<u>Բ๚๖</u>⊾լ<u></u>зк



Parallels

What You'll Learn

Key Ideas

- Describe relationships among lines, parts of lines, and planes. (Lesson 4–1)
- Identify relationships among angles formed by two parallel lines and a transversal.

(Lessons 4–2 and 4–3)

- Identify conditions that produce parallel lines and construct parallel lines. (Lesson 4–4)
- Find the slopes of lines and use slope to identify parallel and perpendicular lines. (Lesson 4–5)
- Write and graph equations of lines. (Lesson 4–6)

Key Vocabulary

linear equation (p. 174) parallel lines (p. 142) skew lines (p. 143) slope (p. 168) transversal (p. 148)

Why It's Important

Sports College football has been an American tradition since the 1800s. American football, soccer, and rugby are all derived from games played in ancient Greece and Rome. Now, college football attracts over 40 million people to games and is watched by millions more on television each year.

Parallel lines are often used as part of building and road construction. You will determine how the parallel lines of a football field can be marked in Lesson 4–4.



Study these lessons to improve your skills.

esson 1–2,

op. 12–17

Algebra Review, p. 719



Use the figure to name examples of each term.

- 1. segment
- **2.** segment with point *H* as an endpoint
- **3.** a point that is *not* in plane *ABF*
- 4. two segments that *do not* intersect



Find the value of each expression.					
5.	$\frac{6-2}{5-3}$	6.	$\frac{12-3}{6-3}$	7.	$\frac{10-9}{9-5}$
8.	$\frac{-3 - (-2)}{4 - 2}$	9.	$\frac{-4-1}{5-3}$	10.	$\frac{-10 - (-7)}{3 - 0}$
11.	$\frac{-6 - (-4)}{-2 - (-1)}$	12.	$\frac{4-(-4)}{4-9}$	13.	$\frac{-6-1}{0-2}$

Draw and label a coordinate plane on a piece of grid paper. Then graph and label each point.

14. <i>A</i> (2, 4)	15. <i>B</i> (-1, -3)	16. C(2, 0)
17. D(-3, 1)	18. <i>E</i> (0, -1)	19. <i>F</i> (4, - 4)



Lesson 2–4, 3p. 68–73

Make this Foldable to help you organize your Chapter 4 notes. Begin with four sheets of plain $8\frac{1}{2}$ " by 11" paper.

• Fold in half along the width.

\leq	\leq		
		4	
		1	

- 8 Repeat steps 1 and 2 three times and glue all four pieces together.
- **Open** and fold up the bottom to form a pocket.



4 Label each pocket with a lesson name. Use the last two pockets for vocabulary. Place an index card in each pocket.

CONTENTS

Parallels

Reading and Writing As you read and study the chapter, write the main ideas, examples of theorems, postulates, and definitions on the index cards.

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Parallel Lines and Planes

What You'll Learn

You'll learn to describe relationships among lines, parts of lines, and planes.

Why It's Important Construction

Carpenters use parallel lines and planes in the construction of furniture. See Exercise 11. Suppose you could measure the distance between the columns of a building at various points. You would find that the distance remains the same at all points. The columns are parallel.

In geometry, two lines in a plane that are always the same distance apart are **parallel lines**. No two parallel lines intersect, no matter how far you extend them.



National Gallery of Art, Washington, D.C.



Since segments and rays are parts of lines, they are considered parallel if the lines that contain them are parallel.



Planes can also be parallel. The shelves in a bookcase are examples of parts of planes. The shelves are the same distance apart at all points, and do not appear to intersect. They are parallel. In geometry, planes that do not intersect are called **parallel planes**.

plane *PSR* || plane *JML* plane *JMS* || plane *KLR* plane *PJK* || plane *SML*



Recall that plane PSR refers to the plane containing points P, S, and R.



Read the symbol || as is parallel to. Arrowheads are often used in figures to indicate parallel lines.



Sometimes lines that do not intersect are not in the same plane. These lines are called **skew lines**.

Definition of Skew Lines Two lines that are not in the same plane are skew if and only if they do not intersect.

Segments and rays can also be skew if they are contained in skew lines. In the figure, \overline{AX} and \overline{BC} are skew segments. They are parts of noncoplanar lines that do not intersect. \overline{AX} and \overline{XZ} intersect at X. They are not skew segments.





CONTENTS

Check for Understanding

Communicating Mathematics

- **1. Draw and label** two parallel lines, *ℓ* and *m*. Indicate that the lines are parallel by using the arrowhead symbol.
- **2. Describe** a real-world example or model of parallel lines.



3. Weiting Matheen Sketch the diagram shown below. Then describe and explain the relationship between lines ℓ and m.



Planes A and B are parallel.

Guided Practice	Describe each pair of segments in the prism as <i>parallel</i> , <i>skew</i> , or <i>intersecting</i> .		
Examples 2–4	4. \overline{KN} , \overline{HL}		
	5. \overline{JM} , \overline{ML}	K	
	6. <i>JM</i> , <i>KH</i>		



Examples 1, 3, & 4 Name the parts of the cube shown at the right.

- **7.** all planes parallel to plane *WXQ*
- **8.** all segments parallel to \overline{PQ}
- **9.** all segments skew to \overline{PS}
- **10.** all pairs of parallel planes



11. Carpentry A carpenter is constructing a chair like the one shown at the left. Describe a pair of parts that are parallel, a pair that intersect, and a pair that are skew.







Exercises

Practice

Homework Help				
For Exercises	See Examples			
12–21, 24–27, 29–31	2-4			
22, 23, 28	1			
32-39	1-4			
Extra Practice				
See page 732.				

Describe each pair of segments in the prism as *parallel*, *skew*, or *intersecting*.

12. <i>BE</i> , <i>CF</i>	13. <i>AD</i> , <i>BE</i>
14. \overline{AD} , \overline{EF}	15. <i>BC</i> , <i>EF</i>
16. \overline{AB} , \overline{DE}	17. <i>AB</i> , <i>CF</i>
18. \overline{AB} , \overline{BC}	19. <i>AD</i> , <i>BC</i>
20. \overline{BE} , \overline{BC}	21. <i>BC</i> , <i>DE</i>



Name the parts of the cube shown at the right.

- 22. six planes
- **23.** all pairs of parallel planes
- **24.** all segments parallel to *EH*
- **25.** all segments skew to \overline{GH}
- **26.** all segments parallel to *AE*
- **27.** all segments skew to *BF*



Name the parts of the pyramid shown at the right.

- **28.** all pairs of intersecting planes
- **29.** all pairs of parallel segments
- **30.** all pairs of skew segments
- **31.** all sets of three segments that intersect in a common point



Draw and label a figure to illustrate each pair.

CONTENTS

- **32.** congruent parallel segments
- **33.** parallel segments not congruent
- 34. segments not parallel or congruent
- **35.** skew segments
- **36.** segments not intersecting or skew
- **37.** parallel planes
- **38.** intersecting planes
- **39.** parallel rays



Complete each sentence with sometimes, always, or never.

- **40.** Skew lines _____ intersect.
- **41.** Skew lines are <u>?</u> parallel.
- **42.** Two parallel lines _____ lie in the same plane.
- **43.** Two lines in parallel planes are _____ skew.
- **44.** Two lines that have no points in common are _____ parallel.
- **45.** If two lines are parallel, then they _____ lie in the same plane.

Applications and Problem Solving

- **46. Interior Design** The shower stall shown in the diagram is formed by a series of intersecting planes. Name two skew segments in the diagram.
- **47. Construction** The Empire State Building, built in 1930–1931, is 102 stories and reaches a height of 1250 feet. Suppose the stories represent parallel planes equal distances apart. What is the approximate distance between floors?



48. Sports Describe how skis on a skier's feet could *intersect*, be *parallel*, or be *skew*.

49. Graphic Arts The comic artist has used parallel lines two ways in the *B.C.* comic below. Explain the two uses of parallel lines in the comic.





