

CHAPTER

8

Quadrilaterals

What You'll Learn

Key Ideas

- Identify parts of quadrilaterals and find the sum of the measures of the interior angles of a quadrilateral. (*Lesson 8-1*)
- Identify and use the properties of parallelograms. (*Lessons 8-2 and 8-3*)
- Identify and use the properties of rectangles, rhombi, squares, trapezoids, and isosceles trapezoids. (*Lessons 8-4 and 8-5*)

Key Vocabulary

parallelogram (*p. 316*)

quadrilateral (*p. 310*)

rectangle (*p. 327*)

rhombus (*p. 327*)

square (*p. 327*)

trapezoid (*p. 333*)

Why It's Important

Art The work of architect Filippo Brunelleschi, designer of the famed cathedral in Florence, Italy, led to a mathematical theory of perspective. He probably developed his theories to help him render architectural drawings. In learning the mathematics of perspective, Renaissance painters were able to depict figures more fully and realistically than artists from the Middle Ages.

Quadrilaterals are used in construction and architecture. You will investigate trapezoids in perspective drawings in Lesson 8-5.

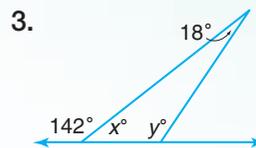
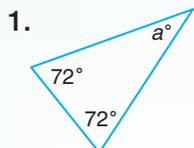


Study these lessons to improve your skills.

✓ Lesson 5-2, pp. 193-197

✓ Check Your Readiness

Find the value of each variable.



✓ Lesson 4-2, pp. 148-153

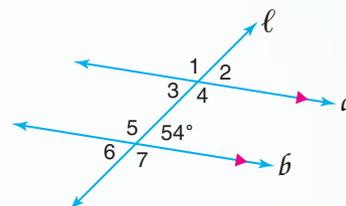
Find the measure of each angle. Give a reason for each answer.

4. $\angle 1$

5. $\angle 2$

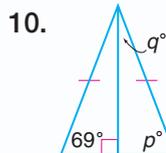
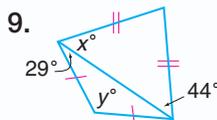
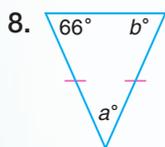
6. $\angle 3$

7. $\angle 4$



✓ Lesson 6-4, pp. 246-250

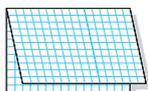
For each triangle, find the values of the variables.



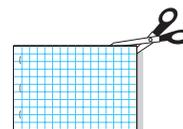
FOLDABLES™ Study Organizer

Make this Foldable to help you organize your Chapter 8 notes. Begin with three sheets of grid paper.

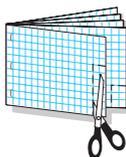
1 **Fold** each sheet in half from top to bottom.



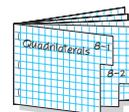
2 **Cut** along each fold. Staple the six half-sheets together to form a booklet.



3 **Cut** five tabs. The top tab is 3 lines wide, the next tab is 6 lines wide, and so on.



4 **Label** each of the tabs with a lesson number.



Reading and Writing As you read and study the chapter, fill the journal with terms, diagrams, and theorems for each lesson.



8-1

Quadrilaterals

What You'll Learn

You'll learn to identify parts of quadrilaterals and find the sum of the measures of the interior angles of a quadrilateral.

Why It's Important

City Planning

City planners use quadrilaterals in their designs. See Exercise 36.

The building below was designed by Laurinda Spear. Different quadrilaterals are used as faces of the building.

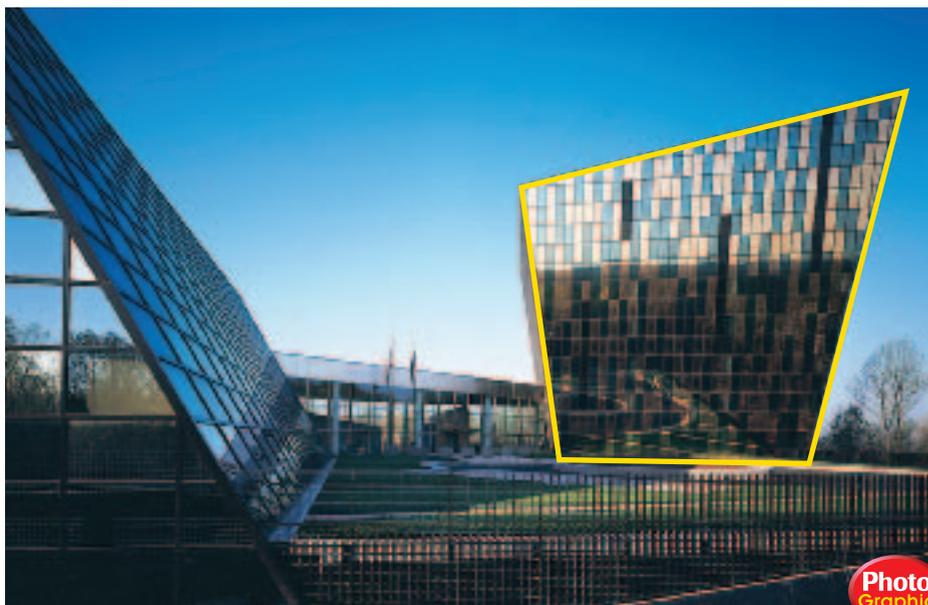


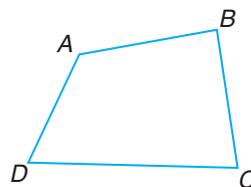
Photo Graphic

Centre for Innovative Technology, Fairfax and Louden Counties, Virginia

A **quadrilateral** is a closed geometric figure with four sides and four vertices. The segments of a quadrilateral intersect only at their endpoints. Special types of quadrilaterals include squares and rectangles.

Quadrilaterals	Not Quadrilaterals

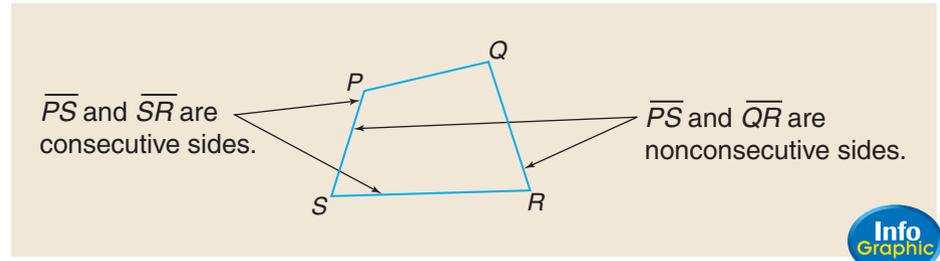
Quadrilaterals are named by listing their vertices in order. There are many names for the quadrilateral at the right. Some examples are quadrilateral $ABCD$, quadrilateral $BCDA$, or quadrilateral $DCBA$.



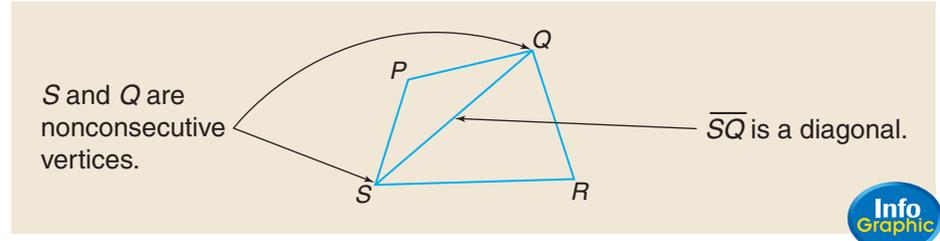
Reading Geometry

In a quadrilateral, nonconsecutive sides, vertices, or angles are also called *opposite* sides, vertices, or angles.

Any two sides, vertices, or angles of a quadrilateral are either **consecutive** or **nonconsecutive**.



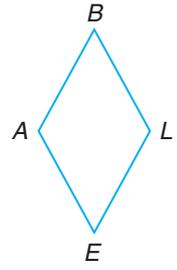
Segments that join nonconsecutive vertices of a quadrilateral are called **diagonals**.



Examples

Refer to quadrilateral **ABLE**.

- 1** Name all pairs of consecutive angles.
 $\angle A$ and $\angle B$, $\angle B$ and $\angle L$, $\angle L$ and $\angle E$, and $\angle E$ and $\angle A$ are consecutive angles.
- 2** Name all pairs of nonconsecutive vertices.
 A and L are nonconsecutive vertices.
 B and E are nonconsecutive vertices.
- 3** Name the diagonals.
 \overline{AL} and \overline{BE} are the diagonals.



Your Turn

Refer to quadrilateral **WXYZ**.

- a. Name all pairs of consecutive sides.
- b. Name all pairs of nonconsecutive angles.
- c. Name the diagonals.



Look Back

Angle Sum Theorem:
Lesson 5-2

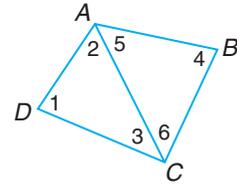
In Chapter 5, you learned that the sum of the measures of the angles of a triangle is 180. You can use this result to find the sum of the measures of the angles of a quadrilateral.

Hands-On Geometry

Materials:  straightedge  protractor

Step 1 Draw a quadrilateral like the one at the right. Label its vertices A , B , C , and D .

