5.** $2y + 4x = 7$     **6.** $3x = -2y + 15$

**Examples 1 & 2**  **Solve each system of equations by graphing.**

**7.** $y = -x + 6$     **8.** $y = x - 1$     **9.** $3 - y = 2$
$y = x - 2$         $x + y = 11$         $4y = 16$

**Example 3**  **10. Pets**  Marieta has 50 feet of fencing to make a dog pen. She wants the length of the pen to be 5 feet longer than its width. The system of equations below represents this problem.

$2w + 2\ell = 50$
$\ell = w + 5$

**a.** Graph the system of equations.
**b.** Determine the dimensions of the pen.
**c.** Explain how the dimensions of the pen are related to the ordered pair solution.

# Exercises

• • • • • • • • • • • • • • • •

**Practice**

**Solve each system of equations by graphing.**

**11.** $y = -3x + 6$     **12.** $x + y = 4$     **13.** $x + y = 6$
$y = x - 6$         $x + y = 2$         $y = x - 2$

**14.** $3x - 2y = 10$     **15.** $x + 2y = 12$     **16.** $y = \frac{1}{2}x - 4$
$x + y = 0$         $y = 2$         $y = -x + 5$

**17.** $2x + y = -7$     **18.** $y = 2x + 1$     **19.** $x + 2y = -9$
$\frac{1}{3}x - y = -7$         $x + 2y = 7$         $x - y = 6$

| Homework Help | |
|---|---|
| For Exercises | See Examples |
| 11, 16, 24, 25 | 1, 2 |
| 12–15, 17–23, 26 | 3 |
| **Extra Practice** | |
| See page 756. | |

**State the letter of the ordered pair that is a solution of both equations.**

**20.** $3x = 15$          **a.** $(5, 0)$   **b.** $(0, 5)$   **c.** $(5, 1)$          **d.** $(5, 8)$
$2x - y = 9$

**21.** $2x - 5y = -1$     **a.** $(0, 5)$   **b.** $(2, 1)$   **c.** $(-0.5, 0)$   **d.** $(-2, -1)$
$x + 2y = 4$

**22.** Find the solution of the system $y = x - 2$ and $x + 2y = -10$ by graphing.

**23.** The graphs of $x + 2y = 6$, $3x - y = 4$, $x + 5y = -4$, and $-3x + 4y = 12$ intersect to form a quadrilateral.

**a.** Graph the system of equations.
**b.** Find the coordinates of the vertices of the quadrilateral.

◀ **CONTENTS** ▶

24. **Games** In 1998, high school sophomore Whitney Braunstein
of Columbus, Ohio, created the board game *Get-a-Pet*, in which
players circle the board trying to collect pets. The equation $y = 80$
represents the number of points needed to buy one pet. The equation
$y = 20 + 20x$ represents the number of points a player can collect by
walking the neighbor's dog once and by mowing the lawn $x$ times.
   **a.** Solve the system of equations by graphing.
   **b.** What does this solution mean?

25. **Business** The number of products that a company must sell so their
cost equals income is called the *break-even point*. For the Gadget
Company, the equation $y = 3x + 150$ represents the weekly cost of
producing $x$ gadgets. The equation that represents the income from
selling the gadgets is $y = 4x$.
   **a.** Graph the system of equations.
   **b.** Find the break-even point and explain what this point means.

26. **Critical Thinking** Graph $x + y = 3$ and $x - y = 4$. Describe the
relationship between the two lines. Explain your reasoning.

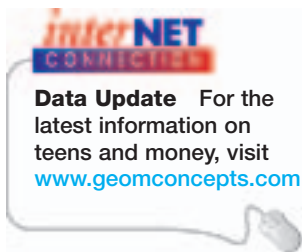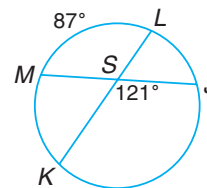27. Position and label a rectangle with length $s$ units and width $t$ units
on a coordinate plane. *(Lesson 15–6)*
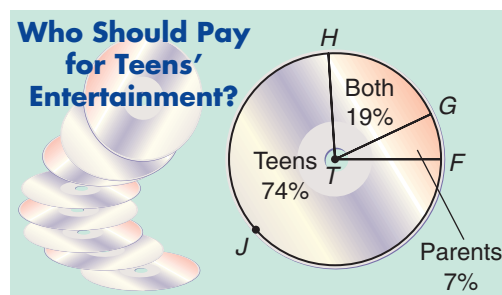
**Find each measure.**

28. $m\angle B$  *(Lesson 14–4)*
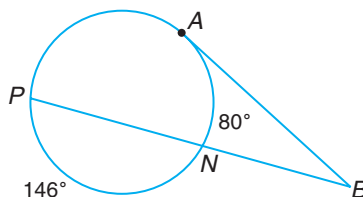
29. $m\widehat{JK}$  *(Lesson 14–3)*

30. **Finances** A recent survey
asked nearly 200,000
students in grades 6–12
the question, "Who should
pay for your movies, CDs,
etc.?" The results are
shown in the graph at the
right. If $m\angle HTG = 68$ and
$m\angle GTF = 25$, find $m\widehat{HJF}$.
*(Lesson 11–2)*



**Who Should Pay
for Teens'
Entertainment?**

Both 19%

Teens 74%

Parents 7%

**Source:** *USA Weekend*

31. **Multiple Choice** Factor $4c^2 - 25d^2$. *(Algebra Review)*
   Ⓐ $(2c - 5d)(2c + 5d)$
   Ⓑ $(5d + 2c)(5d + 2c)$
   Ⓒ $(4c + 25d)(4c - 25d)$
   Ⓓ $(2d - 5c)(2d + 5c)$

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**What You'll Learn**

You'll learn to solve systems of equations by using the substitution or elimination method.

**Why It's Important**

**Consumer Choices**
You can use systems of equations to compare phone rates. *See Exercise 30.*

In the previous lesson, you learned to solve systems of equations by graphing. You can also solve systems of equations by using algebra. One algebraic method is **substitution**. The substitution method for solving a system of equations is illustrated in Example 1.

**1** | **Example**

**Use substitution to solve the system of equations.**

$2x + y = 5$
$3x - 2y = 4$

**Step 1** Solve the first equation for $y$ since the coefficient of $y$ is 1.

$$2x + y = 5$$
$$2x + y - 2x = 5 - 2x \quad \textit{Subtract 2x from each side.}$$
$$y = 5 - 2x$$

**Step 2** In the solution of the system, $y$ must have the same value in both equations. So, substitute $5 - 2x$ for $y$ in the second equation. Then solve for $x$.

$$3x - 2y = 4 \qquad \textit{Original equation}$$
$$3x - 2(5 - 2x) = 4 \qquad \textit{Replace y with 5 − 2x.}$$
$$3x - 10 + 4x = 4 \qquad \textit{Distributive property}$$
$$7x - 10 = 4 \qquad \textit{Add like terms.}$$
$$7x - 10 + 10 = 4 + 10 \qquad \textit{Add 10 to each side.}$$
$$7x = 14 \qquad \textit{Simplify.}$$
$$\frac{7x}{7} = \frac{14}{7} \qquad \textit{Divide each side by 7.}$$
$$x = 2 \qquad \textit{Simplify.}$$

**Step 3** Substitute 2 for $x$ in the first equation and solve for $y$.

$$2x + y = 5 \qquad \textit{Original equation}$$
$$2(2) + y = 5 \qquad \textit{Replace x with 2.}$$
$$4 + y = 5 \qquad \textit{Multiply.}$$
$$4 + y - 4 = 5 - 4 \qquad \textit{Subtract 4 from each side.}$$
$$y = 1 \qquad \textit{Simplify.}$$

*Check your solution by substituting in both equations or by graphing.*

The solution to this system of equations is (2, 1).

**Your Turn**    Use substitution to solve each system of equations.

**a.** $x = y + 1$
$\quad x + y = 8$

**b.** $3x + y = -6$
$\quad -2x + 3y = 4$

**CONTENTS**

Another algebraic method for solving systems of equations is called **elimination**. You can eliminate one of the variables by adding or subtracting the equations.

**Example** ── ❷ **Use elimination to solve the system of equations.**

$$x + y = 9$$
$$-x + 2y = -1$$

$$
\begin{array}{r}
x + \phantom{2}y = 9 \\
(+) \ -x + 2y = -1 \\
\hline
0 + 3y = 8
\end{array}
$$  *Add the equations to eliminate the x terms.*

$$\frac{3y}{3} = \frac{8}{3}$$  *Divide each side by 3.*

$$y = \frac{8}{3}$$  *Simplify.*

The value of $y$ in the solution is $\frac{8}{3}$.

Now substitute in either equation to find the value of $x$. *Choose the equation that is easier for you to solve.*

$$x + y = 9$$  *Original equation*

$$x + \frac{8}{3} = 9$$  *Replace y with $\frac{8}{3}$.*

$$x + \frac{8}{3} - \frac{8}{3} = 9 - \frac{8}{3}$$  *Subtract $\frac{8}{3}$ from each side.*

$$x = \frac{27}{3} - \frac{8}{3} \text{ or } \frac{19}{3}$$  *Simplify.*

The value of $x$ in the solution is $\frac{19}{3}$.

The solution to the system of equations is $\left(\frac{19}{3}, \frac{8}{3}\right)$.

**Your Turn**    **Use elimination to solve each system of equations.**

**c.** $7x - 2y = 20$
   $5x - 2y = 16$

**d.** $x + y = 10$
   $x - y = 5$

When neither the $x$ nor $y$ terms can easily be eliminated by addition or subtraction, you can multiply one or both of the equations by some number.

www.geomconcepts.com/extra_examples

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**Example** ❸

**Business Link**

*The Helping Hearts* is a nonprofit company started by students in Spokane, Washington. The company donated money to charities from the sale of beeswax candles made by the students. Suppose small candles cost $0.60 each to make and package and large candles cost $0.90. The cost of making $x$ small candles and $y$ large candles is $81. A total of 115 candles were made.

Solve the system of equations to find the number of small and large candles that were made.

$$0.60x + 0.90y = 81 \qquad \textit{x represents the number of small candles, and}$$
$$x + y = 115 \qquad \textit{y represents the number of large candles.}$$

*Explore*  Neither of the variables can be eliminated by simply adding or subtracting the equations.

*Plan*  If you multiply the second equation by 0.60 and subtract, the $x$ variables will be eliminated.

*Solve*

$$0.60x + 0.90y = 81 \qquad\qquad 0.60x + 0.90y = 81$$
$$x + y = 115 \quad \boxed{\text{Multiply by 0.60.}} \Rightarrow \quad (-)\ 0.60x + 0.60y = 69$$
$$\overline{\qquad\qquad\qquad\qquad 0 + 0.30y = 12}$$
$$\frac{0.30y}{0.30} = \frac{12}{0.30}$$
$$y = 40$$

Now substitute 40 for $y$ in either of the original equations and find the value of $x$.

$$x + y = 115 \qquad \textit{Original equation}$$
$$x + 40 = 115 \qquad \textit{Replace y with 40.}$$
$$x + 40 - 40 = 115 - 40 \qquad \textit{Subtract 40 from each side.}$$
$$x = 75 \qquad \textit{Simplify.}$$

The solution of this system is (75, 40). Therefore, the students made 75 small candles and 40 large candles.

*Examine*  Check the answer by looking at the original problem.

Are there 115 candles?  $75 + 40 = 115$  ✓

Is the total cost $81?  $0.60(75) + 0.90(40) = \$81$  ✓

The answer checks.

**Your Turn**  Use elimination to solve each system of equations.

e.  $3x + 5y = 12$
    $4x - y = -7$

f.  $6x - 2y = 11$
    $-9x + 5y = -17$

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**Communicating Mathematics**

1. **Explain** how you know that $(3, -3)$ is a solution of the system $x - 3y = 12$ and $2x + y = 3$.