50. Critical Thinking Plane *A* is parallel to plane *B*, and plane *B* is parallel to plane *C*. Is Plane *A* parallel to Plane *C*? Write *yes* or *no*, and explain your answer. Then describe something in your school building that illustrates your response.

Mixed Review

- **51.** In the figure at the right, $BE \perp AD$. (Lesson 3–7) **a.** Name four right angles.
 - **b.** Name a pair of supplementary angles.
 - **c.** Name two pairs of angles whose sum is 90.



52. If $\angle JKL \cong \angle PQR$, $m \angle PQR = 4x + 5$, and $m \angle JKL = 5x - 12$, what is $m \angle PQR$? (Lesson 3–6)

Refer to the figure at the right.

(*Lesson 3–3*)

- **53.** If $m \angle CAE = 78$ and $m \angle DAE = 30$, find $m \angle 1$.
- **54.** Find $m \angle DAF$ if $m \angle DAE = 30$ and $m \angle EAF = 75$.



- **55. Time** Do the hands of a clock at 12:20 P.M. form an acute, obtuse, or right angle? (*Lesson 3–2*)
- **56. Cartography** On a map of Ohio, Cincinnati is located at (3, 5), and Massillon is located at (17, 19). If Columbus is halfway between the two cities, what ordered pair describes its position? (*Lesson* 2–5)
- **57.** It is 4.5 blocks from Carlos' house to Matt's house. From Matt's house to Keisha's house is 10.5 blocks, and from Keisha's house to Carlos' house is 6 blocks. If they live on the same street, whose house is between the other two? (Lesson 2–2)



Cincinnati, Ohio

- **58. Short Response** Name all the planes that can contain three of the points *J*, *K*, *L*, and *M*, if no three points are collinear and the points do not all lie in the same plane. (*Lesson* 1–3)
- **59.** Multiple Choice 12 is what percent of 40? (*Percent Review*) (A) 0.3% (B) $3\frac{1}{3}\%$ (C) 30% (D) 33%

CONTENT

Standardized Test Practice

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Lesson 4–1 Parallel Lines and Planes 147

Parallel Lines and Fransversals

What You'll Learn

You'll learn to identify the relationships among pairs of interior and exterior angles formed by two parallel lines and a transversal.

Why It's Important Construction

Builders use the measures of angle pairs to cut siding for houses. See Exercise 41.

In geometry, a line, line segment, or ray that intersects two or more lines at different points is called a **transversal**. AB is an example of a transversal. It intersects lines ℓ and m. Note all of the different angles that are formed at the points of intersection.





Reading

Read the symbol ∦ as is not parallel to.

The lines cut by a transversal may or may not be parallel.



t is a transversal for ℓ and *m*.



r is a transversal for *b* and *c*.

When a transversal intersects two lines, eight angles are formed, as shown in the figures above. These angles are given special names.

Interior angles lie between the two lines.

 $\angle 3$, $\angle 4$, $\angle 5$, $\angle 6$

Alternate interior angles are on opposite sides of the transversal.

 $\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$

Consecutive interior angles are on the same side of the transversal. $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$

Exterior angles lie outside the two lines.

 $\angle 1$, $\angle 2$, $\angle 7$, $\angle 8$

Alternate exterior angles are on opposite sides of the transversal.

 $\angle 1$ and $\angle 7$, $\angle 2$ and $\angle 8$







Identify each pair of angles as alternate interior, alternate exterior, consecutive interior, or vertical.

$\angle 2$ and $\angle 8$

 $\angle 2$ and $\angle 8$ are exterior angles on opposite sides of the transversal, so they are *alternate exterior* angles.



$\angle 1$ and $\angle 6$

 $\angle 1$ and $\angle 6$ are interior angles on the same side of the transversal, so they are *consecutive interior* angles.



In the following activity, you will investigate the relationships among the angles formed when a transversal intersects two parallel lines.



CONTENTS

Note: Save this drawing to use again in Lesson 4–3.



The results of the activity suggest the theorems stated below.

	Theorem	Words	Models and Symbols
	4–1 Alternate Interior Angles	If two parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent.	$\angle 4 \cong \angle 6$ $\angle 3 \cong \angle 5$ $\begin{pmatrix} 1 & 2 \\ 4 & 3 \\ 5 & 6 \\ 8 & 7 \\ 8 & 7 \\ \hline 8 & 7 $
Look Back Supplementary Angles: Lesson 3–5	4–2 Consecutive Interior Angles	If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is supplementary.	$m \ge 3 + m \ge 6 = 180$ $m \ge 4 + m \ge 5 = 180$
	4–3 Alternate Exterior Angles	If two parallel lines are cut by a transversal, then each pair of alternate exterior angles is congruent.	$\angle 1 \cong \angle 7$ $\angle 2 \cong \angle 8$

You can use these theorems to find the measures of angles.







 $\angle 6$ and $\angle 9$ are alternate interior angles, so by Theorem 4–1 they are congruent. Thus, $m \angle 9 = 112$.

Your Turn

Refer to the figu	ure in Examp	e 3. Find the meas	ure of each angle.
c. ∠1	d. ∠2	e. ∠3	f. ∠4



Check for Understanding

Communicating Mathematics

- **1. Explain** why $\angle 2$ and $\angle 3$ must be congruent.
- **2.** Describe two different methods you could use to find $m \angle 3$ if $m \angle 1 = 130$.

CONTENTS



transversal interior angles alternate interior angles consecutive interior angles

exterior angles

alternate exterior angles

Vocabulary

Lesson 4–2 Parallel Lines and Transversals 151

3. Name each transversal and the lines it intersects in the figure at the right.

Identify each pair of angles as alternate interior, alternate exterior, consecutive

 $a \parallel b$, and h is a transversal. If $m \perp 1 = 48$,

9. ∠3

12. Algebra In the figure at the right, $r \parallel t$,

and *d* is a transversal. Find $m \angle 1$ and $m \angle 2$.

11. ∠7

find the measure of each angle. Give a

6. $\angle 2$ and $\angle 8$ **7.** $\angle 2$ and $\angle 3$

reason for each answer.

8. ∠2

10. ∠4

5. $\angle 1$ and $\angle 5$

interior, or vertical. 4. $\angle 3$ and $\angle 7$ 5.









Exercises

Guided Practice

Examples 1 & 2

Examples 3 & 4

Example 5

Practice

Identify each pair of	angles as altern	ate interior, alte	rnate exterior,
consecutive interior	, or <i>vertical</i> .		

Homework Help				
For Exercises	See Examples			
13-24	1, 2			
25-36	3, 4			
37-39	5			
Extra Practice				
See page 732.				

13. $\angle 1$ and $\angle 5$	14. $\angle 2$ and $\angle 10$
15. $\angle 5$ and $\angle 15$	16. \angle 11 and \angle 3
17. $\angle 1$ and $\angle 4$	18. $\angle 16$ and $\angle 8$
19. $\angle 5$ and $\angle 6$	20. $\angle 10$ and $\angle 14$
21. $\angle 15$ and $\angle 14$	22. $\angle 8$ and $\angle 2$
23. $\angle 12$ and $\angle 14$	24. \angle 9 and \angle 13

Find the measure of each angle. Give a reason for each answer.

25. ∠1	26. ∠4
27. ∠6	28. ∠5







Find the measure of each angle. Give a reason for each answer.

29. ∠9	30. ∠12
31. ∠13	32. ∠15
33. ∠19	34. ∠21
35. ∠22	36. ∠24



Find the values of x and y.



- **40. Road Maps** Trace and label a section of a road map that illustrates two parallel roads intersected by a transversal road or railroad. Use a protractor to measure the angles formed by the intersections on the map. How does this drawing support Theorems 4-1, 4-2, and 4-3?
- **41. Construction** The roof at the right intersects the parallel lines of the siding. Which angles are congruent?
- **42. Critical Thinking** In the figure at the right, explain why you can conclude that $\angle 1 \cong \angle 4$, but you cannot tell whether $\angle 3$ is congruent to $\angle 2$.

