CHAPTER

9

Proportions and Similarity

What You'll Learn

Key Ideas

- Use ratios and proportions to solve problems. *(Lesson 9–1)*
- Identify similar polygons and use similarity tests for triangles. *(Lessons 9–2 and 9–3)*
- Identify and use the relationships between proportional parts of triangles. *(Lessons 9–4 to 9–6)*
- Identify and use proportional relationships of similar triangles. *(Lesson 9–7)*

Key Vocabulary

polygon *(p. 356)*

proportion *(p. 351)*

ratio *(p. 350)*

similar polygons *(p. 356)*

Why It's Important

Mechanics The pit crew of a racing team is responsible for making sure the car is prepared for the driver. To maintain the car, crew members must understand the workings of each part of the complex gear system in the engine.

Proportions are used to compare sizes using ratios. You will investigate gear ratios in Lesson 9–1.

Check Your Readiness

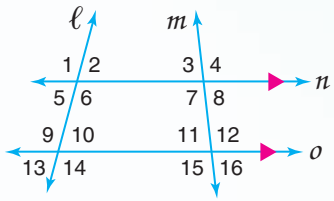
Study these lessons to improve your skills.

- **Algebra Review, p. 720**
- **Algebra Review, p. 721**
- **Lesson 4–3, pp. 156–161**
- **Lesson 1–6, pp. 35–40**

Solve each equation.

1. $y = 12(4.5)$
2. $6.1(3.3) = p$
3. $n = 13.3 \div 3.5$
4. $11.16 \div 0.9 = q$
5. $d = \frac{2}{3} \cdot \frac{3}{4}$
6. $3\left(1\frac{1}{2}\right) = f$
7. $x = \frac{3}{8} \div \frac{1}{4}$
8. $7\frac{1}{2} \div 30 = w$

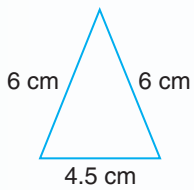
In the figure, $n \parallel o$. Name all angles congruent to the given angle. Give a reason for each answer.



9. $\angle 1$
10. $\angle 4$
11. $\angle 5$
12. $\angle 16$

Find the perimeter of each triangle.

13.



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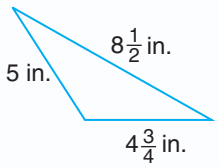

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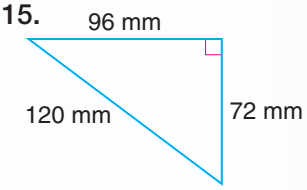
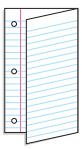


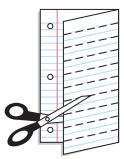
FOLDABLES™ Study Organizer

Make this Foldable to help you organize your Chapter 9 notes. Begin with a sheet of notebook paper.

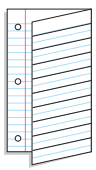
❶ **Fold** lengthwise to the holes.



❷ **Cut** along the top line and then cut 10 tabs.



❸ **Label** each tab with important terms.



Reading and Writing Store the Foldable in a 3–ring binder. As you read and study the chapter, write definitions and examples of important terms under each tab.

9–1 Using Ratios and Proportions

What You'll Learn
You'll learn to use ratios and proportions to solve problems.

Why It's Important
Medicine Nurses solve proportions to determine the amount of medication to give a patient.
See Exercise 45.

In 2000, about 180 million tons of solid waste was created in the United States. Paper made up about 72 million tons of this waste. The **ratio** of paper waste to total solid waste is 72 to 180. This ratio can be written in the following ways.

72 to 180 72:180 $\frac{72}{180}$ $72 \div 180$

Definition of Ratio		
	Words:	A ratio is a comparison of two numbers by division.
	Symbols:	a to b, $a:b$, $\frac{a}{b}$, or $a \div b$, where $b \neq 0$

All ratios should be written in simplest form. Because all fractions can be written as decimals, it is sometimes useful to express ratios in decimal form.

Examples

Write each ratio in simplest form.

1 $\frac{45}{340}$

$$\frac{45}{340} = \frac{45 \div 5}{340 \div 5}$$ *Divide the numerator and denominator by 5.*

$$= \frac{9}{68}$$ *Simplify.*

Reading Geometry
Read 45:340 as *45 to 340*.

2 **six days to two weeks**

To write this as a ratio, the units of measure must be the same. Write both using days. There are seven days in one week, so two weeks equal 14 days. The ratio is $\frac{6}{14}$ or $\frac{3}{7}$.

Your Turn

a. $\frac{18}{24}$

b. 10 kilometers to 20,000 meters

A **proportion** is an equation that shows two equivalent ratios.

$$\frac{20}{30} = \frac{2}{3}$$

Every proportion has two **cross products**. In the proportion above, the terms 20 and 3 are called the **extremes**, and 30 and 2 are called the **means**. The cross products are 20(3) and 30(2). The cross products are always equal in a proportion.

$$\frac{20}{30} = \frac{2}{3}$$
$$20(3) = 30(2)$$
extremes *means*
$$60 = 60$$

Theorem 9–1 Property of Proportions		
	Words:	For any numbers a and c and any nonzero numbers b and d, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.
		Likewise, if $ad = bc$, then $\frac{a}{b} = \frac{c}{d}$.
	Numbers:	If $\frac{5}{10} = \frac{1}{2}$, then $5(2) = 10(1)$.
		If $5(2) = 10(1)$, then $\frac{5}{10} = \frac{1}{2}$.

You can use cross products to solve equations in proportion form. Remember that you should always check your solution in the original proportion.

Example 3

Algebra Link

Algebra Review
Solving Multi-Step Equations, p. 723

Solve $\frac{15}{35} = \frac{3}{2x + 1}$.

$$\frac{15}{35} = \frac{3}{2x + 1}$$ *Original equation*

$$15(2x + 1) = 35(3)$$ *Cross Products*

$$30x + 15 = 105$$ *Distributive Property*

$$30x + 15 - 15 = 105 - 15$$ *Subtract 15 from each side.*

$$30x = 90$$ *Simplify.*

$$\frac{30x}{30} = \frac{90}{30}$$ *Divide each side by 30.*

$$x = 3$$ *Simplify.*

Check:

$$\frac{15}{35} = \frac{3}{2x + 1}$$ *Original equation*

$$\frac{15}{35} \stackrel{?}{=} \frac{3}{2(3) + 1}$$ *Replace x with 3.*

$$\frac{15}{35} \stackrel{?}{=} \frac{3}{7}$$ *2(3) + 1 = 7*

$$15(7) \stackrel{?}{=} 35(3)$$ *Cross Products*

$$105 = 105 \quad \surd$$

The solution is 3.

(continued on the next page)

Your Turn

c. Solve $\frac{3}{b} = \frac{15}{60}$.

d. Solve $\frac{x-2}{4} = \frac{4}{8}$.

Proportions can help you solve real-life problems. The ratios in the proportions must be written in the same order.

Real World

Example 4

Automotive Link

In an engine, the volume of the cylinder changes as the piston moves up and down. The compression ratio is the expanded volume of the cylinder to the compressed volume of the cylinder. One car with a V6 engine has a compression ratio of 9 to 1. If the expanded volume of the cylinder is 22.5 cubic inches, find its compressed volume.

expanded volume → $\frac{9}{1} = \frac{22.5}{x}$ ← *compressed volume*
compressed volume → ← *expanded volume*

$9(x) = 1(22.5)$ *Cross Products*

$x = 2.5$ *Divide each side by 9.*

The cylinder's compressed volume is 2.5 cubic inches.

Check for Understanding

Communicating Mathematics

1. **Write** two ratios that form a proportion and two ratios that do not form a proportion.

2. **Explain** how you would solve the proportion $\frac{14}{21} = \frac{x}{24}$.

3. YOU DECIDE Lawanda says that if $\frac{7}{8} = \frac{x}{y}$, then $\frac{8}{7} = \frac{y}{x}$. Paul disagrees. Who is correct? Explain your reasoning.

Vocabulary

ratio
proportion
cross products
extremes
means

Guided Practice

Getting Ready **Write each ratio as a fraction in simplest form.**

Sample: 6 ounces to 12 ounces **Solution:** $\frac{6 \text{ oz}}{12 \text{ oz}} = \frac{6}{12}$

$= \frac{6 \div 6}{12 \div 6}$ or $\frac{1}{2}$

4. 7 feet to 3 feet
5. 3 grams to 11 grams
6. 16 cm to 5 cm
7. 21 miles to 16 miles
8. 15 km to 5 km
9. 6 meters to 10 meters

Examples 1 & 2 **Write each ratio in simplest form.**

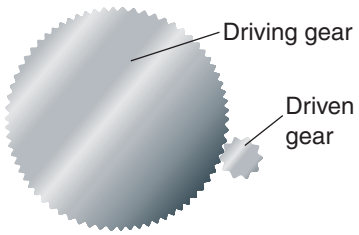
10. $\frac{4}{2}$
11. $\frac{72}{100}$
12. 3 millimeters to 1 centimeter

Example 3 **Solve each proportion.**

13. $\frac{x}{3} = \frac{12}{18}$
14. $\frac{6}{2x} = \frac{15}{30}$
15. $\frac{7}{3} = \frac{3x - 1}{6}$

Example 4

16. **Mechanics** The gear ratio is the number of teeth on the driving gear to the number of teeth on the driven gear. If the gear ratio is 5:2 and the driving gear has 35 teeth, how many teeth does the driven gear have?



Exercises

Practice

Homework Help

For Exercises	See Examples
17–26, 42	1, 2
27–41, 43–46	3, 4

Extra Practice

See page 741.

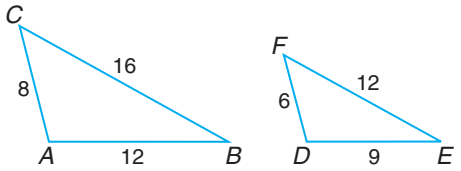
Write each ratio in simplest form.

17. $\frac{2}{10}$
18. $\frac{8}{12}$
19. $\frac{10}{22}$
20. $\frac{18}{36}$
21. $\frac{45}{21}$
22. $\frac{40}{12}$
23. 44 centimeters to 2 meters
24. 6 inches to 2 feet
25. 6 quarts to 1 pint
26. 3 liters to 300 milliliters

Solve each proportion.

27. $\frac{2}{5} = \frac{4}{x}$
28. $\frac{12}{5} = \frac{x}{10}$
29. $\frac{36}{3} = \frac{12}{x}$
30. $\frac{3x}{27} = \frac{2}{9}$
31. $\frac{14}{x - 1} = \frac{7}{4}$
32. $\frac{1}{x + 3} = \frac{3}{29}$
33. $\frac{5}{9} = \frac{5}{x - 3}$
34. $\frac{x + 2}{16} = \frac{7}{4}$
35. $\frac{30 - x}{x} = \frac{3}{2}$
36. If $3{:}x = 18{:}24$, find the value of x.
37. If 3 to 4 and $5x - 1$ to 12 form a proportion, what is the value of x?

38. Refer to the triangles below.



a. Write the ratio of AB to DE.

b. Write the ratio of AC to DF.

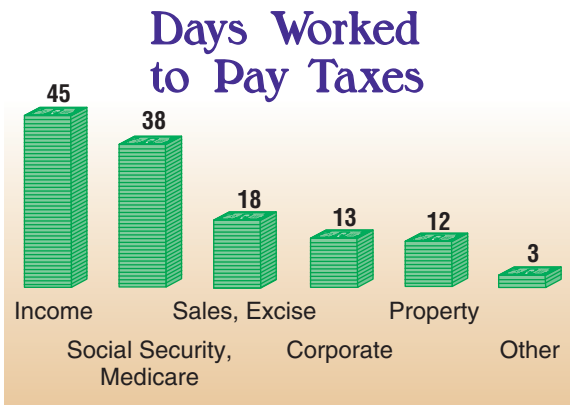
c. Do the two ratios form a proportion? Explain.

If $a = 3$, $b = 2$, $c = 6$, and $d = 4$, determine whether each pair of ratios forms a proportion.

39. $\frac{b}{a} = \frac{d}{c}$ 40. $\frac{c}{b} = \frac{d}{a}$ 41. $\frac{a+b}{b} = \frac{c+d}{d}$

Applications and Problem Solving

42. **Money** The average number of days Americans worked in a recent year to pay taxes is shown below.



Source: Tax Foundation

Data Update For the latest information about the average number of working days needed to pay taxes, visit: www.geomconcepts.com

a. Write a ratio of the number of days worked to pay income taxes to the total number of days in a year.

b. Write a ratio of the number of days worked to pay property taxes to the number of days to pay sales and excise taxes.

43. **Environment** If 2500 square feet of grass supplies enough oxygen for a family of four, how much grass is needed to supply oxygen for a family of five?

44. **Science** Light travels approximately 1,860,000 miles in 10 seconds. How long will it take light to travel the 93,000,000 miles from the sun to Earth?

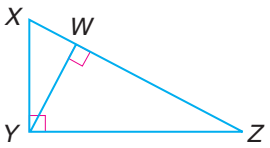
45. **Medicine** Antonio is a nurse. A doctor tells him to give a patient 60 milligrams of acetaminophen. Antonio has a liquid medication that contains 240 milligrams of acetaminophen per 10 milliliters of medication. How many milliliters of the medication should he give the patient?

CONTENTS

46. **Critical Thinking** The **geometric mean** between two positive numbers a and b is the positive number x in $\frac{a}{x} = \frac{x}{b}$.

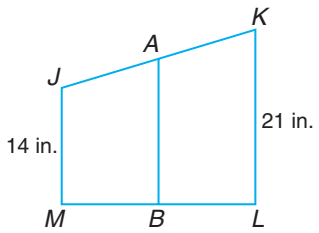
a. Solve $\frac{4}{x} = \frac{x}{9}$ to find the geometric mean between 4 and 9.

b. In any right triangle XYZ, if $\overline{YW}$ is an altitude to the hypotenuse, then XY is the geometric mean between XZ and XW, and YZ is the geometric mean between XZ and WZ. If $XZ = 16$ and $XW = 4$, find XY.

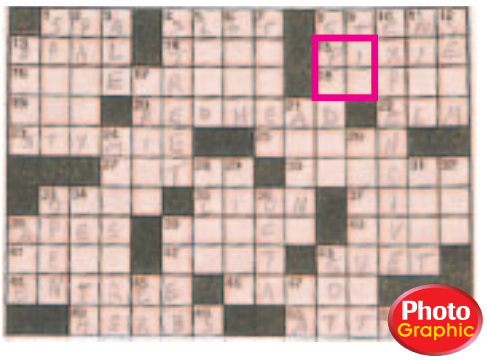


Mixed Review

47. Find the length of median $\overline{AB}$ in trapezoid $JKLM$ if $JM = 14$ inches and $KL = 21$ inches. *(Lesson 8–5)*

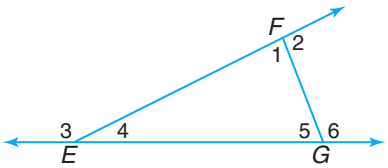


48. **Puzzles** A crossword puzzle is made up of various parallelograms. Identify the parallelogram that is outlined as a *rectangle*, *rhombus*, *square*, or *none of these*. If it is more than one of these, list all that apply. *(Lesson 8–4)*

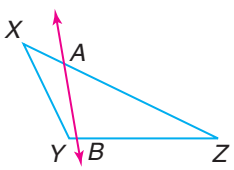


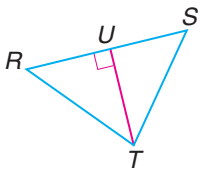
49. Determine if the numbers 8, 15, and 17 can be measures of the sides of a triangle. *(Lesson 7–4)*

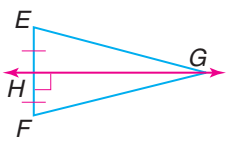
50. In the triangle shown, $m\angle 5 = 9x$, $m\angle 4 = 6x + 2$, and $m\angle 2 = 92$. Find the values of x, $m\angle 5$, and $m\angle 4$. *(Lesson 7–2)*



For each triangle, tell whether the red segment or line is an *altitude*, a *perpendicular bisector*, *both*, or *neither*. *(Lesson 6–2)*

51. 

52. 

53. 

Standardized Test Practice

A B C D

54. **Short Response** According to the building code in Plainfield, Connecticut, the slope of a stairway cannot be steeper than 0.82. The stairs in Troy's home measure 10.5 inches deep and 7.5 inches high. Do the stairs in his home meet the code requirements? Explain. *(Lesson 4–5)*

9–2 Similar Polygons

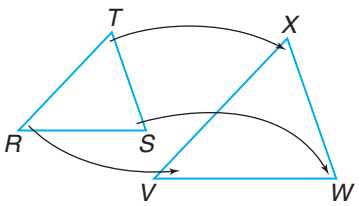
What You'll Learn
You'll learn to identify similar polygons.

