CHAPTER

14 Circle Relationships

What You'll Learn

Key Ideas

- Identify and use properties of inscribed angles and tangents to circles. ***(Lessons 14–1 and 14–2)***
- Find measures of arcs and angles formed by secants and tangents. ***(Lessons 14–3 and 14–4)***
- Find measures of chords, secants, and tangents. ***(Lesson 14–5)***
- Write equations of circles. ***(Lesson 14–6)***

Key Vocabulary

inscribed angle ***(p. 586)***

intercepted arc ***(p. 586)***

secant segment ***(p. 600)***

tangent ***(p. 592)***

Why It's Important

Astronomy Planets and stars were observed by many ancient civilizations. Later scientists like Isaac Newton developed mathematical theories to further their study of astronomy. Modern observatories like Kitt Peak National Observatory in Arizona use optical and radio telescopes to continue to expand the understanding of our universe.

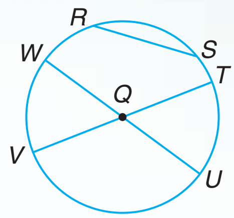
Circle relationships can be applied in many scientific fields. You will use circles to investigate two galaxies in Lesson 14–2.

Study these lessons to improve your skills.

✓ Check Your Readiness

✓ Lesson 11–1, pp. 454–458

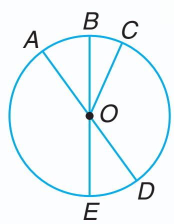
Use $\odot Q$ to determine whether each statement is *true* or *false*.



1. $\overline{WU}$ is a radius of $\odot Q$.
2. $\overline{VT}$ is a diameter of $\odot Q$.
3. $\overline{RS}$ is a chord of $\odot Q$.
4. $\overline{QT}$ is congruent to $\overline{QU}$.

✓ Lesson 11–2, pp. 462–467

Find each measure in $\odot O$ if $m\angle AOB = 36$, $m\widehat{BC} = 24$, and $\overline{AD}$ and $\overline{BE}$ are diameters.



5. $m\angle EOD$
6. $m\widehat{AD}$
7. $m\angle COD$
8. $m\widehat{BD}$
9. $m\widehat{CBE}$
10. $m\angle DOB$

✓ Lesson 6–6, pp. 256–261

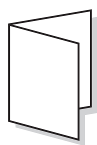
If c is the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.

11. $a = 8, b = 15, c = ?$
12. $a = 12, b = ?, c = 19$
13. $a = 10, b = 10, c = ?$
14. $a = ?, b = 7.5, c = 16.8$

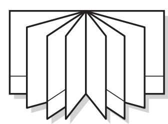
FOLDABLES™ Study Organizer

Make this Foldable to help you organize your Chapter 14 notes. Begin with four sheets of $8\frac{1}{2}$" by 11" paper.

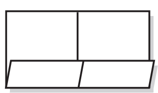
❶ **Fold** each sheet in half along the width.



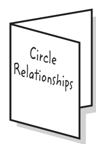
❷ **Open** and fold the bottom edge up to form a pocket. Glue the sides.



❸ **Repeat** Steps 1 and 2 three times and glue all four pieces together.



❹ **Label** each pocket with a lesson title. Use the last two for vocabulary. Place an index card in each pocket.



Reading and Writing As you read and study the chapter, you can write main ideas, examples of theorems, and postulates on the index cards.

14–1 Inscribed Angles

What You'll Learn

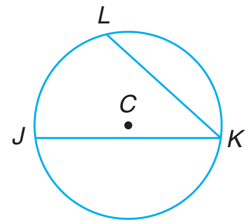
You'll learn to identify and use properties of inscribed angles.

Why It's Important

Architecture

Inscribed angles are important in the overall symmetry of many ancient structures. *See Exercise 8.*

Recall that a polygon can be inscribed in a circle. An angle can also be inscribed in a circle. An **inscribed angle** is an angle whose vertex is on the circle and whose sides contain chords of the circle. Angle JKL is an inscribed angle.

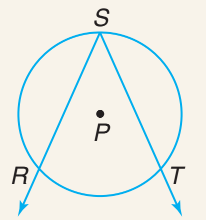


Notice that K, the vertex of $\angle JKL$, lies on $\odot C$. The sides of $\angle JKL$ contain chords LK and JK. Therefore, $\angle JKL$ is an inscribed angle. Each side of the inscribed angle intersects the circle at a point. The two points J and L form an arc. We say that $\angle JKL$ *intercepts* $\widehat{JL}$, or that $\widehat{JL}$ is the **intercepted arc** of $\angle JKL$.

Definition of Inscribed Angle

Words: An angle is inscribed if and only if its vertex lies on the circle and its sides contain chords of the circle.

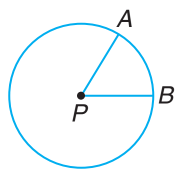
Model:



Symbols: $\angle RST$ is inscribed in $\odot P$.

Example 1

Determine whether $\angle APB$ is an inscribed angle. Name the intercepted arc for the angle.



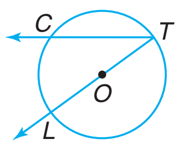
Point P, the vertex of $\angle APB$, is not on $\odot P$. So, $\angle APB$ is not an inscribed angle. The intercepted arc of $\angle APB$ is $\widehat{AB}$.

Look Back

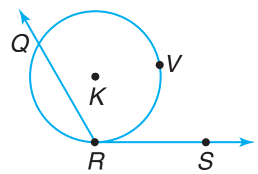
Central Angles, Lesson 11–2

Your Turn **Determine whether each angle is an inscribed angle. Name the intercepted arc for the angle.**

a. $\angle CTL$



b. $\angle QRS$

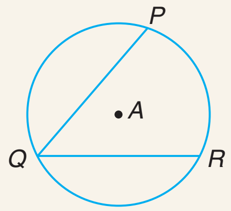


You can find the measure of an inscribed angle if you know the measure of its intercepted arc. This is stated in the following theorem.

Theorem 14–1

Words: The degree measure of an inscribed angle equals one-half the degree measure of its intercepted arc.

Model:

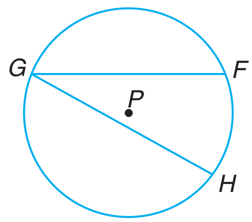


Symbols: $m\angle PQR = \frac{1}{2}m\overset{\frown}{PR}$

You can use Theorem 14–1 to find the measure of an inscribed angle or the measure of its intercepted arc if one of the measures is known.

Examples

2 **If $m\overset{\frown}{FH} = 58$, find $m\angle FGH$.**



$m\angle FGH = \frac{1}{2}(m\overset{\frown}{FH})$ *Theorem 14–1*

$m\angle FGH = \frac{1}{2}(58)$ *Replace $m\overset{\frown}{FH}$ with 58.*

$m\angle FGH = 29$ *Multiply.*

Game Link

Real World

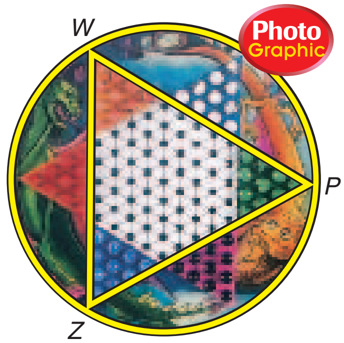
3 **In the game shown at the right, $\triangle WPZ$ is equilateral. Find $m\overset{\frown}{WZ}$.**

$m\angle WPZ = \frac{1}{2}(m\overset{\frown}{WZ})$ *Theorem 14–1*

$60 = \frac{1}{2}(m\overset{\frown}{WZ})$ *Replace $m\angle WPZ$ with 60.*

$2 \cdot 60 = 2 \cdot \frac{1}{2}(m\overset{\frown}{WZ})$ *Multiply each side by 2.*

$120 = m\overset{\frown}{WZ}$ *Simplify.*

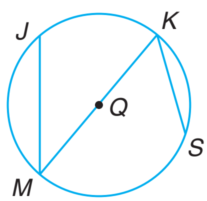


Chinese Checkers

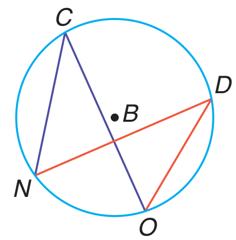
Your Turn

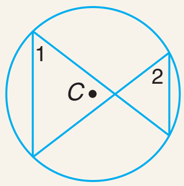
c. If $m\overset{\frown}{JK} = 80$, find $m\angle JMK$.

d. If $m\angle MKS = 56$, find $m\overset{\frown}{MS}$.



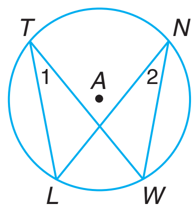
In $\odot B$, if the measure of $\overarc{NO}$ is 74, what is the measure of inscribed angle NCO? What is the measure of inscribed angle NDO? Notice that both of the inscribed angles intercept the same arc, $\overarc{NO}$. This relationship is stated in Theorem 14–2.



Theorem 14–2	
Words:	If inscribed angles intercept the same arc or congruent arcs, then the angles are congruent.
Model:	
Symbols:	$\angle 1 \cong \angle 2$

Example 4 Algebra Link

In $\odot A$, $m\angle 1 = 2x$ and $m\angle 2 = x + 14$. Find the value of x.



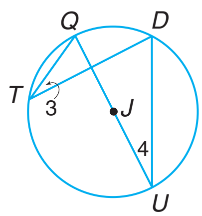
$\angle 1$ and $\angle 2$ both intercept $\overarc{LW}$.

$\angle 1 \cong \angle 2$	*Theorem 14–2*
$m\angle 1 = m\angle 2$	*Definition of congruent angles*
$2x = x + 14$	*Replace $m\angle 1$ with $2x$ and $m\angle 2$ with $x + 14$.*
$2x - x = x + 14 - x$	*Subtract x from each side.*
$x = 14$	*Simplify.*

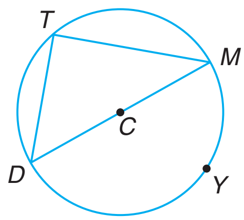
Algebra Review
Solving Equations with the Variable on Both Sides, p. 724

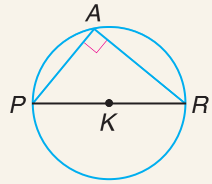
Your Turn

e. In $\odot J$, $m\angle 3 = 3x$ and $m\angle 4 = 2x + 9$. Find the value of x.



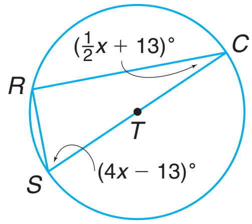
Suppose $\angle MTD$ is inscribed in $\odot C$ and intercepts semicircle $\overarc{MYD}$. Since $m\overarc{MYD} = 180$, $m\angle MTD = \frac{1}{2} \cdot 180$ or 90. Therefore, $\angle MTD$ is a right angle. This relationship is stated in Theorem 14–3.



Theorem 14–3	
	Words: If an inscribed angle of a circle intercepts a semicircle, then the angle is a right angle.
	Model:  **Symbols:** $m\angle PAR = 90$

Example 5 — **Algebra Link**

In $\odot T$, $\overline{CS}$ is a diameter. Find the value of x.



Inscribed angle CRS intercepts semicircle $\overarc{CS}$. By Theorem 14–3, $\angle CRS$ is a right angle. Therefore, $\triangle CRS$ is a right triangle and $\angle C$ and $\angle S$ are complementary.

$$m\angle C + m\angle S = 90 \quad \textit{Definition of complementary angles}$$

$$\left(\frac{1}{2}x + 13\right) + (4x - 13) = 90 \quad \textit{Substitution}$$

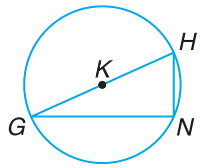
$$\frac{9}{2}x = 90 \quad \textit{Combine like terms.}$$

$$\left(\frac{2}{9}\right)\frac{9}{2}x = \left(\frac{2}{9}\right)90 \quad \textit{Multiply each side by } \frac{2}{9}.$$

$$x = 20 \quad \textit{Simplify.}$$

Your Turn

f. In $\odot K$, $\overline{GH}$ is a diameter and $m\angle GNH = 4x - 14$. Find the value of x.



Check for Understanding

Communicating Mathematics

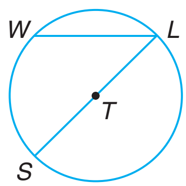
1. **Describe** an intercepted arc of a circle. State how its measure relates to the measure of an inscribed angle that intercepts it.
2. **Draw** inscribed angle QLS in $\odot T$ that has a measure of 100. Include all labels.

Vocabulary

inscribed angle
intercepted arc

Guided Practice
Example 1

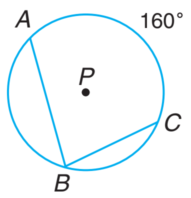
3. Determine whether $\angle WLS$ is an inscribed angle. Name the intercepted arc for the angle.



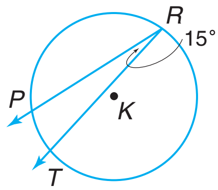
Examples 2 & 3

Find each measure.

4. $m\angle ABC$



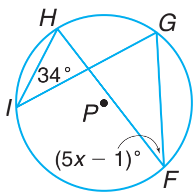
5. $m\overset{\frown}{PT}$



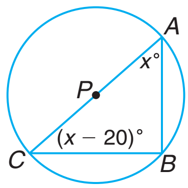
Examples 4 & 5

In each circle, find the value of *x*.

6.



7.



Example 2

8. **Architecture** Refer to $\odot C$ in the application at the beginning of the lesson. If $m\overset{\frown}{JL} = 84$, find $m\angle JKL$.

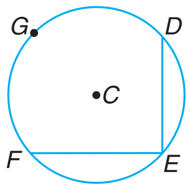
Exercises

Practice

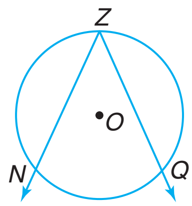
Homework Help	
For Exercises	See Examples
9–11	1
12–17, 26, 27	2, 3
18–21, 24	4
22, 23	5
Extra Practice	
See page 752.	

Determine whether each angle is an inscribed angle. Name the intercepted arc for the angle.