 $m \angle ADB = 23$ and $m \angle BDC = 3y - 2$,

find *y*. (*Lesson* 3–7)



Exercise 41



Draw and label a cube. Name the following pairs. (Lesson 4-1)

43. parallel segments **44.** intersecting

- segments
- **45.** skew segments



Draw and label a coordinate plane on a piece of grid paper. Then graph and label each point. (Lesson 2-4)

48. *H*(3, -2)

CONTENTS

Standardized Test Practice

Mixed Review

Reading

Double arrowheads

of parallel lines.

indicate a second pair

Applications and

Problem Solving

- **50. Short Response** Write a sequence of five numbers that follows the pattern +1, +3, +5, (Lesson 1–1)
 - **Lesson 4–2** Parallel Lines and Transversals 153

49. *J*(4, 0)

www.geomconcepts.com/self_check_quiz

47. *G*(−5, −1)

Chapter 4 Investigation



Materials



Spherical Geometry

The geometry you have been studying in this text is called Euclidean geometry. It was named for a famous Greek mathematician named Euclid (325 B.C.-265 B.C.). There are, however, other types of geometry. Let's take a look at *spherical geometry*. Spherical geometry is one form of non-Euclidean geometry.



Investigate

- 1. Use a globe, two large rubber bands, and the steps below to investigate lines on a sphere.
 - a. In Euclidean and spherical geometry, points are the same. A point is just a location that can be represented by a dot.
 - **b.** In Euclidean geometry, you can represent a plane by a sheet of paper. Remember that the paper is only part of the plane. The plane goes on forever in all directions. In spherical geometry, a plane is a sphere. The sphere is finite, that is, it does not go on forever. The globe will represent a plane in spherical geometry for this investigation.
 - c. If possible, place a large rubber band on the globe covering the equator. The equator is known as a line in spherical geometry. In Euclidean geometry, a line extends without end in both directions. In spherical geometry, a line is finite. In spherical geometry, a line is defined as a great circle, which divides a sphere into two congruent halves. On the globe, the equator is also called a line of latitude.
 - d. Place a second large rubber band on the globe so that it extends over both the North and South Poles. Position the band so that it is also a great circle. On the globe, a line like this is also called a line of longitude.
 - e. In Euclidean geometry you learned that when two lines intersect, they have only one point in common. Look at the rubber bands on your globe. How many points do these two lines have in common?



Latitude Lines

Longitude Lines





- This works best if you cut the tape into strips about one-eighth inch wide.
- **2.** Use the globe, removable tape, and the steps below to investigate angle measures in spherical and Euclidean planes.
 - a. Select two points on the equator. Select another point close to the North Pole. Use three pieces of removable tape to form a triangle as shown. Use a protractor to estimate the measure of each angle of the triangle. Record your results.
 - b. Carefully remove the tape from the sphere. Use the three strips to form a triangle on a sheet of paper. Use a protractor to estimate the measure of each angle of the triangle. Record the results.



c. You have formed two triangles with sides of the same length. The first was on the spherical plane. The second was on the Euclidean plane. How do the angle measures of the two triangles compare?

Extending the Investigation

In this extension, you will investigate lines in both Euclidean and spherical geometry by using a globe or geometry drawing software.

- 1. Determine whether all lines of longitude on the globe are lines in spherical geometry.
- 2. Determine whether all lines of latitude on the globe are lines in spherical geometry.
- **3.** In Chapter 3, you learned the theorem that states: if two lines are perpendicular, then they form four right angles. Is this theorem true for spherical geometry? Explain your reasoning and include a sketch.
- 4. **Make a conjecture** about angle measures of triangles in Euclidean and spherical geometry. Use at least three different-sized triangles to support your idea.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a poster comparing a point, a line, and a plane in Euclidean geometry and spherical geometry. Include diagrams or sketches.
- Research parallel lines in spherical geometry. Write a paragraph to report your findings.
- Make a video demonstrating your findings in the project.
- Pair up with another group. Have a debate in which one group is in favor of Euclidean geometry, and the other is in favor of spherical geometry.

CONTENTS



Investigation For more information on non-Euclidean geometry, visit: www.geomconcepts.com

Transversals and Corresponding Angles

What You'll Learn

You'll learn to identify the relationships among pairs of corresponding angles formed by two parallel lines and a transversal.

Why It's Important

Design City planners use corresponding angles. See Exercise 30. When a transversal crosses two lines, the intersection creates a number of angles that are related to each other. Note $\angle 7$ and $\angle 8$ below. Although one is an exterior angle and the other is an interior angle, both lie on the same side of the transversal.



Angles 7 and 8 are called **corresponding angles**. They have different vertices. In the figure, three other pairs of corresponding angles are formed.

Example



a. Name two other pairs of corresponding angles.

As with interior and exterior angles, there is a special relationship between corresponding angles when the transversal intersects parallel lines.

Recall that in Lesson 4–2, you discovered that if parallel lines are cut by a transversal, then each pair of alternate interior angles is congruent, and that each pair of consecutive interior angles is supplementary. Using the drawings you made for the Hands-On Geometry activity on page 149, measure the corresponding angles. What do you notice?





In the figure, $\ell \parallel m$, and *a* is a transversal. Which angles are congruent to $\angle 1$? Explain your answers.

 $\angle 1 \cong \angle 7$ *Vertical angles are congruent.* $\angle 1 \cong \angle 9$ *Postulate* 4–1 $\angle 7 \cong \angle 15$ *Postulate* 4–1

Therefore, $\angle 1 \cong \angle 7 \cong \angle 9 \cong \angle 15$.



Find the measure of $\angle 10$ if $m \angle 1 = 62$.

 $m \angle 1 = m \angle 9$, so $m \angle 9 = 62$. $\angle 9$ and $\angle 10$ are a linear pair, so they are supplementary.

 $m \angle 9 + m \angle 10 = 180$ 62 + $m \angle 10 = 180$ $m \angle 10 = 118$

Definition of supplementary angles Replace $m \angle 9$ *with* 62. $62 + m \angle 10 - 62 = 180 - 62$ Subtract 62 from each side. Simplify.

Your Turn

In the figure above, assume that *b* is also a transversal.

- **b.** Which angles are congruent to $\angle 1$?
- **c.** Find the measure of $\angle 5$ if $m \angle 14 = 98$.

Thus far, you have learned about the relationships among several kinds of special angles.

	Types of angle pairs formed when a transversal cuts two parallel lines.	
Concept Summary	Congruent alternate interior alternate exterior corresponding	Supplementary consecutive interior

You can use corresponding angles to prove the relationship of a perpendicular transversal to two parallel lines. In the figure, $r \parallel s$ and transversal *c* is perpendicular to *r*.

С $\angle 1$ is a right angle. *Definition of perpendicular lines* $m \angle 1 = 90$ *Definition of right angle* $\angle 1 \cong \angle 2$ *Postulate* 4–1 $m \angle 1 = m \angle 2$ *Definition of congruent angles* $90 = m \angle 2$ Substitution S 2 $\angle 2$ is a right angle. *Definition of right angle* Definition of perpendicular lines $c \perp s$

CONTENTS



Lesson 4–3 Transversals and Corresponding Angles 157 This relationship leads to the following theorem.



Check for Understanding

Communicating Mathematics

- **1a. Identify** two pairs of corresponding angles.
- **b.** Explain why $\angle 6 \cong \angle 4$.
- 2. Net the supplementary. Pedro disagrees. Who is correct, and why?



3. Draw a pair of parallel lines cut by a transversal so that one pair of corresponding angles has the given measure. (Use a straightedge and protractor.)