Why It's Important
Construction
Contractors use drawings that are similar to the actual building.
See Example 3.

A **polygon** is a closed figure in a plane formed by segments called **sides**. It is a general term used to describe a geometric figure with at least three sides. Polygons that are the same shape but not necessarily the same size are called **similar polygons**.

The two triangles shown below are similar. For naming similar polygons, the vertices are written in order to show the corresponding parts. The symbol for similar is $\sim$.

$$\triangle RST \sim \triangle VWX$$



Corresponding Angles	Corresponding Sides
$\angle R \leftrightarrow \angle V$	$\overline{RS} \leftrightarrow \overline{VW}$
$\angle S \leftrightarrow \angle W$	$\overline{ST} \leftrightarrow \overline{WX}$
$\angle T \leftrightarrow \angle X$	$\overline{TR} \leftrightarrow \overline{XV}$

Reading Geometry
Read the symbol $\sim$ as *is similar to*.

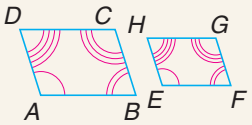
Recall that in congruent figures, corresponding angles and sides are congruent. In similar figures, corresponding angles are congruent, and the measures of corresponding sides have equivalent ratios, or are proportional.

$$\angle R \cong \angle V,\ \angle S \cong \angle W,\ \angle T \cong \angle X, \text{ and } \frac{RS}{VW} = \frac{ST}{WX} = \frac{TR}{XV}$$

Definition of Similar Polygons

Words: Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.

Model:

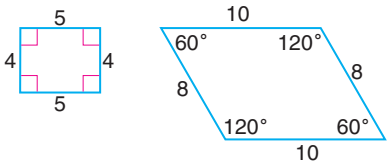


$$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE} \text{ and}$$
$$\angle A \cong \angle E,\ \angle B \cong \angle F,$$
$$\angle C \cong \angle G,\ \angle D \cong \angle H$$

Symbols: polygon $ABCD \sim$ polygon $EFGH$

Example 1

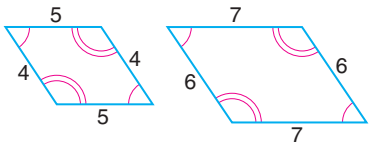
Determine if the polygons are similar. Justify your answer.



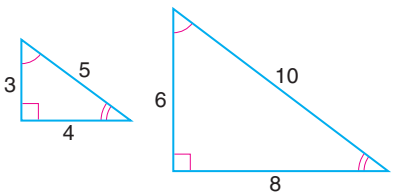
Since $\frac{4}{8} = \frac{5}{10} = \frac{4}{8} = \frac{5}{10}$, the measures of the sides of the polygons are proportional. However, the corresponding angles are not congruent. The polygons are not similar.

Your Turn

a.



b.

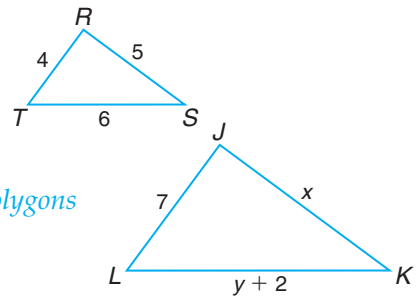


Knowing several measures of two similar figures may allow you to find the measures of missing parts.

Example 2

Algebra Link

Find the values of x and y if $\triangle RST \sim \triangle JKL$.



Use the corresponding order of the vertices to write proportions.

$\frac{RS}{JK} = \frac{ST}{KL} = \frac{TR}{LJ}$ *Definition of similar polygons*

$\frac{5}{x} = \frac{6}{y+2} = \frac{4}{7}$ *Substitution*

Write the proportion that can be solved for x.

$\frac{5}{x} = \frac{4}{7}$

$5(7) = x(4)$ *Cross Products*

$35 = 4x$

$\frac{35}{4} = \frac{4x}{4}$ *Divide each side by 4.*

$8\frac{3}{4} = x$ *Simplify.*

Now write the proportion that can be solved for y.

$\frac{6}{y+2} = \frac{4}{7}$

$6(7) = (y + 2)4$ *Cross Products*

$42 = 4y + 8$ *Distributive Property*

$42 - 8 = 4y + 8 - 8$ *Subtract 8 from each side.*

$34 = 4y$

$\frac{34}{4} = \frac{4y}{4}$ *Divide each side by 4.*

$8\frac{1}{2} = y$ *Simplify.*

Therefore, $x = 8\frac{3}{4}$ and $y = 8\frac{1}{2}$.

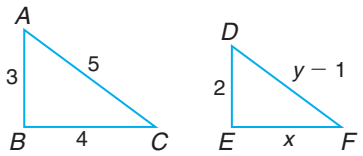
Algebra Review

Solving Multi-Step Equations, p. 723

(continued on the next page)

Your Turn

c. Find the values of x and y if $\triangle ABC \sim \triangle DEF$.



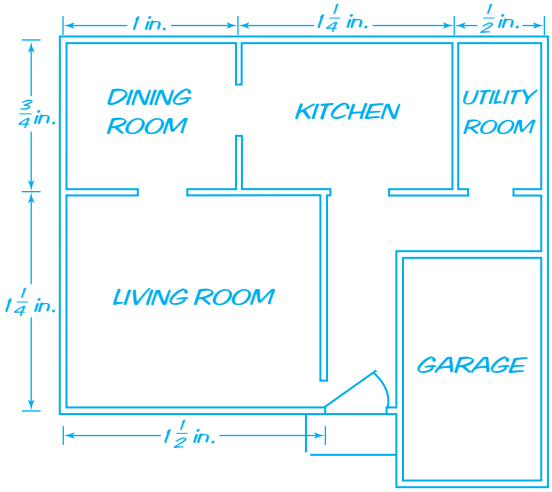
Scale drawings are often used to represent something that is too large or too small to be drawn at actual size. Contractors use scale drawings called *blueprints* to represent the floor plan of a house to be constructed. The blueprint and the floor plan are similar.

Real World

Example 3 **Construction Link**

In the blueprint, 1 inch represents an actual length of 16 feet. Find the actual dimensions of the living room.

On the blueprint, the living room is $1\frac{1}{2}$ inches long and $1\frac{1}{4}$ inches wide. Use proportions to find the actual dimensions.



$$\begin{array}{rl} \text{blueprint} \rightarrow \\ \text{actual} \rightarrow \end{array} \frac{1 \text{ in.}}{16 \text{ ft}} = \frac{1\frac{1}{2} \text{ in.}}{x \text{ ft}} \begin{array}{l} \leftarrow \text{blueprint} \\ \leftarrow \text{actual} \end{array}$$

$$1(x) = 16\left(1\tfrac{1}{2}\right) \quad \text{Cross Products}$$

$$x = 24$$

$$\begin{array}{rl} \text{blueprint} \rightarrow \\ \text{actual} \rightarrow \end{array} \frac{1 \text{ in.}}{16 \text{ ft}} = \frac{1\frac{1}{4} \text{ in.}}{y \text{ ft}} \begin{array}{l} \leftarrow \text{blueprint} \\ \leftarrow \text{actual} \end{array}$$

$$1(y) = 16\left(1\tfrac{1}{4}\right) \quad \text{Cross Products}$$

$$y = 20$$

The actual dimensions of the living room are 24 feet by 20 feet.

Your Turn

d. Use the blueprint to find the actual dimensions of the kitchen.

Check for Understanding

Communicating Mathematics

1. **Compare and contrast** congruent polygons and similar polygons.
2. **Explain** how to find an actual distance using a scale drawing.
3. **Draw** two similar pentagons on grid paper. Label the vertices of the pentagons. Name the corresponding angles and sides. Write a proportion for the measures of the sides.

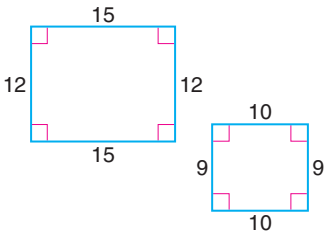
Vocabulary
polygon
sides
similar polygons
scale drawing

Guided Practice

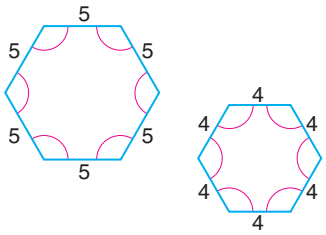
Example 1

Determine whether each pair of polygons is similar. Justify your answer.

4.



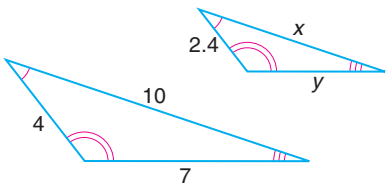
5.



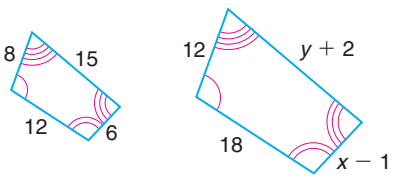
Example 2

Each pair of polygons is similar. Find the values of x and y.

6.



7.



Example 3

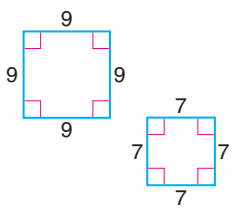
8. **Construction** Refer to Example 3. Find the actual dimensions of the utility room.

Exercises

Practice

Determine whether each pair of polygons is similar. Justify your answer.

9.



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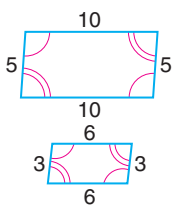

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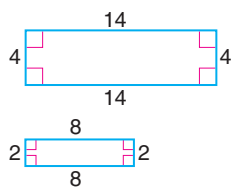


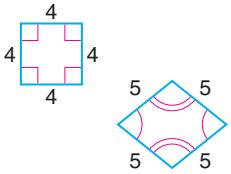
Homework Help

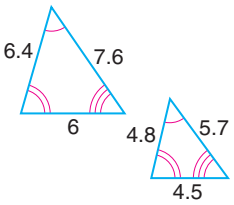
For Exercises	See Examples
9–14, 21, 22	1
15–20	2
23–26	3

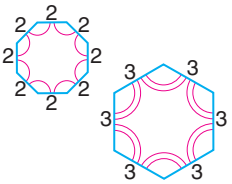
Extra Practice

See page 741.

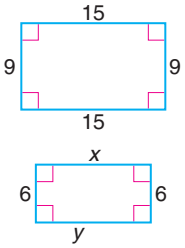
Determine whether each pair of polygons is similar. Justify your answer.

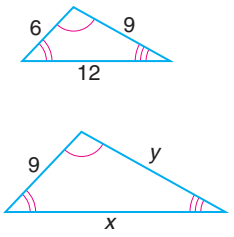
12. 

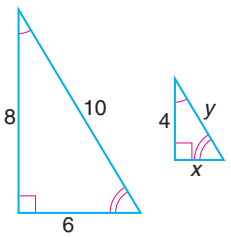
13. 

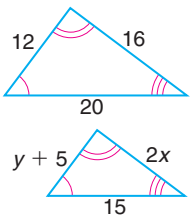
14. 

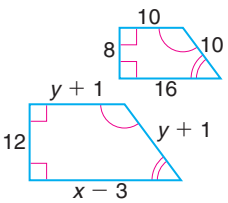
Each pair of polygons is similar. Find the values of *x* and *y*.

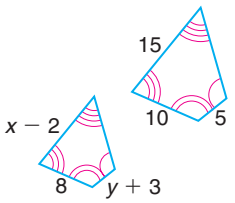
15. 

16. 

17. 

18. 

19. 

20. 

Determine whether each statement is *always*, *sometimes*, or *never* true.

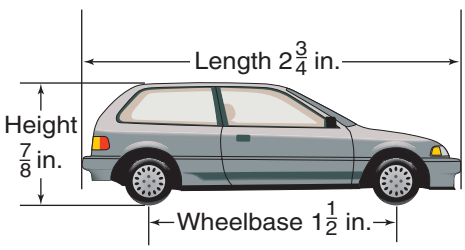
21. Similar polygons are also congruent.
22. Congruent polygons are also similar.

Applications and Problem Solving

23. **Sports** A soccer field is 91 meters by 46 meters. Make a scale drawing of the field if 1 millimeter represents 1 meter.

24. **Publishing** Mi-Ling is working on the school yearbook. She must reduce a photo that is 4 inches wide by 5 inches long to fit in a space 3 inches wide. How long will the reduced photo be?

25. **Automotive Design** Tracy is drawing a scale model of a car she is designing. If $\frac{1}{4}$ inch on the drawing represents 28 inches, find each measurement on the actual car.



a. length
b. height
c. wheelbase

CONTENTS

26. **Travel** Each year, many tourists visit Madurodam in the Netherlands. Madurodam is a miniature town where 1 meter represents 25 meters. How high is a structure in Madurodam that represents a building that is actually 30 meters high?

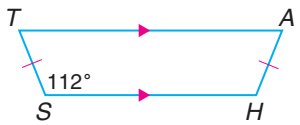
Madurodam, Netherlands

27. **Critical Thinking** Marquis is doing a report on Wyoming. The state is approximately a rectangle measuring 362 miles by 275 miles. If Marquis wishes to draw the largest possible map of Wyoming on an $8\frac{1}{2}$-inch by 11-inch piece of paper, how many miles should one inch represent?

Mixed Review

28. Find the value of y if $4{:}y = 16{:}36$. *(Lesson 9–1)*

29. Trapezoid $TAHS$ is isosceles. Find $m\angle T$, $m\angle H$, and $m\angle A$. *(Lesson 8–5)*

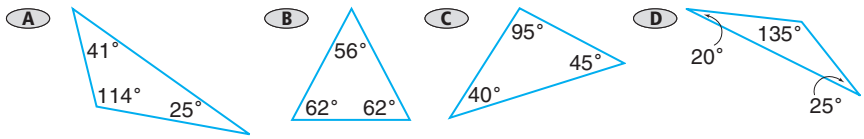


30. In quadrilateral $QRST$, diagonal QS bisects diagonal RT. Is $QRST$ a parallelogram? Explain. *(Lesson 8–3)*

Standardized Test Practice

31. **Short Response** In right triangle ASP, $m\angle S = 90$. Which side has the greatest measure? *(Lesson 7–3)*

32. **Multiple Choice** Which triangle is not obtuse? *(Lesson 5–1)*



Quiz 1 — Lessons 9–1 and 9–2

Solve each proportion. *(Lesson 9–1)*

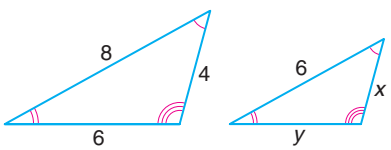
1. $\frac{2}{x} = \frac{8}{12}$

2. $\frac{18}{4x} = \frac{3}{2}$

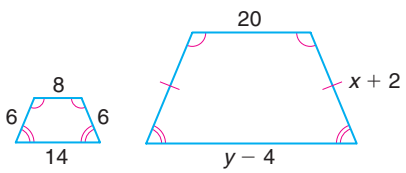
3. $\frac{x+4}{3} = \frac{25}{5}$

Each pair of polygons is similar. Find the values of x and y. *(Lesson 9–2)*

4.



5.



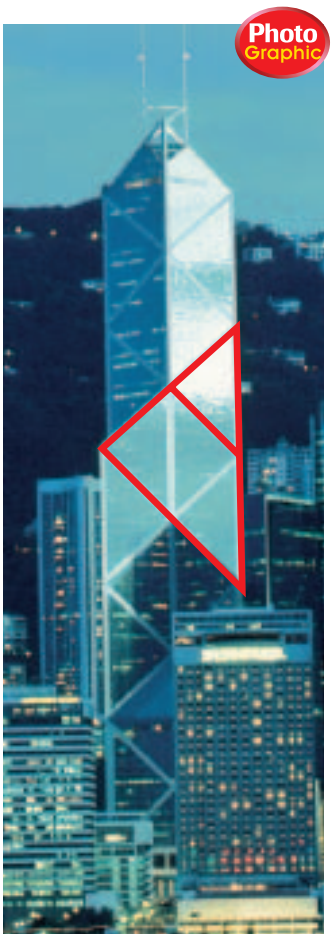
9–3 Similar Triangles

What You'll Learn
You'll learn to use AA, SSS, and SAS similarity tests for triangles.

Why It's Important
Surveying Surveyors use similar triangles to measure distances that cannot be measured directly. *See Exercise 5.*

The Bank of China building in Hong Kong is one of the ten tallest buildings in the world. Designed by American architect I. M. Pei, the outside of the 70-story building is sectioned into triangles, which are meant to resemble the trunk of a bamboo plant. Some of the triangles are similar, as shown below.