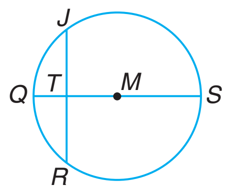
9. $\angle DEF$



10. $\angle NZQ$

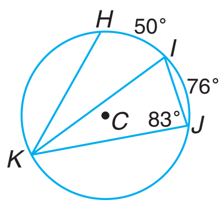


11. $\angle JTS$

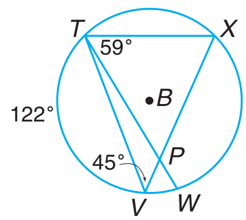


Find each measure.

12. $m\angle HKI$
13. $m\angle IKJ$
14. $m\overset{\frown}{IK}$

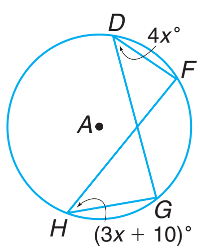


15. $m\overset{\frown}{XW}$
16. $m\angle TXV$
17. $m\overset{\frown}{VW}$

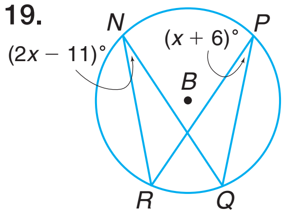


In each circle, find the value of *x*.

18.



19.



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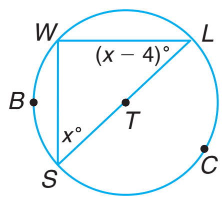

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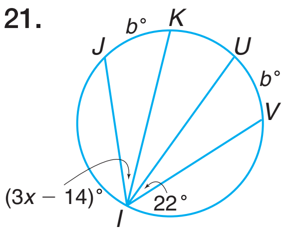

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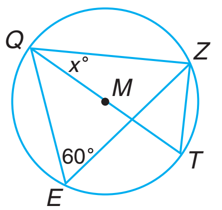
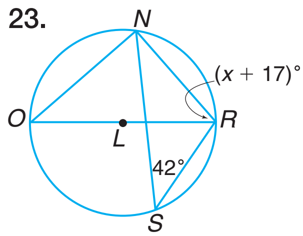


23.

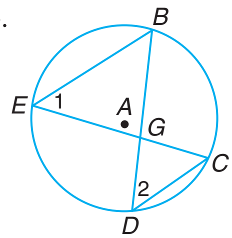


24. In $\odot A$, $m\angle 1 = 13x - 9$ and $m\angle 2 = 27x - 65$.

a. Find the value of x.

b. Find $m\angle 1$ and $m\angle 2$.

c. If $m\angle BGE = 92$, find $m\angle ECD$.



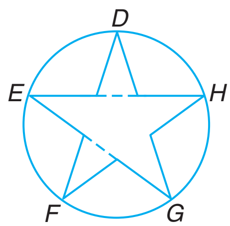
Applications and Problem Solving

25. Literature Is Dante's suggestion in the quote at the right always possible? Explain why or why not.

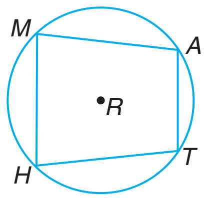
> Or draw a triangle inside a semicircle
> That would have no right angle.
>
> —Dante, *The Divine Comedy*

Visitor Center, Washington, Texas

26. History The symbol at the right appears throughout the Visitor Center in Texas' Washington-on-the-Brazos State Historical Park. If $\overset{\frown}{DH} \cong \overset{\frown}{HG} \cong \overset{\frown}{GF} \cong \overset{\frown}{FE} \cong \overset{\frown}{ED}$, find $m\angle HEG$.

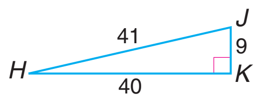


27. Critical Thinking Quadrilateral *MATH* is inscribed in $\odot R$. Show that the opposite angles of the quadrilateral are supplementary.



Mixed Review

28. Use $\triangle HJK$ to find cos H. Round to four decimal places. *(Lesson 13–5)*



29. A right cylinder has a base radius of 4 centimeters and a height of 22 centimeters. Find the lateral area of the cylinder to the nearest hundredth. *(Lesson 12–2)*

30. Find the area of a 20° sector in a circle with diameter 15 inches. Round to the nearest hundredth. *(Lesson 11–6)*

Standardized Test Practice

A B C D

31. Grid In Students are using a slide projector to magnify insects' wings. The ratio of actual length to projected length is 1:25. If the projected length of a wing is 8.14 centimeters, what is the actual length in centimeters? Round to the nearest hundredth. *(Lesson 9–1)*

32. Multiple Choice Solve $\sqrt{2q + 7} = 19$. *(Algebra Review)*

Ⓐ 36 Ⓑ 177 Ⓒ 184 Ⓓ 736

14–2 Tangents to a Circle

What You'll Learn

You'll learn to identify and apply properties of tangents to circles.

Why It's Important

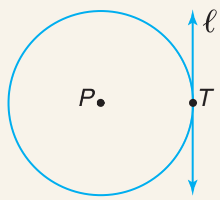
Astronomy Scientists use tangents to calculate distances between stars. *See Example 2.*

A **tangent** is a line that intersects a circle in exactly one point. Also, by definition, a line segment or ray can be tangent to a circle if it is a part of a line that is tangent to the circle. Using tangents, you can find more properties of circles.

Definition of a Tangent

Words: In a plane, a line is a tangent if and only if it intersects a circle in exactly one point.

Model:



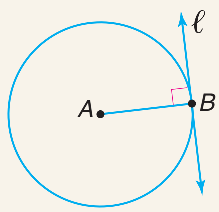
Symbols: Line ℓ is tangent to $\odot P$. T is called the **point of tangency**.

Two special properties of tangency are stated in the theorems below.

Theorem 14–4

Words: In a plane, if a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

Model:



Symbols: If line ℓ is tangent to $\odot A$ at point B, then $\overline{AB} \perp \ell$.

The converse of Theorem 14–4 is also true.

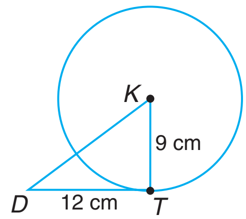
Theorem 14–5

Words: In a plane, if a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is a tangent.

Symbols: If $\overline{AB} \perp \ell$, then ℓ is tangent to $\odot A$ at point B.

Example 1 **Algebra Link**

$\overline{TD}$ is tangent to $\odot K$ at T. Find KD.

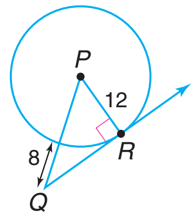


From Theorem 14–4, $\overline{KT} \perp \overline{TD}$. Thus, $\angle KTD$ is a right angle, and $\triangle KTD$ is a right triangle.

$$
\begin{aligned}
(KD)^2 &= (KT)^2 + (TD)^2 && \textit{Pythagorean Theorem}\\
(KD)^2 &= 9^2 + 12^2 && \textit{Replace KT with 9 and TD with 12.}\\
(KD)^2 &= 81 + 144 && \textit{Square 9 and 12.}\\
\sqrt{(KD)^2} &= \sqrt{225} && \textit{Take the square root of each side.}\\
KD &= 15 && \textit{Simplify.}
\end{aligned}
$$

Your Turn

a. $\overrightarrow{QR}$ is tangent to $\odot P$ at R. Find RQ.



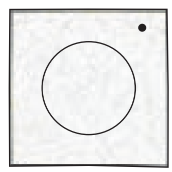
In the following activity, you'll find a relationship between two tangents that are drawn from a point outside a circle.

Hands-On Geometry
Paper Folding

Materials:  compass  patty paper  straightedge

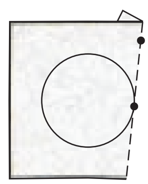
Step 1 Use a compass to draw a circle on patty paper.

Step 2 Draw a point outside the circle.



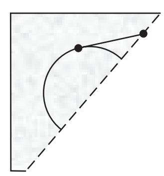
Step 3 Carefully fold the paper so that a tangent is formed from the point to one side of the circle. Use a straightedge to draw the segment. Mark your point of tangency.

Step 4 Repeat Step 3 for a tangent line that intersects the tangent line in Step 3.



Try These

1. Fold the paper so that one tangent covers the other. Compare their lengths.
2. **Make a conjecture** about the relationship between two tangents drawn from a point outside a circle.

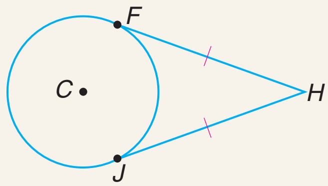


The results of the activity suggest the following theorem.

Theorem 14–6

Words: If two segments from the same exterior point are tangent to a circle, then they are congruent.

Model:



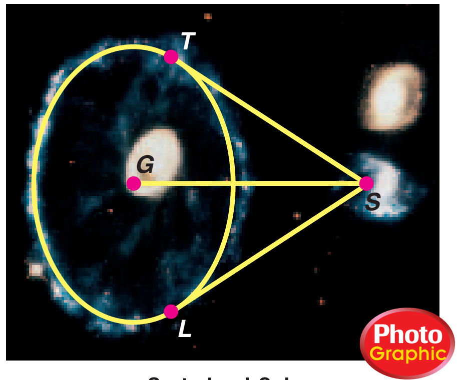
Symbols: If $\overline{HF}$ and $\overline{HJ}$ are tangent to $\odot C$, then $\overline{HF} \cong \overline{HJ}$.

Real World

Example 2

Astronomy Link

The ring of stars in the photograph appeared after the small blue galaxy on the right S crashed through the large galaxy on the left G. The two galaxies are 168 thousand light-years apart ($GS = 168$ thousand light-years), and $\odot G$ has a radius of 75 thousand light-years. Find ST and SL if they are tangent to $\odot G$.



Cartwheel Galaxy

Explore From Theorem 14–6, $\overline{ST} \cong \overline{SL}$, so we only need to find the measure of one of the segments.

Plan By Theorem 14–4, $\overline{GT} \perp \overline{ST}$. Thus, $\angle GTS$ is a right angle and $\triangle GTS$ is a right triangle. We can use the Pythagorean Theorem to find ST.

Solve

$$\begin{aligned}
(GS)^2 &= (GT)^2 + (ST)^2 && \textit{Pythagorean Theorem}\\
168^2 &= 75^2 + (ST)^2 && \textit{Substitution}\\
28{,}224 &= 5625 + (ST)^2 && \textit{Square 168 and 75.}\\
28{,}224 - 5625 &= 5625 + (ST)^2 - 5625 && \textit{Subtract 5625 from each side.}\\
22{,}599 &= (ST)^2\\
\sqrt{22{,}599} &= \sqrt{(ST)^2} && \textit{Take the square root of each side.}\\
150.33 &\approx ST && \textit{Simplify.}
\end{aligned}$$

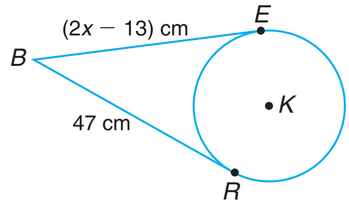
Examine Check your answer by substituting into the original equation.

$$(GS)^2 = (GT)^2 + (ST)^2$$
$$168^2 \stackrel{?}{=} 75^2 + 150.33^2$$
$$28{,}224 \approx 28{,}224.11 \quad \checkmark \quad \text{The answer checks.}$$

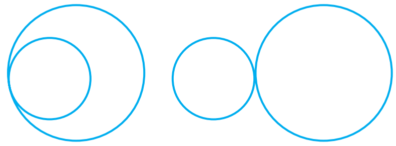
If you round your final answer to the nearest tenth, the measure of $\overline{ST}$ is about 150.3 thousand light-years. By Theorem 14–6, the measure of $\overline{SL}$ is also about 150.3 thousand light-years.

Your Turn

b. $\overline{BE}$ and $\overline{BR}$ are tangent to $\odot K$. Find the value of x.



Two circles can be tangent to each other. If two circles are tangent and one circle is inside the other, the circles are **internally tangent.** If two circles are tangent and neither circle is inside the other, the circles are **externally tangent.**



Check for Understanding

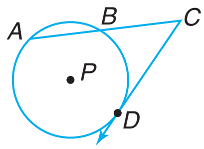
Communicating Mathematics

1. **Determine** how many tangents can be drawn to a circle from a single point outside the circle. Explain why these tangents must be congruent.

Vocabulary

tangent
point of tangency
internally tangent
externally tangent

2. **Explain** why $\overrightarrow{CD}$ is tangent to $\odot P$, but $\overline{CA}$ is not tangent to $\odot P$.



Guided Practice

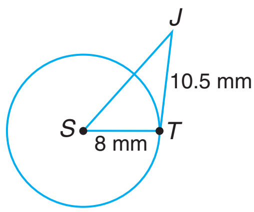
Getting Ready **Evaluate each expression. Round to the nearest tenth.**

Sample: $\sqrt{16^2 - 9^2}$ **Solution:** $\sqrt{16^2 - 9^2} = \sqrt{256 - 81}$
$= \sqrt{175} \approx 13.2$

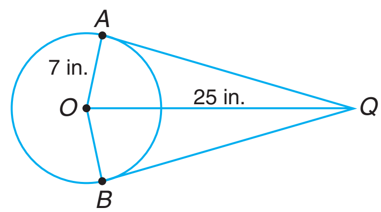
3. $\sqrt{441 - 20^2}$ 4. $\sqrt{7^2 + 10^2}$ 5. $\sqrt{19^2 - 12^2}$

Examples 1 & 2

6. $\overline{JT}$ is tangent to $\odot S$ at T. Find SJ to the nearest tenth.

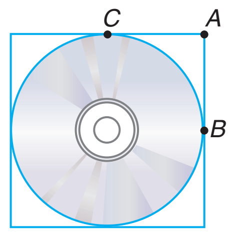


7. $\overline{QA}$ and $\overline{QB}$ are tangent to $\odot O$. Find QB.



Example 2

8. **Music** The figure at the right shows a compact disc (CD) packaged in a square case.
   a. Obtain a CD case and measure to the nearest centimeter from the corner of the disc case to each point of tangency, such as $\overline{AB}$ and $\overline{AC}$.
   b. Which theorem is verified by your measures?



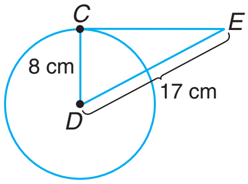
Exercises

Practice

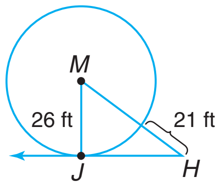
Homework Help	
For Exercises	See Examples
9–11, 15, 16, 26	1
12–14, 17–20, 22, 27	2
Extra Practice	
See page 752.	