2. **Write** a system of equations that has $(0, 1)$ as a solution.

3. **Writing Math** Describe the difference between the substitution and elimination methods. Explain when it is better to use each method.

> **Vocabulary**
> substitution
> elimination

**Guided Practice**

⏱ **Getting Ready** Find the value of *y* for each given value of *x*.

**Sample:** $4x - 5y = 12$; $x = 2y$    **Solution:**
$$4x - 5y = 12$$
$$4(2y) - 5y = 12$$
$$3y = 12 \text{ or } y = 4$$

4. $2x + 7y = 2$;
   $x = 8$

5. $9y = 11 - x$;
   $x = y + 3$

6. $2x - 6y = 4$;
   $x = 2y - 1$

*Example 1*

**Use substitution to solve each system of equations.**

7. $x + 2y = 5$
   $2x + y = 7$

8. $y = x - 1$
   $3x - 6y = 16$

*Examples 2 & 3*

**Use elimination to solve each system of equations.**

9. $5x - 4y = 10$
   $x + 4y = 2$

10. $x - 3y = 3$
    $2x + 9y = 11$

11. **Business** *Food From the 'Hood* is a company in Los Angeles in which students sell their own vegetables and salad dressings. Suppose in one week they sell 250 bottles of creamy Italian and garlic herb dressings. This can be represented by the equation $x + y = 250$. The creamy Italian $x$ is \$3 a bottle, and the garlic herb $y$ is \$2.40 a bottle. If they earn \$668.40 from the sales of these two dressings, this can be represented by the equation $3x + 2.4y = 668.4$.  *Example 1*

    a. Use substitution to solve the system of equations.

    b. How many bottles of each type of salad dressing did they sell?

## Exercises

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**Practice**

**Use substitution to solve each system of equations.**

12. $y = 4$
    $x + y = 9$

13. $x = 1 - 4y$
    $3x + 2y = 23$

14. $9x + y = 20$
    $4x + 3y = 14$

15. $3x + 4y = -7$
    $2x + y = -3$

16. $2x = 5$
    $x + y = 7$

17. $x + 2y = 4$
    $\frac{3}{4}x + \frac{1}{2}y = 2$

◀ **CONTENTS** ▶

**Use elimination to solve each system of equations.**

**18.** $x + y = 2$
$x - y = 6$

**19.** $x - y = 6$
$x + y = 7$

**20.** $2x - y = 32$
$2x + y = 60$

**21.** $y - x = 2$
$3y - 8x = 9$

**22.** $2x + 5y = 13$
$4x - 3y = -13$

**23.** $3x - 2y = 15$
$2x - 5y = -1$

**State whether *substitution* or *elimination* would be better to solve each system of equations. Explain your reasoning. Then solve the system.**

**24.** $y = 7 - x$
$2x - y = 8$

**25.** $2x - 5y = -2$
$x = y - 7$

**26.** $3x + 5y = -16$
$2x - 2y = 0$

**27.** Solve the system of equations by using elimination. Round to the nearest hundredth.

$3x + 14y = 6$
$2x + 3y = 5$

**28.** What is the value of $x$ in the solution of this system of equations?

$x - y = 16$
$\frac{1}{2}x + \frac{1}{2}y = 37$

**Applications and Problem Solving**

**29. Transportation**   Josh is 3 miles from home riding his bike averaging 7 miles per hour. The miles traveled $y$ can be represented by the equation $y = 3 + 7x$, where $x$ represents the time in hours. His mother uses her car to catch up to him because he forgot his water bottle. If she averages 25 miles per hour, the equation $y = 25x$ represents her distance traveled.
  **a.** Explain how you can find the time it takes Josh's mother to catch up to him. (Assume that the speeds are constant.)
  **b.** How many minutes does it take Josh's mother to catch up to him?
  **c.** Explain the meaning of the $y$ value in the solution.

**30. Cellular Phones**   Tonisha is looking for the best cellular phone rate. Discount Cellular's plan costs $7.95 a month plus 36¢ per minute. Maland Communications advertises their Basic Plan for $9.95 a month plus 29¢ per minute.
  **a.** If $x$ represents the number of minutes used in a month and $y$ represents the total monthly cost, write an equation for each cellular phone plan.
  **b.** Solve the system of equations. Round to the nearest tenth.
  **c.** Explain what this solution represents.
  **d.** Which plan should Tonisha choose if she plans on talking 30 minutes per month? Explain how you determined your answer.

31. **Critical Thinking**   If you use elimination to solve each system of equations, what do you get when you add or subtract the equations? Describe what this means in terms of the solution.

   **a.** $5x - 2y = 4$
      $10x - 4y = 8$
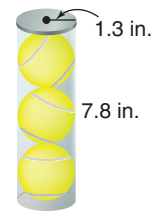
   **b.** $2x + y = 15$
      $4x + 2y = -3$

**Mixed Review**

32. Solve the system of equations by graphing.   *(Lesson 16–1)*
   $y = 4x - 3$
   $x + 2y = 12$

33. Refer to the figures at the right. Write the *Given* statements and the *Prove* statement of a two-column proof showing that angle $S$ is congruent to angle $V$.   *(Lesson 15–3)*
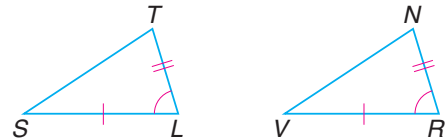
34. $\overline{GI}$ is a diameter of $\odot B$, and $\angle GHI$ is inscribed in $\odot B$. What is the measure of $\angle GHI$?   *(Lesson 14–1)*

35. **Sports**   Find the volume of the tennis ball can. Round to the nearest hundredth.   *(Lesson 12–3)*

1.3 in.

7.8 in.

**Exercise 35**

36. **Short Response**   Determine whether the following statement is *true* or *false*. Every rectangle is a parallelogram.   *(Lesson 8–4)*

37. **Short Response**   Draw two intersecting lines and name a pair of supplementary angles formed by them.   *(Lesson 3–5)*

---

## Quiz 1    Lessons 16–1 and 16–2

▶ **Solve each system of equations by graphing.**   *(Lesson 16–1)*

**1.** $y = \frac{1}{2}x + 3$
   $x = 4$

**2.** $-x + y = 6$
   $-3x + 3y = 3$

**Use substitution or elimination to solve each system of equations.**   *(Lesson 16–2)*

**3.** $x = 5 - y$
   $3y = 3x + 1$

**4.** $2x + 3y = 11$
   $4x - 7y = 35$

**5. Sales**   Used Music sells CDs for $9 and cassette tapes for $4. Suppose $x$ represents the number of CDs sold and $y$ represents the number of cassette tapes sold. Then $x + y = 38$ describes the total number sold in one afternoon and $9x + 4y = 297$ describes the total sales during that time.   *(Lesson 16–2)*

   **a.** Solve the system of equations.

   **b.** How many more CDs than cassette tapes were sold?

The map below shows that the center of a hurricane has moved from 30°N latitude, 75°W longitude to 32°N latitude, 78°W longitude.
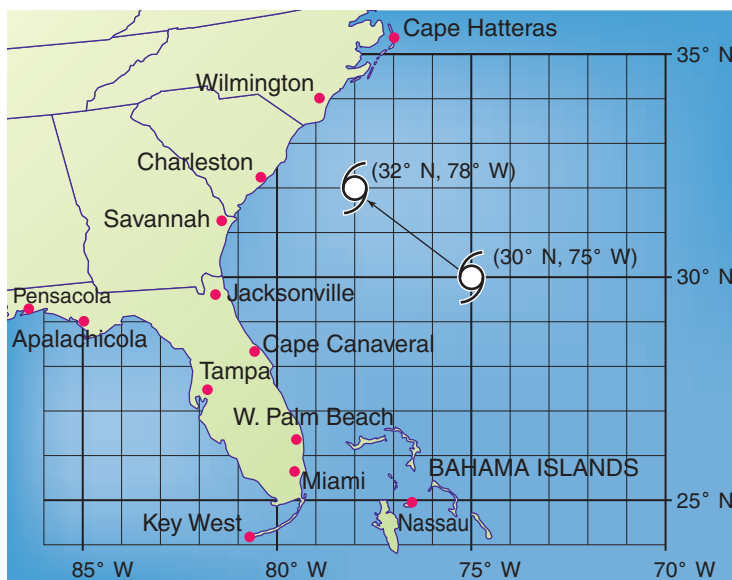


Tracking a hurricane by its latitude and longitude is like the translation of a geometric figure on a coordinate plane. In Lesson 5–3, you learned that a **translation** is a figure that is moved from one position to another without turning. In the figure below, $\triangle ABC$ is translated 3 units right and 5 units up. The image is $\triangle A'B'C'$.

### Reading Geometry

Read the symbol ' as *prime*.

*preimage*

$A(-1, 1)$    +3    +5

$B(5, 4)$    +3    +5

$C(4, 1)$    +3    +5

*image*

$A'(2, 6)$

$B'(8, 9)$

$C'(7, 6)$
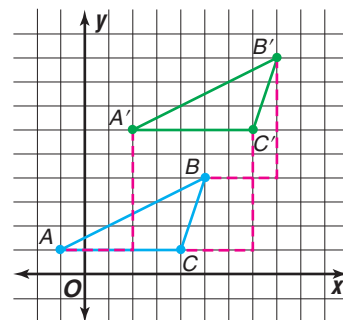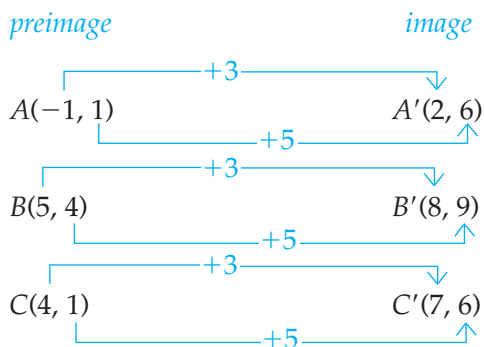


This translation can be written as the ordered pair (3, 5). To find the image of any point of $\triangle ABC$, add 3 to the $x$-coordinate of the ordered pair and add 5 to the $y$-coordinate.

$$(x, y) \rightarrow (x + 3, y + 5)$$

CONTENTS

| **Definition of Translation** | **Words:** | A translation is the sliding of a figure from one position to another. |
|---|---|---|
| | **Model:** |  |
| | **Symbols:** | $(x, y)$ translated by $(a, b)$ is $(x + a, y + b)$. |
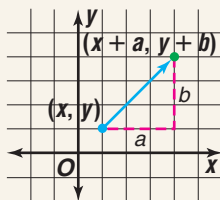
**Example** ❶ Graph $\triangle XYZ$ with vertices $X(3, -2)$, $Y(-1, 0)$, and $Z(4, 1)$. Then find the coordinates of its vertices if it is translated by $(-2, 3)$. Graph the translation image.
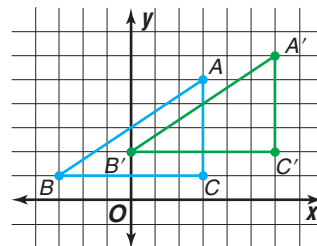
To find the coordinates of the vertices of $\triangle X'Y'Z'$, add $-2$ to each $x$-coordinate and add 3 to each $y$-coordinate of $\triangle XYZ$: $(x - 2, y + 3)$.

$X(3, -2) + (-2, 3) \rightarrow$
$X'(3 + (-2), (-2) + 3)$ or $X'(1, 1)$

$Y(-1, 0) + (-2, 3) \rightarrow$
$Y'(-1 + (-2), 0 + 3)$ or $Y'(-3, 3)$

$Z(4, 1) + (-2, 3) \rightarrow$
$Z'(4 + (-2), 1 + 3)$ or $Z'(2, 4)$
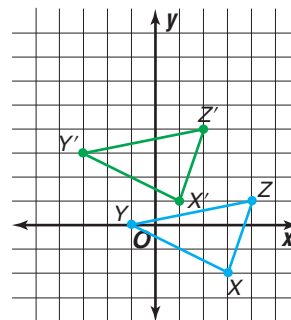


The coordinates of the vertices of $\triangle X'Y'Z'$ are $X'(1, 1)$, $Y'(-3, 3)$, and $Z'(2, 4)$.