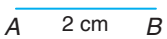
In previous chapters, you learned several basic tests for determining whether two triangles are congruent. Recall that each congruence test involves only three corresponding parts of each triangle. Likewise, there are tests for similarity that will not involve all the parts of each triangle.



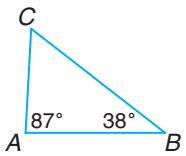
Hands-On Geometry

Materials: ruler protractor

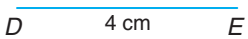
Step 1 On a sheet of paper, use a ruler to draw a segment 2 centimeters in length. Label the endpoints of the segment A and B.



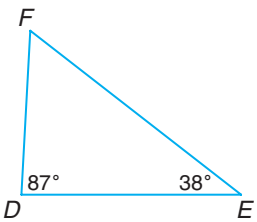
Step 2 Use a protractor to draw an angle at A so that $m\angle A = 87$. Draw an angle at B so that $m\angle B = 38$. Extend the sides of $\angle A$ and $\angle B$ so that they intersect to form a triangle. Label the third vertex C.



Step 3 Now draw a segment 4 centimeters in length. Label the endpoints D and E.



Step 4 Use a protractor to draw an angle at D so that $m\angle D = 87$. Draw an angle at E so that $m\angle E = 38$. Extend the sides of $\angle D$ and $\angle E$ to form a triangle. Label the third vertex F.



Try These

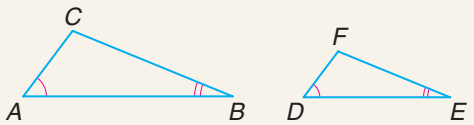
1. What is $m\angle C$? What is $m\angle F$?
2. Use a ruler to find BC, CA, EF, and FD.
3. Find $\frac{AB}{DE}$, $\frac{BC}{EF}$, and $\frac{CA}{FD}$.
4. Are the triangles similar? Why or why not?

The activity suggests Postulate 9–1.

Postulate 9–1 AA Similarity

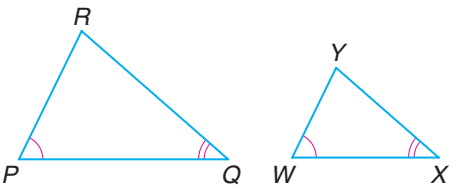
Words: If two angles of one triangle are congruent to two corresponding angles of another triangle, then the triangles are similar.

Model:



Symbols: If $\angle A \cong \angle D$ and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

In $\triangle PQR$ and $\triangle WXY$, if $\angle P \cong \angle W$ and $\angle Q \cong \angle X$, then the triangles are similar. By definition of similar polygons, there are four other parts of the triangles that are related.



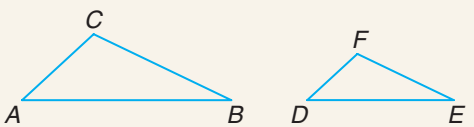
$\angle R \cong \angle Y$ $\quad \frac{PQ}{WX} = \frac{QR}{XY} = \frac{RP}{YW}$ $\quad \triangle PQR \sim \triangle WXY$

Two other tests are used to determine whether two triangles are similar.

Theorem 9–2 SSS Similarity

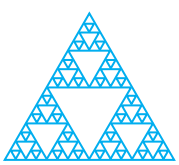
Words: If the measures of the sides of a triangle are proportional to the measures of the corresponding sides of another triangle, then the triangles are similar.

Model:



Symbols: If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, then $\triangle ABC \sim \triangle DEF$.

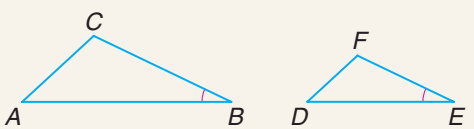
*A **fractal** is a geometric figure that is created by repeating the same process over and over. One characteristic of a fractal is that it has a self-similar shape. The Sierpinski Triangle shown below is an example of a fractal.*



Theorem 9–3 SAS Similarity

Words: If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and their included angles are congruent, then the triangles are similar.

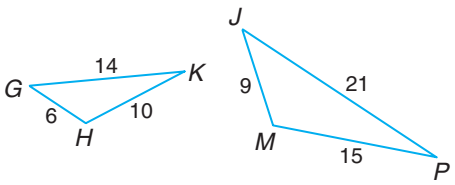
Model:



Symbols: If $\frac{AB}{DE} = \frac{BC}{EF}$, and $\angle B \cong \angle E$, then $\triangle ABC \sim \triangle DEF$.

Examples

1 **Determine whether the triangles are similar. If so, tell which similarity test is used and complete the statement.**

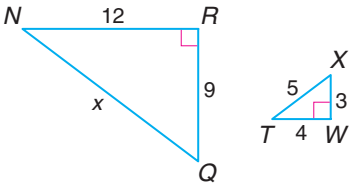


$\triangle GHK \sim \triangle$ ___?___

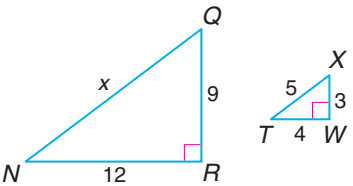
Since $\frac{6}{9} = \frac{10}{15} = \frac{14}{21}$, the triangles are similar by SSS Similarity. Therefore, $\triangle GHK \sim \triangle JMP$.

2 **Find the value of x.**

To see the corresponding parts more easily, flip $\triangle QRN$ so that it is in the same position as $\triangle XWT$.



Since $\angle R$ and $\angle W$ are right angles, they are congruent. We also know that $\frac{12}{4} = \frac{9}{3}$. Therefore, $\triangle QRN \sim \triangle XWT$ by SAS Similarity. Use the definition of similar polygons to find x.



$\frac{QR}{XW} = \frac{NQ}{TX}$ *Definition of Similar Polygons*

$\frac{9}{3} = \frac{x}{5}$ *QR = 9, XW = 3, NQ = x, TX = 5*

$9(5) = 3(x)$ *Cross Products*

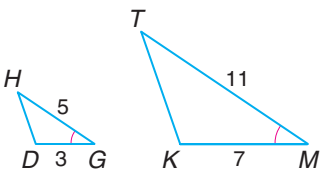
$45 = 3x$ *Simplify.*

$\frac{45}{3} = \frac{3x}{3}$ *Divide each side by 3.*

$15 = x$ *Simplify.*

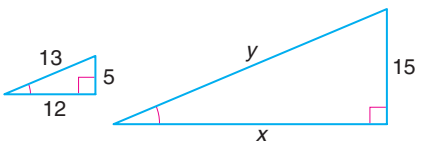
Your Turn

a. Determine whether the triangles are similar. If so, tell which similarity test is used and complete the statement.



$\triangle DGH \sim \triangle$ ___?___

b. Find the values of x and y if the triangles are similar.



CONTENTS

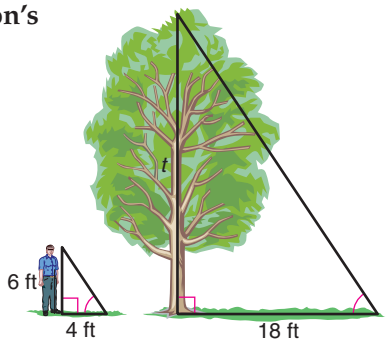
Similar triangles can be used to find the length of an object that is difficult to measure directly.

Real World

Example 3

Landscaping Link

Editon is landscaping a yard. To see how well a tree shades an area of the yard, he needs to know the tree's height. The tree's shadow is 18 feet long at the same time that Editon's shadow is 4 feet long. If Editon is 6 feet tall, how tall is the tree?



Draw a diagram. The rays of the sun form congruent angles with the ground. Both Editon and the tree form right angles with the ground. Therefore, the triangles in the diagram are similar by AA Similarity. Use the similar triangles to find the height of the tree t.

$$\begin{array}{rcl} \textit{Editon's height} \rightarrow & \frac{6}{t} = \frac{4}{18} & \leftarrow \textit{Editon's shadow} \\ \textit{tree's height} \rightarrow & & \leftarrow \textit{tree's shadow} \end{array}$$

$$6(18) = t(4) \quad \textit{Cross Products}$$

$$108 = 4t \quad \textit{Simplify.}$$

$$\frac{108}{4} = \frac{4t}{4} \quad \textit{Divide each side by 4.}$$

$$27 = t \quad \textit{Simplify.}$$

The tree is 27 feet tall.

Check for Understanding

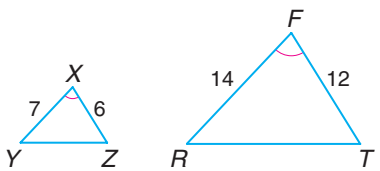
Communicating Mathematics

1. **Sketch and label** two similar right triangles ABC and DEF with right angles at C and F. Let the measures of angles A and D be 30. Name the corresponding sides that are proportional.
2. Refer to Example 2.
   a. If the sides of $\triangle XWT$ form a Pythagorean triple, what is true about the sides of $\triangle QRN$?
   b. Why is this true?

Guided Practice

Example 1

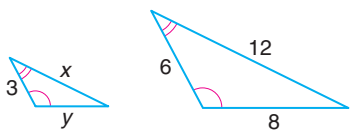
3. Determine whether the triangles are similar. If so, tell which similarity test is used and complete the statement.



$\triangle XYZ \sim \triangle$ ___?___

Example 2

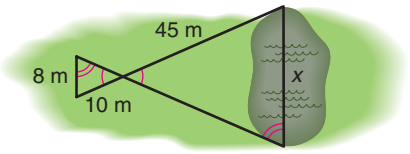
4. Find the values of x and y.



CONTENTS

Example 3

5. **Surveying** Syreeta Coleman is a surveyor. To find the distance across Muddy Pond, she forms similar triangles and measures distances as shown at the right. What is the distance across Muddy Pond?

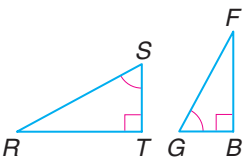


Exercises

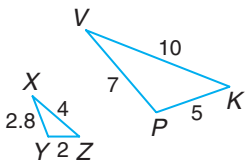
Practice

Homework Help	
For Exercises	See Examples
6–8	1
9–11	2
14–16	2, 3
Extra Practice	
See page 742.	

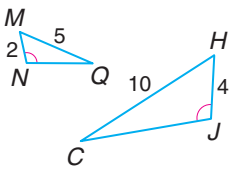
Determine whether each pair of triangles is similar. If so, tell which similarity test is used and complete the statement.

6. 

$\triangle RST \sim \triangle$ ___?___

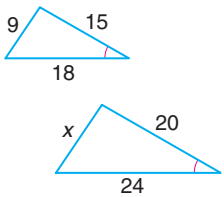
7. 

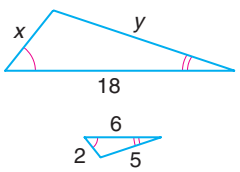
$\triangle XYZ \sim \triangle$ ___?___

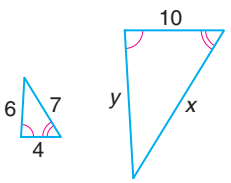
8. 

$\triangle MNQ \sim \triangle$ ___?___

Find the value of each variable.

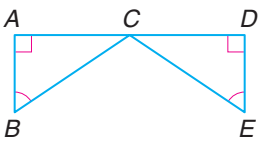
9. 

10. 

11. 

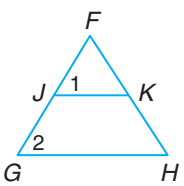
Give a reason for each statement in Exercises 12–13.

12. If $\angle B \cong \angle E$ and $\angle A$ and $\angle D$ are right angles, show that $\frac{BC}{EC} = \frac{AB}{DE}$.



a. $\angle B \cong \angle E$
b. $\angle A$ and $\angle D$ are right angles.
c. $\angle A \cong \angle D$
d. $\triangle ABC \sim \triangle DEC$
e. $\frac{BC}{EC} = \frac{AB}{DE}$

13. If $\overline{JK} \parallel \overline{GH}$, show that $\frac{FJ}{FG} = \frac{FK}{FH}$.

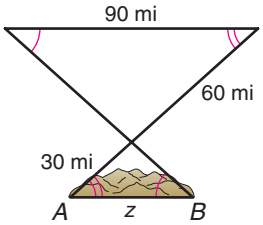


a. $\overline{JK} \parallel \overline{GH}$
b. $\angle 1 \cong \angle 2$
c. $\angle F \cong \angle F$
d. $\triangle FJK \sim \triangle FGH$
e. $\frac{FJ}{FG} = \frac{FK}{FH}$

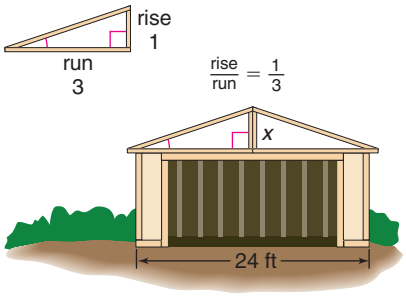
CONTENTS

Applications and Problem Solving

14. Road Construction The state highway department is considering the possibility of building a tunnel through the mountain from point A to point B. Surveyors provided the map at the right. How long would the tunnel be?



15. Construction The pitch of a roof is the ratio of the rise to the run. The Ace Construction Company is building a garage that is 24 feet wide. If the pitch of the roof is to be 1:3, find the rise of the roof.



16. Architecture Maria is visiting Washington, D.C. She wants to know the height of the Washington Monument. The monument's shadow is 111 feet at the same time that Maria's shadow is 1 foot. Maria is 5 feet tall.

a. Draw a figure to represent the problem. Label all known distances.

b. Outline two similar triangles in red.

c. Determine the height of the Washington Monument.

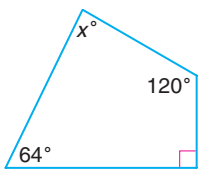
17. Critical Thinking A *primitive Pythagorean triple* is a set of whole numbers that satisfies the equation $a^2 + b^2 = c^2$ and has no common factors except 1. A *family of Pythagorean triples* is a primitive triple and its whole number multiples. How are the triangles represented by a family of Pythagorean triples related? Explain.

Mixed Review

18. Scale Drawings A window measures 8 feet by 3 feet. Make a scale drawing of the window if $\frac{1}{4}$ inch represents 1 foot. *(Lesson 9–2)*

19. Write the ratio *10 months to 5 years* in simplest form. *(Lesson 9–1)*

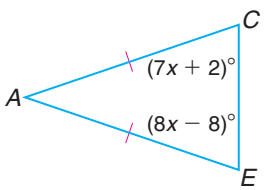
20. Find x in the figure shown. *(Lesson 8–1)*



Standardized Test Practice

A B C D

21. Short Response In $\triangle ACE$, $\overline{AC} \cong \overline{AE}$. If $m\angle C = 7x + 2$, and $m\angle E = 8x - 8$, what is $m\angle C$ and $m\angle E$? *(Lesson 6–4)*



22. Multiple Choice 4 is what percent of 20? *(Percent Review)*

(A) 16% (B) 18% (C) 20% (D) 22%

9–4 Proportional Parts and Triangles

What You'll Learn

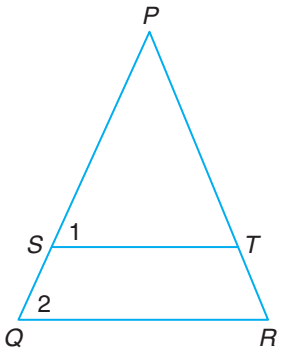
You'll learn to identify and use the relationships between proportional parts of triangles.

Why It's Important

Construction

Carpenters use proportional parts of triangles to determine the length of a brace. *See Exercise 23.*

In $\triangle PQR$, $\overline{ST} \parallel \overline{QR}$, and $\overline{ST}$ intersects the other two sides of $\triangle PQR$. Note the shape of $\triangle PST$. Are $\triangle PQR$ and $\triangle PST$ similar?



Since $\angle 1$ and $\angle 2$ are congruent corresponding angles and $\angle P \cong \angle P$, $\triangle PST \sim \triangle PQR$. Why?

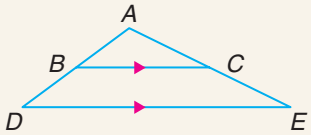
By Postulate 9–1, if two angles are congruent, then the two triangles are similar.

This characteristic of a line parallel to a side of a triangle is expressed in Theorem 9–4.

Theorem 9–4

Words: If a line is parallel to one side of a triangle and intersects the other two sides, then the triangle formed is similar to the original triangle.

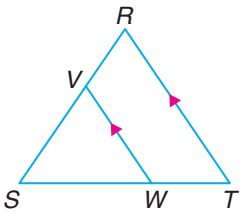
Model:



Symbols: If $\overline{BC} \parallel \overline{DE}$, then $\triangle ABC \sim \triangle ADE$.