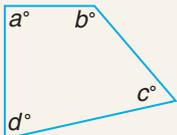
Step 2 Draw diagonal \overline{AC} . Note that two triangles are formed. Label the angles as shown.



Try These

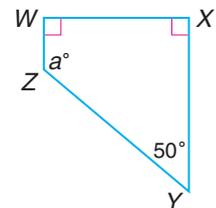
1. Use the Angle Sum Theorem to find $m\angle 1 + m\angle 2 + m\angle 3$.
2. Use the Angle Sum Theorem to find $m\angle 4 + m\angle 5 + m\angle 6$.
3. Find $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6$.
4. Use a protractor to find $m\angle 1$, $m\angle DAB$, $m\angle 4$, and $m\angle BCD$. Then find the sum of the angle measures. How does the sum compare to the sum in Exercise 3?

You can summarize the results of the activity in the following theorem.

Theorem 8-1	<p>Words: The sum of the measures of the angles of a quadrilateral is 360.</p> <p>Model: </p>	<p>Symbols: $a + b + c + d = 360$</p>
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Example

- 4** Find the missing measure in quadrilateral $WXYZ$.



$$\begin{aligned}
 m\angle W + m\angle X + m\angle Y + m\angle Z &= 360 && \text{Theorem 8-1} \\
 90 + 90 + 50 + a &= 360 && \text{Substitution} \\
 230 + a &= 360 && \text{Add.} \\
 230 - 230 + a &= 360 - 230 && \text{Subtract 230 from each side.} \\
 a &= 130 && \text{Simplify.}
 \end{aligned}$$

Therefore, $m\angle Z = 130$.

Your Turn

- d. Find the missing measure if three of the four angle measures in quadrilateral $ABCD$ are 50, 60, and 150.

Example**Algebra Link****Algebra Review**

Solving Multi-Step Equations, p. 723

- 5 Find the measure of $\angle U$ in quadrilateral $KDUC$ if $m\angle K = 2x$, $m\angle D = 40$, $m\angle U = 2x$ and $m\angle C = 40$.

$$m\angle K + m\angle D + m\angle U + m\angle C = 360$$

$$2x + 40 + 2x + 40 = 360$$

$$4x + 80 = 360$$

$$4x + 80 - 80 = 360 - 80$$

$$4x = 280$$

$$\frac{4x}{4} = \frac{280}{4}$$

$$x = 70$$

Since $m\angle U = 2x$, $m\angle U = 2 \cdot 70$ or 140.

Theorem 8-1

Substitution

Add.

Subtract 80 from each side.

Simplify.

Divide each side by 4.

Simplify.

Your Turn

- e. Find the measure of $\angle B$ in quadrilateral $ABCD$ if $m\angle A = x$, $m\angle B = 2x$, $m\angle C = x - 10$, and $m\angle D = 50$.

Check for Understanding**Communicating Mathematics**

- Sketch and label a quadrilateral in which \overline{AC} is a diagonal.
- Writing Math** Draw three figures that are *not* quadrilaterals. Explain why each figure is *not* a quadrilateral.

Vocabulary

quadrilateral
consecutive
nonconsecutive
diagonal

Guided Practice**Getting Ready**

Solve each equation.

Sample: $120 + 55 + 45 + x = 360$

Solution: $220 + x = 360$

$$x = 140$$

3. $130 + x + 50 + 80 = 360$

4. $90 + 90 + x + 55 = 360$

5. $28 + 72 + 134 + x = 360$

6. $x + x + 85 + 105 = 360$

Refer to quadrilateral $MQPN$ for Exercises 7–9.

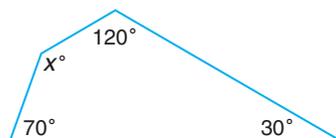
Example 1**Example 2****Example 3**

- Name a pair of consecutive angles.
- Name a pair of nonconsecutive vertices.
- Name a diagonal.

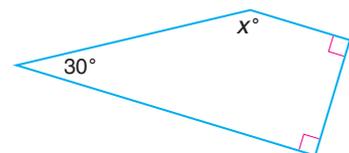
**Example 4**

Find the missing measure in each figure.

10.

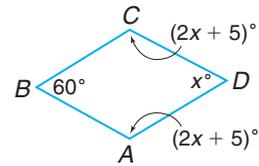


11.



Example 5

12. **Algebra** Find the measure of $\angle A$ in quadrilateral $BCDA$ if $m\angle B = 60$, $m\angle C = 2x + 5$, $m\angle D = x$, and $m\angle A = 2x + 5$.



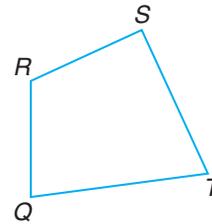
Exercises

Practice

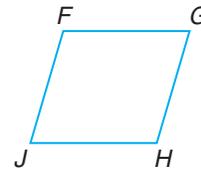
Homework Help	
For Exercises	See Examples
17, 19	3
18	1
21	2
22–27, 35	4, 5
36	1–3
Extra Practice	
See page 739.	

Refer to quadrilaterals $QRST$ and $FGHJ$.

- Name a side that is consecutive with \overline{RS} .
- Name the side opposite \overline{ST} .
- Name a pair of consecutive vertices in quadrilateral $QRST$.
- Name the vertex that is opposite S .
- Name the two diagonals in quadrilateral $QRST$.
- Name a pair of consecutive angles in quadrilateral $QRST$.
- Name a diagonal in quadrilateral $FGHJ$.
- Name a pair of nonconsecutive sides in quadrilateral $FGHJ$.
- Name the angle opposite $\angle F$.

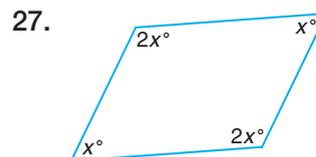
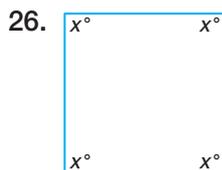
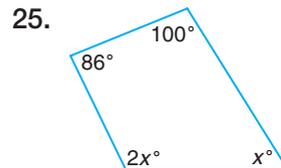
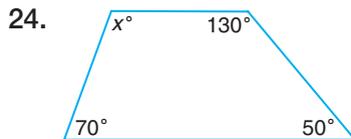
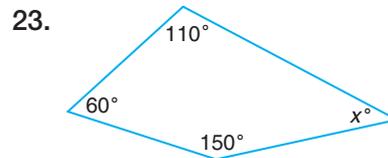
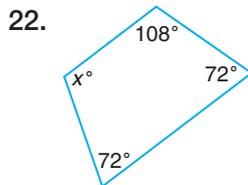


Exercises 13–18



Exercises 19–21

Find the missing measure(s) in each figure.



28. Three of the four angle measures in a quadrilateral are 90, 90, and 125. Find the measure of the fourth angle.

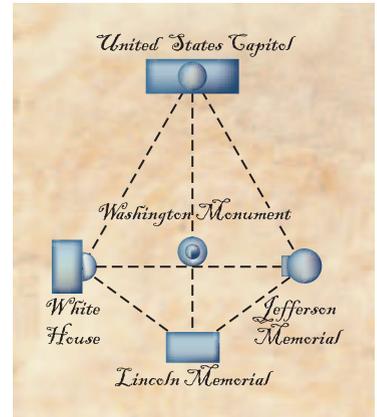
Use a straightedge and protractor to draw quadrilaterals that meet the given conditions. If none can be drawn, write *not possible*.

29. exactly two acute angles 30. exactly four right angles
 31. exactly four acute angles 32. exactly one obtuse angle
 33. exactly three congruent sides 34. exactly four congruent sides

Applications and Problem Solving

35. **Algebra** Find the measure of each angle in quadrilateral $RSTU$ if $m\angle R = x$, $m\angle S = x + 10$, $m\angle T = x + 30$, and $m\angle U = 50$.

36. **City Planning** Four of the most popular tourist attractions in Washington, D.C., are located at the vertices of a quadrilateral. Another attraction is located on one of the diagonals.



- a. Name the attractions that are located at the vertices.
 b. Name the attraction that is located on a diagonal.

37. **Critical Thinking** Determine whether a quadrilateral can be formed with strips of paper measuring 8 inches, 4 inches, 2 inches, and 1 inch. Explain your reasoning.



Mixed Review

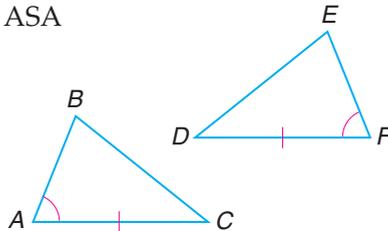
Determine whether the given numbers can be the measures of the sides of a triangle. Write *yes* or *no*. (Lesson 7-4)

38. 6, 4, 10 39. 2.2, 3.6, 5.7 40. 3, 10, 13.6

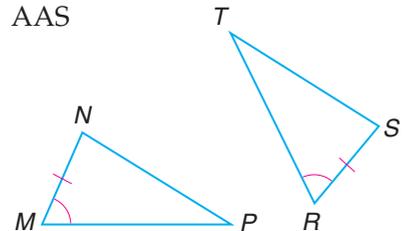
41. In $\triangle LNK$, $m\angle L < m\angle K$ and $m\angle L > m\angle N$. Which side of $\triangle LNK$ has the greatest measure? (Lesson 7-3)

Name the additional congruent parts needed so that the triangles are congruent by the indicated postulate or theorem. (Lesson 5-6)

42. ASA



43. AAS



Standardized Test Practice

- (A) (B) (C) (D)

44. **Multiple Choice** The total number of students enrolled in public colleges in the U.S. is expected to be about 12,646,000 in 2005. This is a 97% increase over the number of students enrolled in 1970. About how many students were enrolled in 1970? (Algebra Review)

- (A) 94,000 (B) 6,419,000 (C) 12,267,000 (D) 24,913,000



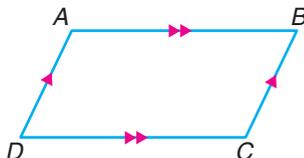
What You'll Learn

You'll learn to identify and use the properties of parallelograms.