**Your Turn**

Graph $\triangle LMN$ with vertices $L(0, 3)$, $M(4, 2)$, and $N(-3, -1)$. Then find the coordinates of its vertices if it is translated by $(-2, -2)$. Graph the translation image.

# Check for Understanding

**Communicating Mathematics**

1. Write a sentence to describe a figure that is translated by $(-2, -4)$.

2. **Compare** the lengths of corresponding sides in $\triangle ABC$ and $\triangle A'B'C'$ on the next page. What can you conclude about the relationship between $\triangle ABC$ and its image? Explain.

**◀ CONTENTS ▶**

3. **You Decide?** Mikasi thinks the figure at the right shows the translation $(3, -1)$. Nicole thinks it shows the translation $(3, 1)$. Who is correct? Explain your reasoning.

4. Find the coordinates of the vertices of $\triangle XYZ$ if it is translated by $(2, 1)$. Then graph the translation image.
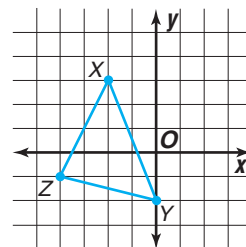
5. Graph $\triangle RST$ with vertices $R(5, 2)$, $S(-2, 4)$, and $T(-1, 1)$. Then find the coordinates of its vertices if it is translated by $(1, -4)$. Graph the translation image.

6. **Engineering** In 1870, the Cape Hatteras Lighthouse was built 1600 feet from the ocean. By July 1999, the shore was just a few feet away. Engineers placed the lighthouse on tracks and moved it inland to protect it from soil erosion. Suppose three of the vertices at its base were $L(16, 7)$, $M(22, 12)$, and $N(30, 7)$. If the lighthouse was moved 540 feet up the shoreline and 1506 feet inland, the translation can be given by $(540, -1506)$. Find the coordinates of $L'$, $M'$, and $N'$ after the move.
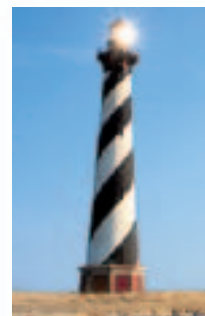
Cape Hatteras Lighthouse, North Carolina

# Exercises

**Practice**

**Find the coordinates of the vertices of each figure after the given translation. Then graph the translation image.**
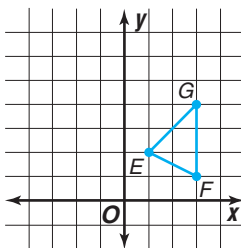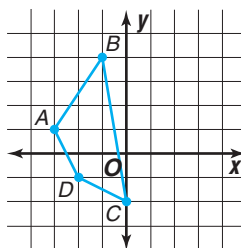
7. $(-2, 2)$
8. $(3, 0)$
9. $(-4, -1)$
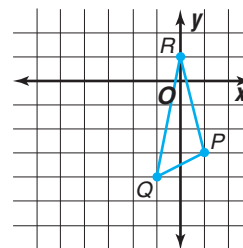
**Homework Help**

| For Exercises | See Examples |
|---|---|
| 7–14 | 1 |

**Extra Practice**
See page 756.

**Graph each figure. Then find the coordinates of the vertices after the given translation and graph the translation image.**

| | Figure | Vertices | Translated By: |
|---|---|---|---|
| **10.** | △JKL | J(4, 0), K(2, −1), L(0, 1) | (0, −4) |
| **11.** | △XYZ | X(−5, −2), Y(−2, 7), Z(1, −6) | (6, 2) |
| **12.** | square QRST | Q(2, 1), R(4, 3), S(2, 5), T(0, 3) | (−3, 5) |

**13.** Quadrilateral $TUVW$ has vertices $T(8, 1)$, $U(0, −7)$, $V(−10, −3)$, and $W(−5, 2)$. Suppose you translate the figure 3 units right and 2 units down. What are the coordinates of its vertices $T'$, $U'$, $V'$, and $W'$? Graph the translation image.

**14.** Triangle $BCD$ has vertices $B(2, 4)$, $C(6, 1)$, and $D(0, 1)$. Suppose $△BCD$ is translated along the $y$-axis until $D'$ has coordinates $(0, −8)$.

    **a.** Describe this translation using an ordered pair.

    **b.** Find the coordinates of $B'$ and $C'$.

**Applications and Problem Solving**

**15.** **Art**  A *frieze* is a pattern running across the upper part of a wall. Make a frieze by drawing a basic pattern on grid paper and then translating it as many times as you can.
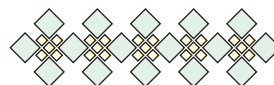
**16.** **Music**  When music is *transposed* to a different key, each note is moved the same distance up or down the musical scale. The music below shows "Deck the Halls" in two different keys.



Key of F



Key of G

Copy the music at the left and translate it into the key of G. The first measure is done for you.

**17.** **Critical Thinking**  Suppose a triangle is translated by $(3, −2)$ and then the image is translated by $(−3, 2)$. Without graphing, what is the final position of the figure? Explain your reasoning.

**Mixed Review**

**18.** Solve the system of equations $y = 4x − 5$ and $2x + 7y = 10$.  *(Lesson 16–2)*

**19.** Write the equation of a circle that has a diameter of 9 units and its center at $(6, −21)$.  *(Lesson 14–6)*

**Find the missing measures. Write all radicals in simplest form.**

**20.**  *(Lesson 13–3)*



**21.**  *(Lesson 13–2)*



**Standardized Test Practice**

Ⓐ Ⓑ Ⓒ Ⓓ

**22.** **Multiple Choice**  Find the surface area of a bowling ball if its diameter is 8.5 inches. Round to the nearest hundredth.  *(Lesson 12–6)*

  Ⓐ  106.81 in$^2$   Ⓑ  226.98 in$^2$   Ⓒ  229.98 in$^2$   Ⓓ  907.92 in$^2$

**CONTENTS**

## Animator

If you loved being scared by Godzilla or by the dinosaurs in *Jurassic Park*, thank an animator. Animators create sequences of motion-based art for the motion picture and television industry. Most animators use computer animation programs.

Computer animation results from thousands of changes that occur on a coordinate plane. Let's start with one change. This graphing calculator program draws a triangle and translates it a certain distance horizontally and vertically.

```
PROGRAM:MAPPING              L2(3))
:Disp "ENTER VERTICES"       :Line(L1(1), L2(1), L1(3),
 For(N, 1, 3)                 L2(3))
:Input "X", X                :Pause
:X→L1(N)                     :Input "HORIZONTAL MOVE", H
:Input "Y", Y                :Input "VERTICAL MOVE", V
:Y→L2(N)                     :Line(L1(1)+H, L2(1)+V,
:End                          L1(2)+H, L2(2)+V)
:ClrDraw                     :Line(L1(2)+H, L2(2)+V,
:ZStandard                    L1(3)+H, L2(3)+V)
:Line(L1(1), L2(1), L1(2),   :Line(L1(1)+H, L2(1)+V,
 L2(2))                       L1(3)+H, L2(3)+V)
:Line(L1(2), L2(2), L1(3),   :Stop
```

1. Use the program to draw $\triangle ABC$ and its translated image.
   a. $A(2, 3)$, $B(5, 9)$, $C(0, 4)$; horizontal move: 2; vertical move: 1
   b. $A(-4, 3)$, $B(1, 7)$, $C(3, -2)$; horizontal move: 4; vertical move: $-3$
   c. $A(3, 6)$, $B(5, 2)$, $C(-2, 8)$; horizontal move: $-4$; vertical move: $-5$
2. Modify the program so that the first triangle is erased before the image is drawn.

### FAST FACTS About Animators

**Working Conditions**
- work as part of a team
- use computers for extended periods of time

**Education**
- internship or college degree in graphic art, computer-aided design, or visual communications
- Knowledge and training in computer techniques are critical.

**Employment**

Animators who stay at least 10 years in the field — 20%

Animators who pursue other careers — 80%

*inter*NET CONNECTION
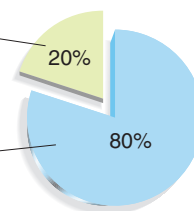
**Career Data** For the latest information on a career as an animator, visit:
www.geomconcepts.com

CONTENTS

# Reflections

## What You'll Learn
You'll learn to investigate and draw reflections on a coordinate plane.

## Why It's Important
**Printing** The use of reflections is important in some types of printing.
*See Exercise 15.*

The photograph at the right shows the reflection of an ancient Roman bridge in the Tiber River. Every point on the bridge has a corresponding point on the water. In mathematics, this type of one-to-one correspondence is also called a **reflection**.

In the following activity, you'll investigate reflections over coordinate axes.

Tiber River, Rome

---

## Hands-On Geometry

**Materials:**  grid paper    tracing paper    straightedge

**Step 1**  On a coordinate graph, use a straightedge to draw a quadrilateral with vertices $A(1, 0)$, $B(2, 3)$, $C(4, 1)$, and $D(3, -3)$.

**Step 2**  Fold a piece of tracing paper twice to create coordinate axes. Unfold the paper and label the $x$- and $y$-axes.

**Step 3**  Place the tracing paper on top of the coordinate graph, lining up the axes on both pieces of paper. Trace quadrilateral $ABCD$.

**Step 4**  Turn over the tracing paper so that the figure is flipped over the $y$-axis.
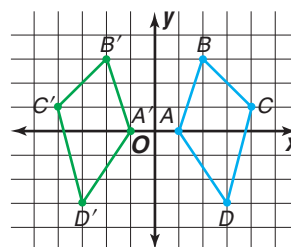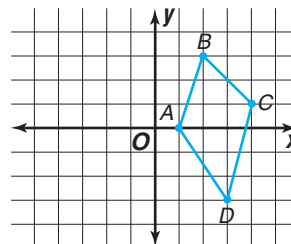
### Try These
1. Name the coordinates for the reflected quadrilateral $A'B'C'D'$ from Step 4.
2. Repeat Step 4, but this time flip $ABCD$ over the $x$-axis. Name the coordinates for this reflected quadrilateral $A''B''C''D''$.
3. Compare the coordinates of the original quadrilateral with the coordinates of each reflection. What do you notice?

---

The results of this activity are stated in the following definition.

| | |
|---|---|
| **Definition of Reflection** | **Words:** A reflection flips a figure over a line. |

**Model:**

$(-x, y)$    $(x, y)$

$O$    $x$

$(x, -y)$
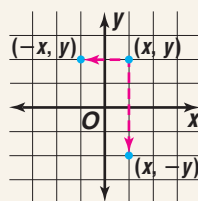
**Symbols:**
reflection over $x$-axis:
$(x, y) \rightarrow (x, -y)$

reflection over $y$-axis:
$(x, y) \rightarrow (-x, y)$

---

**Examples**

**1** Graph $\triangle PQR$ with vertices $P(-5, 3)$, $Q(-4, -1)$, and $R(-2, 2)$. Then find the coordinates of its vertices if it is reflected over the $x$-axis and graph its reflection image.
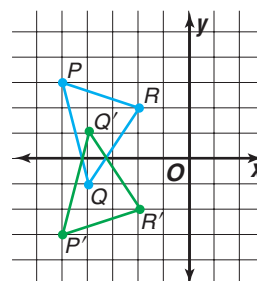
To find the coordinates of the vertices of $\triangle P'Q'R'$, use the definition of reflection over the $x$-axis: $(x, y) \rightarrow (x, -y)$.