You can use Theorem 9–4 to write proportions.

Example 1 **Complete the proportion $\frac{SV}{SR} = \frac{?}{RT}$.**



Since $\overline{VW} \parallel \overline{RT}$, $\triangle SVW \sim \triangle SRT$.

Therefore, $\frac{SV}{SR} = \frac{VW}{RT}$.

Your Turn

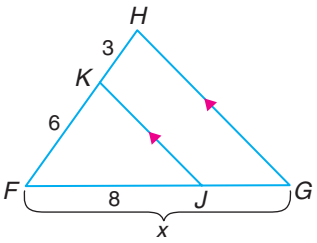
a. Use the triangle above to complete the proportion $\frac{ST}{SW} = \frac{SR}{?}$.

You can use proportions to solve for missing measures in triangles.

Example 2

Algebra Link

In the figure, $\overline{JK} \parallel \overline{GH}$. Find the value of x.



Explore You are given a triangle with a line parallel to one side of the triangle. You need to find the measure of $\overline{FG}$.

Plan You know $\triangle FJK \sim \triangle FGH$ by Theorem 9–4. Use this information to write a proportion and solve for x.

Solve

$\frac{FK}{FH} = \frac{FJ}{FG}$ *Definition of Similar Polygons*

$\frac{6}{9} = \frac{8}{x}$ *FK = 6, FH = 6 + 3 or 9, FJ = 8, FG = x*

$6(x) = 9(8)$ *Cross Products*

$6x = 72$ *Simplify.*

$\frac{6x}{6} = \frac{72}{6}$ *Divide each side by 6.*

$x = 12$ *Simplify.*

Algebra Review
Solving One-Step Equations, p. 722

Examine Check the proportion by substituting 12 for x.

$\frac{6}{9} = \frac{8}{x}$ *Original Proportion*

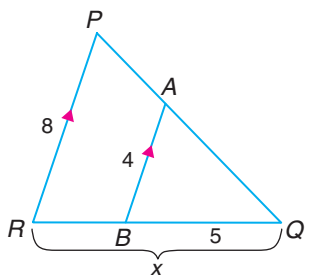
$\frac{6}{9} \stackrel{?}{=} \frac{8}{12}$ *Substitution*

$6(12) \stackrel{?}{=} 9(8)$ *Cross Products*

$72 = 72$ ✓

Your Turn

b. In the figure, $\overline{AB} \parallel \overline{PR}$. Find the value of x.

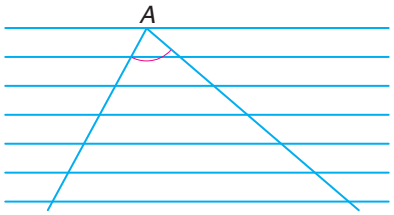


What other relationship occurs when a line is parallel to one side of a triangle and intersects the other two sides?

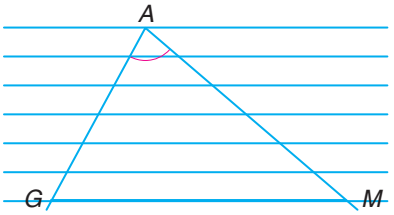
Hands-On Geometry

Materials: lined paper, protractor, ruler

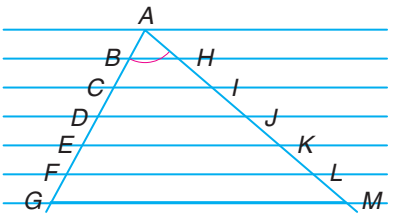
Step 1 On a piece of lined paper, pick a point on one of the lines and label it A. Use a straightedge and protractor to draw $\angle A$ so that $m\angle A < 90$ and only the vertex lies on the line.



Step 2 Extend one side of $\angle A$ down six lines. Label this point G. Do the same for the other side of $\angle A$. Label this point M. Now connect points G and M to form $\triangle AGM$.



Step 3 Label the points where the horizontal rules intersect $\overline{AG}$, B through F, as shown. Label those points where the horizontal rules intersect $\overline{AM}$, H through L.



Try These

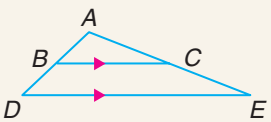
1. Measure $\overline{AC}$, $\overline{CG}$, $\overline{AD}$, $\overline{DG}$, $\overline{AE}$, $\overline{EG}$, $\overline{AI}$, $\overline{IM}$, $\overline{AJ}$, $\overline{JM}$, $\overline{AK}$, and $\overline{KM}$.
2. Calculate and compare the following ratios.
   a. $\frac{AC}{CG}$ and $\frac{AI}{IM}$ b. $\frac{AD}{DG}$ and $\frac{AJ}{JM}$ c. $\frac{AE}{EG}$ and $\frac{AK}{KM}$
3. What can you conclude about the lines through the sides of $\triangle AGM$ and parallel to $\overline{GM}$?

The activity above suggests Theorem 9–5.

Theorem 9–5

Words: If a line is parallel to one side of a triangle and intersects the other two sides, then it separates the sides into segments of proportional lengths.

Model:



Symbols: If $\overline{BC} \parallel \overline{DE}$, then $\frac{AB}{BD} = \frac{AC}{CE}$.

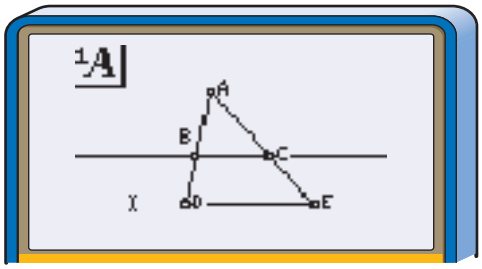
You can use a TI–83 Plus/TI–84 Plus graphing calculator to verify Theorem 9–5.

Graphing Calculator Tutorial
See pp. 782–785.

Graphing Calculator Exploration

Step 1 Select the Triangle tool on F2. Draw and label triangle DAE.

Step 2 Next, use the Point on Object tool on F2 to draw a point between D and A on side $\overline{DA}$. Label the point B.



Step 3 Use the Parallel Line tool on F3 to draw a line through B parallel to side $\overline{DE}$.

Step 4 Use the Intersection Point tool on F2 to mark the point where the line intersects side $\overline{EA}$. Label the point C. You now have a figure that you can use to verify Theorem 9–5.

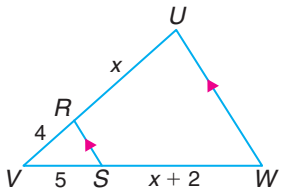
Try These

1. Use the Distance & Length tool on F5 to find the length of $\overline{AB}$, $\overline{BD}$, $\overline{AC}$, and $\overline{CE}$.
2. Use the Calculate tool on F5 to calculate the value of $AB \div BD$ and $AC \div CE$. What can you say about these values?
3. Describe what happens to the segment lengths from Exercise 1 and the ratios from Exercise 2 when you drag point B closer to A.

Example 3

Algebra Link

In the figure, $\overline{RS} \parallel \overline{UW}$. Find the value of x.



$$\frac{VR}{RU} = \frac{VS}{SW} \quad \textit{Theorem 9–5}$$

$$\frac{4}{x} = \frac{5}{x+2} \quad \textit{VR = 4, RU = x, VS = 5, SW = x + 2}$$

$$4(x+2) = x(5) \quad \textit{Cross Products}$$

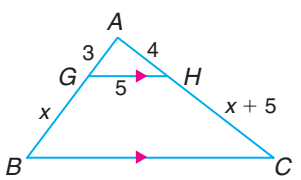
$$4x + 8 = 5x \quad \textit{Distributive Property}$$

$$4x + 8 - 4x = 5x - 4x \quad \textit{Subtract 4x from each side.}$$

$$8 = x \quad \textit{Simplify.}$$

Your Turn

c. In the figure, $\overline{GH} \parallel \overline{BC}$. Find the value of x.

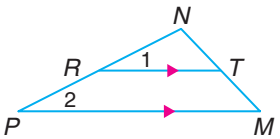


Check for Understanding

Communicating Mathematics

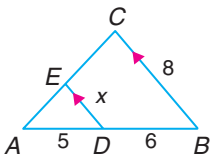
1. **Explain** why $\triangle NRT \sim \triangle NPM$.

2. **Write** four proportions for the figure.



Exercises 1–2

3. **You Decide** Casey uses the proportion $\frac{5}{6} = \frac{x}{8}$ to solve for x in the figure. Jacob says she should use the proportion $\frac{5}{11} = \frac{x}{8}$. Who is correct? Explain your reasoning.



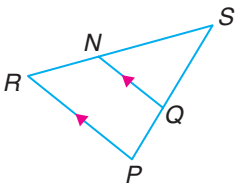
Guided Practice

Example 1

Complete each proportion.

4. $\frac{NQ}{RP} = \frac{?}{SR}$

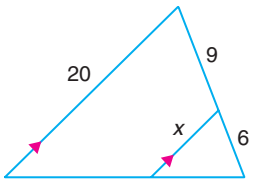
5. $\frac{SN}{?} = \frac{SQ}{QP}$



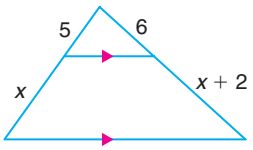
Examples 2 & 3

Find the value of each variable.

6.

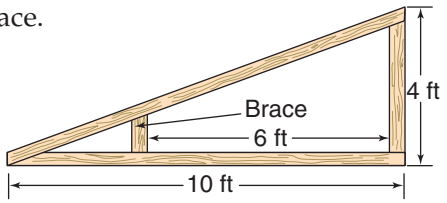


7.



Example 2

8. **Construction** A roof rafter is shown at the right. Find the length of the brace.



Exercises

Practice

Homework Help	
For Exercises	See Examples
9–14	1
15–24	2, 3
Extra Practice	
See page 742.	

Complete each proportion.

9. $\frac{GN}{GH} = \frac{GM}{?}$

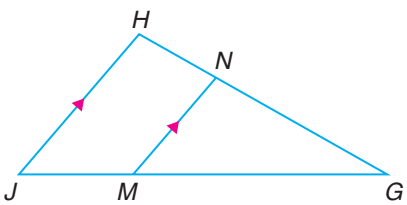
10. $\frac{GN}{NH} = \frac{GM}{?}$

11. $\frac{NM}{HJ} = \frac{?}{GJ}$

12. $\frac{?}{MN} = \frac{GH}{GN}$

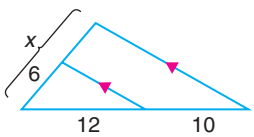
13. $\frac{GJ}{?} = \frac{GH}{GN}$

14. $\frac{?}{NH} = \frac{GM}{MJ}$

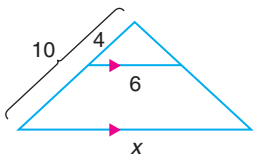


Find the value of each variable.

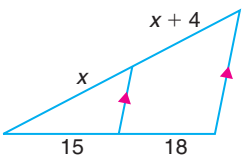
15.



16.



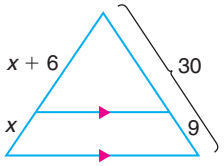
17.



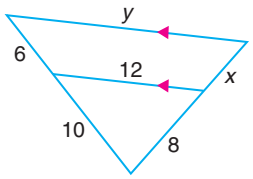
CONTENTS

Find the value of each variable.

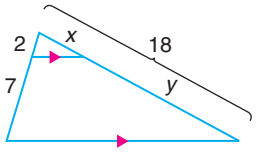
18.



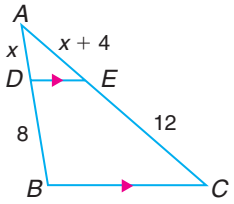
19.



20.

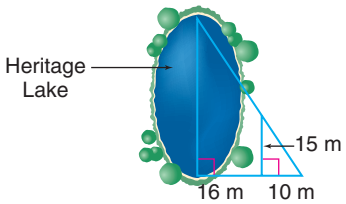


21. Find AD, AB, AE, and AC.

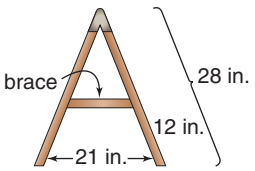


Applications and Problem Solving

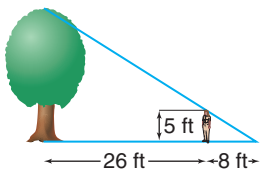
22. **Surveying** Antoine wants to find the distance across Heritage Lake. According to his measurements, what is the distance across the lake?



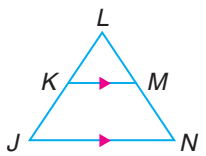
23. **Construction** Hannah is building a sawhorse. According to the diagram, how long should she make the brace?



24. **Forestry** Ranger Lopez wants to know how tall the tree is that she planted five years ago. She walks away from the tree until the end of her shadow and the tree's shadow coincide. Use her measurements to determine the height of the tree.



25. **Critical Thinking** $\triangle JLN$ is equilateral. If $\overline{KM} \parallel \overline{JN}$, is $\triangle KLM$ equilateral? Explain.



Mixed Review

26. Draw and label two triangles that are similar by SAS Similarity. *(Lesson 9–3)*

27. **Sports** A volleyball court measures 30 feet by 60 feet. Make a scale drawing of the court if 1 centimeter represents 12 feet. *(Lesson 9–2)*

28. If $m\angle 1 = 67$ and $\angle 1$ and $\angle 2$ form a linear pair, find $m\angle 2$. *(Lesson 3–5)*

Standardized Test Practice

29. **Short Response** Graph point M with coordinates $(-4, -3)$. *(Lesson 2–4)*

30. **Multiple Choice** Solve $3h - 5 = 13$. *(Algebra Review)*

(A) -4 (B) -2 (C) -7 (D) 6

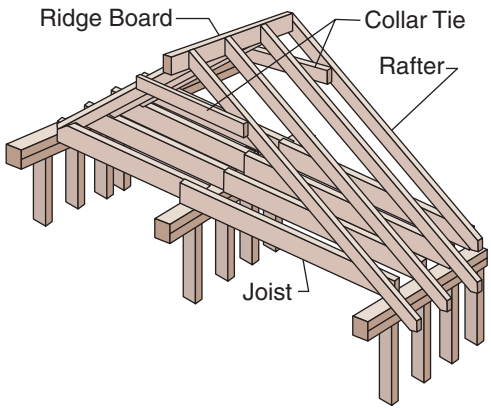
9–5 Triangles and Parallel Lines

What You'll Learn
You'll learn to use proportions to determine whether lines are parallel to sides of triangles.

Why It's Important
Building Carpenters can use proportions to make sure boards are parallel to other boards. *See Exercise 1.*

Jodie Rudberg is a carpenter. She is building the framework for a roof. How can she be sure the collar tie is parallel to the joist?

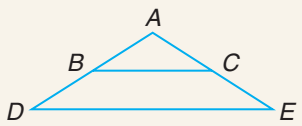
You know that if a line is parallel to one side of a triangle and intersects the other two sides, then it separates the sides into segments of proportional lengths (Theorem 9–5). The converse of this theorem is also true.



Theorem 9–6

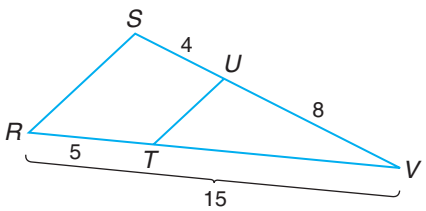
Words: If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

Model:



Symbols: If $\frac{AB}{BD} = \frac{AC}{CE}$, then $\overline{BC} \parallel \overline{DE}$.

Example 1 Determine whether $\overline{TU} \parallel \overline{RS}$.



First, find TV.

$RT + TV = RV$
$5 + TV = 15$ *Replace RT with 5 and RV with 15.*
$5 + TV - 5 = 15 - 5$ *Subtract 5 from each side.*
$TV = 10$

Then, determine whether $\frac{RT}{TV}$ and $\frac{SU}{UV}$ form a proportion.

$\frac{RT}{TV} \stackrel{?}{=} \frac{SU}{UV}$
$\frac{5}{10} \stackrel{?}{=} \frac{4}{8}$ *RT = 5, TV = 10, SU = 4, and UV = 8*
$5(8) \stackrel{?}{=} 10(4)$ *Cross Products*
$40 = 40$ ✓

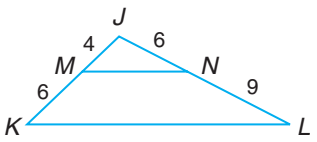
Using Theorem 9–6, $\overline{TU} \parallel \overline{RS}$.