Why It's Important**Carpentry**

Carpenters use the properties of parallelograms when they build stair rails. See *Exercise 28*.

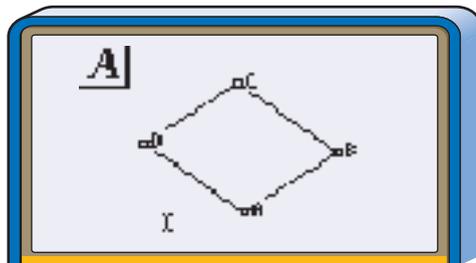
A **parallelogram** is a quadrilateral with two pairs of parallel sides. A symbol for parallelogram $ABCD$ is $\square ABCD$. In $\square ABCD$ below, \overline{AB} and \overline{DC} are parallel sides. Also, \overline{AD} and \overline{BC} are parallel sides. The parallel sides are congruent.

**Graphing Calculator Tutorial**

See pp. 782–785.

**Graphing Calculator Exploration**

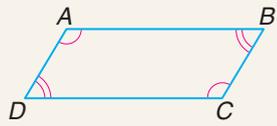
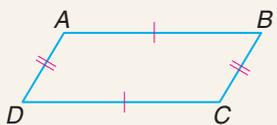
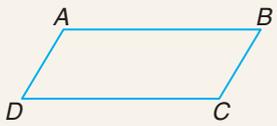
- Step 1** Use the Segment tool on the **F2** menu to draw segments AB and AD that have a common endpoint A . Be sure the segments are not collinear. Label the endpoints.
- Step 2** Use the Parallel Line tool on the **F3** menu to draw a line through point B parallel to \overline{AD} . Next, draw a line through point D parallel to \overline{AB} .
- Step 3** Use the Intersection Point tool on the **F2** menu to mark the point where the lines intersect. Label this point C . Use the Hide/Show tool on the **F5** menu to hide the lines.
- Step 4** Finally, use the Segment tool to draw \overline{BC} and \overline{DC} . You now have a parallelogram whose properties can be studied with the calculator.



Try These

1. Use the Angle tool under Measure on the **F5** menu to **verify** that the opposite angles of a parallelogram are congruent. Describe your procedure.
2. Use the Distance & Length tool under Measure on the **F5** menu to **verify** that the opposite sides of a parallelogram are congruent. Describe your procedure.
3. Measure two pairs of consecutive angles. **Make a conjecture** as to the relationship between consecutive angles in a parallelogram.
4. Draw the diagonals of $\square ABCD$. Label their intersection E . Measure \overline{AE} , \overline{BE} , \overline{CE} , and \overline{DE} . **Make a conjecture** about the diagonals of a parallelogram.

The results of the activity can be summarized in the following theorems.

Theorem	Words	Models and Symbols
8-2	Opposite angles of a parallelogram are congruent.	 $\angle A \cong \angle C, \angle B \cong \angle D$
8-3	Opposite sides of a parallelogram are congruent.	 $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$
8-4	The consecutive angles of a parallelogram are supplementary.	 $m\angle A + m\angle B = 180$ $m\angle A + m\angle D = 180$

Using Theorem 8-4, you can show that the sum of the measures of the angles of a parallelogram is 360.

Examples

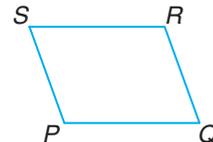
In $\square PQRS$, $PQ = 20$, $QR = 15$, and $m\angle S = 70$.

- 1 Find SR and SP .

$\overline{SR} \cong \overline{PQ}$ and $\overline{SP} \cong \overline{QR}$ *Theorem 8-3*

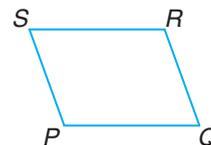
$SR = PQ$ and $SP = QR$ *Definition of congruent segments*

$SR = 20$ and $SP = 15$ *Replace PQ with 20 and QR with 15.*



2 Find $m\angle Q$.

$$\begin{aligned}\angle Q &\cong \angle S && \text{Theorem 8-2} \\ m\angle Q &= m\angle S && \text{Definition of congruent angles} \\ m\angle Q &= 70 && \text{Replace } m\angle S \text{ with } 70.\end{aligned}$$



3 Find $m\angle P$.

$$\begin{aligned}m\angle S + m\angle P &= 180 && \text{Theorem 8-4} \\ 70 + m\angle P &= 180 && \text{Replace } m\angle S \text{ with } 70. \\ 70 - 70 + m\angle P &= 180 - 70 && \text{Subtract } 70 \text{ from each side.} \\ m\angle P &= 110 && \text{Simplify.}\end{aligned}$$

Your Turn

In $\square DEFG$, $DE = 70$, $EF = 45$, and $m\angle G = 68$.

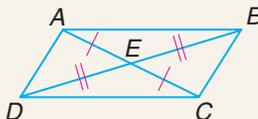
- Find GF .
- Find DG .
- Find $m\angle E$.
- Find $m\angle F$.

The result in Theorem 8-5 was also found in the Graphing Calculator Exploration.

Theorem 8-5

Words: The diagonals of a parallelogram bisect each other.

Model:



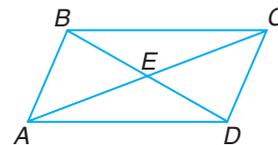
Symbols: $\overline{AE} \cong \overline{EC}$,
 $\overline{BE} \cong \overline{ED}$

Example

4 In $\square ABCD$, if $AC = 56$, find AE .

Theorem 8-5 states that the diagonals of a parallelogram bisect each other. Therefore, $\overline{AE} \cong \overline{EC}$ or $AE = \frac{1}{2}(AC)$.

$$\begin{aligned}AE &= \frac{1}{2}(AC) && \text{Definition of bisect} \\ AE &= \frac{1}{2}(56) \text{ or } 28 && \text{Replace } AC \text{ with } 56.\end{aligned}$$



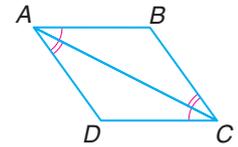
Your Turn

- If $DE = 11$, find DB .

Look Back

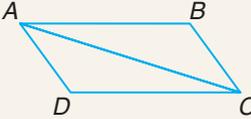
Alternate Interior Angles: Lesson 4–2;
ASA: Lesson 5–6

A diagonal separates a parallelogram into two triangles. You can use the properties of parallel lines to find the relationship between the two triangles. Consider $\square ABCD$ with diagonal \overline{AC} .



- $\overline{DC} \parallel \overline{AB}$ and $\overline{AD} \parallel \overline{BC}$ *Definition of parallelogram*
- $\angle ACD \cong \angle CAB$ and $\angle CAD \cong \angle ACB$ *If two parallel lines are cut by a transversal, alternate interior angles are congruent.*
- $\overline{AC} \cong \overline{AC}$ *Reflexive Property*
- $\triangle ACD \cong \triangle CAB$ *ASA*

This property of the diagonal is illustrated in the following theorem.

Theorem 8–6	Words: A diagonal of a parallelogram separates it into two congruent triangles.
	Model:  Symbols: $\triangle ACD \cong \triangle CAB$

Check for Understanding

Communicating Mathematics

- Name five properties that all parallelograms have.
- Draw parallelogram $MEND$ with diagonals MN and DE intersecting at X . Name four pairs of congruent segments.
- You Decide?** Karen and Tai know that the measure of one angle of a parallelogram is 50° . Karen thinks that she can find the measures of the remaining three angles without a protractor. Tai thinks that is not possible. Who is correct? Explain your reasoning.

Vocabulary

parallelogram

Guided Practice

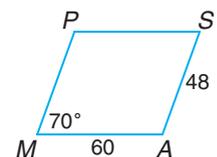
Examples 1–3

Example 4

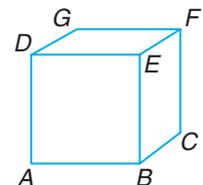
Example 1

Find each measure.

- $m\angle S$
- MP
- Suppose the diagonals of $\square MPSA$ intersect at point T . If $MT = 15$, find MS .
- Three parallelograms are used to produce a three-dimensional view of a cube. Name all of the segments that are parallel to the given segment.
 - \overline{AB}
 - \overline{BE}
 - \overline{DG}



Exercises 4–8



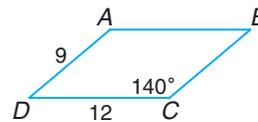
Exercises

Practice

Homework Help	
For Exercises	See Examples
10–23	1–3
24–28	4
Extra Practice	
See page 740.	

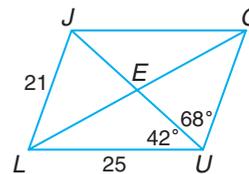
Find each measure.

10. $m\angle A$ 11. $m\angle B$
 12. AB 13. BC



In the figure, $OE = 19$ and $EU = 12$. Find each measure.