Κ





Guided Practice	Getting Ready Fin	d the value of <i>x</i> .	
	Sample: $5x - 9 = 2x$	Solution: 5	x - 2x = 9
			3x = 9 $x = 3$
	4. $12x = 8x + 1$	5. $3x + 6 = 4x - 7$	6. $x - 10 + 7x = 180$
Example 2	 In the figure, s t and c congruent to the given for each answer. 7. ∠1 8. ∠5 	e∦d. Name all angles angle. Give a reason	$\begin{array}{c} 5 & t \\ 1/2 & 3/4 \\ \hline 5/6 & 7/8 \\ \hline 9/10 & 11/12 \\ 13/14 & 15/16 \\ \hline \end{array}$
Examples 3 & 4	Find the measure of ea	ach numbered angle.	
	9. 68° 1 2 3		8 40°
Examples 2 & 3	11. Farming The road so in the diagram divided rectangular parcel of into two parts. If m_{\perp} find $m \ge 1$.	shown des a of land $\angle 6 = 52,$	
Example 4	12. Algebra If $m \angle 10 = m \angle 12 = 3x + 8$, find $m \angle 11$.	$4x - 5$ and $4x$, $m \angle 10$, and	10
Exercises	• • • •	• • • • • • •	• • • • • • •
Practice	In the figure, <i>a</i> ∥ <i>b</i> . Nam to the given angle. Giv answer.	ne all angles congruent e a reason for each	1 <u>2</u> <u>5</u> <u>6</u> 4 <u>3</u> <u>8</u> 7
	13. ∠2	14. ∠3	9 10 13 14
	15. ∠8	16. ∠9	12 11 16 15
	17. ∠12	18. ∠14	а б

Homework Help		
For Exercises	See Examples	
13–18, 29, 30	2	
19-24	3	
25-28	4	
Extra f	Practice	
See page 732.		

Find the measure of each numbered angle.



Applications and Problem Solving

29. Flag Design Trace the drawing of the flag of the Bahamas. Assume segments that appear to be parallel are parallel. Make a conjecture about which angles you can conclude are congruent and explain your reasoning. Check your conjecture by measuring the angles.



- **30. City Planning** In New York City, roads running parallel to the Hudson River are named avenues, and those running perpendicular to the river are named streets. What is the measure of the angle formed at the intersection of a street and an avenue?
- **31. Critical Thinking** In the figure at the right, why can you conclude that $\angle 6$ and $\angle 4$ are congruent, but you cannot state that $\angle 6$ and $\angle 2$ are congruent?
- **32.** Find the measure of $\angle 1$. (Lesson 4-2)
- **33.** Name all pairs of parallel lines. (Lesson 4-1)



Draw and label a figure for each situation described. (Lesson 1–2)

35. *AC*

- **34**. line ℓ
- **36.** plane *FGH*

- **37.** lines *p* and *q* intersecting at point *R*
- **38.** Grid In Austin practices the flute 9 minutes the first day, 10 minutes the second day, 12 minutes the third day, and 15 minutes the fourth day. If he continues this pattern, how many minutes will Austin practice the sixth day? (Lesson 1–1)
- **39.** Multiple Choice Simplify $\frac{4r^9}{2r^3}$. (Algebra Review) **B** $2r^{12}$ **C** $\frac{1}{2}r^{12}$ **D** $2r^6$ (A) $2r^3$

Quiz 1 Lesson 4–1 through 4–3

1. Give examples of parallel lines and transversals as they are а used in home design. (Lesson 4-1) h In the figure, $m \angle 2 = 56$. Find the measure of each angle. (Lesson 4-2) **3.** ∠7 **2.** ∠3 **4.** ∠4 **5.** Algebra In the figure, $a \parallel b$ and transversal *t* is perpendicular to *a*. If $m \angle 9 = 2x + 8$, find *x*. (Lesson 4-3) Exercises 2-5 www.geomconcepts.com/self_check_quiz

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Mixed Review

4.4 Proving Lines Parallel

What You'll Learn

You'll learn to identify conditions that produce parallel lines and to construct parallel lines.

Why It's Important Maintenance

Groundskeepers use parallel lines when marking the yardage lines on football fields. *See Exercise 8.*



Constructing Congruent Angles: Lesson 3–2 We can use geometry to prove that lines are parallel. We can also use geometry to construct parallel lines.



Try These

- **1.** Identify the special angle pair name for $\angle 1$ and $\angle 2$.
- **2.** Use a ruler to measure the distance between lines ℓ and *m* at several places. **Make a conjecture** about the relationship between the lines.

This activity illustrates a postulate that helps to prove two lines are parallel. This postulate is the converse of Postulate 4–1.





You can use Postulate 4–2 to find the angle measures of corresponding angles so that two lines are parallel.



The intersection at the right is called a *trumpet interchange*. If $m \angle 1 = 13x - 8$ and $m \angle 2 =$ 12x + 4, find x so that $\ell \parallel m$.

From the figure, you know that $\angle 1$ and $\angle 2$ are corresponding angles. So, according to Postulate 4–2, if $m \angle 1 = m \angle 2$, then $\ell \parallel m$.

 $m \angle 1 = m \angle 2$ 13x - 8 = 12x + 413x - 8 - 12x = 12x + 4 - 12x Subtract 12x from each side. x - 8 = 4x - 8 + 8 = 4 + 8*x* = 12

 $m \angle 1 = 13x - 8$ = 13(12) - 8 or 148



Corresponding angles Substitution Simplify. Add 8 to each side. Simplify.

 $m \angle 2 = 12x + 4$ = 12(12) + 4 or 148

Your Turn

Refer to the figure in Example 1.

- **a.** Find $m \angle 3$ and name the type of angle pair formed by $\angle 2$ and $\angle 3$.
- **b.** Make a conjecture about the relationship between $\angle 2$ and $\angle 3$ that must be true for ℓ to be parallel to *m*.

Example 1 illustrates four additional methods for proving that two lines are parallel. These are stated as Theorems 4–5, 4–6, 4–7, and 4–8.

Theorem	Words	Models and Symbols
4–5	In a plane, if two lines are cut by a transversal so that a pair of alternate interior angles is congruent, then the two lines are parallel.	a a b If $\angle 1 \cong \angle 2$, then $a \parallel b$.
4–6	In a plane, if two lines are cut by a transversal so that a pair of alternate exterior angles is congruent, then the two lines are parallel.	a 4 b If $\angle 3 \cong \angle 4$, then $a \parallel b$.

CONTENTS



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Theorem	Words	Models and Symbols
4–7	In a plane, if two lines are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the two lines are parallel.	$a = \frac{5}{6} = b$ If $m \angle 5 + m \angle 6 = 180$, then $a \parallel b$.
4-8	In a plane, if two lines are perpendicular to the same line, then the two lines are parallel.	$f a \perp t \text{ and } b \perp t,$ then $a \parallel b.$

So, we now have five ways to prove that two lines are parallel.



Algebra Review – Solving Multi-Step Equations, p. 723

 $m \angle BES + m \angle EST = 180$ (2x + 10) + (5x + 2) = 180 (2x + 5x) + (10 + 2) = 180 7x + 12 = 180 7x + 12 - 12 = 180 - 12 7x + 12 - 12 = 180 -

Thus, if x = 24, then $\overline{BT} \parallel \overline{TS}$.

Your Turn

d. Refer to the figure in Example 3. Find the value of *x* so $\overline{BT} \parallel \overline{ES}$.





Sports The yardage lines on a football field are parallel. Explain how the grounds crew could use Theorem 4–8 to know where to paint the yardage lines.
 Example 3

Exercises

Practice

Homework Help			
For Exercises	See Examples		
9-14	3		
15–20	2		
21	2, 3		
22, 23	1		
Extra Practice			
See page 733.			



Name the pairs of parallel lines or segments.





are parallel.



- **22. Solar Energy** The figure at the right shows how the sun's rays reflect off special mirrors to provide electricity.
 - **a.** Are the rays from the sun, lines *m* and *n*, parallel lines? Explain.
 - **b.** Are the reflected rays, lines *p* and *q*, parallel lines? Explain.