CONTENTS

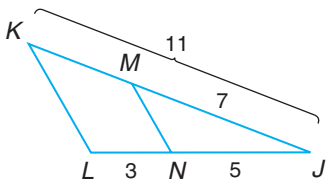
Your Turn

In each figure, determine whether $\overline{MN} \parallel \overline{KL}$.

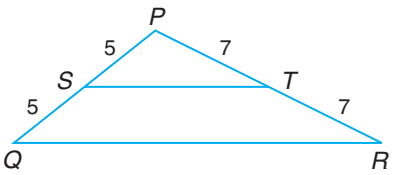
a.



b.



Consider the special case at the right. S is the midpoint of $\overline{PQ}$ and T is the midpoint of $\overline{PR}$. Because $\frac{5}{5} = \frac{7}{7}$, $\overline{ST} \parallel \overline{QR}$ by Theorem 9–6. Therefore, $\triangle PST \sim \triangle PQR$ by Theorem 9–4. Using the definition of similar polygons, $\frac{ST}{QR} = \frac{PS}{PQ}$. But $\frac{PS}{PQ} = \frac{5}{10}$, so $\frac{ST}{QR} = \frac{5}{10}$ or $\frac{1}{2}$.



The general conclusion is stated in Theorem 9–7.

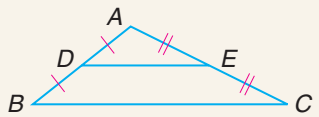
Reading Geometry

Segment DE is called the **midsegment** of the triangle because its endpoints are the midpoints of two sides of the triangle.

Theorem 9–7

Words: If a segment joins the midpoints of two sides of a triangle, then it is parallel to the third side, and its measure equals one-half the measure of the third side.

Model:



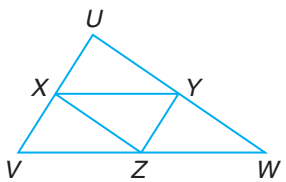
Symbols: If D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{AC}$, then $\overline{DE} \parallel \overline{BC}$ and $DE = \frac{1}{2}BC$.

Examples

Algebra Link

X, Y, and Z are midpoints of the sides of $\triangle UVW$. Complete each statement.

2 **If $VW = 6a$, then $XY =$ ___?___.**



$XY = \frac{1}{2}VW$ *Theorem 9–7*

$XY = \frac{1}{2}(6a)$ *Replace VW with $6a$.*

$XY = 3a$ *Multiply.*

If $VW = 6a$, then $XY = 3a$.

(continued on the next page)

3 **If $m\angle WZY = 2b + 1$, then $m\angle WVU =$ ___?___.**

By Theorem 9–6, $\overline{YZ} \parallel \overline{UV}$. Since $\overline{YZ}$ and $\overline{UV}$ are parallel segments cut by transversal $\overline{VW}$, $\angle WZY$ and $\angle WVU$ are congruent corresponding angles.

If $m\angle WZY = 2b + 1$, then $m\angle WVU = 2b + 1$.

Your Turn

c. $\overline{XZ} \parallel$ ___?___ **d.** If $YZ = c$, then $UV =$ ___?___.

Check for Understanding

Communicating Mathematics

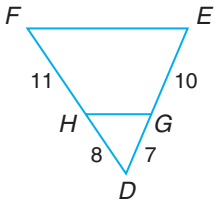
1. **Describe** how Ms. Rudberg can determine if the collar tie is parallel to the joist in the application at the beginning of the lesson.

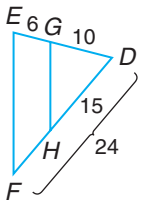
2. **Draw** a triangle. Find the midpoints of two sides of the triangle and draw a segment between the two midpoints. Measure this segment and the third side of the triangle. Which theorem is confirmed?

Guided Practice

Example 1

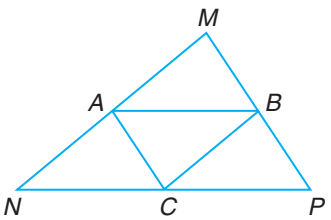
In each figure, determine whether $\overline{GH} \parallel \overline{EF}$.

3. 

4. 

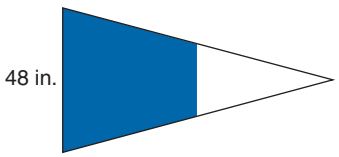
Examples 2 & 3

***A*, *B*, and *C* are the midpoints of the sides of $\triangle MNP$. Complete each statement.**



5. $\overline{MP} \parallel$ ___?___
6. If $BC = 14$, then $MN =$ ___?___.
7. If $m\angle MNP = s$, then $m\angle BCP =$ ___?___.
8. If $MP = 18x$, then $AC =$ ___?___.

9. **Communication** Ships signal each other using an international flag code. There are over 40 signaling flags, including the one at the right. The line that divides the white and blue portions of the flag intersects two sides of the flag at their midpoints. If the longer side of the blue portion is 48 inches long, find the length of the line dividing the white and blue portions of the flag. **Example 2**



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Exercises

Practice

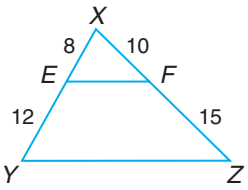
Homework Help	
For Exercises	See Examples
10–17	1
18, 19, 22, 23, 26–31	2
20, 21, 24, 25	3

Extra Practice

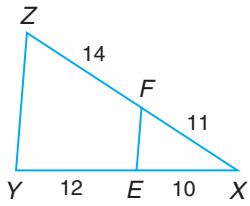
See page 742.

In each figure, determine whether $\overline{EF} \parallel \overline{YZ}$.

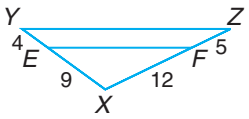
10.



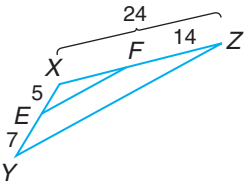
11.



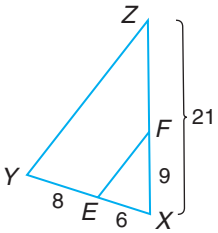
12.



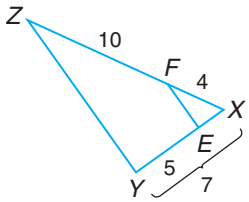
13.



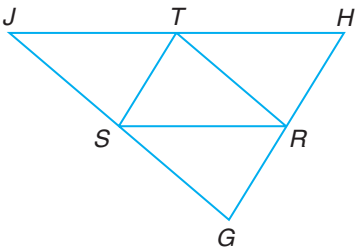
14.



15.



***R*, *S*, and *T* are the midpoints of the sides of $\triangle GHJ$. Complete each statement.**



Exercises 16–27

16. $\overline{RS} \parallel$ ___?___

17. $\overline{GH} \parallel$ ___?___

18. If $RS = 36$, then $HJ =$ ___?___.

19. If $GH = 44$, then $ST =$ ___?___.

20. If $m\angle JST = 57$, then $m\angle JGH =$ ___?___.

21. If $m\angle GHJ = 31$, then $m\angle STJ =$ ___?___.

22. If $GJ = 10x$, then $RT =$ ___?___.

23. If $ST = 20y$, then $GH =$ ___?___.

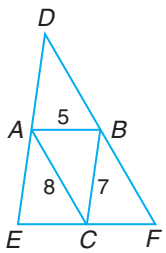
24. If $m\angle HGJ = 4a$, then $m\angle TSJ =$ ___?___.

25. If $m\angle JTS = 8b$, then $m\angle JHG =$ ___?___.

26. If $RS = 12x$, then $HT =$ ___?___.

27. If $GR = x + 5$, then $ST =$ ___?___.

28. A, B, and C are the midpoints of the sides of $\triangle DEF$.



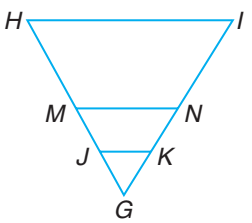
a. Find DE, EF, and FD.

b. Find the perimeter of $\triangle ABC$.

c. Find the perimeter of $\triangle DEF$.

d. Find the ratio of the perimeter of $\triangle ABC$ to the perimeter of $\triangle DEF$.

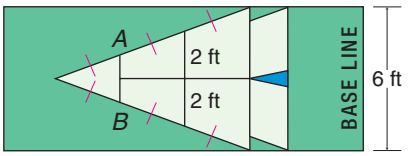
29. M is the midpoint of $\overline{GH}$, J is the midpoint of $\overline{MG}$, N is the midpoint of $\overline{GI}$, and K is the midpoint of $\overline{NG}$. If HI is 24, find MN and JK.



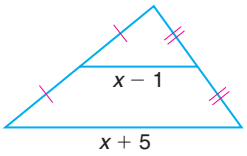
CONTENTS

Applications and Problem Solving

30. Recreation Ryan is painting the lines on a shuffleboard court. Find AB.



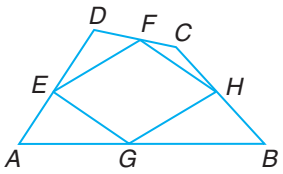
31. Algebra Find the value of x.



32. Critical Thinking $ABCD$ is a quadrilateral. E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{DC}$, G is the midpoint of $\overline{AB}$, and H is the midpoint of $\overline{BC}$.

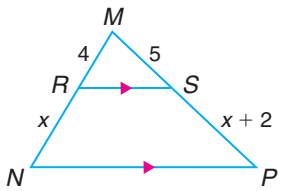
a. What can you say about $\overline{EF}$ and $\overline{GH}$? Explain. (*Hint:* Draw diagonal $\overline{AC}$.)

b. What kind of figure is $EFHG$?



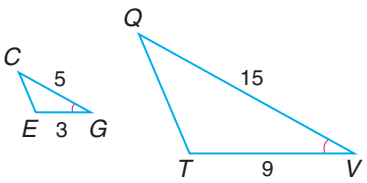
Mixed Review

33. In the figure shown, $\overline{RS} \parallel \overline{NP}$. Find the value of x. *(Lesson 9–4)*



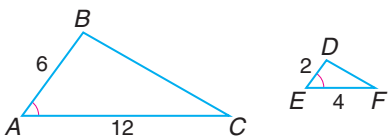
Determine whether the triangles shown are similar. If so, tell which similarity test is used and complete the statement. *(Lesson 9–3)*

34.



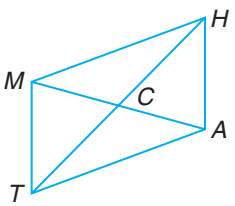
$\triangle CEG \sim \triangle$ ___?___

35.



$\triangle$ ___?___ $\sim \triangle DFE$

36. In $\square MTAH$, $MA = 46$. Find MC. *(Lesson 8–2)*



Standardized Test Practice

37. Grid In Find the measure of $\angle P$ in quadrilateral $GEPN$ if $m\angle G = 130$, $m\angle E = 2x$, $m\angle P = 3x + 10$, and $m\angle N = 95$. *(Lesson 8–1)*

38. Multiple Choice What is the y-intercept of the graph of the equation $y = \frac{1}{3}x + 2$? *(Lesson 4–6)*

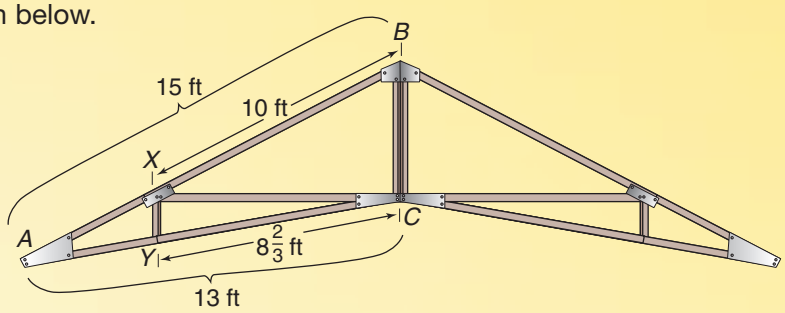
(A) 3 (B) $\frac{1}{3}$ (C) -2 (D) 2

Math In the Workplace

Carpenter

Did you know that carpenters make up the largest group of skilled workers employed in the building trades? Carpenters cut, fit, and assemble wood and other materials in the construction of structures such as houses, buildings, and highways.

When constructing houses, carpenters use trusses to frame the roof. There are many types of trusses. One type, the *scissors truss*, is shown below.



1. Determine whether $\overline{XY} \parallel \overline{BC}$. Explain your reasoning.
2. Complete the following: $\triangle AXY \sim \triangle$ ___?___.

FAST FACTS About Carpenters

Working Conditions

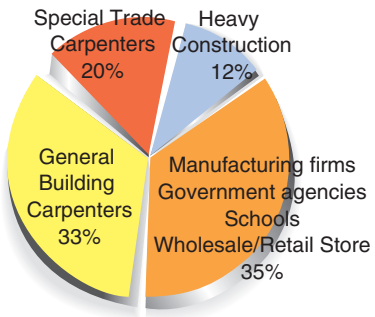
- generally work outdoors
- work may be strenuous
- can change employers each time a job is completed
- may risk injury from slips or falls, working with rough materials, and using tools and power equipment

Education

- high school industrial technology, mechanical drawing, carpentry, and math courses
- on-the-job training or apprenticeships

Employment

Where Carpenters Are Employed



Source: Occupational Outlook Handbook

interNET CONNECTION **Career Data** For the latest information about a career in carpentry, visit:
www.geomconcepts.com

Chapter 9

Investigation

Are Golden Triangles Expensive?

Materials

- tracing paper
- straightedge
- protractor
- compass

Ratios of a Special Triangle

The Taj Mahal (golden rectangle)

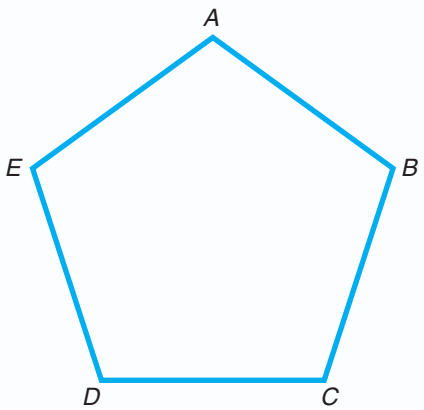
You may have heard of the golden rectangle, whose sides have a special ratio called the **golden ratio**. The golden ratio is approximately 1.618 to 1 or about 1.618. Artists and architects often use the golden rectangle because it is pleasing to the eye. A look at the Parthenon in Greece, the Taj Mahal in India, or the Lincoln Memorial in Washington, D.C., will reveal uses of the golden rectangle in architecture.

Do you think there are golden triangles? How would they be constructed? Let's find out.

Investigate

1. Construct a golden triangle.

a. Trace the regular pentagon below and use a photocopier to enlarge it. Draw $\overline{AC}$ and $\overline{AD}$.



Mona Lisa (golden triangle)

b. Use a protractor to find the measures of the angles of $\triangle ADC$. Classify $\triangle ADC$.

c. Find the ratio of AD to DC. How does this ratio compare to the golden ratio?

Artists often use golden triangles to draw your eye toward the face of the subject. Notice how the folded arms and head of the *Mona Lisa* form a triangle.

2. Use your pentagon and triangle to construct another golden triangle. Follow the steps below.

   a. Using a compass and straightedge, bisect $\angle ADC$. Label the point where the angle bisector intersects $\overline{AC}$ point F.

   b. What are the measures of the angles in $\triangle DCF$? Classify $\triangle DCF$.

   c. Find the ratio of DC to FC. How does this ratio compare to the golden ratio?

   d. What conclusions can you draw from this activity?

Extending the Investigation

In this extension, you will investigate the golden ratio, golden triangles, and golden rectangles.

- Start with a regular pentagon. Draw at least four golden triangles, each one smaller than the previous one. Label the triangles and show the golden ratio in each triangle. You may want to use different colors to outline the different triangles.
- Are the golden triangles you drew within the pentagon similar? Explain.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a bulletin board to display your golden triangles.
- Research the golden ratio. Write a brief paper describing five examples where the golden ratio has been used.
- Research the golden rectangle. Make a poster demonstrating how to construct a golden rectangle.

Investigation For more information on the golden ratio, visit: www.geomconcepts.com

9–6 Proportional Parts and Parallel Lines

What You'll Learn
You'll learn to identify and use the relationships between parallel lines and proportional parts.

Why It's Important
Real Estate Builders can use parallel lines and proportional parts to determine the length of a side of a building site. *See Exercise 24.*

The artistic concept of perspective combines proportion and the properties of parallel lines.

In the figure, points A and B lie on the horizon. $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ represent two lines extending from the horizon to the foreground. The two lines also form transversals for the parallel lines m, n, and p.