14. LE 15. JO
 16. $m\angle OUL$ 17. $m\angle OJL$
 18. $m\angle JLU$ 19. EJ
 20. OL 21. JU

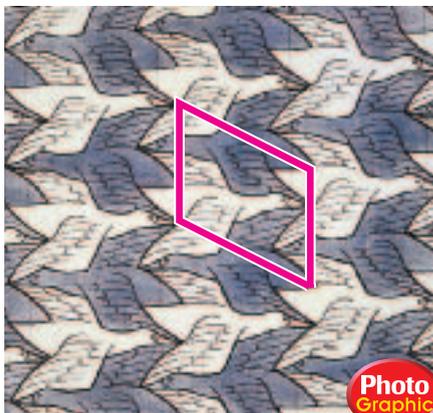


22. In a parallelogram, the measure of one side is 7. Find the measure of the opposite side.
 23. The measure of one angle of a parallelogram is 35. Determine the measures of the other three angles.

Determine whether each statement is *true* or *false*.

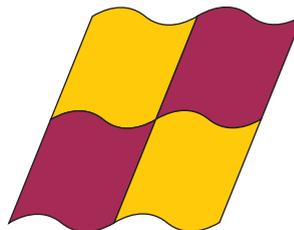
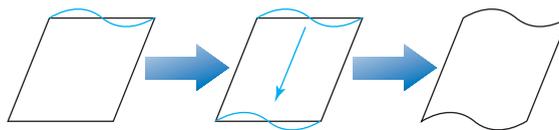
24. The diagonals of a parallelogram are congruent.
 25. In a parallelogram, when one diagonal is drawn, two congruent triangles are formed.
 26. If the length of one side of a parallelogram is known, the lengths of the other three sides can be found without measuring.

Applications and Problem Solving



M. C. Escher, *Study of Regular Division of the Plane with Birds*

27. **Art** The Escher design below is based on a parallelogram. You can use a parallelogram to make a simple Escher-like drawing. Change one side of the parallelogram and then slide the change to the opposite side. The resulting figure is used to make a design with different colors and textures.



Make your own Escher-like drawing.

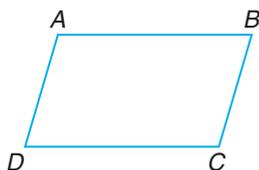
What You'll Learn

You'll learn to identify and use tests to show that a quadrilateral is a parallelogram.

Why It's Important

Crafts Quilters often use parallelograms when designing their quilts. See Exercise 17.

Theorem 8-3 states that the opposite sides of a parallelogram are congruent. Is the converse of this theorem true? In the figure below, \overline{AB} is congruent to \overline{DC} and \overline{AD} is congruent to \overline{BC} .



You know that a parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. If the opposite sides of a quadrilateral are congruent, then is it a parallelogram?

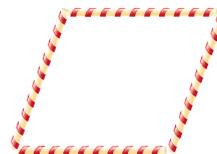
In the following activity, you will discover other ways to show that a quadrilateral is a parallelogram.

Hands-On Geometry

Materials:  straws  scissors  pipe cleaners
 ruler

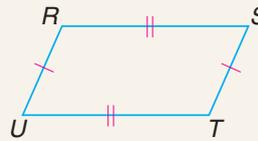
Step 1 Cut two straws to one length and two straws to a different length.

Step 2 Insert a pipe cleaner in one end of each straw. Connect the pipe cleaners at the ends to form a quadrilateral.

**Try These**

- How do the measures of opposite sides compare?
- Measure the distance between the top and bottom straws in at least three places. Then measure the distance between the left and right straws in at least three places. What seems to be true about the opposite sides?
- Shift the position of the sides to form another quadrilateral. Repeat Exercises 1 and 2.
- What type of quadrilateral have you formed? Explain your reasoning.

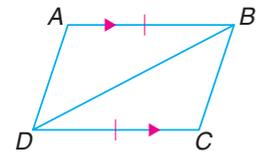
This activity leads to Theorem 8-7, which is related to Theorem 8-3.

Theorem 8-7**Words:** If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.**Model:****Symbols:** $\overline{RS} \cong \overline{UT}$,
 $\overline{RU} \cong \overline{ST}$

You can use the properties of congruent triangles and Theorem 8-7 to find other ways to show that a quadrilateral is a parallelogram.

Example**1**

In quadrilateral $ABCD$, with diagonal BD , $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \cong \overline{CD}$. Show that $ABCD$ is a parallelogram.



Explore You know $\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$. You want to show that $ABCD$ is a parallelogram.

Plan One way to show $ABCD$ is a parallelogram is to show $\overline{AD} \cong \overline{CB}$. You can do this by showing $\triangle ABD \cong \triangle CDB$. Make a list of statements and their reasons.

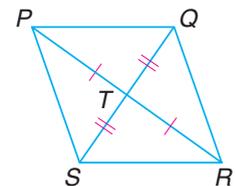
- Solve**
- $\angle ABD \cong \angle CDB$ *If two \parallel lines are cut by a transversal, then each pair of alternate interior angles is \cong .*
 - $\overline{BD} \cong \overline{BD}$ *Reflexive Property*
 - $\overline{AB} \cong \overline{CD}$ *Given*
 - $\triangle ABD \cong \triangle CDB$ *SAS*
 - $\overline{AD} \cong \overline{CB}$ *CPCTC*
 - $ABCD$ is a parallelogram. *Theorem 8-7*

Look Back

Alternate Interior Angles: Lesson 4-2

Your Turn

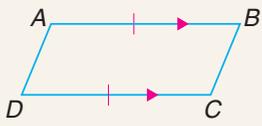
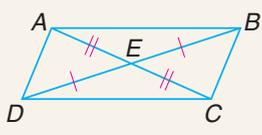
In quadrilateral $PQRS$, \overline{PR} and \overline{QS} bisect each other at T . Show that $PQRS$ is a parallelogram by providing a reason for each step.



- $\overline{PT} \cong \overline{TR}$ and $\overline{QT} \cong \overline{TS}$
- $\angle PTQ \cong \angle RTS$ and $\angle STP \cong \angle QTR$
- $\triangle PQT \cong \triangle RST$ and $\triangle PTS \cong \triangle RTQ$
- $\overline{PQ} \cong \overline{RS}$ and $\overline{PS} \cong \overline{RQ}$
- $PQRS$ is a parallelogram.

These examples lead to Theorems 8-8 and 8-9.

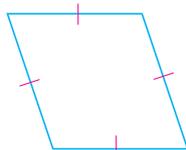


Theorem	Words	Models and Symbols
8-8	If one pair of opposite sides of a quadrilateral is parallel and congruent, then the quadrilateral is a parallelogram.	 $\overline{AB} \cong \overline{DC}, \overline{AB} \parallel \overline{DC}$
8-9	If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	 $\overline{AE} \cong \overline{EC}, \overline{BE} \cong \overline{ED}$

Examples

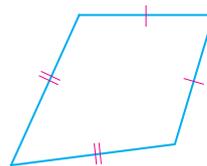
Determine whether each quadrilateral is a parallelogram. If the figure is a parallelogram, give a reason for your answer.

2



The figure has two pairs of opposite sides that are congruent. The figure is a parallelogram by Theorem 8-7.

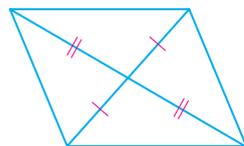
3



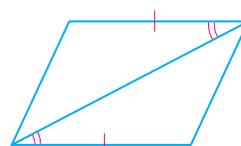
The figure has two pairs of congruent sides, but they are *not* opposite sides. The figure is *not* a parallelogram.

Your Turn

f.



g.



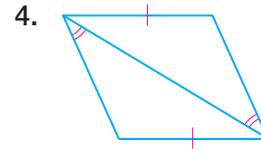
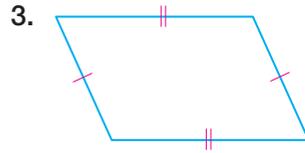
Check for Understanding

Communicating Mathematics

- Draw a quadrilateral that meets each set of conditions and is *not* a parallelogram.
 - one pair of parallel sides
 - one pair of congruent sides
 - one pair of congruent sides and one pair of parallel sides
- Writing Math** List four methods you can use to determine whether a quadrilateral is a parallelogram.

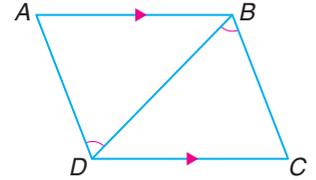
Guided Practice

Examples 2 & 3



Example 1

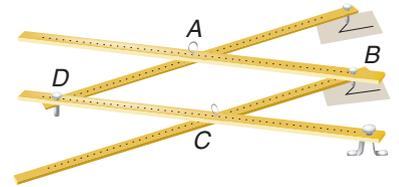
5. In quadrilateral $ABCD$, $\overline{BA} \parallel \overline{CD}$ and $\angle DBC \cong \angle BDA$. Show that quadrilateral $ABCD$ is a parallelogram by providing a reason for each step.



- $\overline{BC} \parallel \overline{AD}$
- $ABCD$ is a parallelogram.

Examples 2 & 3

6. In the figure, $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{DC}$. Which theorem shows that quadrilateral $ABCD$ is a parallelogram?