Find each measure. If necessary, round to the nearest tenth. Assume segments that appear to be tangent are tangent.

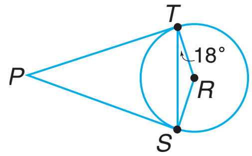
9. *CE*



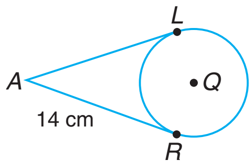
10. *HJ*



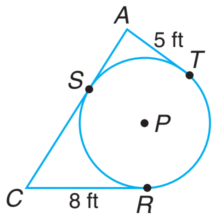
11. $m\angle PTS$



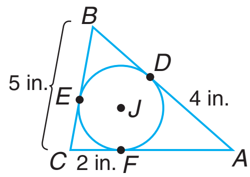
12. *AL*



13. *AC*



14. *BD*



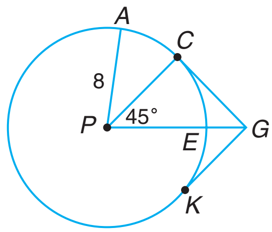
In the figure, $\overline{GC}$ and $\overline{GK}$ are both tangent to ⊙*P*. Find each measure.

15. $m\angle PCG$

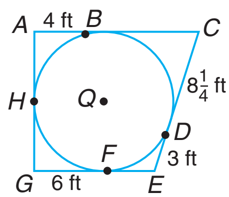
16. $m\angle CGP$

17. *CG*

18. *GK*



19. Find the perimeter of quadrilateral *AGEC*. Explain how you found the missing measures.



$\overline{BI}$ and $\overline{BC}$ are tangent to ⊙*P*.

20. If $BI = 3x - 6$ and $BC = 9$, find the value of x.

21. If $m\angle PIC = x$ and $m\angle CIB = 2x + 3$, find the value of x.

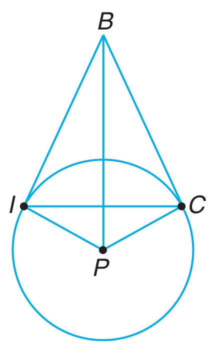
Supply a reason to support each statement.

22. $\overline{BI} \cong \overline{BC}$

23. $\overline{PI} \cong \overline{PC}$

24. $\overline{PB} \cong \overline{PB}$

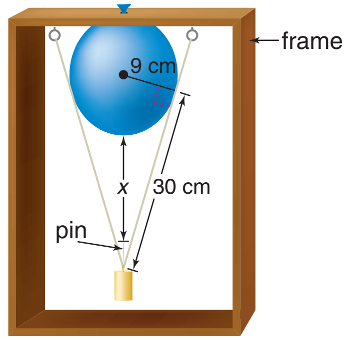
25. $\triangle PIB \cong \triangle PCB$



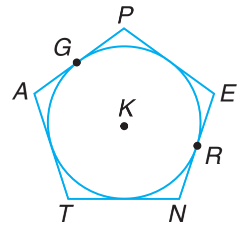
Exercises 20–25

Applications and Problem Solving

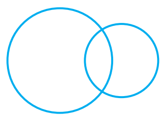
26. **Science** The science experiment at the right demonstrates zero gravity. When the frame is dropped, the pin rises to pop the balloon. If the pin is 2 centimeters long, find x, the distance the pin must rise to pop the balloon. Round to the nearest tenth.

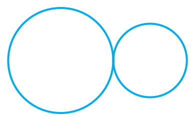


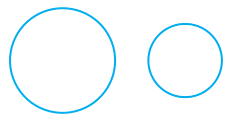
27. **Algebra** Regular pentagon *PENTA* is *circumscribed* about $\odot K$. This means that each side of the pentagon is tangent to the circle.
   a. If $NT = 12x - 30$ and $ER = 2x + 9$, find GP.
   b. Why is the point of tangency the midpoint of each side?



28. **Critical Thinking** How many tangents intersect both circles, each at a single point? Make drawings to show your answers.

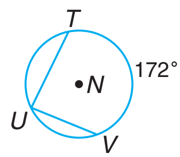
a. 

b. 

c. 

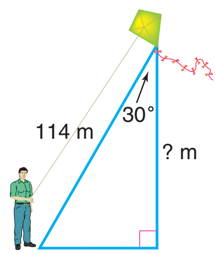
Mixed Review

29. In $\odot N$, find $m\angle TUV$. *(Lesson 14–1)*



30. **Building** A ladder leaning against the side of a house forms a 72° angle with the ground. If the foot of the ladder is 6 feet from the house, find the height that the top of the ladder reaches. Round to the nearest tenth. *(Lesson 13–4)*

31. **Recreation** How far is the kite off the ground? Round to the nearest tenth. *(Lesson 13–3)*



Standardized Test Practice

32. **Grid In** The plans for Ms. Wathen's new sunroom call for a window in the shape of a regular octagon. What is the measure of one interior angle of the window? *(Lesson 10–2)*

33. **Multiple Choice** In parallelogram *RSTV*, $RS = 4p + 9$, $m\angle V = 75$, and $TV = 45$. What is the value of p? *(Lesson 8–2)*

(A) 45 (B) 13.5 (C) 9 (D) 7

Chapter 14 Investigation

The Ins and Outs of Polygons

Materials

 ruler

 compass

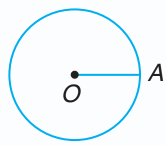
 protractor

Areas of Inscribed and Circumscribed Polygons

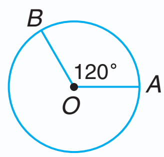
Circles and polygons are paired together everywhere. You can find them in art, advertising, and jewelry designs. How do you think the area of a circle compares to the area of a regular polygon inscribed in it, or to the area of a regular polygon circumscribed about it? Let's find out.

Investigate

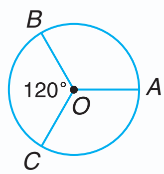
1. Use construction tools to draw a circle with a radius of 2 centimeters. Label the circle *O*.
2. Follow these steps to inscribe an equilateral triangle in $\odot O$.
   a. Draw radius $\overline{OA}$ as shown. Find the area of the circle to the nearest tenth.



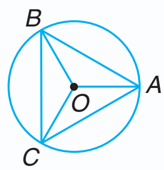
   b. Since there are three sides in a triangle, the measure of a central angle is 360 ÷ 3, or 120. Draw a 120° angle with side $\overline{OA}$ and vertex *O*. Label point *B* on the circle as shown.



   c. Using $\overline{OB}$ as one side of an angle, draw a second 120° angle as shown at the right. Label point *C*.



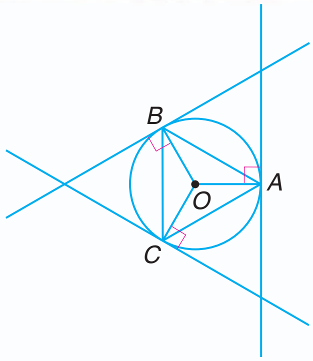
   d. Connect points *A*, *B*, and *C*. Equilateral triangle *ABC* is inscribed in $\odot O$.
   e. Use a ruler to find the measures of one height and base of $\triangle ABC$. Then find and record its area to the nearest tenth.



Look Back

Constructing Perpendicular Line Segments, Lesson 3–7

3. Now circumscribe an equilateral triangle about ⊙O by constructing a line tangent to ⊙O at A, B, and C.
4. Find and record the area of the circumscribed triangle to the nearest tenth.



Extending the Investigation

In this extension, you will compare the areas of regular inscribed and circumscribed polygons to the area of a circle.

- Make a table like the one below. Record your triangle information in the first row.

Regular Polygon	Area of Circle (cm^2)	Area of Inscribed Polygon (cm^2)	Area of Circumscribed Polygon (cm^2)	(Area of Inscribed Polygon) ÷ (Area of Circumscribed Polygon)
triangle	12.6	5.2	20.7	
square				
pentagon				
hexagon				
octagon				

- Use a compass to draw four circles congruent to ⊙O. Record their areas in the table.
- Follow Steps 2 and 3 in the Investigation to inscribe and circumscribe each regular polygon listed in the table.
- Find and record the area of each inscribed and circumscribed polygon. *Refer to Lesson 10–5 to review areas of regular polygons.*
- Find the ratios of inscribed polygon area to circumscribed polygon area. Record the results in the last column of the table. What do you notice?
- **Make a conjecture** about the area of inscribed polygons compared to the area of the circle they inscribe.
- **Make a conjecture** about the area of circumscribed polygons compared to the area of the circle they circumscribe.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a poster displaying your table and the drawings of your circles and polygons.
- Summarize your findings about the areas of inscribed and circumscribed polygons.

interNET CONNECTION **Investigation** For more information on inscribed and circumscribed polygons, visit: www.geomconcepts.com

14–3 Secant Angles

What You'll Learn

You'll learn to find measures of arcs and angles formed by secants.

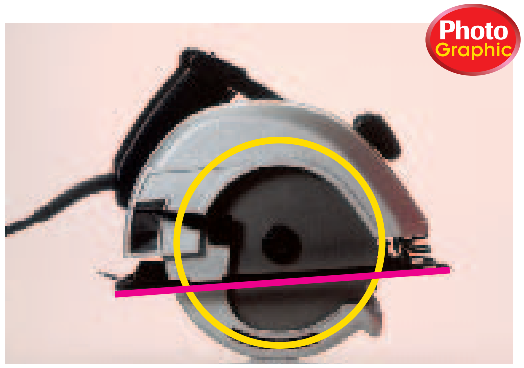
Why It's Important

Marketing

Understanding secant angles can be helpful in locating the source of data on a map.
See Exercise 24.

A circular saw has a flat guide to help cut accurately. The edge of the guide represents a **secant segment** to the circular blade of the saw. A line segment or ray can be a secant of a circle if the line containing the segment or ray is a secant of the circle.

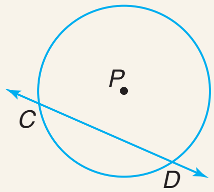
Photo Graphic



Theorem 14–7

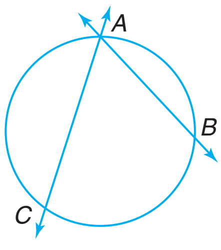
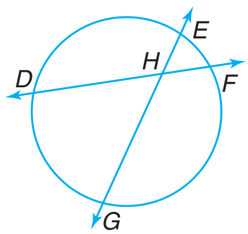
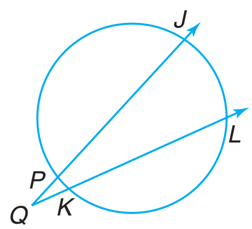
Words: A line or line segment is a secant to a circle if and only if it intersects the circle in two points.

Model:

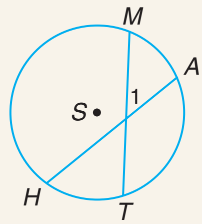


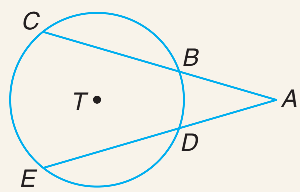
Symbols:
$\overleftrightarrow{CD}$ is a secant of $\odot P$.
Chord CD is a secant segment.

When two secants intersect, the angles formed are called **secant angles**. There are three possible cases.

Case 1 Vertex On the Circle	Case 2 Vertex Inside the Circle	Case 3 Vertex Outside the Circle
		
Secant angle CAB intercepts $\overset{\frown}{BC}$ and is an inscribed angle.	Secant angle DHG intercepts $\overset{\frown}{DG}$, and its vertical angle intercepts $\overset{\frown}{EF}$.	Secant angle JQL intercepts $\overset{\frown}{JL}$ and $\overset{\frown}{PK}$.

When a secant angle is inscribed, as in Case 1, recall that its measure is one-half the measure of the intercepted arc. The following theorems state the formulas for Cases 2 and 3.

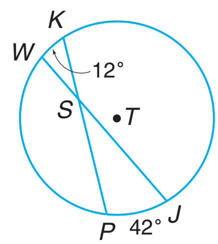
Theorem 14–8	
	Words: If a secant angle has its vertex inside a circle, then its degree measure is one-half the sum of the degree measures of the arcs intercepted by the angle and its vertical angle.
	Model: 
	Symbols: $m\angle 1 = \frac{1}{2}(m\widehat{MA} + m\widehat{HT})$

Theorem 14–9	
	Words: If a secant angle has its vertex outside a circle, then its degree measure is one-half the difference of the degree measures of the intercepted arcs.
	Model: 
	Symbols: $m\angle A = \frac{1}{2}(m\widehat{CE} - m\widehat{BD})$

You can use these theorems to find the measures of arcs and angles formed by secants.

Example 1

Find $m\angle WSK$.



The vertex of $\angle WSK$ is inside $\odot T$. Apply Theorem 14–8.

$m\angle WSK = \frac{1}{2}(m\widehat{WK} + m\widehat{PJ})$ *Theorem 14–8*

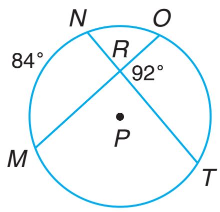
$m\angle WSK = \frac{1}{2}(12 + 42)$ *Replace $m\widehat{WK}$ with 12 and $m\widehat{PJ}$ with 42.*

$m\angle WSK = \frac{1}{2}(54)$ or 27 *Simplify.*

You also could have used this method to find $m\angle PSJ$.

Your Turn

a. Find $m\widehat{OT}$.



Real World

Example 2

Art Link

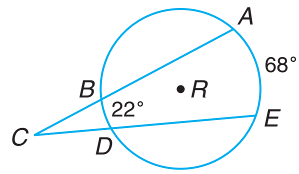
Examine the objects in a student's painting at the right. Since they are difficult to identify, the painting is an example of *non-objective* art. If $m\angle T = 64$ and $m\widehat{NQ} = 19$, find $m\widehat{PR}$.

The vertex of $\angle T$ is outside the circle. Apply Theorem 14–9.

$m\angle T = \frac{1}{2}(m\widehat{PR} - m\widehat{NQ})$	*Theorem 14–9*
$64 = \frac{1}{2}(m\widehat{PR} - 19)$	*Replace $m\angle T$ with 64 and $m\widehat{NQ}$ with 19.*
$2 \cdot 64 = 2 \cdot \frac{1}{2}(m\widehat{PR} - 19)$	*Multiply each side by 2.*
$128 = m\widehat{PR} - 19$	*Simplify.*
$128 + 19 = m\widehat{PR} - 19 + 19$	*Add 19 to each side.*
$147 = m\widehat{PR}$	*Simplify.*

Your Turn

b. Find $m\angle C$.