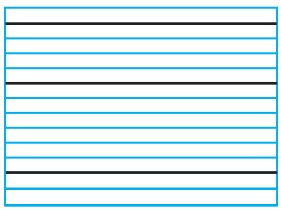
Along the transversals $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$, the parallel lines cut segments of different lengths. Is there a relationship between the lengths of the segments?

The following activity investigates transversals that cross three parallel lines.

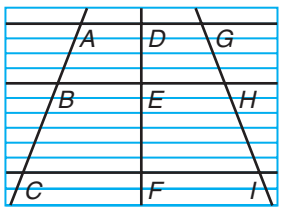
Hands-On Geometry

Materials: lined paper ruler

Step 1 Draw a dark line over the line at the top of your lined paper. Count down four lines and draw a dark line over that line. Count down six more lines and draw a dark line over that line.



Step 2 Draw three different transversals that cross each of the parallel lines you drew. Label the points of intersection as shown.



Try These

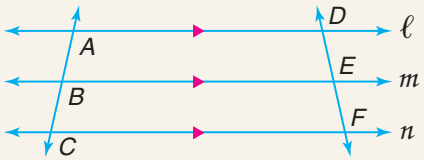
1. Measure $\overline{AB}$, $\overline{BC}$, $\overline{AC}$, $\overline{DE}$, $\overline{EF}$, $\overline{DF}$, $\overline{GH}$, $\overline{HI}$, and $\overline{GI}$.
2. Calculate each set of ratios. Determine whether the ratios in each set are equivalent to each other.
   a. $\frac{AB}{BC}, \frac{DE}{EF}, \frac{GH}{HI}$ b. $\frac{AB}{AC}, \frac{DE}{DF}, \frac{GH}{GI}$ c. $\frac{BC}{AC}, \frac{EF}{DF}, \frac{HI}{GI}$
3. Do the parallel lines divide the transversals proportionally?

The previous activity suggests Theorem 9–8.

Theorem 9–8

Words: If three or more parallel lines intersect two transversals, the lines divide the transversals proportionally.

Model:

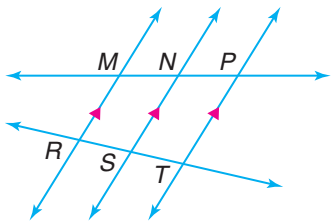


Symbols: If $\ell \parallel m \parallel n$, then $\frac{AB}{BC} = \frac{DE}{EF}$, $\frac{AB}{AC} = \frac{DE}{DF}$, and $\frac{BC}{AC} = \frac{EF}{DF}$.

Example 1

Complete the proportion $\frac{MN}{MP} = \frac{?}{RT}$.

Since $\overleftrightarrow{MR} \parallel \overleftrightarrow{NS} \parallel \overleftrightarrow{PT}$, the transversals are divided proportionally. Therefore, $\frac{MN}{MP} = \frac{RS}{RT}$.



Your Turn

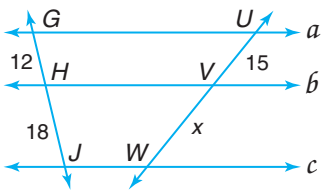
a. Use the figure above to complete the proportion $\frac{RS}{ST} = \frac{MN}{?}$.

You can use proportions from parallel lines to solve for missing measures.

Example 2

Algebra Link

In the figure, $a \parallel b \parallel c$. Find the value of x.



$\frac{GH}{HJ} = \frac{UV}{VW}$ *Theorem 9–8*

$\frac{12}{18} = \frac{15}{x}$ *GH = 12, HJ = 18, UV = 15, VW = x*

$12(x) = 18(15)$ *Cross Products*

$12x = 270$ *Simplify.*

$\frac{12x}{12} = \frac{270}{12}$ *Divide each side by 12.*

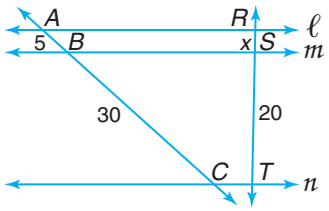
$x = \frac{45}{2}$ or $22\frac{1}{2}$ *Simplify.*

Algebra Review
Solving One-Step Equations, p. 722

(continued on the next page)

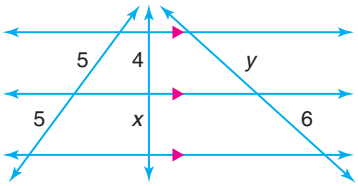
Your Turn

b. In the figure, $\ell \parallel m \parallel n$. Find the value of x.



Suppose three parallel lines intersect a transversal and divide the transversal into congruent segments. Refer to the figure below. The transversal on the left is divided into congruent segments. Is this true of the other two transversals? Find the values of x and y.

$\frac{5}{5} = \frac{4}{x}$	*Theorem 9–8*	$\frac{5}{5} = \frac{y}{6}$
$5(x) = 5(4)$	*Cross Products*	$5(6) = 5(y)$
$5x = 20$	*Simplify.*	$30 = 5y$
$\frac{5x}{5} = \frac{20}{5}$	*Divide each side by 5.*	$\frac{30}{5} = \frac{5y}{5}$
$x = 4$	*Simplify.*	$6 = y$

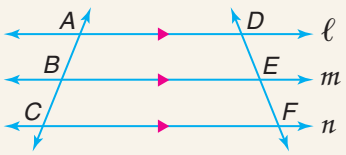


These results suggest Theorem 9–9.

Theorem 9–9

Words: If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

Model:

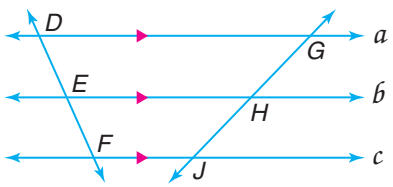


Symbols: If $\ell \parallel m \parallel n$ and $\overline{AB} \cong \overline{BC}$, then $\overline{DE} \cong \overline{EF}$.

Check for Understanding

Communicating Mathematics

1. **Write** at least three proportions if $a \parallel b \parallel c$.
2. **Describe** how you would find GJ if $EF = 15$, $DF = 25$, and $GH = 12$.



Exercises 1–2

3. **Writing Math** Draw a segment and label the endpoints A and B. Use the following steps to divide $\overline{AB}$ into three congruent segments.

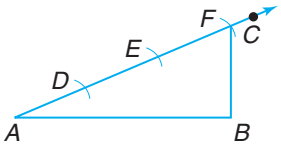
a. Draw $\overrightarrow{AC}$ so that $\angle BAC$ is an acute angle.

b. With a compass, start at A and mark off three congruent segments on $\overrightarrow{AC}$. Label these points D, E, and F.

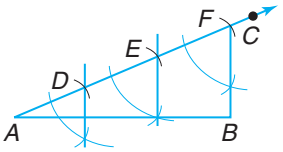
c. Draw $\overline{BF}$.

d. Construct lines through D and E that are parallel to $\overline{BF}$. These parallel lines will divide $\overline{AB}$ into three congruent segments.

e. Explain why this construction works.



Parts a–c



Part d

Look Back

Constructing Parallel Lines, Lesson 4–4

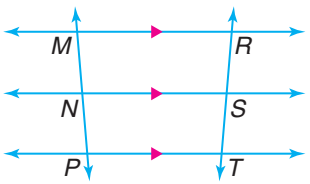
Guided Practice

Example 1

Complete each proportion.

4. $\frac{NP}{MP} = \frac{?}{RT}$

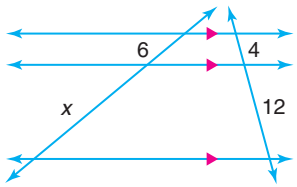
5. $\frac{ST}{?} = \frac{NP}{MN}$



Example 2

Find the value of *x*.

6.



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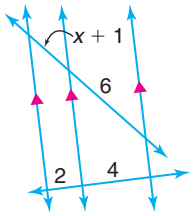
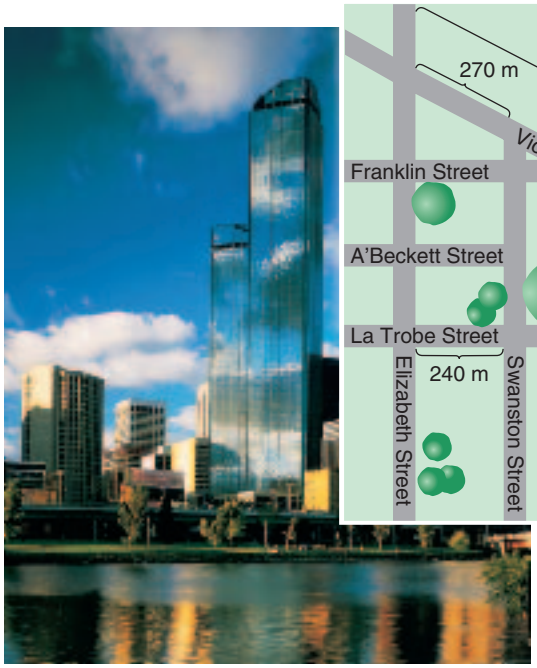
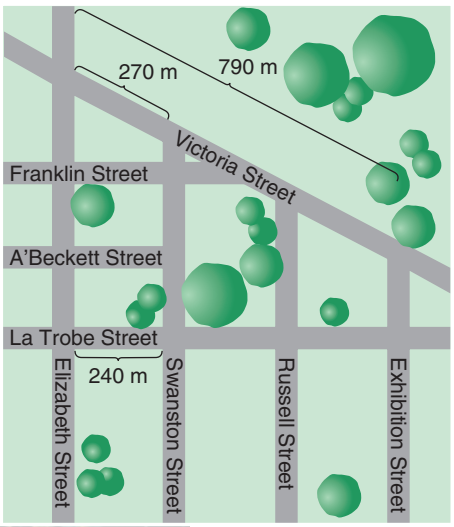



Melbourne, Australia



8. **Travel** Alex is visiting the city of Melbourne, Australia. Elizabeth Street, Swanston Street, Russell Street, and Exhibition Street are parallel. If Alex wants to walk along La Trobe Street from Elizabeth Street to Exhibition Street, approximately how far will he walk? **Example 2**

Exercises

Practice

Homework Help	
For Exercises	See Examples
9–14	1
15–24	2
Extra Practice	
See page 743.	

Complete each proportion.

9. $\frac{DF}{FB} = \frac{?}{CE}$

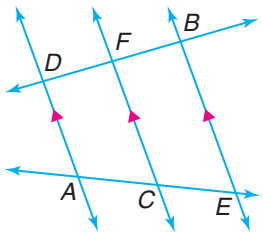
10. $\frac{?}{DF} = \frac{CE}{AC}$

11. $\frac{AC}{?} = \frac{DF}{DB}$

12. $\frac{AC}{AE} = \frac{DF}{?}$

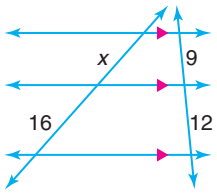
13. $\frac{?}{FB} = \frac{AE}{CE}$

14. $\frac{AE}{?} = \frac{DB}{DF}$



Find the value of *x*.

15.



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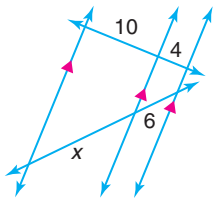

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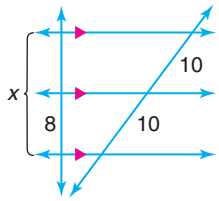

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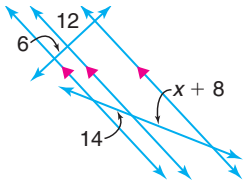

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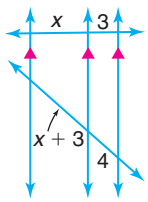

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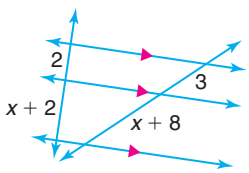
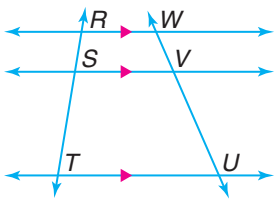


21. If $RT = 12$, $WV = 5$, and $WU = 13$, find RS.

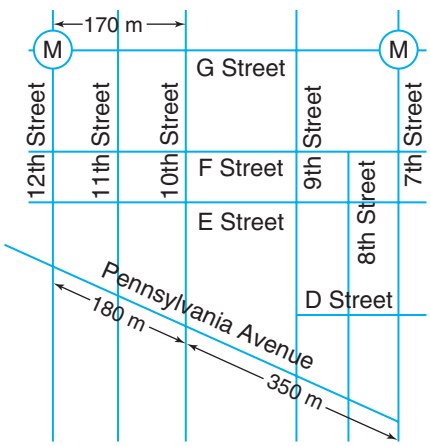
22. If $RT = 20$, $ST = 15$, and $WU = 12$, find VU.



Exercises 21–22

Applications and Problem Solving

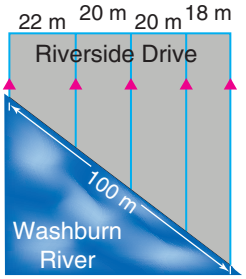
23. **City Planning** Numbered streets in Washington, D.C., run north and south. Lettered name streets run east and west. Other streets radiate out like spokes of a wheel and are named for states. The Metro subway system goes under some of the roads. Find the approximate distance between the two Metro stations indicated on the map by M.



CONTENTS

24. **Real Estate** Hometown Builders are selling four building sites along Washburn River. If the total river frontage of the lots is 100 meters, find the river frontage for each lot.

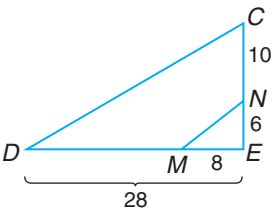
25. **Critical Thinking** Exercise 3 on page 385 describes a construction that divides a segment into three congruent segments. Draw a line segment and describe how to divide it into three segments with the ratio 1:2:3. Then construct the divided segment.



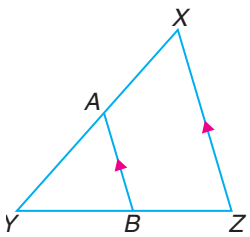
Exercise 24

Mixed Review

26. Triangle CDE is shown. Determine whether $\overline{DC} \parallel \overline{MN}$. *(Lesson 9–5)*



27. Use $\triangle XYZ$ to complete the proportion $\frac{YX}{YA} = \frac{YZ}{?}$. *(Lesson 9–4)*

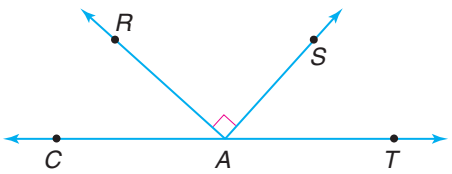


28. Is a triangle with measures 18, 24, and 30 a right triangle? *(Lesson 6–6)*

Standardized Test Practice

A B C D

29. **Grid In** $\overrightarrow{AC}$ and $\overrightarrow{AT}$ are opposite rays and $\overrightarrow{AR} \perp \overrightarrow{AS}$. If $m\angle CAR = 42$, find $m\angle SAT$. *(Lesson 3–7)*

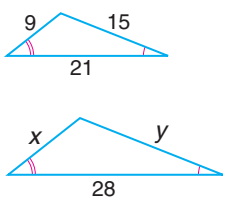


30. **Short Response** Draw and label a rectangle that has an area of 42 square centimeters. *(Lesson 1–6)*

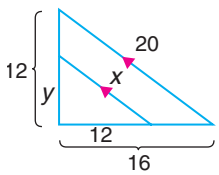
Quiz 2 — Lessons 9–3 through 9–6

Find the value of each variable. *(Lessons 9–3, 9–4, 9–5, and 9–6)*

1.



2.



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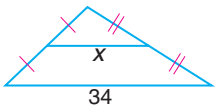
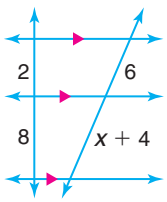


4.



5. Carl is 6 feet tall and casts an 8-foot shadow. At the same time, a flagpole casts a 48-foot shadow. How tall is the flagpole? *(Lesson 9–3)*

9–7 Perimeters and Similarity

What You'll Learn
You'll learn to identify and use proportional relationships of similar triangles.

Why It's Important
Surveying Surveyors use scale factors to estimate distance.
See Exercise 12.

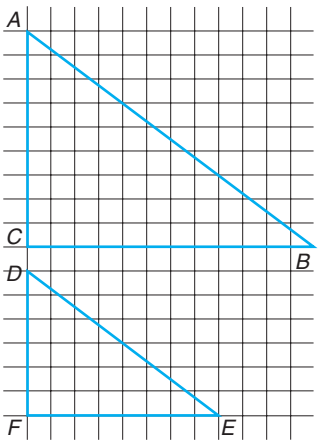
If two triangles are similar, then the measures of their corresponding sides are proportional. Is there a relationship between the measures of the perimeters of the two triangles?