- **23. Construction** Carpenters use parallel wall studs in building supports for walls. Describe three ways a carpenter could guarantee that the wall studs *AB* and *CD* are parallel.
- **24. Critical Thinking** In the Hands-On Geometry activity on page 162, you constructed a line through a point *P* parallel to a line ℓ . In 1795, Scottish mathematician John Playfair (1748–1819) provided the modern version of Euclid's famous **Parallel Postulate**.

If there is a line and a point not on the line, then there exists exactly one line through the point that is parallel to the given line.

Explain the meaning of *exactly one* in the postulate. If you try to draw two lines parallel to a given line through a point not on the line, what happens?

Mixed Review Find the measure of each numbered angle.

- **25.** ∠1 (*Lesson* 4–3)
- **26.** ∠2 (*Lesson* 4–2)



Determine whether each statement is true

or false. Explain your reasoning. (Lesson 2–3)

CONTENTS

- **27.** If LM = MJ, then $\overline{LM} \cong \overline{MJ}$.
- **28.** If $\overline{XY} \cong \overline{YZ}$, $\overline{ST} \cong \overline{PQ}$, and $\overline{YZ} \cong \overline{PQ}$, then $\overline{XY} \cong \overline{ST}$.



29. Multiple Choice The high temperature in Newport on January 12 was 6°C. The low temperature was -7°C. Find the range of the temperatures in Newport on this date. (*Lesson 2–1*)

℃ 7°C

■ 1°C ■ 6.5°C

■ 13°C



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Lesson 4–4 Proving Lines Parallel 167

4-5 Slope

What You'll Learn

You'll learn to find the slopes of lines and use slope to identify parallel and perpendicular lines.

Why It's Important Architecture

Architects use slope to determine the steepness of stairways. *See Exercise 9.* The steepness of a line is called the **slope**. Slope is defined as the ratio of the *rise*, or vertical change, to the *run*, or horizontal change, as you move from one point on the line to another.

You can use two points on a line to find its slope. Consider the slope of \overrightarrow{AE} .



Points Used	A and C	A and D	A and E
Slope	$\frac{\text{rise}}{\text{run}} = \frac{2}{4} \text{ or } \frac{1}{2}$	$\frac{\text{rise}}{\text{run}} = \frac{3}{6} \text{ or } \frac{1}{2}$	$\frac{\text{rise}}{\text{run}} = \frac{4}{8} \text{ or } \frac{1}{2}$

Notice that the slope of \overline{AE} is always $\frac{1}{2}$, regardless of the points chosen. The slope of a line is the same, or constant, everywhere along the line. This means that the choice of points used to find the slope of a line does not affect the slope. You can find the slope of a line as follows.

	Words:	The slope <i>m</i> of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula
Definition of Slope		slope = $\frac{\text{difference of the } y\text{-coordinates}}{\text{difference of the corresponding } x\text{-coordinates}}$.
Symbols: $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_2 \neq x_1$		$m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_2 \neq x_1$

The slope of a vertical line, where $x_1 = x_2$ *, is undefined.*





Examples 1–3 suggest that a line with a negative slope seems to be going downhill, a line with a positive slope seems to be going uphill, and a line with a zero slope is a horizontal line. As shown in Example 4, the slope of a vertical line is undefined.



CONTENTS



This activity illustrates a special characteristic of parallel lines.

Postulate 4–3 Two distinct nonvertical lines are parallel if and only if they have the same slope.

All vertical lines are parallel.

You can use a TI–83/84 Plus calculator to study slopes of perpendicular lines.



Postulate 4–4 Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.





Check for Understanding

Communicating Mathematics

Guided Practice

Examples 1–4

- **1. Describe** a line whose slope is 0 and a line whose slope is undefined.
- **2.** Estimate the slope of line *ℓ* shown at the right. Explain how you determined your estimate.



Vocabulary

slope

3. Sang Hee claims that a line with a slope of 2 is steeper than a line with a slope of $\frac{1}{4}$. Emily claims that a slope of $\frac{1}{4}$ is steeper than a slope of 2. Who is correct? Use a coordinate drawing to support your answer.



CONTENTS

Example 5	Given each set of points, determine if <i>PQ</i> and <i>RS</i> are <i>parallel</i> , <i>perpendicular</i> , or <i>neither</i> .		
	7. <i>P</i> (-9, 2), <i>Q</i> (2, -9), <i>R</i> (9, 5), <i>S</i> (0, -4)		
	8. <i>P</i> (6, 1), <i>Q</i> (4, 0), <i>R</i> (3, -5), <i>S</i> (7, -3)		
Example 5	9. Construction Some building codes require the slope of a stairway to be no steeper than 0.88, or $\frac{22}{25}$. The stairs in Amad's house measure 11 inches deep and 6 inches high. Do the stairs meet the code requirements? Explain.	11 11 11 6 6	
xercises	• • • • • • • • • • • • • • • • • • •		

Practice

Homework Help		
For Exercises	See Examples	
10-15	1, 2	
16-18	3-4	
19-24	5	
25-28	3	
Extra Practice		
See page 733.		

Find the slope of each line.



Given each set of points, determine if \overrightarrow{JK} and \overrightarrow{LM} are *parallel*, *perpendicular*, or *neither*.

- **19.** *J*(-4, 11), *K*(-6, 3), *L*(7, 7), *M*(6, 3)
- **20.** *J*(6, 9), *K*(4, 6), *L*(0, 8), *M*(3, 6)
- **21.** *J*(-8, 1), *K*(-5, -8), *L*(0, 10), *M*(3, 11)
- **22.** *J*(6, 3), *K*(-7, 3), *L*(-4, -5), *M*(1, -5)
- **23.** *J*(-1, 5), *K*(2, -3), *L*(7, 9), *M*(2, 6)
- **24.** *J*(3, -2), *K*(5, -9), *L*(6, 4), *M*(4, -3)
- **25.** Find the slope of the line passing through points at (-7, 4) and (2, 9).
- **26.** C(r, -5) and D(5, 3) are two points on a line. If the slope of the line is $\frac{2}{3}$, find the value of *r*.



Applications and Problem Solving

Mixed Review

- **27. Sports** Refer to the application at the beginning of the lesson. Find the *run* of a hill with a 36-foot *rise* if the hill has a slope of 0.02.
- **28. Construction** To be efficient, gutters should drop $\frac{1}{4}$ inch for every 4 feet that they run toward a downspout. What is the desired slope of a gutter?
- **29. Critical Thinking** Use slope to determine if A(2, 4), B(5, 8), C(13, 2), and D(10, -2) are the vertices of a rectangle. Explain.

30. Name the pairs of parallel segments. (*Lesson* 4-4)





31. Find the measure of each

numbered angle. (Lesson 4-3)

In the figure, \overrightarrow{XA} and \overrightarrow{XD} are opposite rays. (*Lesson* 3–4)

- **32.** Which angle forms a linear pair with $\angle AXB$?
- **33.** Name all pairs of adjacent angles.



34. Multiple Choice What is the hypothesis in the following statement? (Lesson 1-4)

Angles are congruent if they have the same measure.

(A) congruent

C angles are congruent

not congruent

B they have the same measure

Standardized Test Practice

Quiz 2 Lessons 4–4 and 4–5 Find x so that $a \parallel b$. (Lesson 4–4) 2. 3. 1. а а в 137 140° в 85° в $(6x - 2)^{\circ}$ $(4x - 7)^{\circ}$ (x + 15)**4. Music** On the panpipe at the right, find the slope of \overline{GH} . (Lesson 4-5)G(5, -4) **5.** Given *P*(-6, 1), *Q*(6, 4), *R*(3, 4), and *S*(2, 8), determine H(9. -6) if \overrightarrow{PQ} and \overrightarrow{RS} are parallel, perpendicular, or neither. (Lesson 4-5)

CONTENTS



4-6 Equations of Lines

What You'll Learn

You'll learn to write and graph equations of lines.