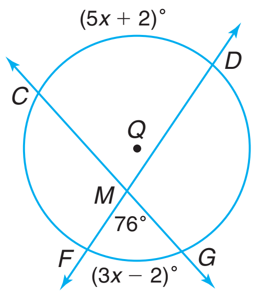


You can also use algebra to solve problems involving secant angles.

Example 3

Algebra Link

Find $m\widehat{FG}$.



Explore First, find the value of x. Then find $m\widehat{FG}$.

Plan The vertex of $\angle FMG$ is inside $\odot Q$. Apply Theorem 14–8.

Solve

$m\angle FMG = \frac{1}{2}(m\widehat{CD} + m\widehat{FG})$	*Theorem 14–8*
$76 = \frac{1}{2}(5x + 2 + 3x - 2)$	*Substitution*
$76 = \frac{1}{2}(8x)$	*Simplify inside the parentheses.*
$76 = 4x$	*Simplify.*
$\frac{76}{4} = \frac{4x}{4}$	*Divide each side by 4.*
$19 = x$	*Simplify.*

Algebra Review
Solving Multi-Step Equations, p. 723

The value of x is 19. Now substitute to find $m\widehat{FG}$.

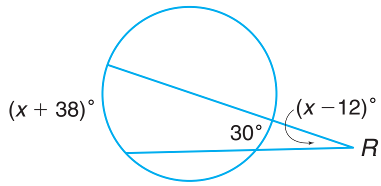
$m\widehat{FG} = 3x - 2$ *Substitution*

$= 3(19) - 2$ or 55 *Replace x with 19.*

Examine Find $m\widehat{CD}$ and substitute into the original equation $m\angle FMG = \frac{1}{2}(m\widehat{CD} + m\widehat{FG})$. The solution checks.

Your Turn

c. Find the value of x. Then find $m\angle R$.



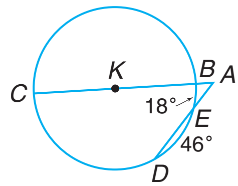
Check for Understanding

Communicating Mathematics

Vocabulary
secant segment
secant angles

1. **Determine** the missing information needed for $\odot K$ if you want to use Theorem 14–9 to find $m\angle A$.

2. **Explain** how to find $m\angle A$ using only the given information.



Exercises 1–2

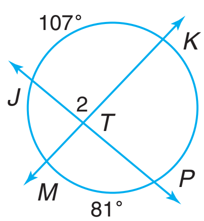
3. Writing Math The word *secant* comes from the Latin word *secare*. Use a dictionary to find the meaning of the word and explain why secant is used for a line that intersects a circle in exactly two points.

Guided Practice

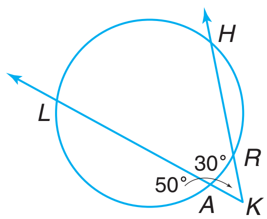
Examples 1 & 2

Find each measure.

4. $m\angle 2$



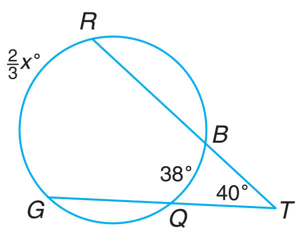
5. $m\widehat{LH}$



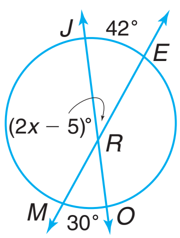
Example 3

In each circle, find the value of *x*. Then find the given measure.

6. $m\widehat{GR}$

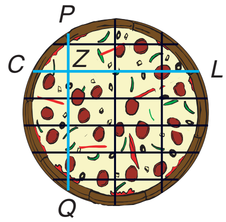


7. $m\angle MRO$



Example 1

8. **Food** A cook uses secant segments to cut a round pizza into rectangular pieces. If $\overline{PQ} \perp \overline{CL}$ and $m\overset{\frown}{QL} = 140$, find $m\overset{\frown}{PC}$.



Exercises

Practice

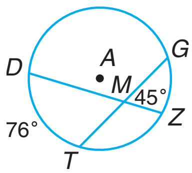
Homework Help	
For Exercises	See Examples
9, 12, 14, 15, 19, 20, 23, 24, 26	1, 3
10, 11, 13, 16–18, 25	2
21, 22	1–3

Extra Practice

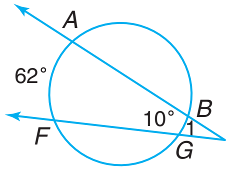
See page 752.

Find each measure.

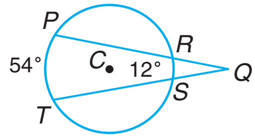
9. $m\overset{\frown}{GZ}$



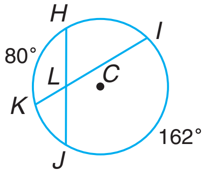
10. $m\angle 1$



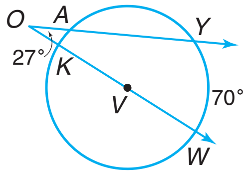
11. $m\angle Q$



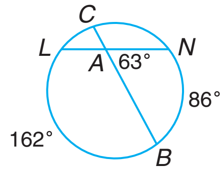
12. $m\angle HLI$



13. $m\overset{\frown}{AK}$

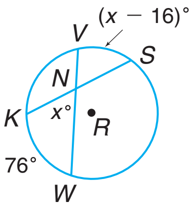


14. $m\overset{\frown}{LC}$

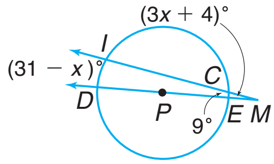


In each circle, find the value of *x*. Then find the given measure.

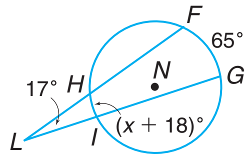
15. $m\overset{\frown}{SV}$



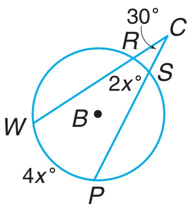
16. $m\angle M$



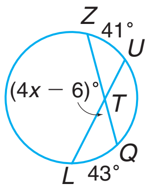
17. $m\overset{\frown}{HI}$



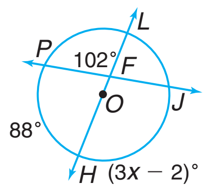
18. $m\overset{\frown}{RS}$



19. $m\angle LTQ$



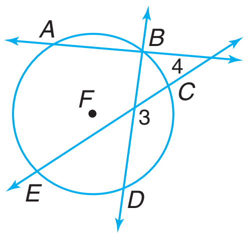
20. $m\overset{\frown}{JH}$



21. If $m\angle 4 = 38$ and $m\overset{\frown}{BC} = 38$, find $m\overset{\frown}{AE}$.

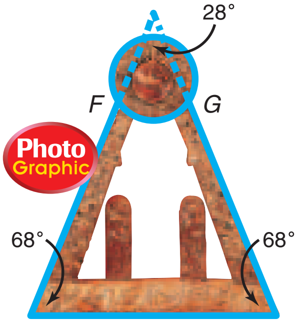
22. If $m\overset{\frown}{BAE} = 198$ and $m\overset{\frown}{CD} = 64$, find $m\angle 3$.

23. In a circle, chords AC and BD meet at P. If $m\angle CPB = 115$, $m\overset{\frown}{AB} = 6x + 16$, and $m\overset{\frown}{CD} = 3x - 12$. Find x, $m\overset{\frown}{AB}$, and $m\overset{\frown}{CD}$.



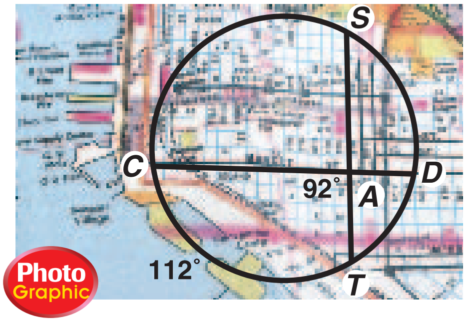
Exercises 21–22

Applications and Problem Solving



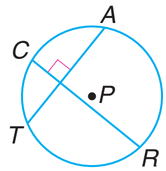
Exercise 25

24. **Marketing** The figure at the right is a "one-mile" circle of San Diego used for research and marketing purposes. What is $m\widehat{SD}$?



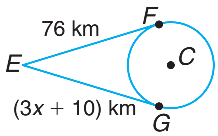
25. **History** The gold figurine at the left was made by the Germanic people in the 8th century. Find $m\widehat{FG}$.

26. **Critical Thinking** In $\odot P$, $\overline{CR} \perp \overline{AT}$. Find $m\widehat{AC} + m\widehat{TR}$.



Mixed Review

27. $\overline{EF}$ and $\overline{EG}$ are tangent to $\odot C$. Find the value of x. *(Lesson 14–2)*



28. A pyramid has a height of 12 millimeters and a base with area of 34 square millimeters. What is its volume? *(Lesson 12–5)*

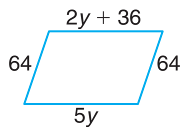
29. Find the circumference of a circle whose diameter is 26 meters. Round to the nearest tenth. *(Lesson 11–5)*

Standardized Test Practice

A B C D

30. **Short Response** Find the area of a trapezoid whose height measures 8 centimeters and whose bases are 11 centimeters and 9 centimeters long. *(Lesson 10–4)*

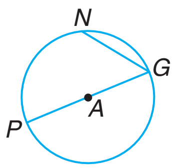
31. **Multiple Choice** Find the value for y that verifies that the figure is a parallelogram. *(Lesson 8–3)*



Ⓐ 4 Ⓑ 12 Ⓒ 12.8 Ⓓ 14

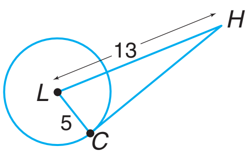
Quiz 1 Lessons 14–1 through 14–3

1. Determine whether $\angle NGP$ is an inscribed angle. Name the intercepted arc. *(Lesson 14–1)*

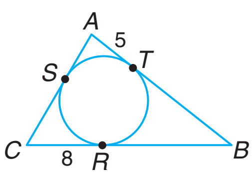


Find each measure. Assume segments that appear to be tangent are tangent. *(Lesson 14–2)*

2. CH

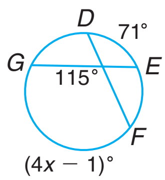


3. AC

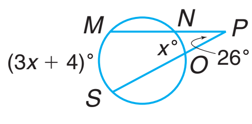


In each circle, find the value of x. Then find the given measure. *(Lesson 14–3)*

4. $m\widehat{GF}$



5. $m\widehat{MS}$



14–4 Secant-Tangent Angles

What You'll Learn

You'll learn to find measures of arcs and angles formed by secants and tangents.

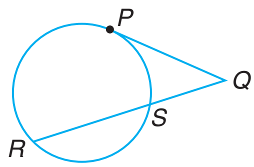
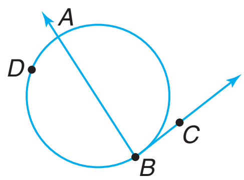
Why It's Important

Archaeology

Scientists can learn a lot about an ancient civilization by using secant-tangent angles to find pottery measurements.
See Exercise 20.

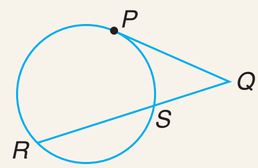
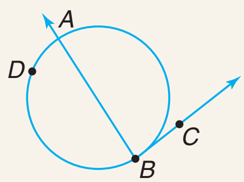
When a secant and a tangent of a circle intersect, a **secant-tangent angle** is formed. This angle intercepts an arc on the circle. The measure of the arc is related to the measure of the secant-tangent angle.

There are two ways that secant-tangent angles are formed, as shown below.

Case 1 Vertex Outside the Circle	Case 2 Vertex On the Circle
 Secant-tangent angle *PQR* intercepts $\overset{\frown}{PR}$ and $\overset{\frown}{PS}$.	 Secant-tangent angle *ABC* intercepts $\overset{\frown}{AB}$.

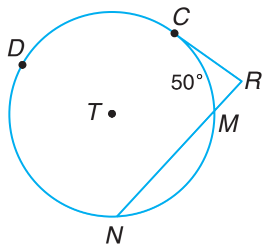
Notice that the vertex of a secant-tangent angle cannot lie inside the circle. This is because the tangent always lies outside the circle, except at the single point of contact.

The formulas for the measures of these angles are shown in Theorems 14–10 and 14–11.

Theorem	Words	Models and Symbols
14–10	If a secant-tangent angle has its vertex outside the circle, then its degree measure is one-half the difference of the degree measures of the intercepted arcs.	 $m\angle PQR = \frac{1}{2}(m\overset{\frown}{PR} - m\overset{\frown}{PS})$
14–11	If a secant-tangent angle has its vertex on the circle, then its degree measure is one-half the degree measure of the intercepted arc.	 $m\angle ABC = \frac{1}{2}(m\overset{\frown}{AB})$

Examples

1 **$\overline{CR}$ is tangent to $\odot T$ at C. If $m\overparen{CDN} = 200$, find $m\angle R$.**

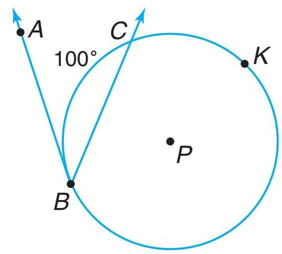


Vertex R of the secant-tangent angle is outside of $\odot T$. Apply Theorem 14–10.

$m\angle R = \frac{1}{2}(m\overparen{CDN} - m\overparen{CM})$	*Theorem 14–10*
$m\angle R = \frac{1}{2}(200 - 50)$	*Substitution*
$m\angle R = \frac{1}{2}(150)$ or 75	*Simplify.*

Algebra Review
Evaluating Expressions, p. 718

2 **$\overrightarrow{BA}$ is tangent to $\odot P$ at B. Find $m\angle ABC$.**



Vertex B of the secant-tangent angle is on $\odot P$. Apply Theorem 14–11.