Hands-On Geometry

Materials: grid paper

Step 1 On grid paper, draw right triangle ABC with legs 9 units and 12 units long as shown.

Step 2 On grid paper, draw right triangle DEF with legs 6 units and 8 units long as shown.



Try These

1. Use the Pythagorean Theorem to find AB and DE.
2. Find the ratios $\frac{AB}{DE}$, $\frac{BC}{EF}$, and $\frac{CA}{FD}$.
3. Are the triangles similar? Explain.
4. Find the perimeters of $\triangle ABC$ and $\triangle DEF$.
5. Find the ratio $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF}$.
6. Compare the ratios $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF}$, $\frac{AB}{DE}$, $\frac{BC}{EF}$, and $\frac{CA}{FD}$. Describe your results.

Look Back
Pythagorean Theorem: Lesson 6–6

The activity above suggests Theorem 9–10.

Theorem 9–10

Words: If two triangles are similar, then the measures of the corresponding perimeters are proportional to the measures of the corresponding sides.

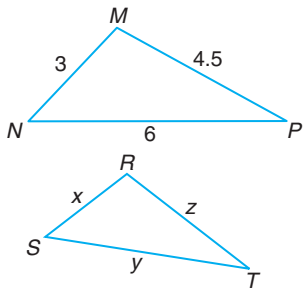
Symbols: If $\triangle ABC \sim \triangle DEF$, then
$$\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}.$$

You can find the measures of all three sides of a triangle when you know the perimeter of the triangle and the measures of the sides of a similar triangle.

Example 1

Algebra Link

The perimeter of $\triangle RST$ is 9 units, and $\triangle MNP \sim \triangle RST$. Find the value of each variable.



$$\frac{MN}{RS} = \frac{\text{perimeter of } \triangle MNP}{\text{perimeter of } \triangle RST} \quad \textit{Theorem 9–10}$$

$$\frac{3}{x} = \frac{13.5}{9} \quad \textit{The perimeter of } \triangle MNP \textit{ is } 3 + 6 + 4.5 \textit{ or } 13.5.$$

$$3(9) = x(13.5) \quad \textit{Cross Products}$$

$$27 = 13.5x \quad \textit{Simplify.}$$

$$\frac{27}{13.5} = \frac{13.5x}{13.5} \quad \textit{Divide each side by 13.5.}$$

$$2 = x \quad \textit{Simplify.}$$

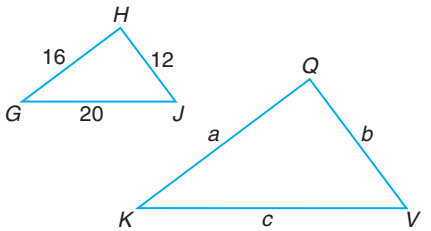
Algebra Review
Solving One-Step Equations, p. 722

Because the triangles are similar, two other proportions can be written to find the value of the other two variables.

$\frac{MN}{RS} = \frac{NP}{ST}$	*Definition of Similar Polygons*	$\frac{MN}{RS} = \frac{PM}{TR}$
$\frac{3}{2} = \frac{6}{y}$	*Substitution*	$\frac{3}{2} = \frac{4.5}{z}$
$3(y) = 2(6)$	*Cross Products*	$3(z) = 2(4.5)$
$3y = 12$	*Simplify.*	$3z = 9$
$\frac{3y}{3} = \frac{12}{3}$	*Divide each side by 3.*	$\frac{3z}{3} = \frac{9}{3}$
$y = 4$	*Simplify.*	$z = 3$

Your Turn

a. The perimeter of $\triangle KQV$ is 72 units, and $\triangle GHJ \sim \triangle KQV$. Find the value of each variable.

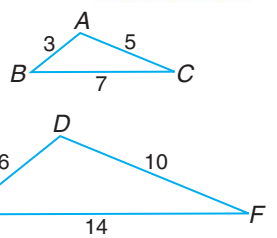


The ratio found by comparing the measures of corresponding sides of similar triangles is called the *constant of proportionality* or the **scale factor**.

If $\triangle ABC \sim \triangle DEF$, then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ or $\frac{3}{6} = \frac{7}{14} = \frac{5}{10}$. Each ratio is equivalent to $\frac{1}{2}$.

The scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{1}{2}$.

The scale factor of $\triangle DEF$ to $\triangle ABC$ is $\frac{2}{1}$.

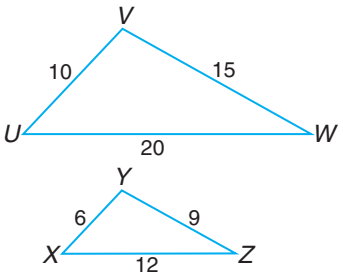


Example 2 **Determine the scale factor of $\triangle UVW$ to $\triangle XYZ$.**

$\frac{UV}{XY} = \frac{10}{6}$ or $\frac{5}{3}$ $\quad \frac{VW}{YZ} = \frac{15}{9}$ or $\frac{5}{3}$

$\frac{WU}{ZX} = \frac{20}{12}$ or $\frac{5}{3}$

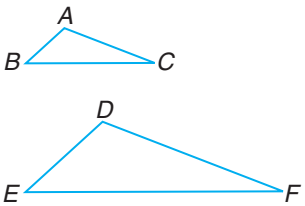
The scale factor is $\frac{5}{3}$.



Your Turn

b. Determine the scale factor of $\triangle XYZ$ to $\triangle UVW$.

In the figure, $\triangle ABC \sim \triangle DEF$. Suppose the scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{1}{2}$.



$$\frac{P \text{ of } \triangle ABC}{P \text{ of } \triangle DEF} = \frac{1}{2} \qquad \textit{P represents perimeter.}$$

$$\frac{P \text{ of } \triangle ABC}{\cancel{P \text{ of } \triangle DEF}}(\cancel{P \text{ of } \triangle DEF}) = \frac{1}{2}(P \text{ of } \triangle DEF) \qquad \textit{Multiply each side by P of } \triangle DEF.$$

$$P \text{ of } \triangle ABC = \frac{1}{2}(P \text{ of } \triangle DEF) \qquad \textit{Simplify.}$$

In general, if the scale factor of $\triangle ABC$ to $\triangle DEF$ is s, P of $\triangle ABC = s(P$ of $\triangle DEF)$.

Example 3 **Suppose $\triangle SLC \sim \triangle GFR$ and the scale factor of $\triangle SLC$ to $\triangle GFR$ is $\frac{2}{3}$. Find the perimeter of $\triangle GFR$ if the perimeter of $\triangle SLC$ is 14 inches.**

$\frac{\triangle SLC}{\triangle GFR} \quad \frac{2}{3} = \frac{14}{x} \quad \frac{\textit{Perimeter of } \triangle SLC}{\textit{Perimeter of } \triangle GFR}$

$2x = 42$ *Find cross products.*

$\frac{2x}{2} = \frac{42}{2}$ *Divide each side by 2.*

$x = 21$ *Simplify.*

The perimeter of $\triangle GFR$ is 21 inches.

Your Turn

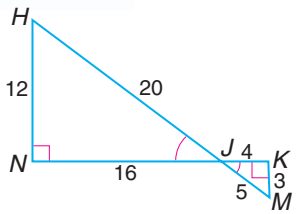
c. Suppose $\triangle PQR \sim \triangle XYZ$ and the scale factor of $\triangle PQR$ to $\triangle XYZ$ is $\frac{5}{6}$. Find the perimeter of $\triangle XYZ$ if the perimeter of $\triangle PQR$ is 25 centimeters.

Check for Understanding

Communicating Mathematics

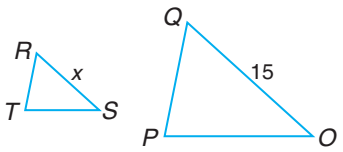
1. **Confirm** that the ratio of the measures of the corresponding sides is the same as the ratio of the measures of the corresponding perimeters.

Vocabulary
scale factor



2. **Identify** the additional information needed to solve for x.

$\triangle TRS \sim \triangle PQO$
perimeter of $\triangle TRS = 20$
$QO = 15$



Guided Practice

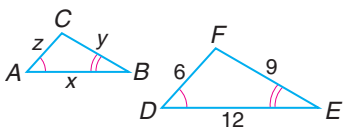
Getting Ready **Write each fraction in simplest form.**

Sample: $\frac{18}{21}$ **Solution:** $\frac{18}{21} = \frac{18 \div 3}{21 \div 3}$ or $\frac{6}{7}$

3. $\frac{6}{42}$ 4. $\frac{10}{24}$ 5. $\frac{63}{18}$ 6. $\frac{91}{13}$

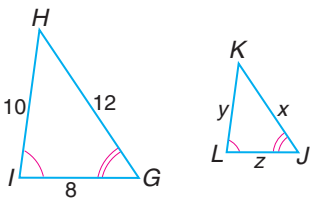
Example 1 **For each pair of similar triangles, find the value of each variable.**

7.



P of $\triangle ABC = 9$

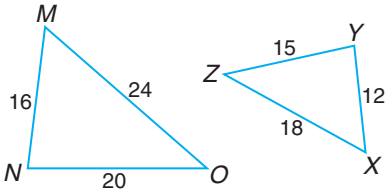
8.



P of $\triangle JKL = 7.5$

Example 2 **Determine the scale factor for each pair of similar triangles.**

9. $\triangle MNO$ to $\triangle XYZ$
10. $\triangle XYZ$ to $\triangle MNO$

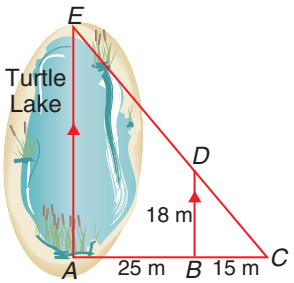


Example 3

11. Suppose $\triangle RST \sim \triangle UVW$ and the scale factor of $\triangle RST$ to $\triangle UVW$ is $\frac{3}{2}$. Find the perimeter of $\triangle UVW$ if the perimeter of $\triangle RST$ is 57 inches.

Example 2

12. **Surveying** Heather is using similar triangles to find the distance across Turtle Lake. What is the scale factor of $\triangle BCD$ to $\triangle ACE$?

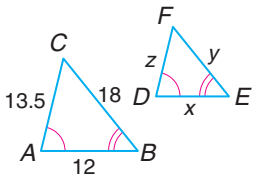


Exercises

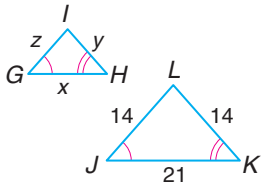
Practice

Homework Help	
For Exercises	See Examples
13–18	1
19–24, 28	2
25–27	3
29	1, 2
Extra Practice	
See page 743.	

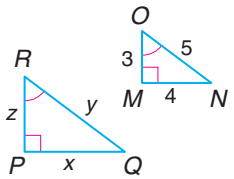
For each pair of similar triangles, find the value of each variable.

13. 

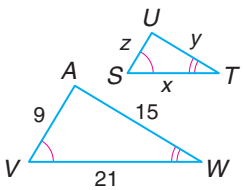
P of $\triangle DEF = 29$

14. 

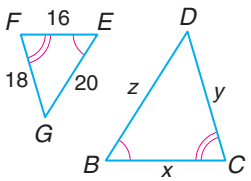
P of $\triangle GHI = 21$

15. 

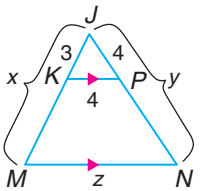
P of $\triangle PQR = 36$

16. 

P of $\triangle STU = 30$

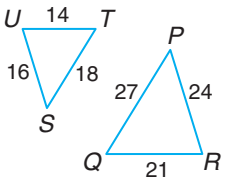
17. 

P of $\triangle BCD = 81$

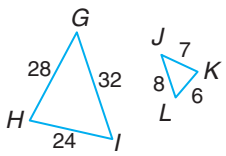
18. 

P of $\triangle MNJ = 44$

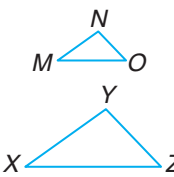
Determine the scale factor for each pair of similar triangles.



19. $\triangle STU$ to $\triangle PQR$
20. $\triangle PQR$ to $\triangle STU$



21. $\triangle GHI$ to $\triangle JKL$
22. $\triangle JKL$ to $\triangle GHI$



Perimeter of $\triangle MNO = 15$

Perimeter of $\triangle XYZ = 27$

23. $\triangle XYZ$ to $\triangle MNO$
24. $\triangle MNO$ to $\triangle XYZ$

25. The perimeter of $\triangle RST$ is 57 feet. If $\triangle RST \sim \triangle HKN$ and the scale factor of $\triangle RST$ to $\triangle HKN$ is $\frac{3}{2}$, find the perimeter of $\triangle HKN$.

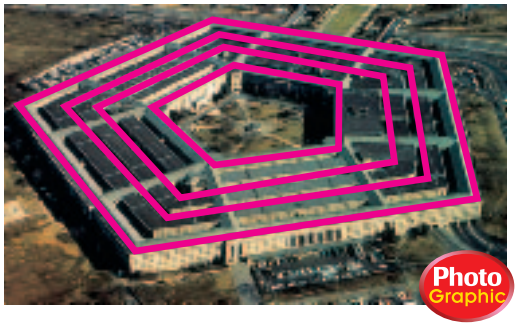
26. Suppose $\triangle JKL \sim \triangle MVW$ and the scale factor of $\triangle JKL$ to $\triangle MVW$ is $\frac{4}{3}$. The lengths of the sides of $\triangle JLK$ are 12 meters, 10 meters, and 10 meters. Find the perimeter of $\triangle MVW$.

27. Suppose $\triangle ABC \sim \triangle DEF$ and the scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{5}{3}$. Find the perimeter of $\triangle DEF$ if the perimeter of $\triangle ABC$ is 25 meters.

Applications and Problem Solving

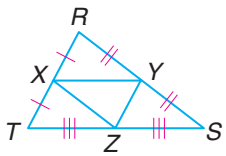
28. **Drafting** In a blueprint of a house, 1 inch represents 3 feet. What is the scale factor of the blueprint to the actual house? (*Hint*: Change feet to inches.)

29. **Architecture** A bird's-eye view of the Pentagon reveals five similar pentagons. Each side of the outside pentagon is about 920 feet. Each side of the innermost pentagon is about 360 feet.



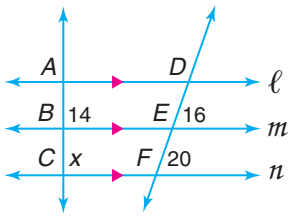
   a. Find the scale factor of the outside pentagon to the innermost pentagon.
   b. Find the perimeter of the outside pentagon.
   c. Find the perimeter of the innermost pentagon.
   d. Find the ratio of the perimeter of the outside pentagon to the perimeter of the innermost pentagon.
   e. Tell how the ratio in part d compares to the scale factor of the pentagons.

30. **Critical Thinking** The perimeter of $\triangle RST$ is 40 feet. Find the perimeter of $\triangle XYZ$.

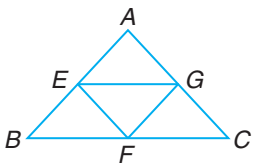


Mixed Review

31. In the figure shown, $\ell \parallel m \parallel n$. Find the value of x. *(Lesson 9–6)*



32. **Algebra** The midpoints of the sides of $\triangle ABC$ are E, F, and G. Find the measure of BC if $EG = 4b$. *(Lesson 9–5)*

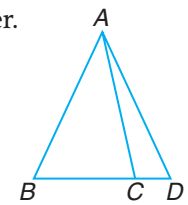


33. *True* or *false*: The diagonals of a square bisect each other. *(Lesson 8–4)*

34. If $MN = 47$, $PQ = 63 - 4c$, and $MN \leq PQ$, what is the value of c? *(Lesson 7–1)*

Standardized Test Practice

35. **Short Response** Suppose $AB = AD = 4$, $m\angle B = m\angle D = 65$, and $AC = 3.5$. Is $\triangle ABC \cong \triangle ADC$? *(Lesson 5–5)*



Exercise 35

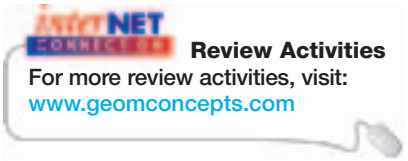
36. **Multiple Choice** In $\triangle RST$, $m\angle R = 72$ and $m\angle S = 37$. What is $m\angle T$? *(Lesson 5–2)*

   (A) 19 (B) 37 (C) 71 (D) 72

CHAPTER 9 Study Guide and Assessment

Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.



cross products *(p. 351)*
extremes *(p. 351)*
geometric mean *(p. 355)*
golden ratio *(p. 380)*
means *(p. 351)*
polygon *(p. 356)*
proportion *(p. 351)*
ratio *(p. 350)*
scale drawing *(p. 358)*
scale factor *(p. 389)*
sides *(p. 356)*
similar polygons *(p. 356)*

Choose the correct term to complete each sentence.