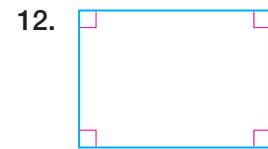
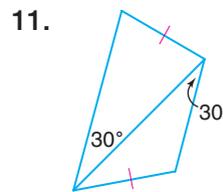
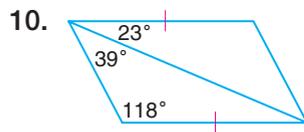
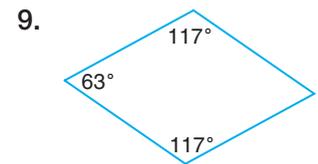
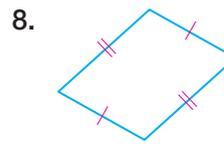
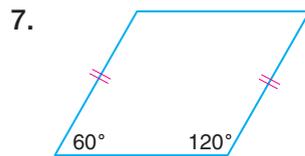


Exercises

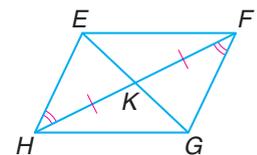
Practice

Determine whether each quadrilateral is a parallelogram. Write *yes* or *no*. If *yes*, give a reason for your answer.

Homework Help	
For Exercises	See Examples
7–12, 14–16	2, 3
13	1
17	2
Extra Practice	
See page 740.	

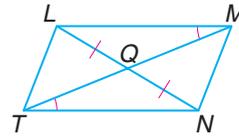


13. In quadrilateral $EFGH$, $\overline{HK} \cong \overline{KF}$ and $\angle KHE \cong \angle KFG$. Show that quadrilateral $EFGH$ is a parallelogram by providing a reason for each step.

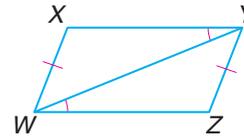


- $\angle EKH \cong \angle FKG$
- $\triangle EKH \cong \triangle FKG$
- $\overline{EH} \cong \overline{FG}$
- $\overline{EH} \parallel \overline{FG}$
- $EFGH$ is a parallelogram.

14. Explain why quadrilateral $LMNT$ is a parallelogram. Support your explanation with reasons as shown in Exercise 13.

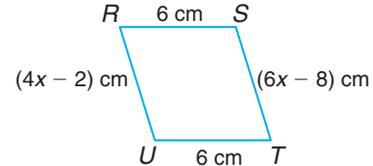


15. Determine whether quadrilateral $XYZW$ is a parallelogram. Give reasons for your answer.

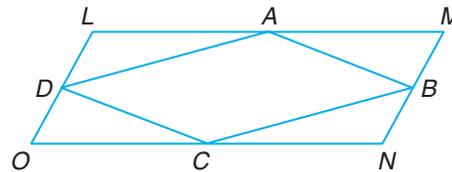


Applications and Problem Solving

16. **Algebra** Find the value for x that will make quadrilateral $RSTU$ a parallelogram.



17. **Quilting** Faith Ringgold is an African-American fabric artist. She used parallelograms in the design of the quilt at the left. What characteristics of parallelograms make it easy to use them in quilts?
18. **Critical Thinking** Quadrilateral $LMNO$ is a parallelogram. Points A , B , C , and D are midpoints of the sides. Is $ABCD$ a parallelogram? Explain your reasoning.



Faith Ringgold, #4 *The Sunflowers Quilting Bee at Arles*

Mixed Review

In $\square ABCD$, $m\angle D = 62$ and $CD = 45$. Find each measure. (*Lesson 8-2*)

19. $m\angle B$ 20. $m\angle C$ 21. AB

22. **Drawing** Use a straightedge and protractor to draw a quadrilateral with exactly two obtuse angles. (*Lesson 8-1*)

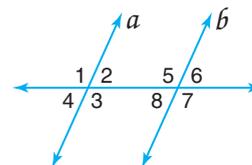
23. Find the length of the hypotenuse of a right triangle whose legs are 7 inches and 24 inches. (*Lesson 6-6*)

Standardized Test Practice

A B C D

24. **Grid In** In order to “curve” a set of test scores, a teacher uses the equation $g = 2.5p + 10$, where g is the curved test score and p is the number of problems answered correctly. How many points is each problem worth? (*Lesson 4-6*)

25. **Short Response** Name two different pairs of angles that, if congruent, can be used to prove $a \parallel b$. Explain your reasoning. (*Lesson 4-4*)



8-4

Rectangles, Rhombi, and Squares

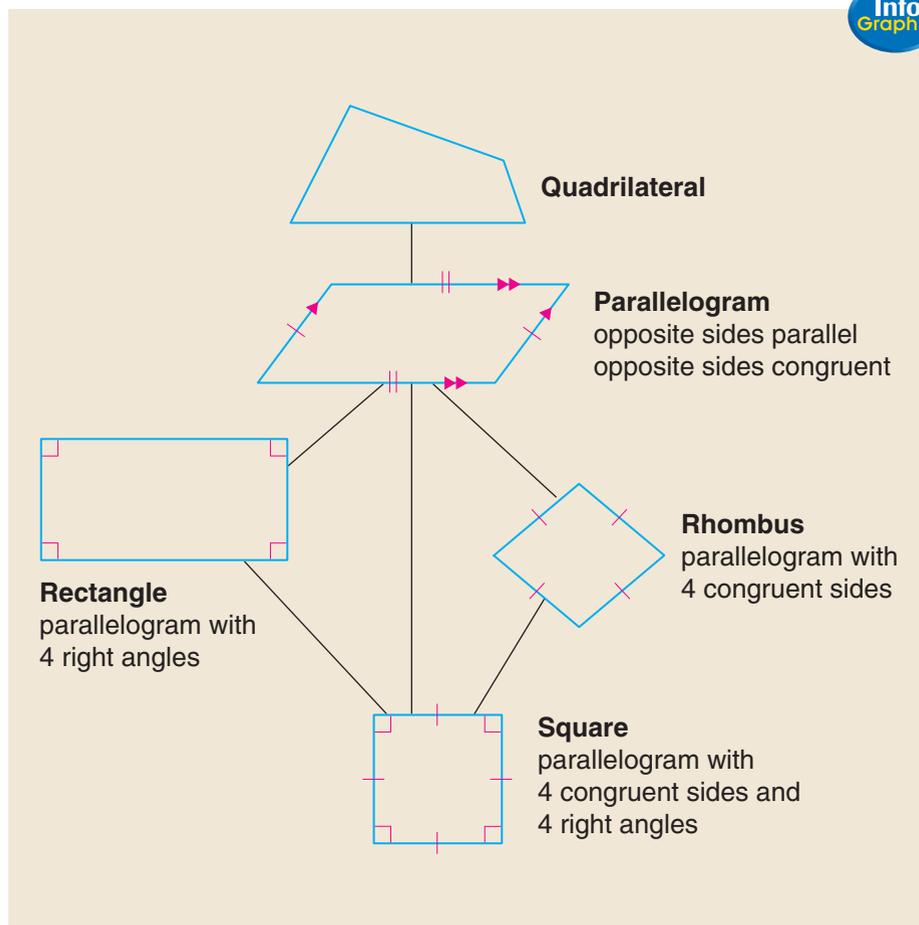
What You'll Learn

You'll learn to identify and use the properties of rectangles, rhombi, and squares.

Why It's Important

Carpentry Carpenters use the properties of rectangles when they build rectangular decks. See Exercise 46.

In previous lessons, you studied the properties of quadrilaterals and parallelograms. Now you will learn the properties of three other special types of quadrilaterals: **rectangles**, **rhombi**, and **squares**. The following diagram shows how these quadrilaterals are related.



Notice how the diagram goes from the most general quadrilateral to the most specific one. Any four-sided figure is a quadrilateral. But a parallelogram is a special quadrilateral whose opposite sides are parallel. The opposite sides of a square are parallel, so a square is a parallelogram. In addition, the four angles of a square are right angles, and all four sides are equal. A rectangle is also a parallelogram with four right angles, but its four sides are not equal.

Both squares and rectangles are special types of parallelograms. The best description of a quadrilateral is the one that is the most specific.

Example

Art Link

1 Identify the parallelogram that is outlined in the painting at the right.

Parallelogram $ABCD$ has four right angles, but the four sides are not congruent. It is a rectangle.

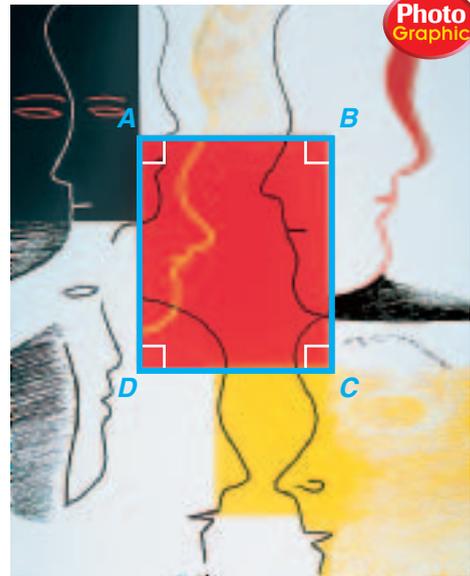
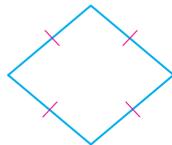


Photo Graphic

Diana Ong, *Blue, Red, and Yellow Faces*

Your Turn

a. Identify the parallelogram.



Reading Geometry

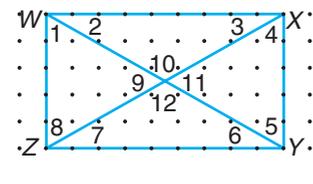
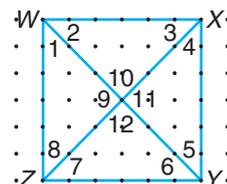
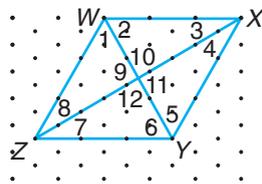
Rhombi is the plural of rhombus.

