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A study of the relationship between Exclusive-OR and Equivalence, Inhibition and Implication Operations

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Abstract:

Some logic operations in digital design are still not deeply explored or researched. There are several relationships, such as the inverse (complement) relationship, between the different logic operations. In this paper the inverse relationship between some logic operations will be demonstrated and completely discussed. This will pave the way for a better understanding of the logic operations and lay the foundation for developing new computer related algorithms.

Keywords: Exclusive -OR, Equivalence, Implication, Inhibition, and Inverse.

1. Introduction

Boolean algebra is the backbone of computer software and hardware systems. Such a phenomenon may be exploited for the purpose of developing new computer applications.

In this paper several inverse relationships between logic operations will be investigated, namely, the inverse relationship between the exclusive-OR and the equivalence, previously discussed by other authors [1-14], and the inverse between the inhibition and the implication [4, 5, 8,10].

2. Exclusive-OR and Equivalence Functions:

These two functions are both commutative and associative and are the complements of each other for two-variable functions [1-6] as follows:

The exclusive-OR (EXOR): $x \oplus y = x^{/} y + xy^{/}$

$$\text{The complement: } \begin{aligned}(x \oplus y)^{/} &= \left(x^{/} y + xy^{/}\right)^{/} \\ &= \left(x + y^{/}\right)\left(x^{/} + y\right) \\ &= x^{/} y^{/} + xy\end{aligned}$$

The equivalence: $x \equiv y = x^{/} y^{/} + xy$

Therefore, the complement of EXOR is equal to the equivalence for two-variable functions. Furthermore, the EXOR is an odd function (is equal to one when the total number of 1's in the input variables is odd) [4,5], and the equivalence is an even functions for two variables (is equal to one when the total number of 1's in the input variables is even), as depicted by the following truth tables:

X	Y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

X	Y	$x \equiv y$
0	0	1
0	1	0
1	0	0
1	1	1

But for more than two variables the relationship between the EXOR and the equivalence will be something different as follows:

a- Three-variable function:

$$
\begin{aligned}
(x \oplus y) \oplus z &= \left(x'y + xy'\right) \oplus z = \left(x'y + xy'\right)' z + \left(x'y + xy'\right) z' \\
&= \left(x'y' + xy\right) z + x'yz' + xy'z' \\
&= x'y'z + xyz + x'yz' + xy'z' \\
&= m_1 + m_2 + m_4 + m_7 \\
&= \sum(1,2,4,7) \quad in \quad sum \quad of \quad \min terms \quad form[4,5]
\end{aligned}
$$

$$
\begin{aligned}
(x \equiv y) \equiv z &= \left(x'y' + xy\right) \equiv z = \left(x'y' + xy\right)' z' + \left(x'y' + xy\right) z \\
&= \left(x'y + xy'\right) z' + x'y'z + xyz \\
&= x'yz' + xy'z' + x'y'z + xyz \\
&= m_1 + m_2 + m_4 + m_7 = \sum(1,2,4,7)
\end{aligned}
$$

The truth table for these functions is the following:

x	y	z	$x \oplus y \oplus z$	$x \equiv y \equiv z$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

It is clear that the EXOR and equivalence are equal for three-variable functions, and they are odd functions.

b- Four-variable function

$$
\begin{aligned}
F_1 = (w \oplus x) \oplus (y \oplus z) &= \left(w'x + wx'\right) \oplus \left(y'z + yz'\right) \\
&= \left(w'x + wx'\right)'\left(y'z + yz'\right) + \left(w'x + wx'\right)\left(y'z + yz'\right)' \\
&= \left(w'x' + wx\right)\left(y'z + yz'\right) + \left(w'x + wx'\right)\left(y'z' + yz\right) \\
&= w'x'y'z + w'x'yz' + wxy'z + wxyz' + w'xy'z' + w'xyz + wx'y'z' + wx'yz \\
&= m_1 + m_2 + m_4 + m_7 + m_8 + m_{11} + m_{13} + m_{14} \\
&= \sum(1,2,4,7,8,11,13,14) \qquad [4,5]
\end{aligned}
$$

$$
\begin{aligned}
F_2 &= (w \equiv x) \equiv (y \equiv z) = (w'x' + wx) \equiv (y'z' + yz) \\
&= (w'x' + wx)'(y'z' + yz)' + (w'x' + wx)(y'z' + yz) \\
&= (w'x + wx')(y'z + yz') + (w'x' + wx)(y'z' + yz) \\
&= w'xy'z + w'xyz' + wx'y'z + wx'yz' + w'x'y'z' + w'x'yz + wxy'z' + wxyz \\
&= m_0 + m_3 + m_5 + m_6 + m_9 + m_{10} + m_{12} + m_{15} \\
&= \sum(0,3,5,6,9,10,12,15)
\end{aligned}
$$

$$F_2' = \sum(1,2,4,7,8,11,13,14)$$

Therefore, the complement of the equivalence is equal to the EXOR for 4-variable functions, and the EXOR (F_1) is an odd function, but equivalence (F_2) is an even function.