Why It's Important Communication

Telephone company representatives can use linear equations to determine the cost of telephone calls. *See Exercise 35.* The equation y = 2x - 1 is called a **linear equation** because its graph is a straight line. We can substitute different values for *x* in the equation to find corresponding values for *y*, as shown in the table at the right.

X	2x - 1	у	(<i>X</i> , <i>Y</i>)
0	2(<mark>0</mark>) - 1	-1	(0, -1)
1	2(<mark>1</mark>) - 1	1	(1, 1)
2	2(<mark>2</mark>) - 1	3	(2, 3)
3	2(<mark>3</mark>) - 1	5	(3, 5)

We can then graph the ordered pairs (x, y). Notice that there is one line that contains all four points. There are many more points whose ordered pairs are solutions of y = 2x - 1. These points also lie on the line.



To find the slope of this line, choose any two points on the line, such as B(1, 1) and C(2, 3).

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad Definition \text{ of slope}$$
$$= \frac{3 - 1}{2 - 1} \qquad \begin{array}{l} \text{Replace } y_2 \text{ with } 3, \text{ and } y_1 \text{ with } 1.\\ \text{Replace } x_2 \text{ with } 2, \text{ and } x1 \text{ with } 1.\\ \end{array}$$
$$= \frac{2}{1} \text{ or } 2$$

Now look at the graph of y = 2x - 1. The *y*-value of the point where the line crosses the *y*-axis is -1. This value is called the *y*-intercept of the line.

$$y = 2x - 1$$

slope $rightarrow y - y$ -intercept

Most linear equations can be written in the form y = mx + b. This form is called the **slope-intercept form**.

Slope-Intercept Form An equation of the line having slope *m* and *y*-intercept *b* is y = mx + b.



Examples

Name the slope and *y*-intercept of the graph of each equation.

 $y = \frac{1}{2}x + 5$ **2** y = 3x = -2The slope is $\frac{1}{2}$. y = 0x + 3The graph is a vertical line. The *y*-intercept The slope is 0. is 5. The *y*-intercept The slope is undefined. is 3. There is no *y*-intercept. 2x - 3y = 18Rewrite the equation in slope-intercept form by solving for *y*. 2x - 3y = 18Original equation 2x - 3y - 2x = 18 - 2x Subtract 2x from each side. -3y = 18 - 2x Simplify. $\frac{-3y}{-3} = \frac{18 - 2x}{-3}$ Divide each side by -3. $y = -6 + \frac{2}{3}x$ Simplify. $y = \frac{2}{2}x - 6$ Write in slope-intercept form. The slope is $\frac{2}{3}$. The *y*-intercept is -6. Your Turn **a.** y = -x + 8**b.** x + 4y = 24**c.** y = -1**d.** *x* = 10 Graph 2x + y = 3 using the slope and *y*-intercept. First, rewrite the equation in slope-intercept form. 2x + y = 3 Original equation 2x + y - 2x = 3 - 2x Subtract 2x from each side. y = 3 - 2x Simplify. y = -2x + 3 Write in slope-intercept form. The *y*-intercept is 3. So, the point at (0, 3) must be on the line. Since the slope is -2, or $\frac{-2}{1}$, plot a point by using a *rise* of -2 units (0, 3) (down) and a run of 1 unit (right). Draw a line through the points. χ Your Turn

e. Graph -x + 3y = 9 using the slope and *y*-intercept.

CONTENTS

The graphs of lines ℓ , *m*, and *t* are shown at the right.

$$\ell: \ y = 2x - 3$$

m: y = 2x + 6
t: y = $-\frac{1}{2}x + 3$



Notice that ℓ appears to be parallel to *m*. You can verify this by Postulate 4–3, since the slopes of ℓ and *m* are the same. Also, *t* appears to be perpendicular to ℓ and *m*. You can verify this by Postulate 4–4, since the product of the slope of *t* and the slope of ℓ or *m* is –1. Thus, the slope-intercept form can be used to write the equation of another line.

Example 6	Write an passes th	equation of the line rough the point at (3	parallel to the graph of $y = 2x - 5$ that 3, 7).		
	Explore	The equation of the need to find the slo	line will be in the form $y = mx + b$. You pe <i>m</i> and the <i>y</i> -intercept <i>b</i> .		
	Plan	Because the lines are parallel, they must have the same slope. The slope of the given line is 2, so the slope <i>m</i> of the line parallel to the graph of $y = 2x - 5$ must be 2.			
		To find b , use the ordered pair (3, 7) and substitute for m , x , and y in the slope-intercept form.			
	Solve	y = mx + b 7 = 2(3) + b 7 = 6 + b 7 - 6 = 6 + b - 6 1 = b	Slope-intercept form Replace m with 2 and (x, y) with $(3, 7)$. Multiply. Subtract 6 from each side. Simplify.		
		The value of <i>b</i> is 1. So, the equation of the line is $y = 2x + 1$.			
	Examine	<i>Examine</i> The graphs of $y = 2x - 5$ and $y = 2x + 1$ both have the same slope, 2, so the lines are parallel. Since $7 = 2(3) + 1$ the graph of $y = 2x + 1$ passes through the point at (3, 7) The solution satisfies the conditions given.			
	Your Tu	rn			
	Write eac	h equation in slope	-intercept form.		
	f. Write a that pa	an equation of the lin asses through the po	The parallel to the graph of $3x + y = 6$ int at (1, 4).		
	g. Write a $y = \frac{1}{4}x$	an equation of the line $+ 5$ that passes through the third passes through the theorem of theorem of the theor	The perpendicular to the graph of bugh the point at $(-3, 8)$.		



Check for Understanding

Communicating Mathematics

- **1.** Explain why y = mx + b is called the *slope-intercept form* of the equation of a line.
- **2.** Explain how you know that lines ℓ, *m*, and *n* are parallel. Then name the *y*-intercept for each line.



Guided Practice	Getting Ready Solve each equation for <i>y</i> .	
	Sample: $8x + 4y = 1$ Solution: $4y = 1 - 8x$ $\frac{4y}{4} = \frac{1 - 8x}{4}$ or $y = \frac{1}{4}$	– 2 <i>x</i>
	3. $y - 6x = -3$ 4. $x + 7y = 14$ 5. $5x - 3y = 9$	
Examples 1–4	Name the slope and y-intercept of the graph of each equation. 6. $y = 2x + 6$ 7. $3x + 2y = 8$	I
Example 5	Graph each equation using the slope and y-intercept.	
	8. $y = -x + 4$ 9. $2x - 5y = 10$	
Example 6	Write an equation of the line satisfying the given conditions.	
	10. slope = 4, goes through the point at $(-1, 3)$	
	11. parallel to the graph of $y = -2x + 6$, passes through the point at $(-4, 4)$	
	12. Nutrition In the equation $C = 15f + 68$, <i>C</i> repret the number of Calories in a strawberry breakfas and <i>f</i> represents the number of fat grams. Example 1	sents t bar,
Sec Total	a. What is the slope of the line? What does it represent?	
	b. What is the <i>y</i> -intercept? What does it represe	nt?
	Lesson 4–6 Equations of Lines	s 177

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Vocabulary

linear equation y-intercept slope-intercept form

Exercises

Practice

Homewo	ork Help
For Exercises	See Examples
13–18	1-3
19–24, 31–34	5
25-30	6
35	1, 5
36	1, 4
Extra f	Practice
See pag	ge 733.

Name the slope and *y*-intercept of the graph of each equation.

· y = 12

4y = 2

13.	y = 9x + 1	14.	$7x + \frac{1}{2}$
15.	3x - 2y = 18	16.	<i>x</i> = 6
17.	y = 5	18.	3x + 4

Graph each equation using the slope and y-intercept.