1. Every proportion has two (similar figures, cross products).
2. A (proportion, ratio) is a comparison of two numbers by division.
3. The cross products are always equal in a (proportion, scale drawing).
4. In (proportions, similar figures), corresponding angles are congruent, and the measures of corresponding sides have equivalent ratios.
5. In the proportion $\frac{2}{5} = \frac{4}{10}$, the terms 2 and 10 are called the (extremes, means).
6. (Scale drawings, Proportions) are used to represent something that is too large or too small to be drawn at actual size.
7. A proportion has two cross products called the extremes and the (ratios, means).
8. The constant of proportionality is also called the (scale factor, scale drawing).
9. Knowing several measures of two (similar figures, scale drawings) may allow you to find the measures of missing parts.
10. The symbols a to b, $a{:}b$, and $\frac{a}{b}$, where $b \neq 0$ represent (ratios, cross products).

Skills and Concepts

Objectives and Examples

- **Lesson 9–1** Use ratios and proportions to solve problems.

Solve $\frac{6}{45} = \frac{2}{3x}$.

$$\frac{6}{45} = \frac{2}{3x}$$
$$6(3x) = 45(2) \quad \textit{Cross Products}$$
$$18x = 90 \quad \textit{Multiply.}$$
$$\frac{18x}{18} = \frac{90}{18} \quad \textit{Divide each side by 18.}$$
$$x = 5 \quad \textit{Simplify.}$$

Review Exercises

Write each ratio in simplest form.

11. $\frac{3}{9}$ **12.** $\frac{45}{100}$ **13.** $\frac{55}{22}$

Solve each proportion.

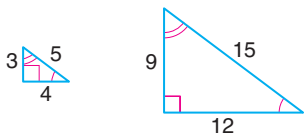
14. $\frac{3}{10} = \frac{9}{x}$ **15.** $\frac{3}{2x} = \frac{12}{16}$

16. $\frac{84}{49} = \frac{12}{17 - x}$ **17.** $\frac{16}{20} = \frac{x + 3}{10}$

Objectives and Examples

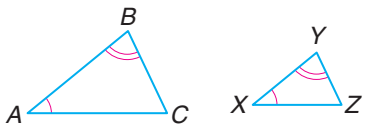
- **Lesson 9–2** Identify similar polygons.

Determine whether the polygons are similar.



Since $\frac{3}{9} = \frac{4}{12} = \frac{5}{15}$, the sides of the polygons are proportional. The corresponding angles are congruent. So, the polygons are similar.

- **Lesson 9–3** Use AA, SSS, and SAS similarity tests for triangles.

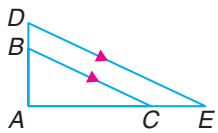


Since $\angle A \cong \angle X$ and $\angle B \cong \angle Y$, the triangles are similar by AA Similarity.

- **Lesson 9–4** Identify and use the relationships between proportional parts of triangles.

Since $\overline{BC} \parallel \overline{DE}$, $\triangle ABC \sim \triangle ADE$.

So, $\frac{AB}{AD} = \frac{BC}{DE}$.



- **Lesson 9–5** Use proportions to determine whether lines are parallel to sides of triangles.

Determine whether $\overline{LM} \parallel \overline{JK}$.

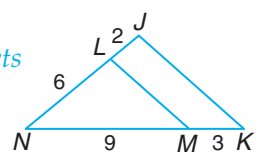
$\frac{LJ}{NJ} \stackrel{?}{=} \frac{MK}{NK}$

$\frac{2}{8} \stackrel{?}{=} \frac{3}{12}$ *Substitute.*

$2(12) \stackrel{?}{=} 8(3)$ *Cross Products*

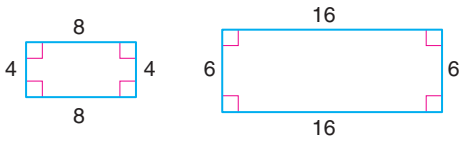
$24 = 24$ ✓

So, $\overline{LM} \parallel \overline{JK}$.

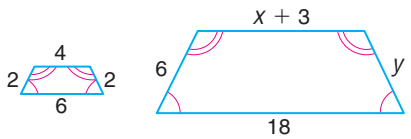


Review Exercises

18. Determine whether the polygons are similar. Justify your answer.

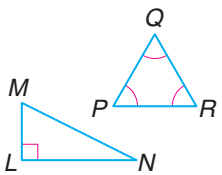


19. The polygons are similar. Find x and y.

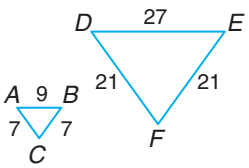


Determine whether each pair of triangles is similar. If so, tell which similarity test is used.

20.



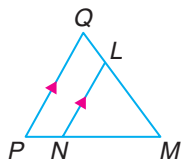
21.



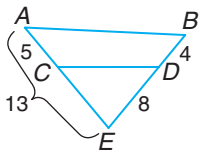
Complete each proportion.

22. $\frac{?}{PQ} = \frac{MN}{MP}$

23. $\frac{ML}{MQ} = \frac{?}{MP}$



24. Determine whether $\overline{AB} \parallel \overline{CD}$.

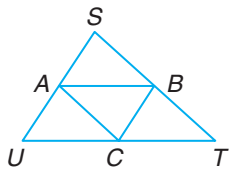


***A*, *B*, and *C* are the midpoints of the sides of $\triangle STU$. Complete.**

25. $\overline{AC} \parallel$ ___?___

26. If $UT = 16$, then $AB =$ ___?___

27. If $BC = 6$, then $SU =$ ___?___

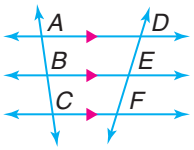


Mixed Problem Solving
See pages 758–765.

Objectives and Examples

- **Lesson 9–6** Identify and use the relationships between parallel lines and proportional parts.

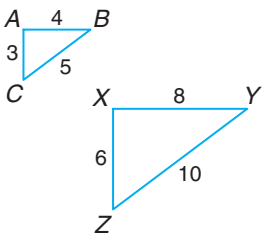
Since $\overleftrightarrow{AD} \parallel \overleftrightarrow{BE} \parallel \overleftrightarrow{CF}$, $\frac{BC}{AC} = \frac{EF}{DF}$.



- **Lesson 9–7** Identify and use proportional relationships of similar triangles.

Determine the scale factor of $\triangle ABC$ to $\triangle XYZ$.

$\frac{AB}{XY} = \frac{AC}{XZ} = \frac{BC}{YZ} = \frac{1}{2}$

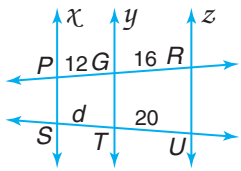


The scale factor of $\triangle ABC$ to $\triangle XYZ$ is $\frac{1}{2}$.

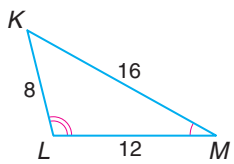
Review Exercises

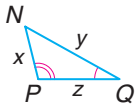
28. Complete the proportion $\frac{AB}{AC} = \frac{?}{DF}$ using the figure at the left.

29. Find the value of d if $x \parallel y \parallel z$.

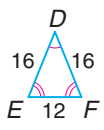


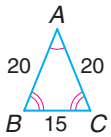
30. The triangles below are similar, and the perimeter of $\triangle NPQ$ is 27. Find the value of each variable.





31. Determine the scale factor of $\triangle DEF$ to $\triangle ABC$.

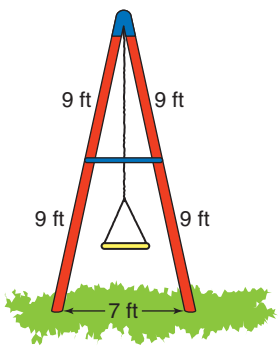




Applications and Problem Solving

32. Sailing The sail on John's boat is shaped like a right triangle. Its hypotenuse is 24 feet long, one leg is 16 feet long, and the other is about 18 feet long. The triangular sail on a model of the boat has a hypotenuse of 3 feet. If the two triangular sails are similar, how long are the legs of the model's sail? *(Lesson 9–3)*

33. Recreation The ends of the swing set at Parkdale Elementary School look like the letter A, as shown in the diagram. If the horizontal bar is parallel to the ground, how long is the horizontal bar? *(Lesson 9–5)*



Exercise 33

CONTENTS

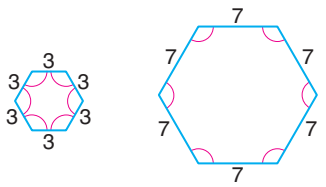
CHAPTER 9 Test

1. **Name** three tests used to determine whether triangles are similar.
2. **Describe** how a scale drawing could be used by an architect.

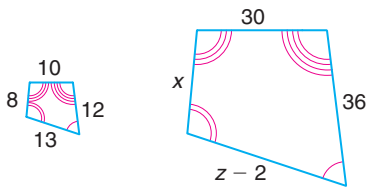
Solve each proportion.

3. $\frac{x-2}{7} = \frac{20}{35}$
4. $\frac{5}{3} = \frac{x+7}{9}$
5. $\frac{27}{x} = \frac{36}{9}$

6. Determine if the polygons are similar. Justify your answer.

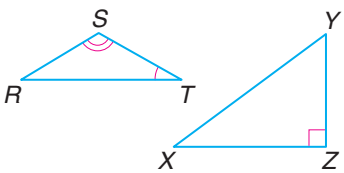


7. The polygons are similar. Find the values of x and z.



Determine whether each pair of triangles is similar. If so, tell which similarity test is used.

8.



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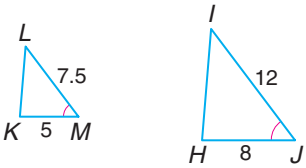

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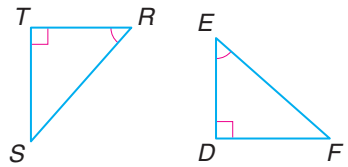
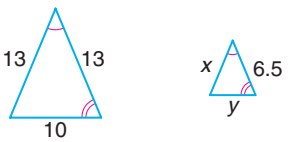
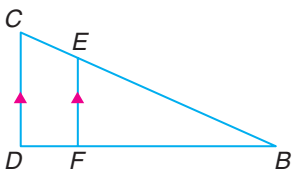


11. Find the values of x and y.





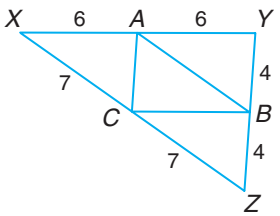
Exercises 12–13

Complete each proportion.

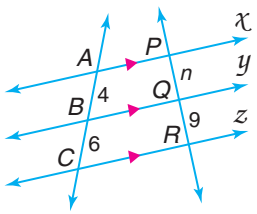
12. $\frac{BF}{?} = \frac{BE}{BC}$
13. $\frac{BE}{BC} = \frac{?}{CD}$

Complete each statement.

14. $AC =$ ___?___.
15. $\overline{BC} \parallel$ ___?___.
16. $\triangle ABC \sim$ ___?___.



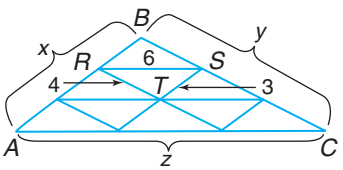
Exercises 14–16



Exercises 17–18

17. List three proportions, given $x \parallel y \parallel z$.
18. Find the value of n.

19. A 42-foot tree casts a shadow of 63 feet. How long is the shadow of a 5-foot girl who is standing near it?
20. A puzzle consists of several triangles that are all similar. If the perimeter of $\triangle ABC$ is 39, find the value of each variable.



Exercise 20

CHAPTER 9

Preparing for Standardized Tests

Ratio and Proportion Problems

Standardized tests almost always include ratio and proportion problems. Remember that ratios can be written in several ways.

$$a \text{ to } b,\ a{:}b,\ \frac{a}{b},\ \text{or } a \div b, \text{ where } b \neq 0$$

Test-Taking Tip

A ratio often compares a part to a part. A fraction compares a part to a whole.

Example 1

A grocery store sells oranges at 3 for $1.29. How much do 10 oranges cost?

Hint Write prices like 3 for $1.29 as a ratio.

Solution Write a proportion. Let one ratio represent the cost of the 3 oranges. Use x for the cost of 10 oranges. Let the second ratio represent the cost of the 10 oranges. Find the cross products. Solve the equation for x.

cost of 3 oranges $\rightarrow$ $\frac{1.29}{3} = \frac{x}{10}$ $\leftarrow$ *cost of 10 oranges*
number of oranges $\rightarrow$ $\leftarrow$ *number of oranges*

$$1.29(10) = 3(x)$$
$$12.9 = 3x$$
$$\frac{12.9}{3} = \frac{3x}{3}$$
$$4.3 = x$$

The cost of 10 oranges is $4.30.

Example 2

A bakery uses a special flour mixture that contains corn, wheat, and rye in the ratio of 3:5:2. If a bag of the mixture contains 5 pounds of rye, how many pounds of wheat does it contain?

(A) 2 (B) 5 (C) 7.5 (D) 10 (E) 12.5

Hint Be on the lookout for extra information that is not needed to solve the problem.

Solution Read the question carefully. It contains a ratio of three quantities. Notice that the amount of corn is *not* part of the question. So you can ignore the part of the ratio that involves corn.

The ratio of wheat to rye is 5:2. The amount of rye is 5 pounds. Create a proportion. Let x represent the amount of wheat. Find the cross products. Solve the equation for x.

$\frac{5}{2} = \frac{x}{5}$ $\leftarrow$ *wheat*
$\leftarrow$ *rye*

$5(5) = 2(x)$ *Cross products*

$25 = 2x$ *Multiply.*

$\frac{25}{2} = \frac{2x}{2}$ *Divide each side by 2.*

$12.5 = x$ *Simplify.*

The bag contains 12.5 pounds of wheat. The answer is E.

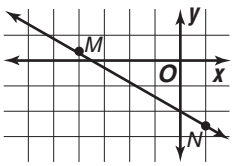
After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

1. On a map, the distance from Springfield to Ames is 6 inches. The map scale is $\frac{1}{2}$ inch = 20 miles. How many miles is it from Springfield to Ames? *(Lesson 9–2)*

(A) 6.67 mi (B) 60 mi
(C) 120 mi (D) 240 mi

2. Which ordered pair represents the *y*-intercept of line *MN*? *(Lesson 4–6)*



(A) (0, −2)
(B) (−2, 0)
(C) (0, −4)
(D) (−4, 0)

3. If 2 packages contain a total of 12 doughnuts, how many doughnuts are there in 5 packages? *(Algebra Review)*

(A) 60 (B) 36 (C) 30 (D) 24

4. Nathan earns $24,000 in salary and 8% commission on his sales. If he needs a total annual income of at least $30,000, how much does he need to sell? *(Percent Review)*

(A) at least $480 (B) at least $6000
(C) at least $26,400 (D) at least $75,000

5. Point *B*(4, 3) is the midpoint of line segment *AC*. If point *A* has coordinates (0, 1), what are the coordinates of point *C*? *(Lesson 2–5)*

(A) (−4, −1) (B) (4, 1) (C) (4, 4)
(D) (8, 5) (E) (8, 9)

6. The ratio of girls to boys in a science class is 4 to 3. If the class has a total of 35 students, how many more girls are there than boys? *(Algebra Review)*

(A) 1 (B) 5 (C) 7 (D) 15

7. Jessica served cheese, peanut butter, and cucumber sandwiches at a luncheon. She also served iced tea and lemonade. Each guest chose one sandwich and one drink. Of the possible combinations of sandwich and drink, how many included iced tea? *(Statistics Review)*

(A) 1 (B) 2 (C) 3 (D) 6

8. The average of five numbers is 20. If one of the numbers is 18, then what is the sum of the other four numbers? *(Statistics Review)*

(A) 2 (B) 20.5 (C) 82
(D) 90 (E) 100

Grid In

9. An average of 3 out of every 10 students are absent from school because of illness during flu season. If there are normally 600 students attending a school, about how many students can be expected to attend during flu season? *(Algebra Review)*

Extended Response

10. The chart shows the average height for males ages 8 to 18. *(Statistics Review)*

Average Height for Males	
Age (yr)	Height (cm)
8	124
9	130
10	135
11	140
12	145
13	152
14	161
15	167
16	172
17	174
18	178

Part A Graph the information on a coordinate plane.

Part B Describe the relationship between age and height.