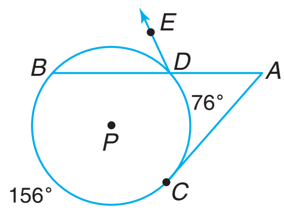
$m\angle ABC = \frac{1}{2}(m\overparen{BC})$	*Theorem 14–11*
$m\angle ABC = \frac{1}{2}(100)$ or 50	*Substitution*

Your Turn

$\overline{AC}$ is tangent to $\odot P$ at C and $\overrightarrow{DE}$ is tangent to $\odot P$ at D.

a. Find $m\angle A$.

b. Find $m\angle BDE$.

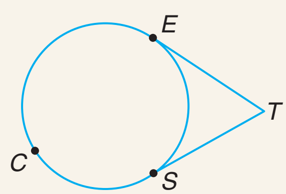


A **tangent-tangent angle** is formed by two tangents. The vertex of a tangent-tangent angle is always outside the circle.

Theorem 14–12

Words: The degree measure of a tangent-tangent angle is one-half the difference of the degree measures of the intercepted arcs.

Model:



Symbols: $m\angle ETS = \frac{1}{2}(m\overparen{ECS} - m\overparen{ES})$

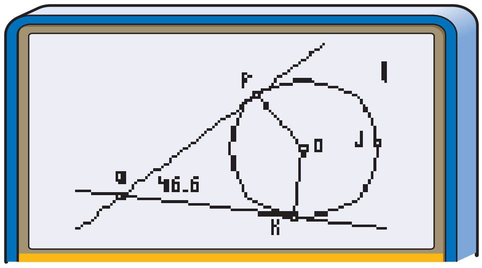
You can use a TI–83 Plus/TI–84 Plus calculator to verify the relationship stated in Theorem 14–12.

Graphing Calculator Tutorial

See pp. 782–785.

Graphing Calculator Exploration

The calculator screen at the right shows an acute angle, $\angle Q$. To verify Theorem 14–12, you can measure $\angle Q$, find the measures of the intercepted arcs, and then perform the calculation.



Try These

1. Use the calculator to construct and label a figure like the one shown above. Then use the Angle tool on the F5 menu to measure $\angle Q$. What measure do you get?
2. How can you use the Angle tool on F5 to find $m\overset{\frown}{PK}$ and $m\overset{\frown}{PJK}$? Use the calculator to find these measures. What are the results?
3. Use the Calculate tool on F5 to find $\frac{1}{2}(m\overset{\frown}{PJK} - m\overset{\frown}{PK})$. How does the result compare with $m\angle Q$ from Exercise 1? Is your answer in agreement with Theorem 14–12?
4. Move point P along the circle so that Q moves farther away from the center of the circle. Describe how this affects the arc measures and the measure of $\angle Q$.
5. Suppose you change $\angle Q$ to an obtuse angle. Do the results from Exercises 1–3 change? Explain your answer.

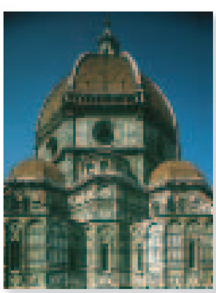
You can use Theorem 14–12 to solve problems involving tangent-tangent angles.

Real World

Example 3

Architecture Link

In the 15th century, Brunelleschi, an Italian architect, used his knowledge of mathematics to create a revolutionary design for the dome of a cathedral in Florence. A close-up of one of the windows is shown at the right. Find $m\angle B$.



The Duomo, Florence, Italy

$\angle B$ is a tangent-tangent angle. Apply Theorem 14–12.

In order to find $m\angle B$, first find $m\overset{\frown}{HLN}$.

$m\overset{\frown}{HLN} + m\overset{\frown}{HN} = 360$ *The sum of the measures of a minor arc and its major arc is 360.*

$m\overset{\frown}{HLN} + 90 = 360$

$m\overset{\frown}{HLN} = 270$ *Subtract 90 from each side.*

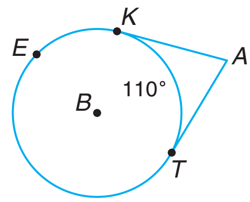
$$m\angle B = \frac{1}{2}(m\overset{\frown}{HLN} - m\overset{\frown}{HN})$$ *Theorem 14–12*

$$m\angle B = \frac{1}{2}(270 - 90)$$ *Substitution*

$$m\angle B = \frac{1}{2}(180) \text{ or } 90$$ *Simplify.*

Your Turn

c. Find $m\angle A$.



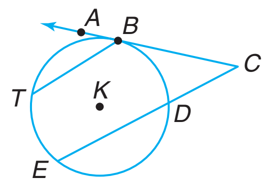
Check for Understanding

Communicating Mathematics

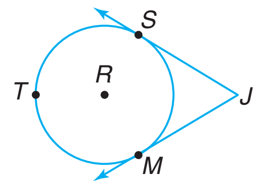
1. **Explain** how to find the measure of a tangent-tangent angle.

Vocabulary
secant-tangent angle
tangent-tangent angle

2. **Name** three secant-tangent angles in $\odot K$.



3. **You Decide?** In $\odot R$, $\overrightarrow{JS}$ and $\overrightarrow{JM}$ are tangents. Maria says that if $m\angle J$ increases, $m\overset{\frown}{STM}$ increases. Is she correct? Make some drawings to support your conclusion.

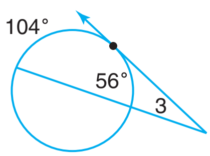


Guided Practice

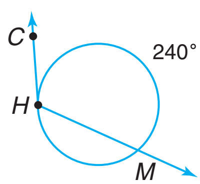
Find the measure of each angle. Assume segments that appear to be tangent are tangent.

Examples 1–3

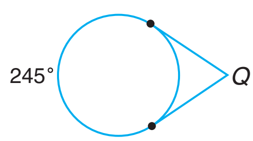
4. $\angle 3$



5. $\angle CHM$

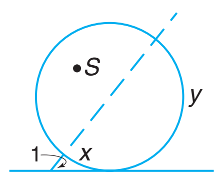


6. $\angle Q$



Example 1

7. **Billiards** Refer to the application at the beginning of the lesson. If $x = 31$ and $y = 135$, find $m\angle 1$, the angle measure of the cue ball's spin.

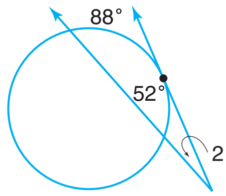


Exercises

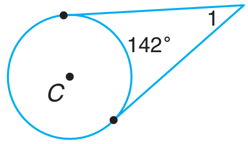
Practice

Find the measure of each angle. Assume segments that appear to be tangent are tangent.

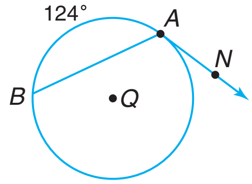
8. $\angle 2$



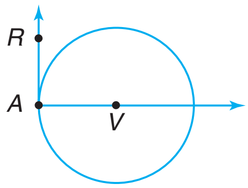
9. $\angle 1$



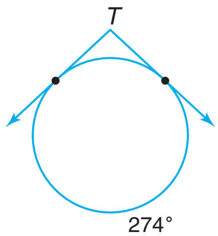
10. $\angle BAN$



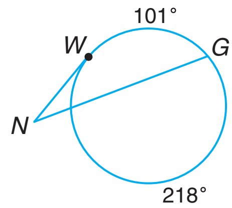
11. $\angle RAV$



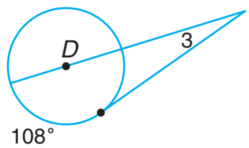
12. $\angle T$



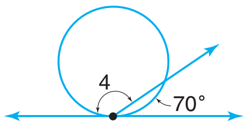
13. $\angle WNG$



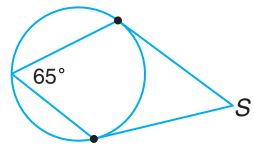
14. $\angle 3$



15. $\angle 4$



16. $\angle S$



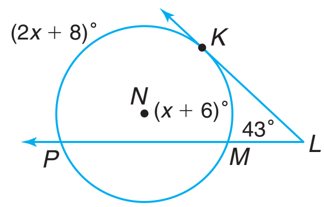
Homework Help	
For Exercises	See Examples
8, 13, 14, 17–20	1
9, 12, 16, 22	3
10, 11, 15, 21	2

Extra Practice

See page 753.

17. In $\odot N$, find the value of x.

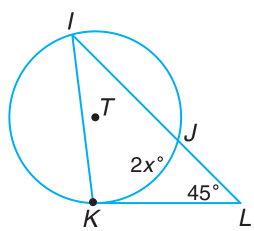
18. What is $m\overset{\frown}{PK}$?



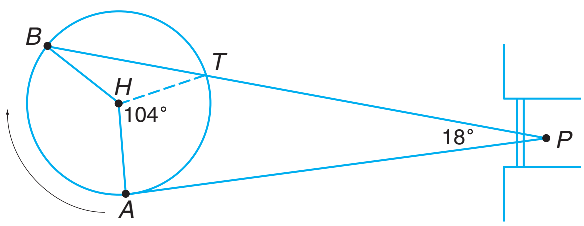
Exercises 17–18

Applications and Problem Solving

19. **Algebra** $\overline{IL}$ is a secant segment, and $\overline{LK}$ is tangent to $\odot T$. Find $m\overset{\frown}{IJ}$ in terms of x. (*Hint*: First find $m\overset{\frown}{IK}$ in terms of x.)



20. **Mechanics** In the piston and rod diagram at the right, the throw arm moves from position A to position B. Find $m\overset{\frown}{AB}$.



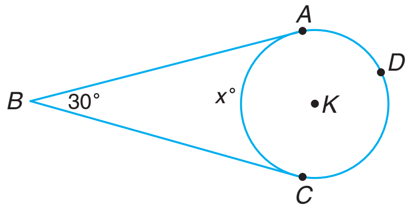
CONTENTS

21. **Archaeology** The most commonly found artifact on an archaeological dig is a pottery shard. Many clues about a site and the group of people who lived there can be found by studying these shards. The piece at the right is from a round plate.

a. If $\overrightarrow{HD}$ is a tangent at H, and $m\angle SHD = 60$, find $m\overset{\frown}{SH}$.

b. Suppose an archaeologist uses a tape measure and finds that the distance along the outside edge of the shard is 8.3 centimeters. What was the circumference of the original plate? Explain how you know.

22. **Critical Thinking** $\overline{AB}$ and $\overline{BC}$ are tangent to $\odot K$.



a. If x represents $m\overset{\frown}{AC}$, what is $m\overset{\frown}{ADC}$ in terms of x?

b. Find $m\overset{\frown}{AC}$.

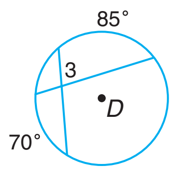
c. Find $m\angle B + m\overset{\frown}{AC}$.

d. Is the sum of the measures of a tangent-tangent angle and the smaller intercepted arc always equal to the sum in part c? Explain.

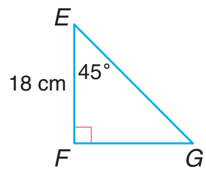
Mixed Review

Find each measure.

23. $m\angle 3$ *(Lesson 14–3)*



24. FG and GE *(Lesson 13–2)*



25. **Museums** A museum of miniatures in Los Angeles, California, has 2-inch violins that can actually be played. If the 2-inch model represents a 2-foot violin, what is the scale factor of the model to the actual violin? (*Hint*: Change feet to inches.) *(Lesson 12–7)*

Standardized Test Practice

26. **Short Response** The perimeter of $\triangle QRS$ is 94 centimeters. If $\triangle QRS \sim \triangle CDH$ and the scale factor of $\triangle QRS$ to $\triangle CDH$ is $\frac{4}{3}$, find the perimeter of $\triangle CDH$. *(Lesson 9–7)*

27. **Multiple Choice** Find the solution to the system of equations. *(Algebra Review)*

$y = 3x + 5$
$5x + 3y = 43$

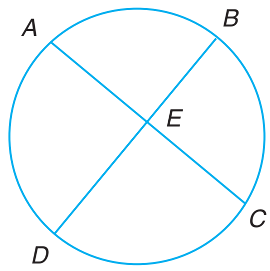
(A) (2, 11) (B) (−11, 2) (C) (−2, 11) (D) (11, 2)

14–5 Segment Measures

What You'll Learn
You'll learn to find measures of chords, secants, and tangents.

Why It's Important
Art The Hopi Indians used special circle segments in their designs and artwork. *See Exercise 20.*

In the circle at the right, chords AC and BD intersect at E. Notice the two pairs of segments that are formed by these intersecting chords.



$\overline{AE}$ and $\overline{EC}$ are segments of $\overline{AC}$.

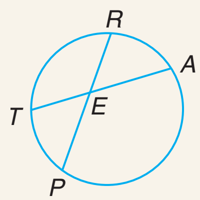
$\overline{BE}$ and $\overline{ED}$ are segments of $\overline{BD}$.

There exists a special relationship for the measures of the segments formed by intersecting chords. This relationship is stated in the following theorem.

Theorem 14–13

Words: If two chords of a circle intersect, then the product of the measures of the segments of one chord equals the product of the measures of the segments of the other chord.

Model:

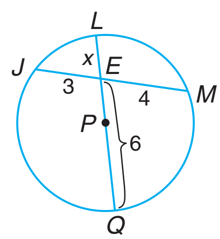


Symbols: $TE \cdot EA = RE \cdot EP$

Example 1
Algebra Link

Algebra Review
Solving One-Step Equations, p. 722

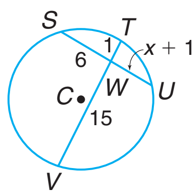
In $\odot P$, find the value of x.



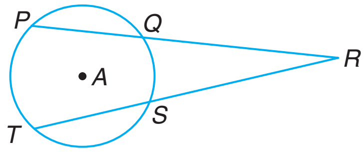
$LE \cdot EQ = JE \cdot EM$	*Theorem 14–13*
$x \cdot 6 = 3 \cdot 4$	*Substitution*
$6x = 12$	*Multiply.*
$\frac{6x}{6} = \frac{12}{6}$	*Divide each side by 6.*
$x = 2$	*Simplify.*

Your Turn

a. In $\odot C$, find UW.



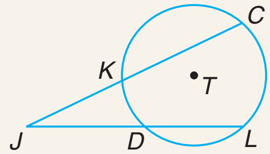
$\overline{RP}$ and $\overline{RT}$ are secant segments of $\odot A$. $\overline{RQ}$ and $\overline{RS}$ are the parts of the segments that lie outside the circle. They are called **external secant segments**.