*preimage*    *image*

$P(-5, 3) \rightarrow P'(-5, -3)$

$Q(-4, -1) \rightarrow Q'(-4, 1)$

$R(-2, 2) \rightarrow R'(-2, -2)$

The vertices of $\triangle P'Q'R'$ are $P'(-5, -3)$, $Q'(-4, 1)$, and $R'(-2, -2)$.

**Art Link**

*Real World*

**2** A *dyrnak gul* is a symmetrical motif found in oriental carpets. Reflect the design over the $y$-axis below to find the coordinates of $J'$, $K'$, $L'$, and $M'$. Then graph the reflection points.

To find the coordinates of $J'$, $K'$, and $L'$, use the definition of reflection over the $y$-axis: $(x, y) \rightarrow (-x, y)$.

*preimage*    *image*

$J(-3, 2) \rightarrow J'(3, 2)$

$K(-5, 0) \rightarrow K'(5, 0)$

$L(-8, 0) \rightarrow L'(8, 0)$

$M(-3, -2) \rightarrow M'(3, -2)$
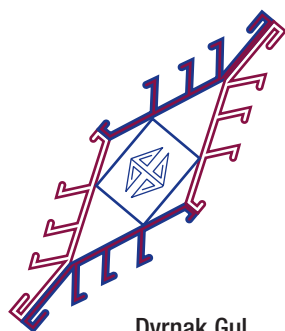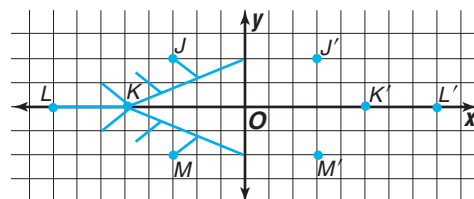
**Dyrnak Gul**

**Your Turn**

Graph each figure. Then find the coordinates of the vertices after a reflection over the given axis and graph the reflection image.

**a.** $\triangle HIJ$: $H(3, 1)$, $I(4, 4)$, $J(-2, 3)$, $x$-axis

**b.** trapezoid $LMNP$: $L(-3, -3)$, $M(-3, 2)$, $N(-1, 2)$, $P(1, -3)$, $y$-axis

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## Check for Understanding

1. **Explain** whether the translated image of a figure can ever be the same as its reflected image. Show an example to support your answer.

2. Suppose you reflect a figure over the $x$-axis and then reflect that image over the $y$-axis. Is this double reflection the same as a translation? Explain why or why not.

3. **You Decide?**   Manuel says that if a figure with parallel sides is reflected over the $x$-axis, the reflected figure will also have parallel sides. Natalie says that such a reflected figure may not always have parallel sides. Who is correct? Explain why.

4. Graph $\triangle ABC$ with vertices $A(-4, -4)$, $B(0, 2)$, and $C(1, -3)$. Then find the coordinates of its vertices if it is reflected over the $x$-axis and graph the reflection image.

5. Find the coordinates of the vertices of quadrilateral $HIJK$ if it is reflected over the $y$-axis. Then graph the reflection image.
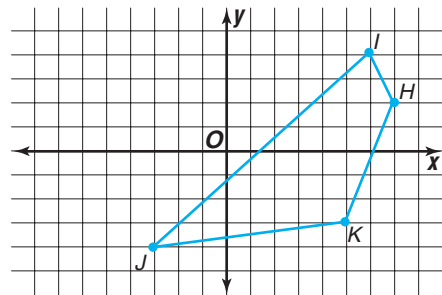
6. **Architecture**   To preserve the symmetry of his house, George Washington had the second window from the left, upstairs, painted on. If this fake window has coordinates $A(-22, -0.5)$, $B(-18, -0.5)$, $C(-18, -4)$, and $D(-22, -4)$, then what are the coordinates of its reflection over the $y$-axis, window $A'B'C'D'$?
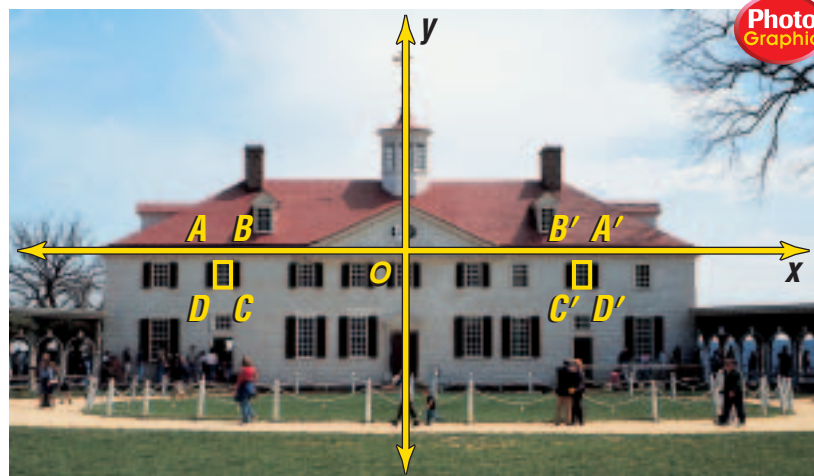
George Washington's home, Mount Vernon, Virginia

# Exercises • • • • • • • • • • • • • • • • • • • • •

**Find the coordinates of the vertices of each figure after a reflection over the given axis. Then graph the reflection image.**
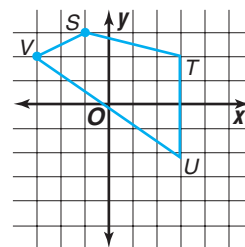
**7.** $y$-axis
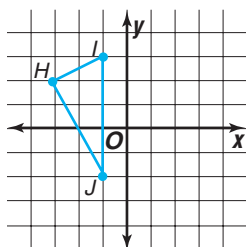
**8.** $x$-axis
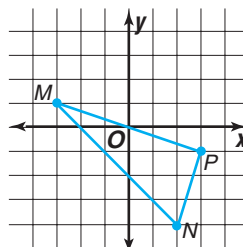
**9.** $x$-axis

**Graph each figure. Then find the coordinates of the vertices after a reflection over the given axis and graph the reflection image.**

| | Figure | Vertices | Reflected Over: |
|---|---|---|---|
| **10.** | $\triangle EFG$ | $E(-1, 2)$, $F(2, 4)$, $G(2, -4)$ | $y$-axis |
| **11.** | $\triangle PQR$ | $P(1, 2)$, $Q(4, 4)$, $R(2, -3)$ | $x$-axis |
| **12.** | quadrilateral $VWXY$ | $V(0, -1)$, $W(1, 1)$, $X(4, -1)$, $Y(1, -6)$ | $y$-axis |

**13.** Suppose $\triangle CDE$ is reflected over the line $x = 2$.

   **a.** Find the coordinates of the vertices after the reflection.

   **b.** Graph the reflection image.
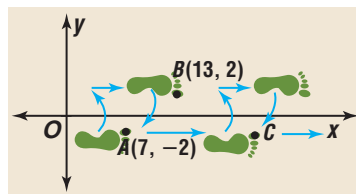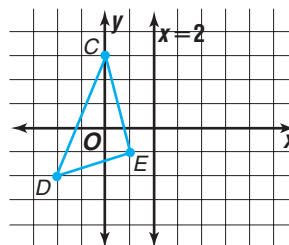
**14.** The combination of a reflection and a translation is called a *glide reflection*. An example of this is a set of footprints.

   **a.** State the steps for finding the coordinates of each footprint, given points $A(7, -2)$ and $B(13, 2)$.

   **b.** Find the coordinates for point $C$.

## Applications and Problem Solving

**15.** **Printing**  In *lithographic printing*, a printed image is a reflection of an inked surface. In the letter A shown at the right, suppose points $H(-5, -7)$, $J(0, -4)$, and $K(7, -12)$ lie on the inked surface. Name the coordinates of the corresponding points on the printed image if the inked surface is reflected over the $x$-axis.

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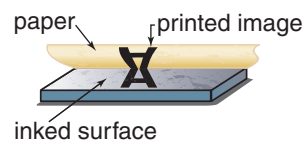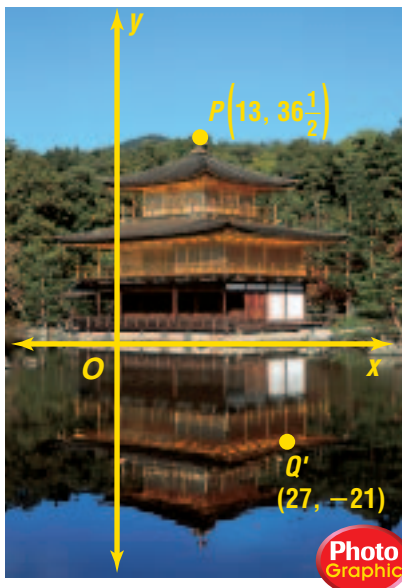
16. **Nature** The *pagoda*, or Far Eastern tower, shown at the left is reflected in the lake.

   a. Suppose point $P\left(13, 36\frac{1}{2}\right)$ lies on the pagoda. Name the coordinates of this point reflected in the water.

   b. If the reflected point $Q'(27, -21)$ appears in the water, what are the coordinates of the original point $Q$ on the pagoda?

17. **Critical Thinking** Triangle *CDE* is the preimage of a double reflection over line $j$ and then line $k$. Copy the figure at the right. Label the vertices of the first image $C'$, $D'$, and $E'$. Then label the vertices of the second image $C''$, $D''$, and $E''$.
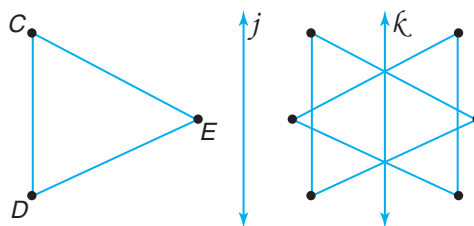


**Mixed Review**

18. Graph $\triangle JKL$ with vertices $J(-4, 4)$, $K(-1, 1)$, and $L(-3, -2)$. Then find the coordinates of its vertices if it is translated by $(3, -2)$. Graph the translation image. *(Lesson 16–3)*

19. **Logic** Write a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, write *no valid conclusion*. *(Lesson 15–1)*

   (1) If two angles are complementary to the same angle, then they are congruent.

   (2) $m\angle A + m\angle R = 90$ and $m\angle D + m\angle R = 90$

20. Find sin $A$. Round to four decimal places. *(Lesson 13–5)*

21. **Short Response** Simplify $\sqrt{3} \cdot \sqrt{18}$. *(Lesson 13–1)*

22. **Multiple Choice** What is the value of $x$? *(Lesson 6–4)*

   Ⓐ 62

   Ⓑ 7

   Ⓒ 56

   Ⓓ 110
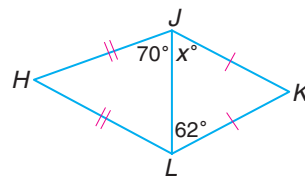
CONTENTS

# 16–5 Rotations

## What You'll Learn

You'll learn to investigate and draw rotations on a coordinate plane.