Rectangles, rhombi, and squares have all of the properties of parallelograms. In addition, they have their own properties.

Hands-On Geometry

Materials: dot paper ruler protractor

Step 1 Draw a rhombus on isometric dot paper. Draw a square and a rectangle on rectangular dot paper. Label each figure as shown below.



Step 2 Measure \overline{WY} and \overline{XZ} for each figure.

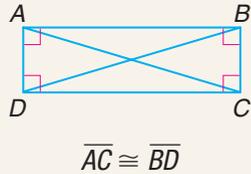
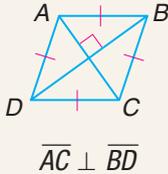
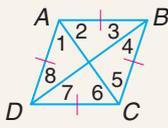
Step 3 Measure $\angle 9$, $\angle 10$, $\angle 11$, and $\angle 12$ for each figure.

Step 4 Measure $\angle 1$ through $\angle 8$ for each figure.

Try These

1. For which figures are the diagonals congruent?
2. For which figures are the diagonals perpendicular?
3. For which figures do the diagonals bisect a pair of opposite angles?

The results of the previous activity can be summarized in the following theorems.

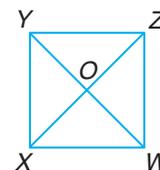
Theorem	Words	Models and Symbols
8-10	The diagonals of a rectangle are congruent.	 $\overline{AC} \cong \overline{BD}$
8-11	The diagonals of a rhombus are perpendicular.	 $\overline{AC} \perp \overline{BD}$
8-12	Each diagonal of a rhombus bisects a pair of opposite angles.	 $m\angle 1 = m\angle 2, m\angle 3 = m\angle 4,$ $m\angle 5 = m\angle 6, m\angle 7 = m\angle 8$

A square is defined as a parallelogram with four congruent angles and four congruent sides. This means that a square is not only a parallelogram, but also a rectangle and a rhombus. Therefore, all of the properties of parallelograms, rectangles, and rhombi hold true for squares.

Examples

- 2** Find XZ in square $XYZW$ if $YW = 14$.

A square has all of the properties of a rectangle, and the diagonals of a rectangle are congruent. So, \overline{XZ} is congruent to \overline{YW} , and $XZ = 14$.



- 3** Find $m\angle YOX$ in square $XYZW$.

A square has all the properties of a rhombus, and the diagonals of a rhombus are perpendicular. Therefore, $m\angle YOX = 90$.

Your Turn

- Name all segments that are congruent to \overline{WO} in square $XYZW$. Explain your reasoning.
- Name all the angles that are congruent to $\angle XYO$ in square $XYZW$. Explain your reasoning.

Check for Understanding

Communicating Mathematics

1. Draw a quadrilateral that is a rhombus but not a rectangle.
2. Compare and contrast the definitions of rectangles and squares.
3. **You? Decide?** Eduardo says that every rhombus is a square. Teisha says that every square is a rhombus. Who is correct? Explain your reasoning.

Vocabulary

rectangle
rhombus
square

Guided Practice

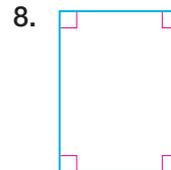
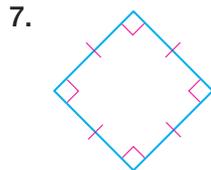
Getting Ready Which quadrilaterals have each property?

Sample: All angles are right angles. **Solution:** square, rectangle

4. The opposite angles are congruent.
5. The opposite sides are congruent.
6. All sides are congruent.

Example 1

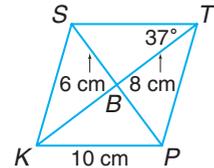
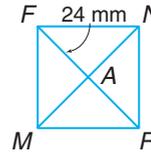
Identify each parallelogram as a *rectangle*, *rhombus*, *square*, or *none of these*.



Examples 2 & 3

Use square *FNRM* or rhombus *STPK* to find each measure.

9. AR
10. MA
11. $m\angle FAN$
12. TP
13. PB
14. $m\angle KTP$



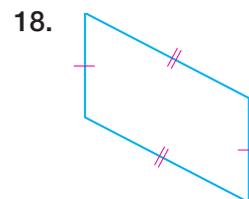
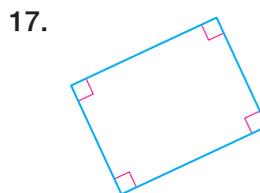
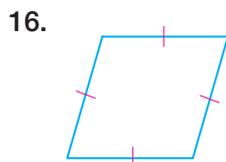
Example 1

15. **Sports** Basketball is played on a court that is shaped like a rectangle. Name two other sports that are played on a rectangular surface and two sports that are played on a surface that is not rectangular.

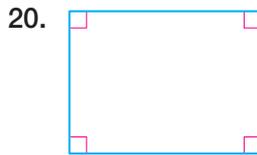
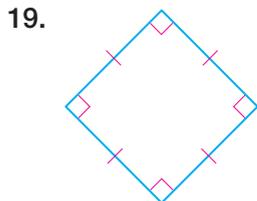
Exercises

Practice

Identify each parallelogram as a *rectangle*, *rhombus*, *square*, or *none of these*.

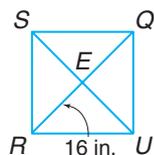


Homework Help	
For Exercises	See Examples
16–21	1
22–38	2, 3
Extra Practice	
See page 740.	

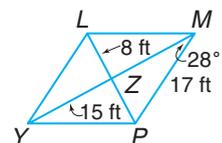


Use square **SQUR** or rhombus **LMPY** to find each measure.

- | | |
|-------------------|-------------------|
| 22. EQ | 23. EU |
| 24. SU | 25. RQ |
| 26. $m\angle SEQ$ | 27. $m\angle SQU$ |
| 28. $m\angle SQE$ | 29. $m\angle RUE$ |
| 30. ZP | 31. YM |
| 32. $m\angle LMP$ | 33. $m\angle MLY$ |
| 34. $m\angle YZP$ | 35. YL |
| 36. YP | 37. $m\angle LPM$ |



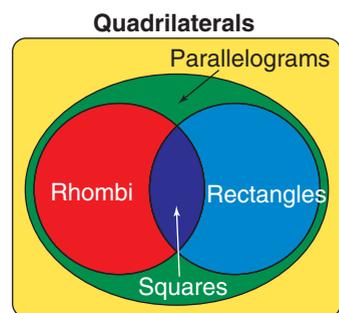
Exercises 22–29



Exercises 30–37

38. Which quadrilaterals have diagonals that are perpendicular?

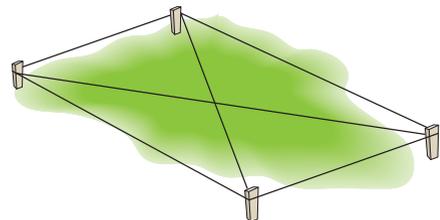
The Venn diagram shows relationships among some quadrilaterals. Use the Venn diagram to determine whether each statement is **true** or **false**.



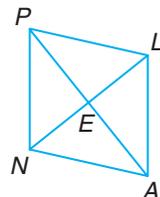
39. Every square is a rhombus.
 40. Every rhombus is a square.
 41. Every rectangle is a square.
 42. Every square is a rectangle.
 43. All rhombi are parallelograms.
 44. Every parallelogram is a rectangle.

Applications and Problem Solving

45. **Algebra** The diagonals of a square are $(x + 8)$ feet and $3x$ feet. Find the measure of the diagonals.
46. **Carpentry** A carpenter is starting to build a rectangular deck. He has laid out the deck and marked the corners, making sure that the two longer lengths are congruent, the two shorter lengths are congruent, and the corners form right angles. In addition, he measures the diagonals. Which theorem guarantees that the diagonals are congruent?



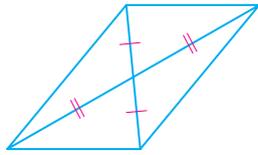
47. **Critical Thinking** Refer to rhombus $PLAN$.
- Classify $\triangle PLA$ by its sides.
 - Classify $\triangle PEN$ by its angles.
 - Is $\triangle PEN \cong \triangle AEL$? Explain your reasoning.



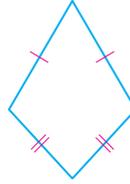
Mixed Review

Determine whether each quadrilateral is a parallelogram. State yes or no. If yes, give a reason for your answer. (Lesson 8-3)

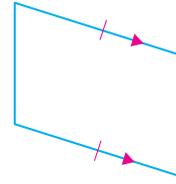
48.



49.



50.



Determine whether each statement is true or false. (Lesson 8-2)

51. If the measure of one angle of a parallelogram is known, the measures of the other three angles can be found without using a protractor.
52. The diagonals of every parallelogram are congruent.
53. The consecutive angles of a parallelogram are complementary.
54. **Extended Response** Write the converse of this statement. (Lesson 1-4) *If a figure is a rectangle, then it has four sides.*

55. **Multiple Choice** If x represents the number of homes with televisions in Dallas, which expression represents the number of homes with televisions in Atlanta?