Definition of External Secant Segment

Words: A segment is an external secant segment if and only if it is the part of a secant segment that is outside a circle.

Model:



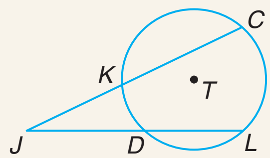
$\overline{JK}$ and $\overline{JD}$ are external secant segments.

A special relationship between secant segments and external secant segments is stated in the following theorem.

Theorem 14–14

Words: If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment equals the product of the measures of the other secant segment and its external secant segment.

Model:

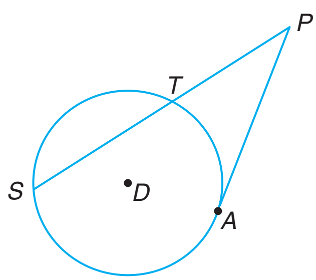


Symbols: $JC \cdot JK = JL \cdot JD$

In $\odot D$, a similar relationship exists if one segment is a secant and one is a tangent. $\overline{PA}$ is a tangent segment.

$$PA \cdot PA = PS \cdot PT$$

$$(PA)^2 = PS \cdot PT$$

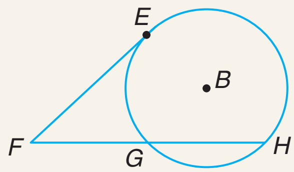


This result is formally stated in the following theorem.

Theorem 14–15

Words: If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment equals the product of the measures of the secant segment and its external secant segment.

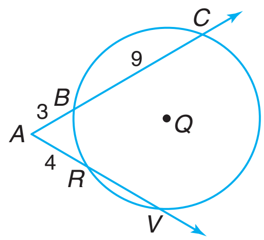
Model:



Symbols: $(FE)^2 = FH \cdot FG$

Examples

2 **Find *AV* and *RV*.**

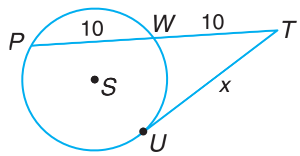


$AC \cdot AB = AV \cdot AR$	*Theorem 14–14*
$(3 + 9) \cdot 3 = AV \cdot 4$	*Substitution*
$12 \cdot 3 = AV \cdot 4$	*Add.*
$36 = 4(AV)$	*Multiply.*
$\frac{36}{4} = \frac{4(AV)}{4}$	*Divide each side by 4.*
$9 = AV$	*Simplify.*

$AR + RV = AV$	*Segment Addition Property*
$4 + RV = 9$	*Substitution*
$4 + RV - 4 = 9 - 4$	*Subtract 4 from each side.*
$RV = 5$	*Simplify.*

Algebra Link

3 **Find the value of *x* to the nearest tenth.**

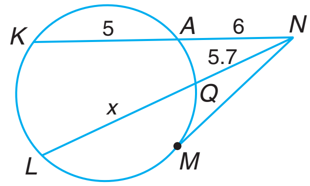


$(TU)^2 = TP \cdot TW$	*Theorem 14–15*
$x^2 = (10 + 10) \cdot 10$	*Substitution*
$x^2 = 20 \cdot 10$	*Add.*
$x^2 = 200$	*Multiply.*
$\sqrt{x^2} = \sqrt{200}$	*Take the square root of each side.*
$x \approx 14.1$	*Use a calculator.*

Your Turn

b. Find the value of x to the nearest tenth.

c. Find MN to the nearest tenth.



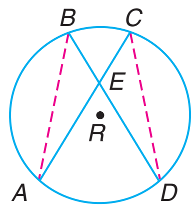
Check for Understanding

Communicating Mathematics

Vocabulary
external secant segments

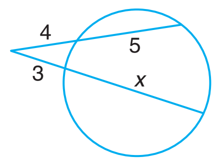
1. **Draw** and label a circle that fits the following description.
   - Has center K.
   - Contains secant segments AM and AL.
   - Contains external secant segments AP and AN.
   - $\overleftrightarrow{JM}$ is tangent to the circle at M.

2. **Complete** the steps below to prove Theorem 14–13. Refer to $\odot R$ shown at the right.



   a. $\angle BAE \cong \angle CDE$ and
      $\angle ABE \cong \angle DCE$ *Theorem* ____
   b. $\triangle ABE \sim \triangle$____   *AA Similarity Postulate*
   c. $\frac{AE}{DE} =$ ___   *Definition of Similar Polygons*
   d. $AE \cdot CE = DE \cdot BE$ ________

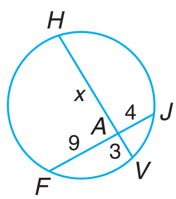
3. **You Decide?** Leon wrote the equation $4 \cdot 5 = 3x$ to find the value of x in the figure at the right. Yoshica wrote the equation $9 \cdot 4 = (3 + x) \cdot 3$. Who wrote the correct equation? Explain.



Guided Practice

Example 1

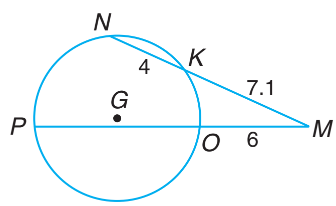
4. Find the value of x.



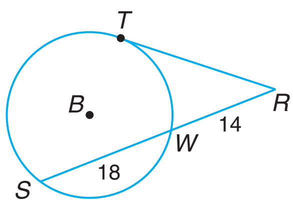
Examples 2 & 3

Find each measure. If necessary, round to the nearest tenth.

5. OP

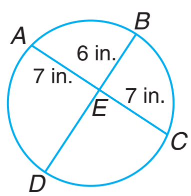


6. TR



Example 1

7. Find DE to the nearest tenth.



Exercises

Practice

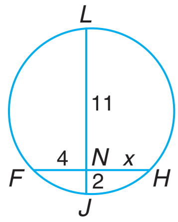
Homework Help	
For Exercises	**See Examples**
8–10, 20	1
11, 14, 16, 21	2
12, 13, 15, 19	3
17, 18	1, 2

Extra Practice

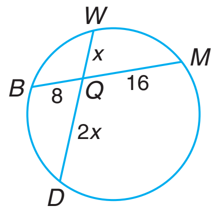
See page 753.

In each circle, find the value of *x*. If necessary, round to the nearest tenth.

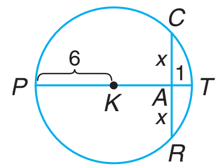
8.



9.

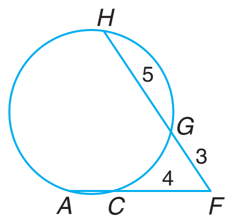


10.

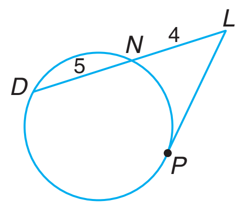


Find each measure. If necessary, round to the nearest tenth.

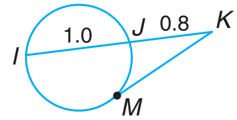
11. AC



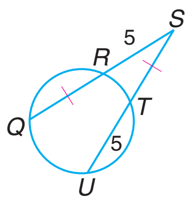
12. LP



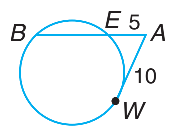
13. KM



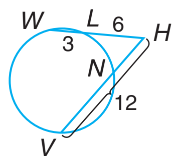
14. QR



15. BE

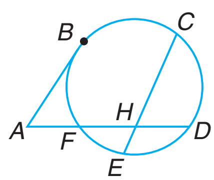


16. NV



17. If $CH = 13$, $EH = 3.2$, and $DH = 6$, find FH to the nearest tenth.

18. If $AF = 7.5$, $FH = 7$, and $DH = 6$, find BA to the nearest tenth.



Applications and Problem Solving

19. **Space** The space shuttle *Discovery D* is 145 miles above Earth. The diameter of Earth is about 8000 miles. How far is its longest line of sight $\overline{DA}$ to Earth?

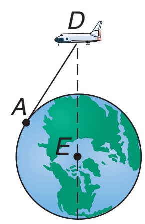
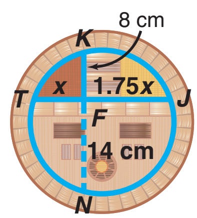


Figure is not drawn to scale.

20. **Native American Art** The traditional sun design appears in many phases of Hopi art and decoration. Find the length of $\overline{TJ}$.

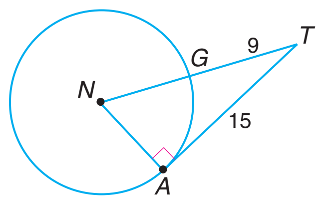


21. **Critical Thinking** Find the radius of $\odot N$:

a. using the Pythagorean Theorem.

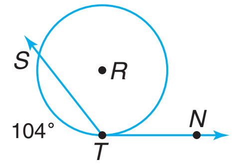
b. using Theorem 14–4. (*Hint*: Extend $\overline{TN}$ to the other side of $\odot N$.)

c. Which method seems more efficient? Explain.



Mixed Review

22. In $\odot R$, find the measure of $\angle STN$. *(Lesson 14–4)*



Exercise 22

23. Simplify $\frac{\sqrt{8}}{\sqrt{36}}$. *(Lesson 13–1)*

24. In a circle, the measure of chord JK is 3, the measure of chord LM is 3, and $m\widehat{JK} = 35$. Find $m\widehat{LM}$. *(Lesson 11–3)*

Standardized Test Practice

25. **Short Response** Determine whether the face of the jaguar has *line symmetry*, *rotational symmetry*, *both*, or *neither*. *(Lesson 10–6)*

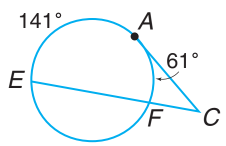
26. **Short Response** Sketch and label isosceles trapezoid $CDEF$ and its median ST. *(Lesson 8–5)*


Exercise 25

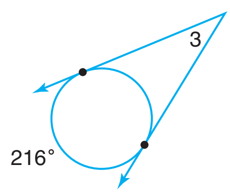
Quiz 2 Lessons 14–4 and 14–5

Find the measure of each angle. *(Lesson 14–4)*

1. $\angle C$

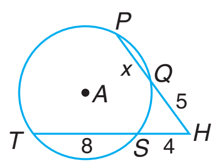


2. $\angle 3$

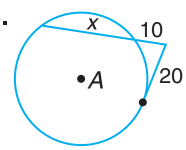


In each circle, find the value of *x*. *(Lesson 14–5)*

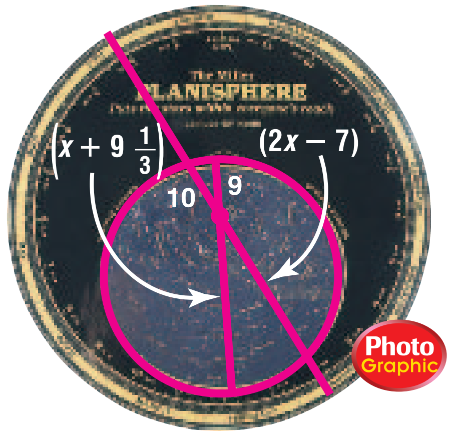
3.



4.



5. **Astronomy** A *planisphere* is a "flattened sphere" that shows the whole sky. The smaller circle inside the chart is the area of sky that is visible to the viewer. Find the value of x. *(Lesson 14–5)*



14–6 Equations of Circles

What You'll Learn

You'll learn to write equations of circles using the center and the radius.

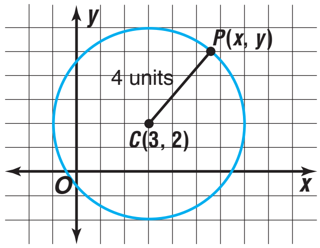
Why It's Important

Meteorology

Equations of circles are important in helping meteorologists track storms shown on radar.
See Exercise 30.

In Lesson 4–6, you learned that the equation of a straight line is linear. In slope-intercept form, this equation is written as $y = mx + b$. A circle is not a straight line, so its equation is not linear. You can use the Distance Formula to find the equation of any circle.

Circle C has its center at $C(3, 2)$. It has a radius of 4 units. Let $P(x, y)$ represent any point on $\odot C$. Then d, the measure of the distance between P and C, must be equal to the radius, 4.



$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d \quad \textit{Distance Formula}$$

$$\sqrt{(x - 3)^2 + (y - 2)^2} = 4 \quad \textit{Replace } (x_1, y_1) \textit{ with } (3, 2) \textit{ and } (x_2, y_2) \textit{ with } (x, y).$$

$$\left(\sqrt{(x - 3)^2 + (y - 2)^2}\right)^2 = 4^2 \quad \textit{Square each side of the equation.}$$

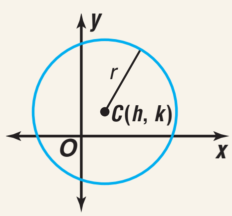
$$(x - 3)^2 + (y - 2)^2 = 16 \quad \textit{Simplify.}$$

Therefore, the equation of the circle with center at (3, 2) and a radius of 4 units is $(x - 3)^2 + (y - 2)^2 = 16$. This result is generalized in the equation of a circle given below.

Theorem 14–16 General Equation of a Circle

Words: The equation of a circle with center at (h, k) and a radius of r units is $(x - h)^2 + (y - k)^2 = r^2$.

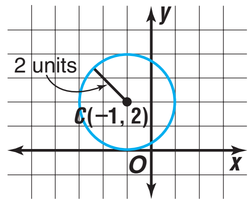
Model:



CONTENTS

Example 1

Write an equation of a circle with center $C(-1, 2)$ and a radius of 2 units.



$$(x - h)^2 + (y - k)^2 = r^2 \quad \textit{General Equation of a Circle}$$
$$[x - (-1)]^2 + (y - 2)^2 = 2^2 \quad (h, k) = (-1, 2), r = 2$$
$$(x + 1)^2 + (y - 2)^2 = 4 \quad \textit{Simplify.}$$

The equation for the circle is $(x + 1)^2 + (y - 2)^2 = 4$.