## Why It's Important

**Art**  Rotations are often used to create patterns for stained glass windows. *See Exercise 15.*
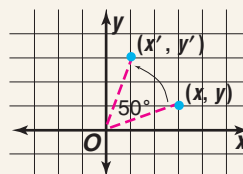
A trapeze artist swings on the trapeze in a circular motion. This type of movement around a fixed point is called a **turn** or a **rotation**. The fixed point is called the **center of rotation**. This point may be in the center of an object, as in a spinner, or outside an object, as with the swinging trapeze artist whose center of rotation is at the top of the trapeze.
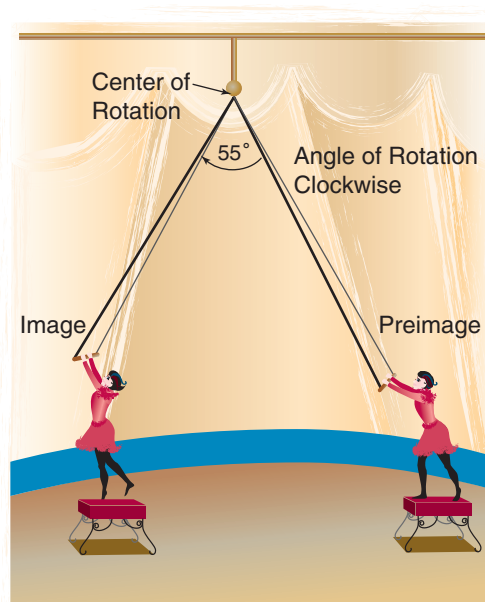


| | |
|---|---|
| **Definition of Rotation** | **Words:**  A rotation is the turning of a figure about a fixed point. |
| | **Model:**  |
| | **Symbols:**  $(x, y)$ rotated 50° about the origin is $(x', y')$. |

Rotations can be either clockwise or counterclockwise. As with translations and reflections, there is a one-to-one correspondence between the preimage and the image, and the resulting image after a rotation is congruent to the original figure.
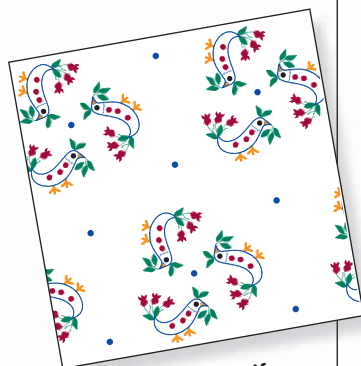
In each case below, $\overline{JK}$ is rotated 60° counterclockwise about the origin.

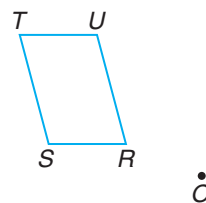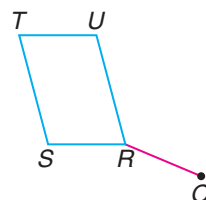You can use tracing paper to rotate figures whose centers of rotation are on the figure or outside of the figure.

**Real World**
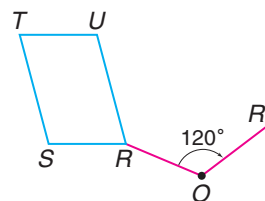
**Example** ──① **Art Link**

**The pattern in the Hungarian needlework at the left is formed by 120°-rotations. Rotate quadrilateral *RSTU* 120° clockwise about point *O* by tracing the figure.**

**Step 1** Draw a segment from point *O* to *R*.

**Step 2** Use a protractor to draw an angle 120° clockwise about point *O*. Draw segment $\overline{OR'}$ congruent to $\overline{OR}$.

**Step 3** Trace the figure on a sheet of tracing paper. Label the corresponding vertices *R'*, *S'*, *T'*, and *U'*.

**Step 4** Place a straight pin through the two pieces of paper at point *O*. Rotate the top paper clockwise, keeping point *O* in the same position, until the figure is rotated 120°.
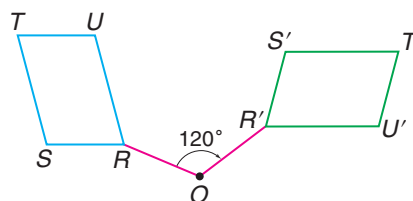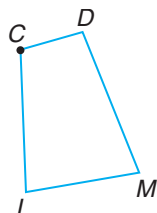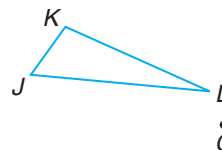
*R'S'T'U'* is the rotation image of *RSTU*.

Bird Motif, Northern Hungary

**Your Turn**

**Rotate each figure about point *C* by tracing the figure. Use the given angle of rotation.**

**a.** 110° clockwise

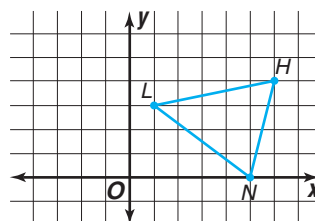**b.** 60° counterclockwise

www.geomconcepts.com/extra_examples

**CONTENTS**

You can also draw rotations on the coordinate plane without using tracing paper.

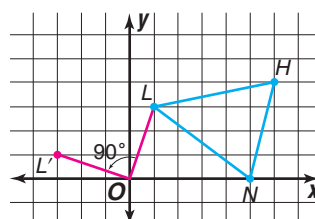**2** **Find the coordinates of the vertices of △LNH if it is rotated 90° counterclockwise about the origin. Graph the rotation image.**
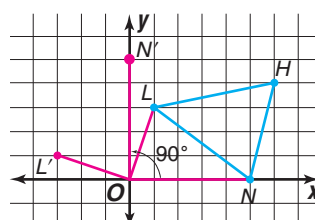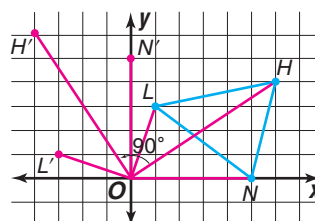


**Step 1** Draw a segment from the origin to point L.

**Step 2** Use a protractor to draw ∠LOL′ so that its measure is 90 and OL = OL′.
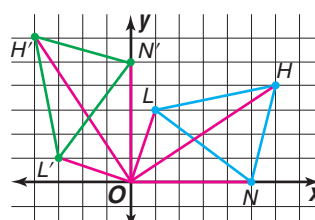


**Step 3** Draw a segment from the origin to point N.

**Step 4** Use a protractor to draw ∠NON′ so that its measure is 90 and ON = ON′.



**Step 5** Draw a segment from the origin to point H.

**Step 6** Use a protractor to draw ∠HOH′ so that its measure is 90 and OH = OH′.



**Step 7** Connect the points to form △L′N′H′.

The vertices of △L′N′H′ are L′(−3, 1), N′(0, 5), and H′(−4, 6).



*Your Turn*

**Graph each figure. Then find the coordinates of the vertices after the given rotation about the origin and graph the rotation image.**

**c.** segment *LM* with vertices *L*(−2, 4) and *M*(0, 6) rotated 90° clockwise

**d.** △*STR* with vertices *S*(0, 0), *T*(0, −4), and *R*(3, −2), rotated 180° counterclockwise

You can use a TI–83/84 Plus calculator to rotate figures about a selected point.

## Graphing Calculator Exploration

**Step 1** Construct a triangle by using the Triangle tool on the [F2] menu. Label this triangle *ABC*.

**Step 2** Draw the point about which you want to rotate the triangle. Label it Point *P*.

**Step 3** Use the segment tool on [F2] to draw two segments with a common endpoint. Then use the Angle Measure tool on [F5] to find the measure of the angle. This will be your angle of rotation.

**Step 4** Use the Rotation tool on [F4] to rotate the triangle about the point using the angle. Label the corresponding vertices of the rotated image as *X*, *Y*, and *Z*.

**Step 5** Draw segments from *P* to the corresponding vertices *B* and *Y*. Use the Display tool on [F5] to make these segments stand out.
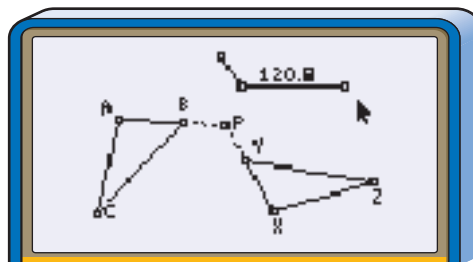
### Try These

**1.** Use the angle tool on [F6] to measure ∠*BPY* and ∠*CPZ*.

**2.** What is the relation of the angle measures from Exercise 1 to the angle measure you used for the rotation?

**3.** Describe what happens to the angle measures if you drag point *P* to a different location.

**4.** Change the angle measure for the rotation from 120° to 110°. What happens to the image of the original triangle and the angle measures?

## Check for Understanding

**Communicating Mathematics**

**1. Study** Example 2. What can you conclude about the coordinates of the image and preimage in a 90° rotation?

**2. Describe** two techniques for finding the rotation of a figure about a fixed point.

**3.** **You Decide** Diem rotated quadrilateral *HIJK* 305° in a clockwise direction about the origin. Lakesha said she could have rotated the quadrilateral 55° in a counterclockwise direction and found the same image. Is Lakesha correct? Explain why or why not.
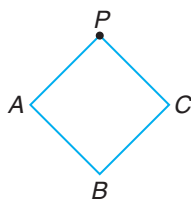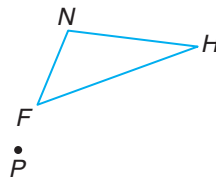
*Vocabulary*
turn
rotation
center of rotation

**Rotate each figure about point *P* by tracing the figure. Use the given angle of rotation.**

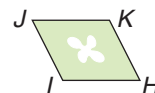Example 1

**4.** 130° counterclockwise

**5.** 85° clockwise

Example 2

**6.** Graph △*QRS* with vertices *Q*(1, 1), *R*(4, −3), and *S*(1, −3). Then find the coordinates of its vertices if the figure is rotated clockwise 180° about the origin. Graph the rotation image.

Example 1

**7. Art** Rotate *HIJK* 120° clockwise about *H* and 120° counterclockwise about *H* to complete the Islamic mosaic pattern.
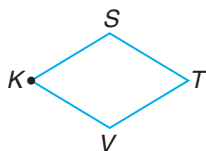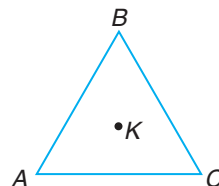
# Exercises

**Rotate each figure about point *K* by tracing the figure. Use the given angle of rotation.**

**8.** 90° clockwise
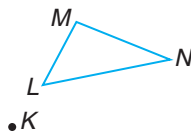
**9.** 120° counterclockwise

### Homework Help

| For Exercises | See Examples |
|---|---|
| 8, 10–12, 14 | 1 |
| 9, 13, 17 | 2 |

**Extra Practice**
See page 757.

**10.** 60° clockwise

**11.** 180° clockwise

**Find the coordinates of the vertices of each figure after the given rotation about the origin. Then graph the rotation image.**

**12.** 90° clockwise
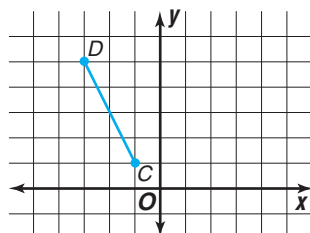
**13.** 180° counterclockwise
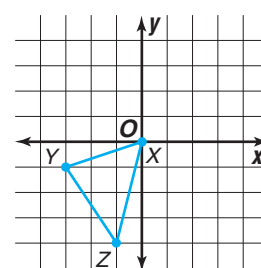
**14.** Segment $QR$ has endpoints $Q(-4, 3)$ and $R(0, 1)$. Find the coordinates of the vertices of the segment if it is rotated 90° clockwise about the origin.

**Applications and Problem Solving**

**15. Art** In the stained glass window at the right, how are rotations used to create the pattern? Use tracing paper and label the figures to show your answer.

**16. Art** The design at the left is by Canadian scientist Francois Brisse. Describe the transformations he could have used to create the design.

Notre Dame Cathedral, Paris

**17. Critical Thinking** Triangle $ABC$ has been rotated in a counterclockwise direction about point $D$. Find the angle of rotation.