- (A) $x - 221$
- (B) $x + 221$
- (C) $x - 2035$
- (D) $x + 2035$

Homes with Televisions (thousands)

Area	Homes
New York	7376
Los Angeles	5402
Chicago	3399
Philadelphia	2874
San Francisco	2441
Boston	2392
Dallas-Ft. Worth	2256
Washington, DC	2224
Atlanta	2035
Detroit	1923

Source: Nielsen Media Research

Standardized Test Practice

(A) (B) (C) (D)

Data Update For the latest information on homes with televisions, visit:

www.geomconcepts.com

Quiz 2

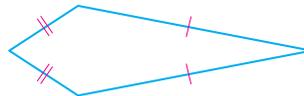
Lessons 8-3 and 8-4

Determine whether each quadrilateral is a parallelogram. State yes or no. If yes, give a reason for your answer. (Lesson 8-3)

1.

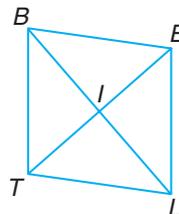


2.



Refer to rhombus *BTLE*. (Lesson 8-4)

3. Name all angles that are congruent to $\angle BIE$.
4. Name all segments congruent to \overline{IE} .
5. Name all measures equal to BE .



8-5

Trapezoids

What You'll Learn

You'll learn to identify and use the properties of trapezoids and isosceles trapezoids.

Why It's Important

Art Trapezoids are used in perspective drawings. See Example 1.

Many state flags use geometric shapes in their designs. Can you find a quadrilateral in the Maryland state flag that has exactly one pair of parallel sides?



Maryland state flag

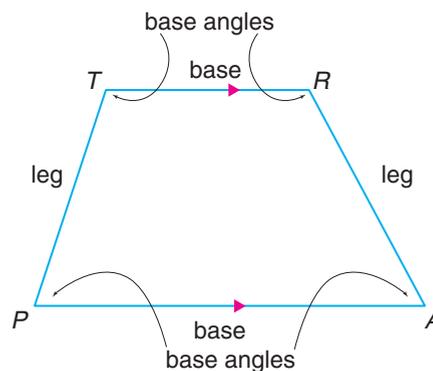
A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. The parallel sides are called **bases**. The nonparallel sides are called **legs**.

Study trapezoid $TRAP$.

$\overline{TR} \parallel \overline{PA}$ \overline{TR} and \overline{PA}
are the bases.

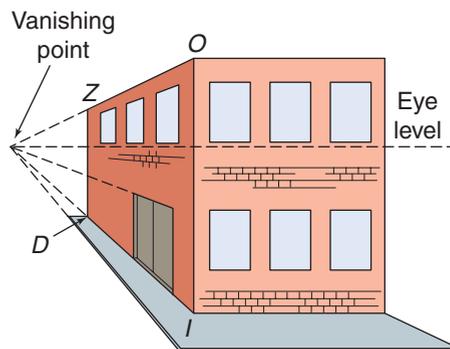
$\overline{TP} \not\parallel \overline{RA}$ \overline{TP} and \overline{RA}
are the legs.

Each trapezoid has two pairs of **base angles**. In trapezoid $TRAP$, $\angle T$ and $\angle R$ are one pair of base angles; $\angle P$ and $\angle A$ are the other pair.



Example Art Link

1 Artists use *perspective* to give the illusion of depth to their drawings. In perspective drawings, vertical lines remain parallel, but horizontal lines gradually come together at a point. In trapezoid $ZOID$, name the bases, the legs, and the base angles.



Bases \overline{ZD} and \overline{OI} are parallel segments.

Legs \overline{ZO} and \overline{DI} are nonparallel segments.

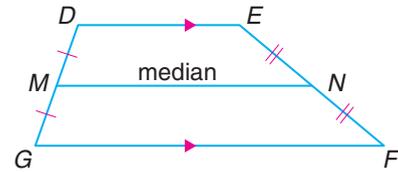
Base Angles $\angle Z$ and $\angle D$ are one pair of base angles;
 $\angle O$ and $\angle I$ are the other pair.



Reading Geometry

Another name for the median of a trapezoid is the **midsegment** of the trapezoid.

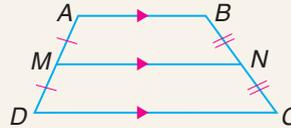
The **median** of a trapezoid is the segment that joins the midpoints of its legs. In the figure, \overline{MN} is the median.



Theorem 8-13

Words: The median of a trapezoid is parallel to the bases, and the length of the median equals one-half the sum of the lengths of the bases.

Model:



Symbols:

$$\overline{AB} \parallel \overline{MN}, \overline{DC} \parallel \overline{MN}$$

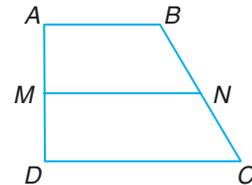
$$MN = \frac{1}{2}(AB + DC)$$

Example

2

Find the length of median MN in trapezoid $ABCD$ if $AB = 12$ and $DC = 18$.

$$\begin{aligned} MN &= \frac{1}{2}(AB + DC) && \text{Theorem 8-13} \\ &= \frac{1}{2}(12 + 18) && \text{Replace } AB \text{ with } 12 \\ & && \text{and } DC \text{ with } 18. \\ &= \frac{1}{2}(30) \text{ or } 15 && \text{Simplify.} \end{aligned}$$



The length of the median of trapezoid $ABCD$ is 15 units.

Your Turn

- a. Find the length of median MN in trapezoid $ABCD$ if $AB = 20$ and $DC = 16$.

Look Back

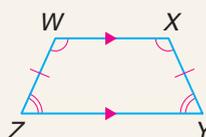
Isosceles Triangle:
Lesson 6-4

If the legs of a trapezoid are congruent, the trapezoid is an **isosceles trapezoid**. In Lesson 6-4, you learned that the base angles of an isosceles triangle are congruent. There is a similar property for isosceles trapezoids.

Theorem 8-14

Words: Each pair of base angles in an isosceles trapezoid is congruent.

Model:

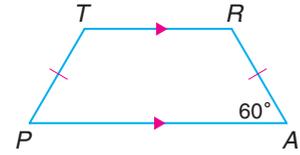


Symbols:

$$\angle W \cong \angle X, \angle Z \cong \angle Y$$

Example**3**

Find the missing angle measures in isosceles trapezoid $TRAP$.



Find $m\angle P$.

$$\angle P \cong \angle A \quad \text{Theorem 8-14}$$

$$m\angle P = m\angle A \quad \text{Definition of congruent}$$

$$m\angle P = 60 \quad \text{Replace } m\angle A \text{ with } 60.$$

Find $m\angle T$. Since $TRAP$ is a trapezoid, $\overline{TR} \parallel \overline{PA}$.

$$m\angle T + m\angle P = 180 \quad \text{Consecutive interior angles are supplementary.}$$

$$m\angle T + 60 = 180 \quad \text{Replace } m\angle P \text{ with } 60.$$

$$m\angle T + 60 - 60 = 180 - 60 \quad \text{Subtract } 60 \text{ from each side.}$$

$$m\angle T = 120 \quad \text{Simplify.}$$

Find $m\angle R$.

$$\angle R \cong \angle T \quad \text{Theorem 8-14}$$

$$m\angle R = m\angle T \quad \text{Definition of congruent}$$

$$m\angle R = 120 \quad \text{Replace } m\angle T \text{ with } 120.$$

Look Back

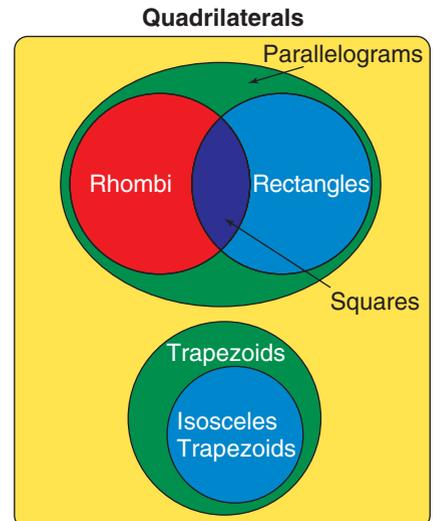
Consecutive Interior
Angles:
Lesson 4-2

Your Turn

- b. The measure of one angle in an isosceles trapezoid is 48. Find the measures of the other three angles.

In this chapter, you have studied quadrilaterals, parallelograms, rectangles, rhombi, squares, trapezoids, and isosceles trapezoids. The Venn diagram illustrates how these figures are related.

- The Venn diagram represents all quadrilaterals.
- Parallelograms and trapezoids do not share any characteristics except that they are both quadrilaterals. This is shown by the nonoverlapping regions in the Venn diagram.
- Every isosceles trapezoid is a trapezoid. In the Venn diagram, this is shown by the set of isosceles trapezoids contained in the set of trapezoids.
- All rectangles and rhombi are parallelograms. Since a square is both a rectangle and a rhombus, it is shown by overlapping regions.



Check for Understanding

Communicating Mathematics

1. Draw an isosceles trapezoid and label the legs and the bases.
2. Explain how the length of the median of a trapezoid is related to the lengths of the bases.
3. **Writing Math** Copy and complete the following table. Write *yes* or *no* to indicate whether each quadrilateral always has the given characteristics.

Vocabulary

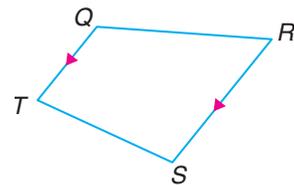
trapezoid
bases
legs
base angles
median
isosceles trapezoid

Characteristics	Parallelogram	Rectangle	Rhombus	Square	Trapezoid
Opposite sides are parallel.					
Opposite sides are congruent.					
Opposite angles are congruent.					
Consecutive angles are supplementary.					
Diagonals bisect each other.					
Diagonals are congruent.					
Diagonals are perpendicular.					
Each diagonal bisects two angles.					