c- Five-variable function

$$F_1 = v \oplus w \oplus x \oplus y \oplus z$$

Where, $w \oplus x \oplus y \oplus z = \sum(1,2,4,7,8,11,13,14)$

Therefore,

$$
\begin{aligned}
F_1 &= v \oplus (m_1 + m_2 + m_4 + m_7 + m_8 + m_{11} + m_{13} + m_{14}) \\
&= v(m_1 + m_2 + m_4 + m_7 + m_8 + m_{11} + m_{13} + m_{14})' + v'(m_1 + m_2 + m_4 + m_7 + m_8 + m_{11} + m_{13} + m_{14}) \\
&= v(m_0 + m_3 + m_5 + m_6 + m_9 + m_{10} + m_{12} + m_{15}) + v'(m_1 + m_2 + m_4 + m_7 + m_8 + m_{11} + m_{13} + m_{14}) \\
&= (m_{16} + m_{19} + m_{21} + m_{22} + m_{25} + m_{26} + m_{28} + m_{31}) + (m_1 + m_2 + m_4 + m_7 + m_8 + m_{11} + m_{13} + m_{14}) \\
&= \sum(1,2,4,7,8,11,13,14,16,19,21,22,25,26,28,31)
\end{aligned}
$$

Where, $vm_0 = vw'x'y'z' = m_{16}$

And $v'm_1 = v'w'x'y'z = m_1$

$F_2 = v \equiv w \equiv x \equiv y \equiv z$

Where, $w \equiv x \equiv y \equiv z = \sum(0,3,5,6,9,10,12,15)$

Therefore,

$$
\begin{aligned}
F_2 &= v \equiv (m_0 + m_3 + m_5 + m_6 + m_9 + m_{10} + m_{12} + m_{15}) \\
&= v'(m_0 + m_3 + m_5 + m_6 + m_9 + m_{10} + m_{12} + m_{15})' + v(m_0 + m_3 + m_5 + m_6 + m_9 + m_{10} + m_{12} + m_{15}) \\
&= v'(m_1 + m_2 + m_4 + m_7 + m_8 + m_{11} + m_{13} + m_{14}) + v(m_0 + m_3 + m_5 + m_6 + m_9 + m_{10} + m_{12} + m_{15}) \\
&= (m_1 + m_2 + m_4 + m_7 + m_8 + m_{11} + m_{13} + m_{14}) + (m_{16} + m_{19} + m_{21} + m_{22} + m_{25} + m_{26} + m_{28} + m_{31}) \\
&= \sum(1,2,4,7,8,11,13,14,16,19,21,22,25,26,28,31)
\end{aligned}
$$

It is clear that the EXOR and equivalence are equal for five-variable functions, and they are odd functions.

3. Inhibition and Implication Functions:

The inhibition and implication functions are the complements of each other for two-variable functions [4,5,8,10] as follows:

The inhibition: $x / y = x.y'$

The complement: $(x/y)' = (x.y')' = x' + y$

The implication: $x \supset y = x' + y$

Therefore, the complement of inhibition is equal to the implication for two-variable functions. But for more than two variables, the inhibition and implication are not the complements of each other as follows:

a- Three-variable function:

$$(x/y)/z = (xy')/z = xy'z'$$

The complement of inhibition: $(xy'z')' = x' + y + z$

The implication: $(x \supset y) \supset z = (x' + y) \supset z = (x' + y)' + z = xy' + z$

Since, the inhibition and implication are not commutative or associative [4,5], the different combinations of x,y,z will show also that the complement of inhibition is not equal to the implication.

b- Four-variable function:

$$\begin{aligned}(w/x)/(y/z) &= (wx')/(yz') = (wx')\,(yz')' \\ &= (wx')\,(y' + z) = wx'y' + wx'z\end{aligned}$$

The complement of inhibition:

$$\begin{aligned}(wx'y' + wx'z)' &= (w' + x + y)\,(w' + x + z') \\ &= w' + w'x + w'z' + w'x + x + xz' + w'y + xy + yz' \\ &= w'(1 + x) + w'z' + x(1 + z') + w'y + xy + yz' \\ &= w' + w'z' + x + w'y + xy + yz' \\ &= w'(1 + z') + x(1 + y) + w'y + yz' \\ &= w' + x + w'y + yz' = w'(1 + y) + x + yz' \\ &= w' + x + yz'\end{aligned}$$

The implication: $(w \supset x) \supset (y \supset z) = (w' + x) \supset (y' + z)$

$$\begin{aligned}&= (w' + x)' + (y' + z) \\ &= wx' + y' + z\end{aligned}$$

Therefore, the complement of inhibition and the implication are not equal, and we can conclude that the inhibition and implication functions are the complements of each other only for two variables.

4. Conclusions

- The EXOR and equivalence functions are the complements of each other when the number of variables in the function is even (two, four, ...).
- The EXOR and equivalence functions are equal when the number of variables in the function is odd (three, five,...).
- The EXOR is always an odd function, but the equivalence is an even function when the number of variables is even, and is an odd function when the number of variables is odd.
- The inhibition and implication functions are the complements of each other only for two-variable functions.

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