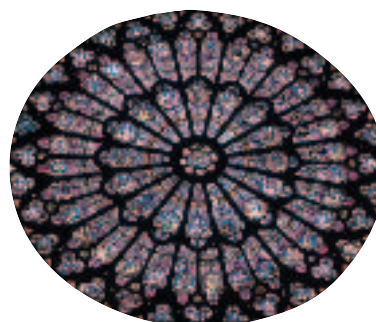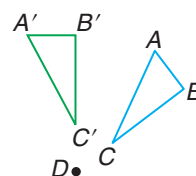**Mixed Review**

**18.** Triangle $QRS$ has vertices $Q(-2, 6)$, $R(1, 1)$, and $S(4, -3)$. Find the coordinates of its vertices if it is reflected over the $x$-axis. *(Lesson 16–4)*

**19.** Solve the system of equations $2x - y = 8$ and $6x - y = 7$. *(Lesson 16–2)*

**20.** Draw a figure and write a two-column proof to show that opposite angles of a rhombus are congruent. *(Lesson 15–4)*

**Standardized Test Practice**
Ⓐ Ⓑ Ⓒ Ⓓ

**21. Short Response** If $\overline{ML}$ and $\overline{MN}$ are tangent to $\odot E$ at $L$ and $N$ respectively, then $\overline{ML} \cong$ ___?___ . *(Lesson 14–2)*

**22. Multiple Choice** A purse has an original price of $45. If the purse is on sale for 25% off, what is the sale price? *(Percent Review)*

Ⓐ $11.25　　Ⓑ $56.25　　Ⓒ $20.00　　Ⓓ $33.75

---

## Quiz 2　　Lessons 16–3 through 16–5

▶ **Graph each figure. Then find the coordinates of the vertices after the given transformation and graph the image.**

| Figure | Vertices | Transformation |
|---|---|---|
| **1.** $\triangle HIJ$ | $H(-2, 1)$, $I(2, 3)$, $J(0, 0)$ | translated by $(2, 4)$ *(Lesson 16–3)* |
| **2.** $\triangle EFG$ | $E(2, 0)$, $F(-1, -1)$, $G(1, 3)$ | translated by $(-3, 1)$ *(Lesson 16–3)* |
| **3.** $\triangle ABC$ | $A(-4, -2)$, $B(-1, -4)$, $C(2, -2)$ | reflected over $x$-axis *(Lesson 16–4)* |
| **4.** quadrilateral $QRST$ | $Q(1, 0)$, $R(2, -3)$, $S(0, -3)$, $T(-3, -1)$ | reflected over $y$-axis *(Lesson 16–4)* |

**5. Design** Describe how rotation was used to create the design at the right and list the angles of rotation that were used. *(Lesson 16–5)*
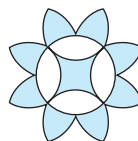
www.geomconcepts.com/self_check_quiz

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# 16-6 Dilations

**12**

## What You'll Learn

You'll learn to investigate and draw dilations on a coordinate plane.

## Why It's Important

**Publishing** Artists must understand dilations when sizing art for textbooks. *See Exercise 11.*

Alexis went to the camera shop to have a picture enlarged from a 4 × 6 to an 8 × 12. This transformation is called a **dilation**.
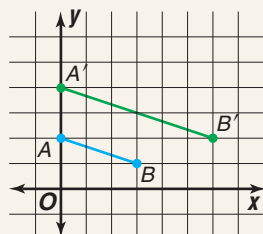
**8**

**Photo Graphic**

In previous lessons, we learned that in translations, reflections, and rotations, the image and preimage are congruent. Dilations are different because they alter the size of an image, but not its shape. The ratio of a dilated image to its preimage is called the *scale factor*.

**6**

**4**
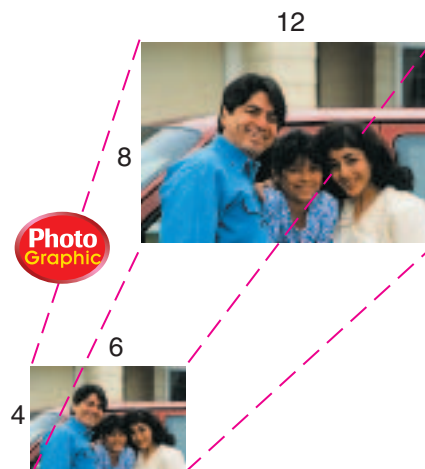
| Definition of Dilation | **Words:** A dilation reduces or enlarges a figure by a scale factor $k$. |
|---|---|
| | **Model:** |
| | **Symbols:** $A'B' = k(AB)$ |

A figure is enlarged if the scale factor is greater than 1, and reduced if the scale factor is between 0 and 1. Note this in the following examples.

**Real World**

**Example 1**

**Animal Link**

$\overline{CM}$ with endpoints $C(4, 2)$ and $M(2, 6)$ represents the length of a baby dolphin. If the mother dolphin is one and a half times the size of her baby, find the coordinates of the dilation image of $\overline{CM}$ with a scale factor of 1.5, and graph its dilation image.

Since $k > 1$, this is an enlargement. To find the dilation image, multiply each coordinate in the ordered pairs by 1.5.

*preimage*    *image*      *preimage*    *image*

     — × 1.5 —         — × 1.5 —

$C(4, 2)$     $C'(6, 3)$     $M(2, 6)$     $M'(3, 9)$

     — × 1.5 —         — × 1.5 —
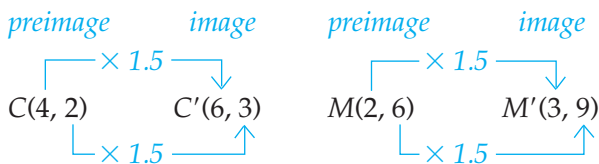
The coordinates of the endpoints of the dilation image are $C'(6, 3)$ and $M'(3, 9)$.
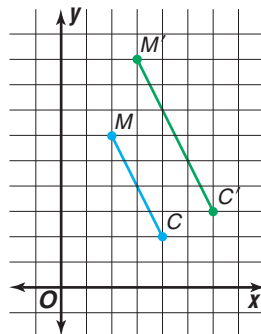
**Graph △QRS with vertices Q(−2, 6) and R(8, 0), and S(6, 5). Then find the coordinates of the dilation image with a scale factor of $\frac{1}{2}$ and graph its dilation image.**

Since $k < 1$, this is a reduction. To find the dilation image, multiply each coordinate in the ordered pairs by $\frac{1}{2}$.

*preimage*          *image*

$Q(-2, 6)$  $\times \frac{1}{2}$  $Q'(-1, 3)$

$R(8, 0)$  $\times \frac{1}{2}$  $R'(4, 0)$
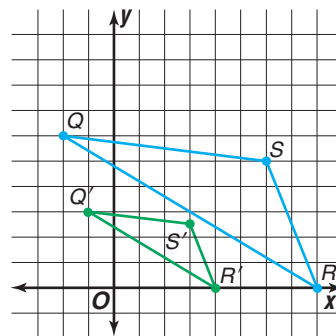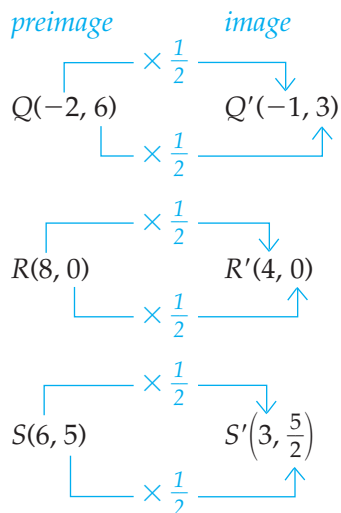
$S(6, 5)$  $\times \frac{1}{2}$  $S'\left(3, \frac{5}{2}\right)$



The coordinates of the vertices of the dilation image are $Q'(-1, 3)$, $R'(4, 0)$, and $S'\left(3, \frac{5}{2}\right)$.

### Your Turn

**Graph each figure. Then find the coordinates of the dilation image for the given scale factor $k$, and graph the dilation image.**

**a.** △GHI with vertices $G(0, -2)$, $H(1, 3)$, and $I(4, 1)$; $k = 2$

**b.** $\overline{LN}$ with endpoints $L(8, 8)$ and $N(4, 5)$; $k = \frac{1}{4}$

## Check for Understanding

**Communicating Mathematics**

1. **Explain** how you can determine whether a dilation is a reduction or an enlargement.

2. **Compare and contrast** the difference between a dilation and the other transformations that you have studied.

3. **Write** about dilations that you can find in everyday life. Explain whether they are reductions or enlargements and estimate what the scale factors might be.

*Vocabulary*

dilation

www.geomconcepts.com/extra_examples

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⊕ **Getting Ready** **Find each product.**

**Sample:** $(2, -1) \times 7$          **Solution:** $(2 \times 7, -1 \times 7)$ or $(14, -7)$

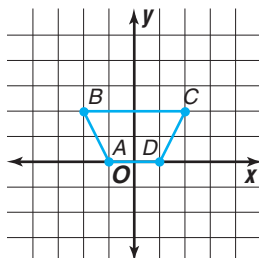**4.** $(0, 7) \times 4$          **5.** $(-8, 2) \times \frac{1}{4}$          **6.** $(10, -6.5) \times 0.5$
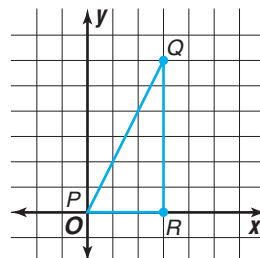
**Examples 1 & 2**

**Find the coordinates of the dilation image for the given scale factor $k$, and graph the dilation image.**

**7.** 2                    **8.** $\frac{1}{3}$



**Graph each figure. Then find the coordinates of the dilation image for the given scale factor $k$, and graph the dilation image.**

**Example 1**    **9.** $\overline{FG}$ with endpoints $F(-2, 1)$ and $G(1, -2)$; $k = 3$

**Example 2**    **10.** $\triangle STW$ with vertices $S(4, 8)$, $T(10, 0)$, and $W(0, -4)$; $k = \frac{1}{4}$

**Example 2**    **11.** **Publishing** When an artist receives art specifications for a textbook, the artist must size the art so that it fits the space on the page. In order for the triangle at the right to fit the given space, the artist must make it $\frac{3}{4}$ its original size.

     **a.** What will be the coordinates of the vertices of the new image?

     **b.** Graph the new image.
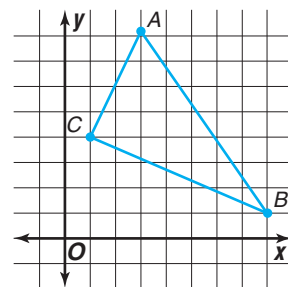


## Exercises

• • • • • • • • • • • • • • • • •

**Practice**

**Find the coordinates of the dilation image for the given scale factor $k$, and graph the dilation image.**

**12.** 2          **13.** $\frac{1}{3}$          **14.** $\frac{1}{4}$

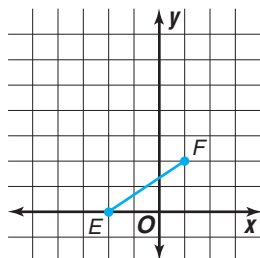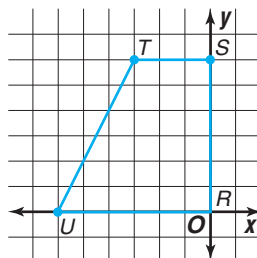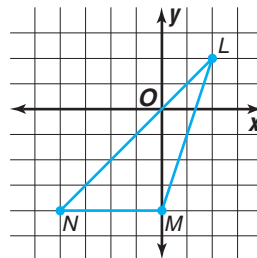| Homework Help | |
|---|---|
| For Exercises | See Examples |
| 12, 15, 17, 20, 23, 26 | 1 |
| 13, 14, 16, 18, 19, 21 | 2 |
| **Extra Practice** | |
| See page 757. | |

◄ **CONTENTS** ►

**Find the coordinates of the dilation image for the given scale factor *k*, and graph the dilation image.**

**15.** 3



**16.** $\frac{1}{2}$



**17.** 4
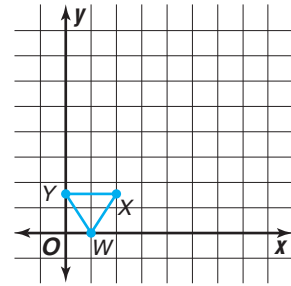


**Graph each figure. Then find the coordinates of the dilation image for the given scale factor *k*, and graph the dilation image.**
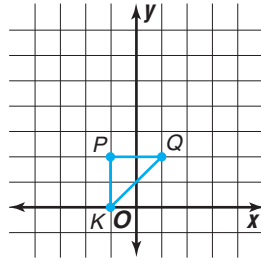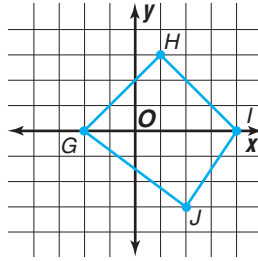
**18.** $\overline{ST}$ with endpoints $S(-2, 4)$ and $T(4, 0)$; $k = \frac{1}{2}$

**19.** $\triangle ABC$ with vertices $A(2, 0)$, $B(0, -6)$, and $C(-4, -4)$; $k = \frac{1}{4}$

**20.** quadrilateral $NPQR$ with vertices $N(1, -2)$, $P(1, 0)$, $Q(2, 2)$, and $R(3, 0)$; $k = 2$

**21.** Graph quadrilateral $JKLM$ with vertices $J(0, 0)$, $K(5, 3)$, $L(7, -2)$, and $M(4, -4)$. Then find the coordinates of the dilation image for the scale factor $\frac{3}{4}$, and graph the dilation image.