19. $y = 3x - 5$	20. $y = -x + 6$
21. $y + 7x = 4$	22. $-\frac{1}{2}x + 2y = 9$
23. $\frac{1}{3}x - y = 2$	24. $4x - 3y = -6$

Write an equation of the line satisfying the given conditions.

- **25.** slope = 3, goes through the point at (-1, 4)
- **26.** parallel to the graph of y = -2x 6, passes through the point at (4, 4)
- **27.** parallel to the graph of 4x + y = 9, passes through the point at (0, -5)
- **28.** parallel to the *x*-axis, passes through the point at (-3, -6)
- **29.** slope is undefined, passes through the point at (-3, 7)
- **30.** perpendicular to the graph of $y = \frac{1}{2}x 3$, passes through the point at (5, -4)

Choose the correct graph of lines a, b, c, or d for each equation. 31. y = 2x + 132. y = 2x + 333. y = -2x - 1

34. y = -2x + 3



Applications and Problem Solving



Data Update For the latest information on telephone rates, visit: www.geomconcepts.com

- **35. Communication** One telephone company's charges are given by the equation y = 0.40x + 0.99, where *y* represents the total cost in dollars for a telephone call and *x* represents the length of the call in minutes.
 - **a.** Make a table of values showing what a telephone call will cost after 0, 1, 2, 3, 4, and 5 minutes.
 - **b.** Graph the values in your table.
 - **c.** What is the slope of the line? What does it represent?
 - d. What is the *y*-intercept of the line? What does it represent?



- **36. Sports** The graph at the right shows the distance a baseball can be hit when it is pitched at different speeds.
 - **a.** What is the *y*-intercept? What does this value represent?
 - **b.** Estimate the slope.
 - **c.** Write an equation of the line.



37. Critical Thinking Explain how you could find an equation of a line if you are only given the coordinates of two points on the line.



the estimated cost of wireless phone use from 1998 to 2003. (Lesson 4-5) **38.** Which section of the graph

- shows when the greatest change occurred? How does its slope compare to the rest of the graph?
- **39.** Describe the slope of a graph showing an increase in cost.
- **40.** Find *x* so that *a* || *b*. (Lesson 4-4)





41. Haloke used the design shown below in a quilt she made for her grandmother. Name all the angles with *Q* as a vertex. (Lesson 3-1)



- **42.** Algebra What is the length of a rectangle with area 108 square inches and width 8 inches? (Lesson 1–6)
- **43. Extended Response** Just by looking, which segment appears to be longer, AJ or KR? Use a ruler to measure the two segments. What do you discover? (Lesson 1-5)

CONTENTS

- **44. Multiple Choice** Which is *not* a plane represented in the figure? (Lesson 1–3)
 - $\bigcirc ABD$ **B** CDF

 \bigcirc BFE \bigcirc ADF R В



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Study Guide and Assessment

Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

Geometry

alternate exterior angles (p. 148)alternate interior angles (p. 148)consecutive interior angles (p. 148)corresponding angles (p. 156)exterior angles (p. 148)finite (p. 154)great circle (p. 154) interior angles (p. 148)line (p. 154)line of latitude (p. 154)line of longitude (p. 154)parallel lines (p. 142)parallel planes (p. 142)skew lines (p. 143)transversal (p. 148)



For more review activities, visit: www.geomconcepts.com

Algebra

linear equation (*p.* 174) slope (*p.* 168) slope-intercept form (*p.* 174) *y*-intercept (*p.* 174)

Choose the letter of the term that best describes each set of angles or lines.

- **1.** $\angle 2$ and $\angle 7$
- **2.** ∠1, ∠3, ∠6, ∠8
- **3.** $\angle 5$ and $\angle 1$
- **4.** lines *m* and *n*
- **5.** $\angle 7$ and $\angle 4$
- **6.** line *q*
- **7.** ∠2, ∠4, ∠5, ∠7
- **8.** $\angle 1$ and $\angle 6$
- **9.** lines *q* and *p*
- **10.** lines *p* and *m*



- **a.** alternate exterior angles
- **b.** alternate interior angles
- **c.** consecutive interior angles
- d. corresponding angles
- **e.** exterior angles
- **f.** interior angles
- g. parallel lines
- **h.** skew lines
- i. perpendicular lines
- j. transversal

Skills and Concepts

Objectives and Examples

• Lesson 4–1 Describe relationships among lines, parts of lines, and planes.



In the prism, plane *QRS* is parallel to plane *TUV*.

 \overline{TW} and \overline{UV} are parallel to \overline{RS} . \overline{QR} , \overline{TU} , \overline{RS} , \overline{UV} intersect \overline{RU} .

 \overline{RS} , \overline{SP} , \overline{UV} , and \overline{WV} are skew to \overline{QT} .

Review Exercises

Describe each pair of segments in the prism as *parallel*, *skew*, or *intersecting*.

- **11.** *HB*, *FD*
- **12.** \overline{BC} , \overline{AG}
- **13.** *EC*, *HE*

Name the parts of the prism. G

14. six planes

CONTENTS

- **15.** all segments skew to \overline{GE}
- **16.** all segments parallel to \overline{GF}



Exercises 11-16

www.geomconcepts.com/vocabulary_review

Objectives and Examples

• Lesson 4–2 Identify the relationships among pairs of interior and exterior angles formed by two parallel lines and a transversal.

 $\angle 3$ and $\angle 8$ are alternate interior angles, so they are congruent.



 $\angle 1$ and $\angle 8$ are consecutive interior angles, so they are supplementary.

 $\angle 4$ and $\angle 7$ are alternate exterior angles, so they are congruent.

• Lesson 4–3 Identify the relationships among pairs of corresponding angles formed by two parallel lines and a transversal.

If $s \parallel t$, which angles are congruent to $\angle 2$?



 $\begin{array}{l} \angle 2 \cong \angle 5 & Vertical \ angles \ are \ congruent. \\ \angle 2 \cong \angle 3 & Postulate \ 4-1 \\ \angle 3 \cong \angle 7 & Vertical \ angles \ are \ congruent. \end{array}$

Therefore, $\angle 5$, $\angle 3$, and $\angle 7$ are congruent to $\angle 2$.

• Lesson 4–4 Identify conditions that produce parallel lines and construct parallel lines.

Find *x* so that $\overline{JK} \parallel \overline{MN}$.



CONTENTS

 $\angle JLM \text{ and } \angle LMN \text{ are alternate interior}$ $angles. If <math>\angle JLM \cong \angle LMN$, then $\overline{JK} \parallel \overline{MN}$. $m \angle JLM = m \angle LMN$ 63 = 5x - 2 Substitution 65 = 5x Add 2 to each side. 13 = x Divide each side by 5. **Review Exercises**

Identify each pair of angles as alternate interior, alternate exterior, consecutive interior, or vertical.

17.	$\angle 1$ and $\angle 6$
18.	$\angle 4$ and $\angle 2$
19.	$\angle 3$ and $\angle 8$
20.	/7 and $/3$



Exercises 17–24

If $m \angle 1 = 124$, find the measure of each angle. Give a reason for each answer.

21.	∠3	22.	$\angle 4$
23.	∠6	24.	∠8

Name all angles congruent to the given angle. Give a reason for each answer.



Find the measure of each numbered angle.





Name the pairs of parallel lines or segments.



Objectives and Examples

• Lesson 4–5 Find the slopes of lines and use slope to identify parallel and perpendicular lines.

Given A(9, 2), D(6, 3), F(-5, 7), and S(-4, 10), is $\overrightarrow{AD} \perp \overrightarrow{FS}$, $\overrightarrow{AD} \parallel \overrightarrow{FS}$, or neither?

slope of \overrightarrow{AD}	slope of \overrightarrow{FS}
$m = \frac{3-2}{6-9}$	$m = \frac{10 - 7}{-4 - (-5)}$
$=\frac{1}{-3}$ or $-\frac{1}{3}$	$=\frac{3}{1}$ or 3
Since $\left(-\frac{1}{3}\right)\left(\frac{3}{1}\right) =$	$-1, \overleftarrow{AD} \perp \overleftarrow{FS}.$

Review Exercises

Find the slope of each line.