Your Turn

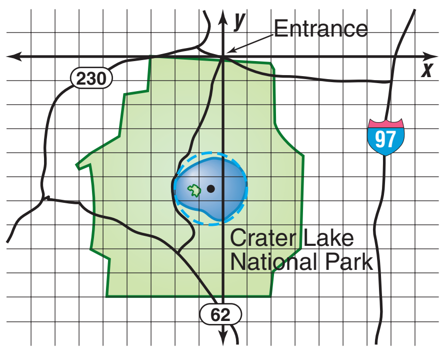
a. Write an equation of a circle with center at $(3, -2)$ and a diameter of 8 units.

You can also use the equation of a circle to find the coordinates of its center and the measure of its radius.

Real World Example 2

Geography Link

The lake in Crater Lake Park was formed thousands of years ago by the explosive collapse of Mt. Mazama. If the park entrance is at (0, 0), then the equation of the circle representing the lake is $(x + 1)^2 + (y + 11)^2 = 9$. Find the coordinates of its center and the measure of its diameter. Each unit on the grid represents 2 miles.




Crater Lake, Oregon

Rewrite the equation in the form $(x - h)^2 + (y - k)^2 = r^2$.

$$[(x - (-1)]^2 + [(y - (-11)]^2 = 3^2$$

Since $h = -1$, $k = -11$, and $r = 3$, the center of the circle is at $(-1, -11)$. Its radius is 3 miles, so its diameter is 6 miles.

Your Turn

b. Find the coordinates of the center and the measure of the radius of a circle whose equation is $x^2 + \left(y - \frac{3}{4}\right)^2 = \frac{25}{4}$.

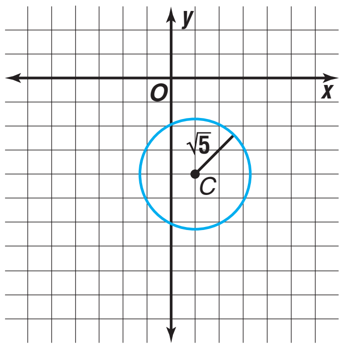
Check for Understanding

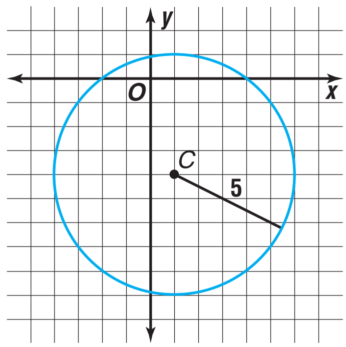
Communicating Mathematics

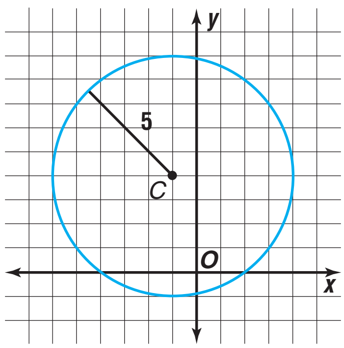
1. **Draw** a circle on a coordinate plane. Use a ruler to find its radius and write its general equation.

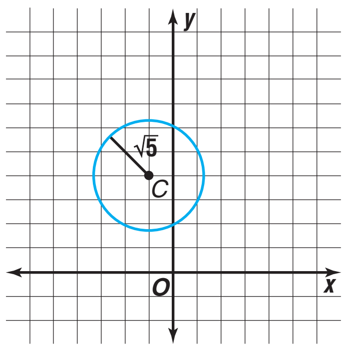
2. **Match** each graph below with one of the equations at the right.

(1) $(x + 1)^2 + (y - 4)^2 = 5$
(2) $(x - 1)^2 + (y + 4)^2 = 5$
(3) $(x + 1)^2 + (y - 4)^2 = 25$
(4) $(x - 1)^2 + (y + 4)^2 = 25$

a.


b.


c.


d.


3. **Explain** how you could find the equation of a line that is tangent to the circle whose equation is $(x - 4)^2 + (y + 6)^2 = 9$.

4. Writing Math How could you find the equation of a circle if you are given the coordinates of the endpoints of a diameter? First, make a sketch of the problem and then list the information that you need and the steps you could use to find the equation.

Guided Practice

Getting Ready **If r represents the radius and d represents the diameter, find each missing measure.**

Sample: $d = \frac{1}{3}$, $r^2 = \underline{\quad ? \quad}$ **Solution:** $r^2 = \left(\frac{1}{2} \cdot \frac{1}{3}\right)^2$ or $\frac{1}{36}$

5. $r^2 = 169$, $d = \underline{\quad ? \quad}$

6. $d = 2\sqrt{18}$, $r^2 = \underline{\quad ? \quad}$

7. $d = \frac{2}{5}$, $r^2 = \underline{\quad ? \quad}$

8. $r^2 = \frac{16}{49}$, $d = \underline{\quad ? \quad}$

Example 1

Write an equation of a circle for each center and radius or diameter measure given.

9. $(1, -5)$, $d = 8$

10. $(3, 4)$, $r = \sqrt{2}$

Example 2

Find the coordinates of the center and the measure of the radius for each circle whose equation is given.

11. $(x - 7)^2 + (y + 5)^2 = 4$

12. $(x - 6)^2 + y^2 = 64$

Example 1

13. Botany Scientists can tell what years had droughts by studying the rings of bald cypress trees. If the radius of a tree in 1612 was 14.5 inches, write an equation that represents the cross section of the tree. Assume that the center is at (0, 0).

Exercises

Practice

Write an equation of a circle for each center and radius or diameter measure given.

14. $(2, -11)$, $r = 3$

15. $(-4, 2)$, $d = 2$

16. $(0, 0)$, $r = \sqrt{5}$

17. $(6, 0)$, $r = \frac{2}{3}$

18. $(-1, -1)$, $d = \frac{1}{4}$

19. $(-5, 9)$, $d = 2\sqrt{20}$

Homework Help

For Exercises	See Examples
14–19, 28, 29, 31, 32	1
20–27, 30	2

Extra Practice

See page 753.

Find the coordinates of the center and the measure of the radius for each circle whose equation is given.

20. $(x - 9)^2 + (y - 10)^2 = 1$

21. $x^2 + (y + 5)^2 = 100$

22. $(x + 7)^2 + (y - 3)^2 = 25$

23. $\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{1}{3}\right)^2 = \frac{16}{25}$

24. $(x - 19)^2 + y^2 = 20$

25. $(x - 24)^2 + (y + 8.1)^2 - 12 = 0$

Graph each equation on a coordinate plane.

26. $(x + 5)^2 + (y - 2)^2 = 4$

27. $x^2 + (y - 3)^2 = 16$

28. Write an equation of the circle that has a diameter of 12 units and its center at $(-4, -7)$.

29. Write an equation of the circle that has its center at $(5, -13)$ and is tangent to the y-axis.

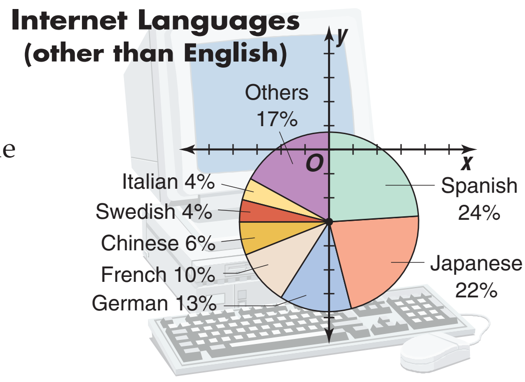
Applications and Problem Solving

30. **Meteorology** Often when a hurricane is expected, all people within a certain radius are evacuated. Circles around a radar image can be used to determine a safe radius. If an equation of the circle that represents the evacuated area is given by $(x + 42)^2 + (y - 11)^2 = 1024$, find the coordinates of the center and measure of the radius of the evacuated area. Units are in miles.

interNET CONNECTION

Data Update For the latest information on the percents of international internet users, visit: www.geomconcepts.com

31. **Technology** Although English is the language used by more than half the Internet users, over 56 million people worldwide use a different language, as shown in the circle graph at the right. If the circle displaying the information has a center $C(0, -3)$ and a diameter of 7.4 units, write an equation of the circle.

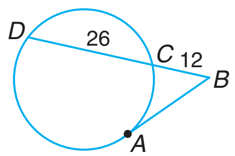


Source: Euro-Marketing Associates

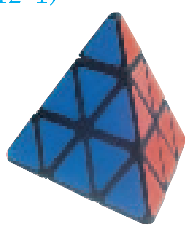
32. **Critical Thinking** The graphs of $x = 4$ and $y = -1$ are both tangent to a circle that has its center in the fourth quadrant and a diameter of 14 units. Write an equation of the circle.

Mixed Review

33. Find AB to the nearest tenth. *(Lesson 14–5)*



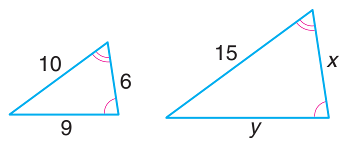
34. **Toys** Describe the basic shape of the toy as a geometric solid. *(Lesson 12–1)*



35. Find the area of a regular pentagon whose perimeter is 40 inches and whose apothems are each 5.5 inches long. *(Lesson 10–5)*

Standardized Test Practice

36. **Short Response** Find the values of x and y. *(Lesson 9–3)*



37. **Multiple Choice** Find the length of the diagonal of a rectangle whose length is 12 meters and whose width is 4 meters. *(Lesson 6–6)*

(A) 48 m (B) 160 m (C) 6.9 m (D) 12.6 m

Math In the Workplace

Meteorologist

Do you enjoy watching storms? Have you ever wondered why certain areas of the country have more severe weather conditions such as hurricanes or tornadoes? If so, you may want to consider a career as a meteorologist. In addition to forecasting weather, meteorologists apply their research of Earth's atmosphere in areas of agriculture, air and sea transportation, and air-pollution control.

1. Suppose your home is located at (0, 0) on a coordinate plane. If the "eye of the storm," or the storm's center, is located 25 miles east and 12 miles south of you, what are the coordinates of the storm's center?
2. If the storm has a 7-mile radius, write an equation of the circle representing the storm.
3. Graph the equation of the circle in Exercise 2.

FAST FACTS About Meteorologists

Working Conditions

- may report from radio or television station studios
- must be able to work as part of a team
- those not involved in forecasting work regular hours, usually in offices
- may observe weather conditions and collect data from aircraft

Education

- high school math and physical science courses
- bachelor's degree in meteorology
- A master's or Ph.D. degree is required for research positions.

Employment

4 out of 10 meteorologists have federal government jobs.

Government Position	Tasks Performed
Beginning Meterologist	collect data, perform computations or analysis
Entry-Level Intern	learn about the Weather Service's forecasting equipment and procedures
Permanent Duty	handle more complex forecasting jobs

interNET CONNECTION **Career Data** For the latest information about a career as a meteorologist, visit: www.geomconcepts.com

Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

InterNET CONNECTION Review Activities
For more review activities, visit:
www.geomconcepts.com

external secant segment *(p. 613)*
externally tangent *(p. 595)*
inscribed angle *(p. 586)*
intercepted arc *(p. 586)*
internally tangent *(p. 595)*
point of tangency *(p. 592)*
secant angle *(p. 600)*
secant-tangent angle *(p. 606)*
secant segment *(p. 600)*
tangent *(p. 592)*
tangent-tangent angle *(p. 607)*

Choose the term or terms from the list above that best complete each statement.

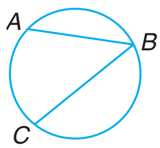
1. When two secants intersect, the angles formed are called ___?___.
2. The vertex of a(n) ___?___ is on the circle and its sides contain chords of the circle.
3. A tangent-tangent angle is formed by two ___?___.
4. A tangent intersects a circle in exactly one point called the ___?___.
5. The measure of an inscribed angle equals one-half the measure of its ___?___.
6. A(n) ___?___ is the part of a secant segment that is outside a circle.
7. A(n) ___?___ is formed by a vertex outside the circle or by a vertex on the circle.
8. A ___?___ is a line segment that intersects a circle in exactly two points.
9. The measure of a(n) ___?___ is always one-half the difference of the measures of the intercepted arcs.
10. If a line is tangent to a circle, then it is perpendicular to the radius drawn to the ___?___.

Skills and Concepts

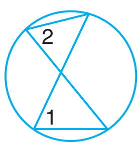
Objectives and Examples

- **Lesson 14–1** Identify and use properties of inscribed angles.

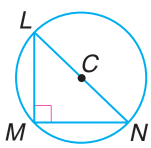
$m\angle ABC = \frac{1}{2}m\widehat{AC}$



$\angle 1 \cong \angle 2$



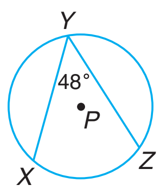
$m\angle LMN = 90$



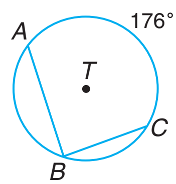
Review Exercises

Find each measure.

11. $m\widehat{XZ}$

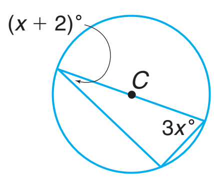


12. $m\angle ABC$

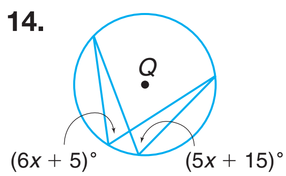


In each circle, find the value of *x*.

13.



14.



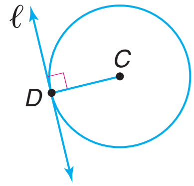
www.geomconcepts.com/vocabulary_review

CONTENTS

Objectives and Examples	Review Exercises

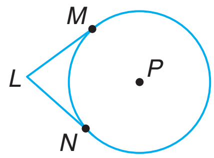
- **Lesson 14–2** Identify and apply properties of tangents to circles.

If line ℓ is tangent to $\odot C$, then $\overline{CD} \perp \ell$.



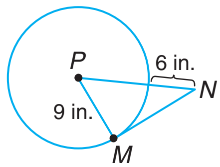
If $\overline{CD} \perp \ell$, then ℓ must be tangent to $\odot C$.

If $\overline{LM}$ and $\overline{LN}$ are tangent to $\odot P$, then $\overline{LM} \cong \overline{LN}$.

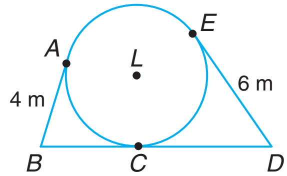


Find each measure. Assume segments that appear to be tangent are tangent.