**22.** Triangle $J'L'M'$ is the dilation image of $\triangle JLM$. Find the scale factor.
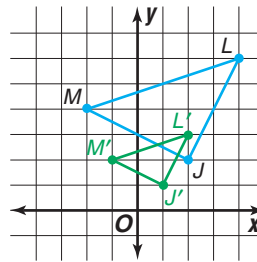
**23.** **Technology**  Mr. Hernandez wants to project a $2 \times 2$-inch slide onto a wall to create an image 64 inches $\times$ 64 inches. If the slide projector makes the image twice as large each yard that it is moved away from the wall, how far away from the wall should Mr. Hernandez place the projector?

**24.** **Photography**  Refer to the application at the beginning of the lesson. Can a $5 \times 7$ picture be directly enlarged to an $11 \times 14$ poster? If so, state the scale factor. If not, explain why.

**25. Art** What type of transformations did artist Norman Rockwell use in his painting at the right? List the transformations and explain how each one was used.

Norman Rockwell,
*Triple Self-Portrait*

**26. Critical Thinking** Suppose rectangle *HIJK* is dilated with a scale factor of 2.

a. Graph the dilation image.

b. Compare the perimeter of the dilation image with the perimeter of the preimage.

c. Compare the area of the dilation image with the area of the preimage.
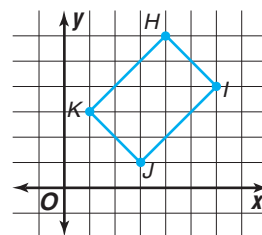
**27.** Rotate △*JLN* 35° counterclockwise about point *K* by tracing the figure. *(Lesson 16–5)*

**28.** Solve the system of equations by graphing. *(Lesson 16–1)*
$$y = 2x$$
$$\frac{1}{2}x - y = 3$$

**29.** Find *GM*. Round to the nearest tenth. *(Lesson 14–5)*

**30.** Find tan *B*. Round to four decimal places. *(Lesson 13–4)*
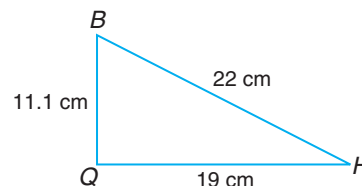
**31. Multiple Choice** The length of the diagonal of a square is $15\sqrt{2}$ feet. Find the length of a side. *(Lesson 13–2)*
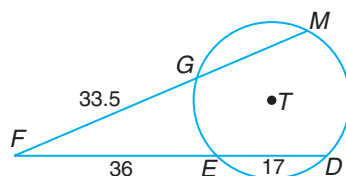
Ⓐ 7.5 ft    Ⓑ 10.6 ft    Ⓒ 30 ft    Ⓓ 15 ft

CONTENTS

# Artists Do Math, Don't They?

## Materials

straightedge

compass

protractor

grid paper

## Composition of Transformations

You can find many examples of transformations in art—repetitions of congruent or similar figures. You can also find **composition of transformations**.

This is when artists use more than one transformation, like in the figure at the right. Can you name two transformations that the artist used?

## Investigate

1. Use a pencil, straightedge, and grid paper to explore composition of transformations.

   a. Draw and label the *x*-axis and *y*-axis on the grid paper. Use a straightedge to draw a triangle with vertices *A*(3, 1), *B*(5, 2), and *C*(4, 6).

   b. Draw the reflection of △*ABC* over the *x*-axis and label it △*A′B′C′*.

   c. Draw the reflection of △*A′B′C′* over the *y*-axis and label it △*A″B″C″*. Record the coordinates of the vertices of △*A″B″C″* in a table.

   d. Rotate △*ABC* 180° clockwise about the origin. Record the coordinates of the vertices in a table. How do these coordinates compare to the coordinates in part c? What can you conclude?

2. Repeat Exercise 1, this time reflecting the triangle over the $x$-axis, and then over the line $y = -7$. Is there a way to get to this image by performing just one transformation? Explain.

3. **Fractals** Mathematician Benoit Mandelbrot used the term *fractal* to describe things in nature that are irregular in shape, such as clouds, coastlines, and trees. Fractals are self-similar; that is, the smaller details of the shape have the same characteristics as the original form. Describe how transformations and compositions of transformations could be used to draw a picture of a fractal.

## Extending the Investigation

In this extension, you will create your own artwork using composition of transformations.

- Use paper and construction tools or geometry software to construct any figure with three or more vertices on a coordinate grid.
- Make a table and record the coordinates for each vertex of the figure.
- Perform two transformations on the figure. This can be any combination of translations, reflections, rotations, and dilations. After each transformation, record the coordinates of the vertices in your table.
- Perform the two transformations again, and record the coordinates of the vertices.
- Repeat this procedure several times.
- Are any of your combinations the same as a single rotation? Explain.

## Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a poster displaying your table and your final artwork.
- Write a paper summarizing how you used multiple transformations to create your artwork.

**NET** **Investigation** For more information on transformations and art, visit: www.geomconcepts.com

CONTENTS

## Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

**NET CONNECTION** **Review Activities**
For more review activities, visit:
**www.geomconcepts.com**

### Geometry

center of rotation *(p. 697)*
composition of transformations *(p. 708)*
dilation *(p. 703)*
reflection *(p. 692)*

rotation *(p. 697)*
translation *(p. 687)*
turn *(p. 697)*

### Algebra

elimination *(p. 682)*
substitution *(p. 681)*
system of equations *(p. 676)*

**State whether each sentence is *true* or *false*. If false, replace the underlined word(s) to make a true statement.**

1. A dilation alters the size of a figure but does not change its shape.
2. Substitution and elimination are methods for solving translations.
3. A reflection is the turning of a figure about a fixed point.
4. A figure is reduced in a dilation if the scale factor is between 0 and 1.
5. In a reflection, a figure is moved from one position to another without turning.
6. Systems of equations can be solved algebraically by elimination.
7. The fixed point about which a figure is rotated is called the center of rotation.
8. A rotation flips a figure over a line.
9. It is better to use substitution to solve a system of equations when one of the equations is already solved for a variable.
10. Another name for a rotation is a reflection.

# Skills and Concepts

## Objectives and Examples

## Review Exercises
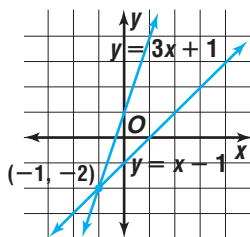
• **Lesson 16–1** Solve systems of equations by graphing.

Solve the system of equations by graphing.
$y = 3x + 1$
$y = x - 1$



The solution is
$(-1, -2)$.

**Solve each system of equations by graphing.**

11. $x + y = 4$
    $y = 2x + 1$
12. $y = x - 5$
    $y = 5x + 7$
13. $3y = 9$
    $15 - y = 9$
14. $y = -x + 6$
    $-2x + y = 6$

www.geomconcepts.com/vocabulary_review

**CONTENTS**

## Objectives and Examples

- **Lesson 16–2**  Solve systems of equations by using the substitution or elimination method.

$$x + 2y = 4 \text{ and } 3x - 2y = 4$$

**Substitution:**
1. Solve the first equation for $x$.
2. Substitute the result into the second equation and solve for $y$.
3. Substitute the value of $y$ into the first equation and solve for $x$.

**Elimination:**
1. Add the equations to eliminate the $y$ terms.
2. Solve the resulting equation for $x$.
3. Substitute the value of $x$ into either original equation and solve for $y$.

## Review Exercises

**State whether *substitution* or *elimination* would be better to solve each system of equations. Explain your reasoning. Then solve the system.**

**15.** $5x + y = 11$
$x - y = 7$

**16.** $-2x + y = 40$
$3x + 7y = 195$

**17.** $7x - y = -10$
$2x + 2y = 52$
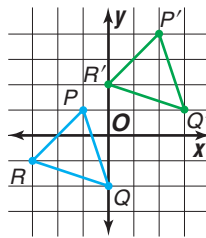
**18.** $-x + 6y = 8$
$x - y = 2$

**19.** $4x + 12y = -4$
$-x - 5y = 3$

---

- **Lesson 16–3**  Investigate and draw translations on a coordinate plane.

Find the coordinates of the vertices of $\triangle PQR$ if it is translated by (3, 3).



$P(-1, 1) + (3, 3) \rightarrow P'(-1 + 3, 1 + 3)$ or $P'(2, 4)$
$Q(0, -2) + (3, 3) \rightarrow Q'(0 + 3, -2 + 3)$ or $Q'(3, 1)$
$R(-3, -1) + (3, 3) \rightarrow R'(-3 + 3, -1 + 3)$ or $R'(0, 2)$

**Graph each figure. Then find the coordinates of the vertices after the given translation and graph its translation image.**

**20.** quadrilateral $ABCD$ with vertices $A(1, 1)$, $B(1, 5)$, $C(7, 5)$, and $D(7, 1)$ translated by $(-2, -3)$

**21.** triangle $LMN$ with vertices $L(-3, 1)$, $M(0, 3)$, and $N(1, -1)$ translated by $(1, 1)$

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- **Lesson 16–4**  Investigate and draw reflections on a coordinate plane.

Find the coordinates of the vertices of $\triangle XYZ$ if it is reflected over the $y$-axis.



$X(-3, 1) \rightarrow X'(3, 1)$
$Y(-2, 3) \rightarrow Y'(2, 3)$
$Z(-1, 0) \rightarrow Z'(1, 0)$

**Graph each figure. Then find the coordinates of the vertices after a reflection over the given axis and graph the reflection image.**

**22.** triangle $DEF$ with vertices $D(0, 3)$, $E(3, 3)$, and $F(1, 1)$ reflected over the $x$-axis

**23.** quadrilateral $STUV$ with vertices $S(0, 2)$, $T(4, 1)$, $U(2, -1)$, and $V(-1, -2)$ reflected over the $y$-axis
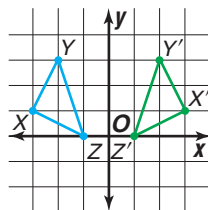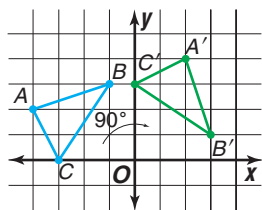
## Objectives and Examples

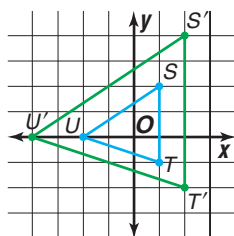- **Lesson 16–5** Investigate and draw rotations on a coordinate plane.