Given each set of points, determine if \overrightarrow{GH} and \overrightarrow{PQ} are *parallel*, *perpendicular*, or *neither*.

37. *G*(-4, 3), *H*(10, -9), *P*(8, 6), *Q*(1, 0) **38.** *G*(11, 0), *H*(3, -7), *P*(4, 16), *Q*(-4, 9)

• Lesson 4–6 Write and graph equations of lines.

Slope-intercept form:

$$y = mx + b$$
slope $- - - - - y$ -intercept

To graph a linear equation using slope and *y*-intercept, rewrite the equation by solving for *y*.

Name the slope and *y*-intercept of the graph of each equation. Then graph the equation.

39. $y = -\frac{3}{5}x + 1$ **40.** 2x + 5y = 20

41. Write an equation of the line perpendicular to the graph of y = -2x + 4 that passes through the point at (-8, 3).

Applications and Problem Solving

42. Gymnastics Describe bars ℓ and m on the equipment below as *parallel*, *skew*, or *intersecting*. (*Lesson* 4–1)

ł				
- C	1			
m				
				8
				0
			[[

43. Construction A flight of stairs is parallel to the overhead ceiling. If x = 122, find y. (Lesson 4–4)





Choose the letter on the right that identifies each angle pair.

Test

1. $\angle 5$ and $\angle 16$

CHAPTER

- **2.** \angle 9 and \angle 3
- **3.** $\angle 10$ and $\angle 13$
- **4.** $\angle 11$ and $\angle 9$
- **5.** $\angle 2$ and $\angle 7$
- b. consecutive interior anglesc. alternate interior angles
 - **d.** alternate exterior angles

a. corresponding angles

e. vertical angles

Describe each pair of segments or planes in the prism as *parallel*, *skew*, or *intersecting*.

6. \overline{IJ} and \overline{EF}

- **7.** plane *JBH* and plane *IGH*
- **8.** \overline{CG} and \overline{AB} **9.** plane *ADF* and plane *HBC*
- **10.** Name all segments parallel to *AD*.
- **11.** Name all segments that intersect *BH*.

In the figure, $q \parallel r$.

12. If $m \angle 7 = 88$, find the measure of each numbered angle.

13. If $\angle 1$ is a right angle, name two pairs of perpendicular lines.

14. If $m \angle 5 = 5x$ and $m \angle 7 = 2x + 63$, find $m \angle 5$.

Name the parallel segments formed if the following angles are congruent.

15. $\angle 2$ and $\angle 6$ **16.** $\angle 3$ and $\angle 4$ **17.** If $m \angle DCB = 12x - 3$ and $m \angle 7 = 8x + 3$, find x so that $\overline{AD} \parallel \overline{BC}$.

Given each set of points, find the slopes of \overrightarrow{MN} and \overrightarrow{UV} . Then determine whether the two lines are *parallel*, *perpendicular*, or *neither*.

- **18.** *M*(-3, -5), *N*(-3, 4), *U*(7, -2), *V*(4, -5)
- **19.** *M*(2, 12), *N*(-8, 9), *U*(8, 4), *V*(-2, 1)
- **20.** *M*(4, 8), *N*(9, 2), *U*(0, 3), *V*(-6, -2)
- **21.** Graph the equation 3x + y = 1 using the slope and *y*-intercept.
- **22.** Find and graph the equation of the line with slope -1 passing through the point at (2, 3).
- **23.** Find the equation of the line parallel to the *y*-axis and passing through the point at (-4, 7).
- **24. Meteorology** The weather symbol at the right represents a heavy thunderstorm. If \overline{AB} and \overline{CD} are parallel segments and $m \angle B = 85$, find $m \angle C$.
- **25. Civil Engineering** A public parking lot is constructed so that all parking stalls in an aisle are parallel. If $m \angle KLA = 43$, find $m \angle PKR$.













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Data Analysis Problems

Proficiency tests usually include several problems on interpreting graphs and creating graphs from data. The SAT and ACT may include a few questions on interpreting graphs. Often a graph is used to answer two or more questions.

You'll need to understand these ways of representing data: bar graphs, circle graphs, line graphs, stem-and-leaf plots, histograms, and frequency tables.

Test-Taking Tip

If a problem includes a graph, first look carefully at its axis labels and units. Then read the question(s) about the graph.

Example 1

The table below shows the number of employees who earn certain hourly wages at two different companies.

Hourly	Number of Employees			
Wage (\$)	Company 1	Company 2		
6.00	3	4		
6.50	5	6		
7.00	6	7		
7.50	4	5		
8.00	1	2		

Construct a double-bar graph to show the number of employees at each company who earn the given hourly wages.

Hint Decide what quantities will be shown on each axis of your graph. Label the axes.

Solution Draw the double-bar graph. Put the wages on the horizontal axis and the number of employees on the vertical axis.





55

Example 2

Sara's Driving Speed, Saturday Afternoon



For what percent of the time was Sara driving 40 miles per hour or faster?

A	20%	B	25%	$33\frac{1}{3}\%$
D	40%	E	50%	5

Hint Don't mix units, like *hours* and *minutes*. In this case, use hours.

Solution The total time Sara drove is 3 hours (from 1:00 until 4:00). She drove 40 miles per hour or faster from 2:30 to 4:00, or $1\frac{1}{2}$ hours. The fraction of the time she drove 40 mph or faster is $\frac{1\frac{1}{2}}{3}$ or $\frac{1}{2}$. The equivalent percent is 50%. So, the answer is E.



Preparing for Standardized Tests For test-taking strategies and more practice, see pages 766–781.

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

1. What is a reasonable conclusion from the information given? (*Statistics Review*)



- Fewer than 20 customers purchased classical concert tickets last month.
- Plays are more popular than country music concerts at Bob's.
- C Rock concerts are less popular than country music concerts at Bob's.
- **D** Bob doesn't sell rap concert tickets.
- **2.** $(-2)^3 + (3)^{-2} + \frac{8}{9} =$ (Algebra Review)

(A) -7 (B) $-1\frac{7}{9}$ (C) $\frac{8}{9}$ (D) $1\frac{7}{9}$ (E) 12

3. Kendra has 80 CDs. If 40% are jazz CDs and the rest are blues CDs, how many blues CDs does she have? (*Percent Review*)

A	32	B	40	\bigcirc	42
D	48	E	50		

- **4.** Cody has 3 pairs of jeans and 4 sweatshirts. How many combinations of 1 pair of jeans and 1 sweatshirt can he wear? (*Statistics Review*)
 - (A) 3 (B) 4 (C) 7 (D) 12
- **5.** If 3x + 7 = 28, what is x? (Algebra Review) (A) 4 (B) 5 (C) 6 (D) 7

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- 6. \overrightarrow{FB} and \overrightarrow{EC} are parallel, and the measure of $\angle CXD$ is 62. Find the measure of $\angle FYX$. (Lesson 4–2) (A) 28 (B) 62 (C) 90 (D) 118
- **7.** In a stem-and-leaf plot of the data in the chart, which numbers would be the best choice for the stems? *(Statistics Review)*

Average Temperature (°F)					
52	55	49	51	57	66
62	61	61	73	78	76
69	58	59	74	78	79

8. What is the y-intercept of line MN? (Lesson 4-6)
A -4 B 2
-2 D 0

Grid In

CONTENTS

9. Six cards are numbered 0 through 5. Two are selected without replacement. What is the probability that their sum is 4? *(Statistics Review)*

Extended Response

10. The numbers of visitors each day to a history museum during the month of June are shown below. (*Statistics Review*)
11 19 8 7 18 43 22 18 14 21 19
41 61 36 16 16 14 24 31 64 29 24
27 33 31 71 89 61 41 34

Part A Construct a frequency table for the data.

Part B Construct a histogram that represents the data.