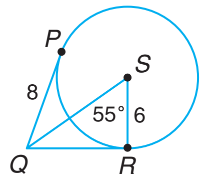
15. MN



16. BD



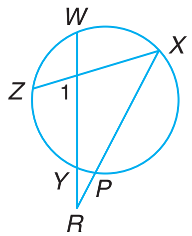
17. Find $m\angle RQS$ and QS.



- **Lesson 14–3** Find measures of arcs and angles formed by secants.

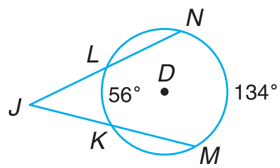
$$m\angle 1 = \frac{1}{2}(m\widehat{WX} + m\widehat{YZ})$$

$$m\angle R = \frac{1}{2}(m\widehat{WX} - m\widehat{YP})$$

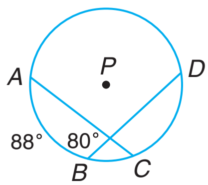


Find each measure.

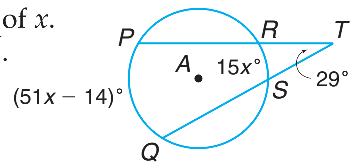
18. $m\angle J$



19. $m\widehat{CD}$

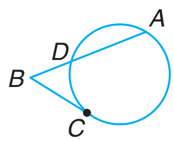


20. Find the value of x. Then find $m\widehat{RS}$.

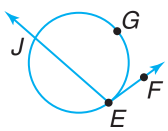


- **Lesson 14–4** Find measures of arcs and angles formed by secants and tangents.

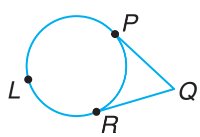
$$m\angle ABC = \frac{1}{2}(m\widehat{AC} - m\widehat{CD})$$



$$m\angle JEF = \frac{1}{2}(m\widehat{JGE})$$

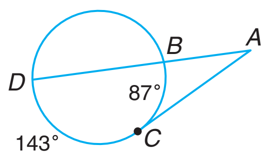


$$m\angle PQR = \frac{1}{2}(m\widehat{PLR} - m\widehat{PR})$$

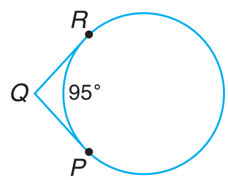


Find the measure of each angle. Assume segments that appear to be tangent are tangent.

21. $\angle CAD$



22. $\angle PQR$

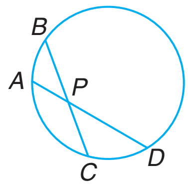


Mixed Problem Solving
See pages 758–765.

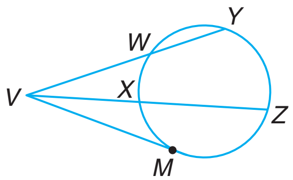
Objectives and Examples

- **Lesson 14–5** Find measures of chords, secants, and tangents.

$AP \cdot PD = BP \cdot PC$



$VY \cdot VW = VZ \cdot VX$

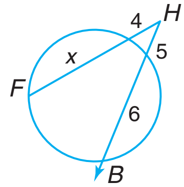


$(VM)^2 = VZ \cdot VX$

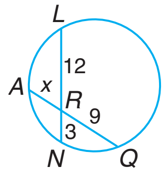
Review Exercises

In each circle, find the value of *x*. If necessary, round to the nearest tenth.

23.

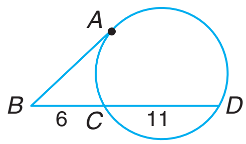


24.

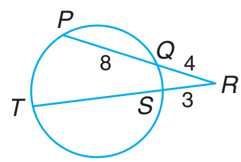


Find each measure. If necessary, round to the nearest tenth.

25. AB

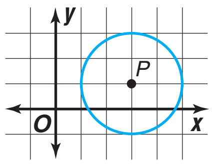


26. ST



- **Lesson 14–6** Write equations of circles using the center and the radius.

Write the equation of a circle with center $P(3, 1)$ and a radius of 2 units.



$(x - h)^2 + (y - k)^2 = r^2$ *General equation*
$(x - 3)^2 + (y - 1)^2 = 2^2$ *$(h, k) = (3, 1); r = 2$*

The equation is $(x - 3)^2 + (y - 1)^2 = 4$.

Write the equation of a circle for each center and radius or diameter measure given.

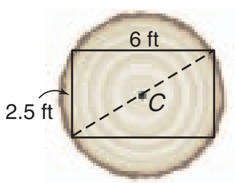
27. $(-3, 2)$, $r = 5$
28. $(6, 1)$, $r = 6$
29. $(5, -5)$, $d = 4$

Find the coordinates of the center and the measure of the radius for each circle whose equation is given.

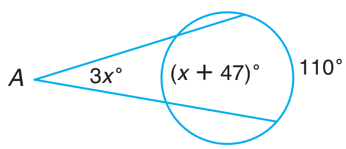
30. $(x + 2)^2 + (y + 3)^2 = 36$
31. $(x - 9)^2 + (y + 6)^2 = 16$
32. $(x - 5)^2 + (y - 7)^2 = 169$

Applications and Problem Solving

33. Lumber A lumber yard receives perfectly round logs of raw lumber for further processing. Determine the diameter of the log at the right. *(Lesson 14–1)*



34. Algebra Find x. Then find $m\angle A$. *(Lesson 14–3)*

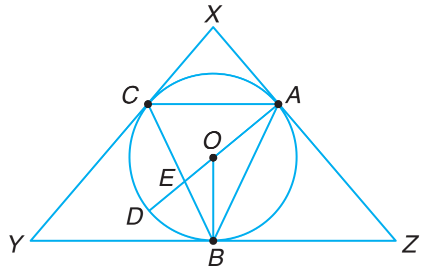


CHAPTER 14 Test

1. **Compare and contrast** a tangent to a circle and a secant of a circle.
2. **Draw** a circle with the equation $(x - 1)^2 + (y + 1)^2 = 4$.
3. **Define** the term *external secant segment*.

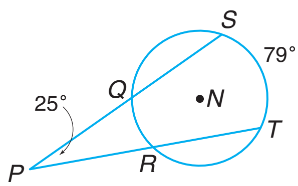
$\odot O$ is inscribed in $\triangle XYZ$, $m\widehat{AB} = 130$, $m\widehat{AC} = 100$, and $m\angle DOB = 50$. Find each measure.

4. $m\angle YXZ$
5. $m\angle CAD$
6. $m\angle XZY$
7. $m\angle AEC$
8. $m\angle OBZ$
9. $m\angle ACB$

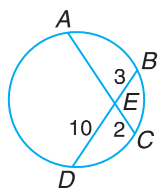


Find each measure. If necessary, round to the nearest tenth. Assume segments that appear to be tangent are tangent.

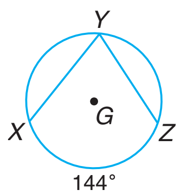
10. $m\widehat{QR}$



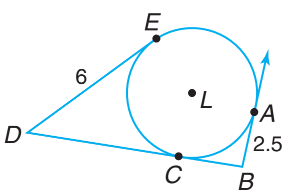
11. AE



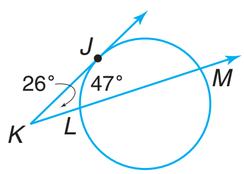
12. $m\angle XYZ$



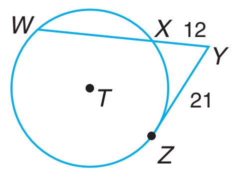
13. BD



14. $m\widehat{JM}$

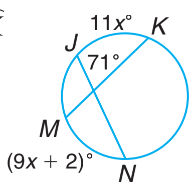


15. WX

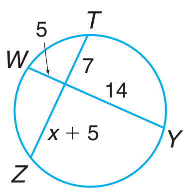


Find each value of x. Then find the given measure.

16. $m\widehat{JK}$



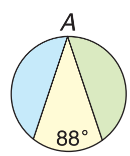
17. TZ



Write the equation of a circle for each center and radius or diameter measure given.

18. $(6, -1)$, $d = 12$
19. $(3, 7)$, $r = 1$

20. **Antiques** A round stained-glass window is divided into three sections, each a different color. In order to replace the damaged middle section, an artist must determine the exact measurements. Find the measure of $\angle A$.



Right Triangle and Trigonometry Problems

Many geometry problems on standardized tests involve right triangles and the Pythagorean Theorem.

The ACT also includes trigonometry problems. Memorize these ratios.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}},\ \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}},\ \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Standardized tests often use the Greek letter θ (*theta*) for the measure of an angle.

Test-Taking Tip

The 3-4-5 right triangle and its multiples, like 6-8-10 and 9-12-15, occur frequently on standardized tests. Other Pythagorean triples, like 5-12-13 and 7-24-25, also occur often. Memorize them.

Example 1

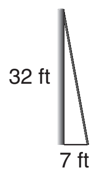
A 32-foot telephone pole is braced with a cable that runs from the top of the pole to a point 7 feet from the base. What is the length of the cable rounded to the nearest tenth?

(A) **31.2 ft** **(B)** **32.8 ft**

(C) **34.3 ft** **(D)** **36.2 ft**

Hint If no diagram is given, draw one.

Solution Draw a sketch and label the given information.



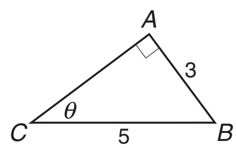
You can assume that the pole makes a right angle with the ground. In this right triangle, you know the lengths of the two sides. You need to find the length of the hypotenuse. Use the Pythagorean Theorem.

$c^2 = a^2 + b^2$	*Pythagorean Theorem*
$c^2 = 32^2 + 7^2$	*$a = 32$ and $b = 7$*
$c^2 = 1024 + 49$	*$32^2 = 1024$ and $7^2 = 49$*
$c^2 = 1073$	*Add.*
$c = \sqrt{1073}$	*Take the square root of each side.*
$c \approx 32.8$	*Use a calculator.*

To the nearest tenth, the hypotenuse is 32.8 feet. The answer is B.

Example 2

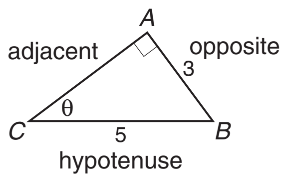
In the figure at the right, $\angle A$ is a right angle, $\overline{AB}$ is 3 units long, and $\overline{BC}$ is 5 units long. If the measure of $\angle C$ is θ, what is the value of $\cos\theta$?



(A) $\frac{3}{5}$ **(B)** $\frac{3}{4}$ **(C)** $\frac{4}{5}$ **(D)** $\frac{5}{4}$ **(E)** $\frac{5}{3}$

Hint In trigonometry problems, label the triangle with the words *opposite*, *adjacent*, and *hypotenuse*.

Solution



To find $\cos\theta$, you need to know the length of the adjacent side. Notice that the hypotenuse is 5 and one side is 3, so this is a 3-4-5 right triangle. The adjacent side is 4 units.

Use the ratio for $\cos\theta$.

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	*Definition of cosine*
$= \frac{4}{5}$	*Substitution*

The answer is C.

CONTENTS

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 766–781.

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

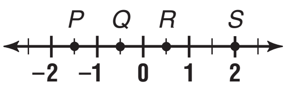
1. Fifteen percent of the coins in a piggy bank are nickels and 5% are dimes. If there are 220 coins in the bank, how many are not nickels or dimes? *(Percent Review)*

(A) 80 (B) 176 (C) 180 (D) 187 (E) 200

2. A bag contains 4 red, 10 blue, and 6 yellow balls. If three balls are removed at random and no ball is returned to the bag after removal, what is the probability that all three balls will be blue? *(Statistics Review)*

(A) $\frac{1}{2}$ (B) $\frac{1}{8}$ (C) $\frac{3}{20}$ (D) $\frac{2}{19}$ (E) $\frac{3}{8}$

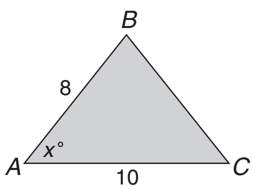
3. Which point represents a number that could be the product of two negative numbers and a positive number greater than 1? *(Algebra Review)*



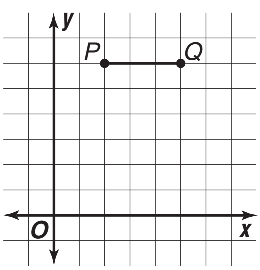
(A) *P* and *Q* (B) *P* only
(C) *R* and *S* (D) *S* only

4. What is the area of $\triangle ABC$ in terms of x? *(Lesson 13–5)*

(A) $10 \sin x$
(B) $40 \sin x$
(C) $80 \sin x$
(D) $40 \cos x$
(E) $80 \cos x$



5. Suppose $\triangle PQR$ is to have a right angle at *Q* and an area of 6 square units. Which could be coordinates of point *R*? *(Lesson 10–4)*

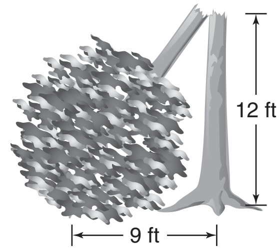


(A) (2, 2) (B) (5, 8)
(C) (5, 2) (D) (2, 8)

6. What is the diagonal distance across a rectangular yard that is 20 yd by 48 yd? *(Lesson 6–6)*

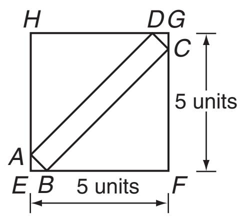
(A) 52 yd (B) 60 yd
(C) 68 yd (D) 72 yd

7. What was the original height of the tree? *(Lesson 6–6)*



(A) 15 ft
(B) 20 ft
(C) 27 ft
(D) 28 ft

8. Points *A*, *B*, *C*, and *D* are on the square. *ABCD* is a rectangle, but not a square. Find the perimeter of *ABCD* if the distance from *E* to *A* is 1 and the distance from *E* to *B* is 1. *(Lesson 6–6)*



(A) 64 units (B) $10\sqrt{2}$ units
(C) 10 units (D) 8 units

Grid In

9. Segments *AB* and *BD* are perpendicular. Segments *AB* and *CD* bisect each other at *x*. If $AB = 8$ and $CD = 10$, what is *BD*? *(Lesson 2–3)*

Extended Response

10. The base of a ladder should be placed 1 foot from the wall for every 3 feet of length. *(Lesson 6–6)*

Part A How high can a 15-foot ladder safely reach? Draw a diagram.

Part B How long a ladder is needed to reach a window 24 feet above the ground?