Find the coordinates of the vertices of $\triangle ABC$ if it is rotated 90° clockwise about the origin.

The coordinates of $\triangle A'B'C'$ are $A'(2, 4)$, $B'(3, 1)$, and $C'(0, 3)$.

- **Lesson 16–6** Investigate and draw dilations on a coordinate plane.

Find the coordinates of the dilation image of $\triangle STU$ with a scale factor of 2.
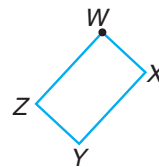
Multiply each coordinate by 2.

$S(1, 2) \longrightarrow S'(2, 4)$

$T(1, -1) \longrightarrow T'(2, -2)$

$U(-2, 0) \longrightarrow U'(-4, 0)$

## Review Exercises

24. Rotate quadrilateral $WXYZ$ 45° counterclockwise about point $W$ by tracing.

25. Find the coordinates of $\triangle STR$ if the figure is rotated 90° clockwise about the origin. Then graph the rotation image.

**Graph each figure. Then find the coordinates of the dilation image for the given scale factor $k$, and graph the dilation image.**

26. $\triangle DEF$ with vertices $D(0, 0)$, $E(2, -4)$, and $F(-2, -2)$; $k = \frac{1}{2}$

27. quadrilateral $QRST$ with vertices $Q(0, 1)$, $R(0, 0)$, $S(-1, -1)$, and $T(-2, 1)$; $k = 3$

28. quadrilateral $ABCD$ with vertices $A(-2, 2)$, $B(2, 2)$, $C(2, -1)$, and $D(-2, -1)$; $k = 2$

# Applications and Problem Solving

29. **Fund-raiser** The Central Middle School chorale sold hot dogs for $2 and cookies for $1 to raise funds for a new piano. If $x$ represents the number of hot dogs sold and $y$ represents the number of cookies sold, then $x + y = 220$ describes the total number of items sold and $2x + y = 368$ describes the money they made. How many hot dogs and cookies were sold? *(Lesson 16–2)*

30. **School Spirit** Gina is designing a large banner for an after-school pep rally. If the scale factor of her design to the actual banner is $\frac{1}{8}$ and the dimensions of the design are 2 feet by 3 feet, what will be the dimensions of the completed banner? *(Lesson 16–6)*

1. **List** the four types of transformations in this chapter and give a brief description and an example of each.

2. **Describe** three methods for solving a system of equations.

**Solve each system of equations by graphing.**

3. $x + 4y = 0$
   $y = -x - 3$

4. $y = -\frac{1}{2}x - 2$
   $x - 2y = 12$

5. $y = 5x + 1$
   $x + y = 7$

**Solve each system of equations by substitution or elimination.**

6. $-x + y = 2$
   $3x - 2y = 5$

7. $x - 4y = 1$
   $3x + 4y = 7$

8. $2x + 2y = -18$
   $6x - y = 2$

**Graph each figure. Then find the coordinates of the vertices after the given transformation and graph the image.**

| Figure | Vertices | Transformation |
|---|---|---|
| 9. $\triangle PQR$ | $P(-2, 0)$, $Q(0, -1)$, $R(-3, -3)$ | reflected over the $y$-axis |
| 10. quadrilateral $HIJK$ | $H(-2, 4)$, $I(-1, 2)$, $J(-2, 1)$, $K(-4, 2)$ | rotated 90° clockwise about the origin |
| 11. $\triangle ABC$ | $A(-2, 3)$, $B(-1, 2)$, $C(-4, 1)$ | translated by $(5, -2)$ |
| 12. square $WXYZ$ | $W(1, 2)$, $X(3, 2)$, $Y(3, 0)$, $Z(1, 0)$ | reflected over the $x$-axis |
| 13. $\triangle EFG$ | $E(-1, 0)$, $F(0, -3)$, $G(-3, -1)$ | rotated 90° counterclockwise about the origin |

14. Find the coordinates of the dilation image for a scale factor of 2. Then graph the dilation image.
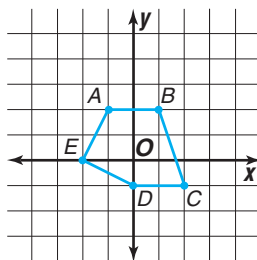


15. **Landscaping** Mr. Collins is landscaping his yard. He plans to move a storage shed from the side of his house to the corner of the yard. Suppose the vertices at the corners of the base of the shed are $S(32, 35)$, $H(42, 35)$, $E(42, 27)$, and $D(32, 27)$. If the shed is moved 15 feet across the yard and 81 feet back, the translation can be given by $(15, 81)$. Find the coordinates of the vertices after the shed is moved.

16. **Office Equipment** Most copy machines can reduce and enlarge images. It is necessary to reduce a rectangular image from 16 cm by 20 cm to 12 cm by 15 cm. What scale factor should be used?

## Systems of Equations and Polynomial Problems

Standardized tests often include problems on systems of equations. You usually can add or subtract the equations to solve them.

The ACT and SAT also contain several problems that ask you to simplify rational and polynomial expressions.

> **Test-Taking Tip**
>
> Memorize these polynomial relationships.
> $a^2 - b^2 = (a + b)(a - b)$
> $a^2 + 2ab + b^2 = (a + b)^2$
> $a^2 - 2ab + b^2 = (a - b)^2$

### Example 1

At the school store, Zoe spent $13.05 for 2 pens and 3 notebooks, and Kenji spent $9.94 for 5 pens and 1 notebook. Which system of equations would allow you to determine the cost of each pen and each notebook?

- **A** $2x + 3y = 13.05$
     $5x + y = 9.94$
- **B** $3x + 2y = 13.05$
     $5x + y = 9.94$
- **C** $2x + 3y = 9.94$
     $5x + y = 13.05$
- **D** $2x + 3y = 9.94$
     $5x + y = 13.05$

> **Hint** Write the equations and then look to see which answer choice matches your equations.

**Solution** Since the answer choices use the variables $x$ and $y$, let $x$ represent the cost of a pen, and let $y$ represent the cost of a notebook.

Translate Zoe's purchase into an equation.

| cost of 2 pens | plus | cost of 3 notebooks | equals | total cost |
|---|---|---|---|---|
| $2x$ | $+$ | $3y$ | $=$ | $13.05$ |

Translate Kenji's purchase into an equation.

| cost of 5 pens | plus | cost of 1 notebook | equals | total cost |
|---|---|---|---|---|
| $5x$ | $+$ | $y$ | $=$ | $9.94$ |

Compare your equations to the choices. The answer is A.

### Example 2

If $4x + 2y = 24$ and $\frac{7y}{2x} = 7$, then $x = ?$

> **Hint** Simplify equations before solving.

**Solution** There are two equations and two variables, so this is a system of equations. First simplify the equations, if possible. Start with the first equation. Divide each side by 2.

$$4x + 2y = 24 \boxed{\div 2} \Rightarrow 2x + y = 12$$

Now solve the second equation for $y$.

$$\frac{7y}{2x} = 7 \qquad \textit{Original equation}$$
$$2x \cdot \frac{7y}{2x} = 2x \cdot 7 \qquad \textit{Multiply each side by } 2x.$$
$$7y = 14x \qquad \textit{Simplify.}$$
$$\frac{7y}{7} = \frac{14x}{7} \qquad \textit{Divide each side by 7.}$$
$$y = 2x \qquad \textit{Simplify.}$$

You need to find the value of $x$. Substitute $2x$ for $y$ in the first equation.

$$2x + y = 12 \qquad \textit{Original equation}$$
$$2x + 2x = 12 \qquad \textit{Replace } y \textit{ with } 2x.$$
$$4x = 12 \qquad \textit{Add like terms.}$$
$$\frac{4x}{4} = \frac{12}{4} \qquad \textit{Divide each side by 4.}$$
$$x = 3 \qquad \textit{Simplify.}$$

The answer is 3.

www.geomconcepts.com/standardized_test

**CONTENTS**

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.
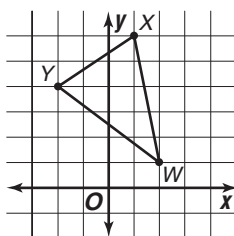
## Multiple Choice

**1.** Rachel has $100 in her savings account and deposits an additional $25 per week. Nina has $360 in her account and is saving $5 per week. After how many weeks will the girls have the same amount? *(Algebra Review)*

   Ⓐ 10   Ⓑ 11   Ⓒ 12   Ⓓ 13

**2.** If $\triangle WXY$ is translated 5 units right and 3 units up to become $\triangle W'X'Y'$, where is $Y'$?
*(Lesson 16–3)*
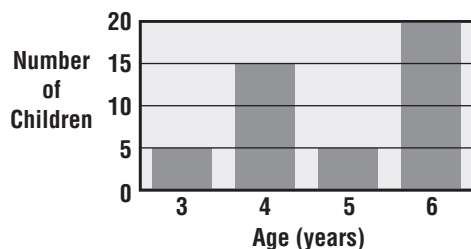
   Ⓐ $(6, 3)$  Ⓑ $(7, 3)$
   Ⓒ $(7, 4)$  Ⓓ $(3, 7)$

**3.** For all $y \neq 3$, $\dfrac{y^2 - 9}{3y - 9} = ?$    *(Algebra Review)*

   Ⓐ $y$   Ⓑ $y + 1$  Ⓒ $\dfrac{y}{3}$   Ⓓ $\dfrac{y + 3}{3}$

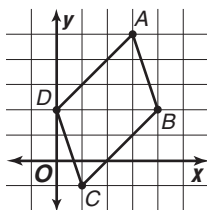**4.** What is the mode of the ages?
*(Statistics Review)*

**Puppet Show Attendance**



   Ⓐ 3   Ⓑ 4   Ⓒ 5   Ⓓ 6

**5.** If $\square ABCD$ is reflected over the $y$-axis to become $\square A'B'C'D'$, what are the coordinates of $C'$?
*(Lesson 16–4)*

   Ⓐ $(1, -1)$
   Ⓑ $(-1, -1)$
   Ⓒ $(1, 1)$
   Ⓓ $(-1, 1)$

**6.** $(10x^4 - x^2 + 2x - 8) - (3x^4 + 3x^3 + 2x + 8) =$
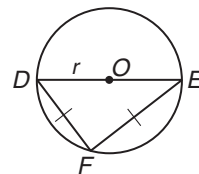*(Algebra Review)*

   Ⓐ $7x^4 - 3x^3 - x^2 - 16$
   Ⓑ $7x^4 - 4x^2 - 16$
   Ⓒ $7x^4 + 3x^3 - x^2 + 4x$
   Ⓓ $7x^4 + 2x^2 + 4x$
   Ⓔ $13x^4 - 3x^3 + x^2 + 4x$

**7.** A two-digit number is 7 times its unit digit. If 18 is added to the number, its digits are reversed. Find the number.
*(Algebra Review)*

   Ⓐ 24   Ⓑ 30   Ⓒ 32   Ⓓ 35

**8.** What is the area of the triangle in terms of the radius $r$ of the circle?
*(Lesson 14–1)*

   Ⓐ $\frac{1}{2}r^2$ units$^2$

   Ⓑ $r^2$ units$^2$

   Ⓒ $\frac{\pi r^2}{2}$ units$^2$

   Ⓓ $2r^2$ units$^2$

*Note:* Figure not drawn to scale.

## Grid In

**9. Grid-In** The average of $x$ and $y$ is 100, and the ratio of $x$ to $y$ is 3 to 2. What is the value of $x - y$?   *(Statistics Review)*

## Extended Response

**10.** A student group is taking a field trip on a bus that holds at most 40 people. Student tickets cost $4, and chaperone tickets cost $7. The group has $196.   *(Algebra Review)*

**Part A** Write a system of two inequalities to find the number of students $s$ and number of chaperones $c$ that can go on the trip.

**Part B** Graph the inequalities in the first quadrant. Label the region that is the solution. Give one example of a solution.