Guided Practice

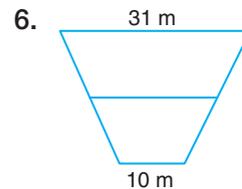
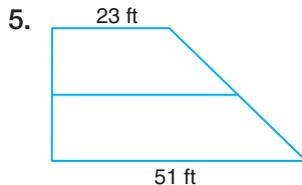
Example 1

4. In trapezoid $QRST$, name the bases, the legs, and the base angles.



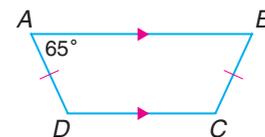
Example 2

- Find the length of the median in each trapezoid.



Example 3

7. Trapezoid $ABCD$ is isosceles. Find the missing angle measures.



Example 3

8. **Construction** A hip roof slopes at the ends of the building as well as the front and back. The front of this hip roof is in the shape of an isosceles trapezoid. If one angle measures 30° , find the measures of the other three angles.



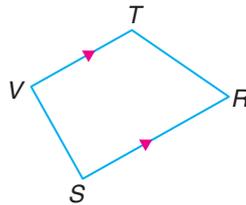
Exercises

Practice

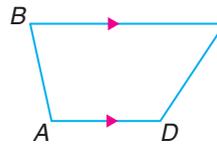
For each trapezoid, name the bases, the legs, and the base angles.

Homework Help	
For Exercises	See Examples
8–20, 22	3
9–11, 29	1
12–17, 21, 30	2
Extra Practice	
See page 741.	

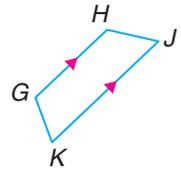
9.



10.

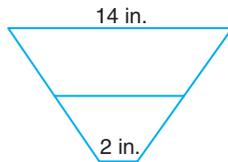


11.

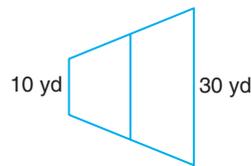


Find the length of the median in each trapezoid.

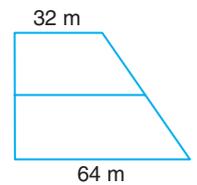
12.



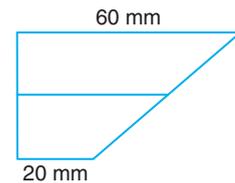
13.



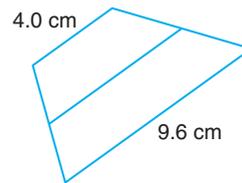
14.



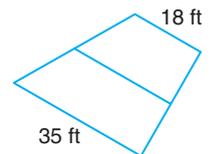
15.



16.

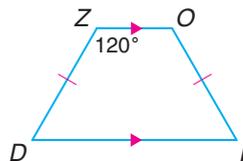


17.

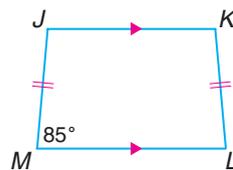


Find the missing angle measures in each isosceles trapezoid.

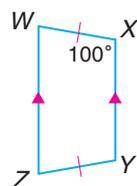
18.



19.



20.



21. Find the length of the shorter base of a trapezoid if the length of the median is 34 meters and the length of the longer base is 49 meters.
22. One base angle of an isosceles trapezoid is 45° . Find the measures of the other three angles.

Determine whether it is possible for a trapezoid to have the following conditions. Write yes or no. If yes, draw the trapezoid.

23. three congruent sides
24. congruent bases
25. four acute angles
26. two right angles
27. one leg longer than either base
28. two congruent sides, but not isosceles

Applications and Problem Solving

29. **Bridges** Explain why the figure outlined on the Golden Gate Bridge is a trapezoid.



Photo Graphic

30. **Algebra** If the sum of the measures of the bases of a trapezoid is $4x$, find the measure of the median.
31. **Critical Thinking** A sequence of trapezoids is shown. The first three trapezoids in the sequence are formed by 3, 5, and 7 triangles.



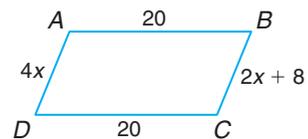
- a. How many triangles are needed for the 10th trapezoid?
- b. How many triangles are needed for the n th trapezoid?

Mixed Review

Name all quadrilaterals that have each property. (Lesson 8-4)

32. four right angles
33. congruent diagonals

34. **Algebra** Find the value for x that will make quadrilateral $ABCD$ a parallelogram. (Lesson 8-3)



Standardized Test Practice

- (A) (B) (C) (D)

35. **Extended Response** Draw and label a figure to illustrate that \overline{JN} and \overline{LM} are medians of $\triangle JKL$ and intersect at I . (Lesson 6-1)

36. **Multiple Choice** In the figure, $AC = 60$, $CD = 12$, and B is the midpoint of \overline{AD} . Choose the correct statement. (Lesson 2-5)

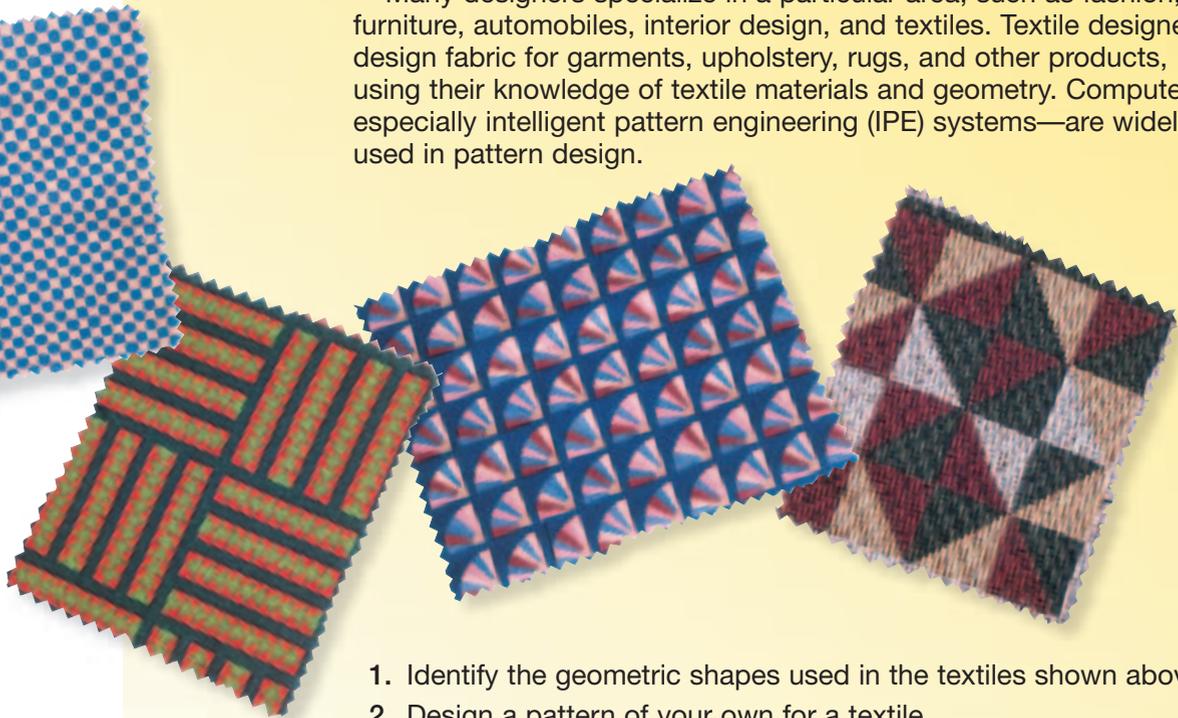


- (A) $BC > CD$
- (B) $BC < CD$
- (C) $BC = CD$
- (D) There is not enough information.

Designer

Are you creative? Do you find yourself sketching designs for new cars or the latest fashion trends? Then you may like a career as a designer. Designers organize and design products that are visually appealing and serve a specific purpose.

Many designers specialize in a particular area, such as fashion, furniture, automobiles, interior design, and textiles. Textile designers design fabric for garments, upholstery, rugs, and other products, using their knowledge of textile materials and geometry. Computers—especially intelligent pattern engineering (IPE) systems—are widely used in pattern design.



1. Identify the geometric shapes used in the textiles shown above.
2. Design a pattern of your own for a textile.

FAST FACTS About Fashion Designers

Working Conditions

- vary by places of employment
- overtime work sometimes required to meet deadlines
- keen competition for most jobs

Education

- a 2- or 4-year degree is usually needed
- computer-aided design (CAD) courses are very useful
- creativity is crucial

Earnings



Source: Bureau of Labor Statistics



Career Data For the latest information on a career as a designer, visit:

www.geomconcepts.com



Materials

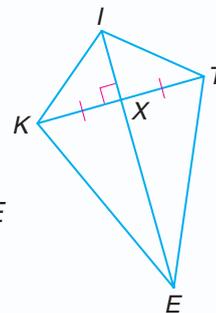
-  unlined paper
-  compass
-  straightedge
-  protractor
-  ruler

Kites

A kite is more than just a toy to fly on a windy day. In geometry, a **kite** is a special quadrilateral that has its own properties.

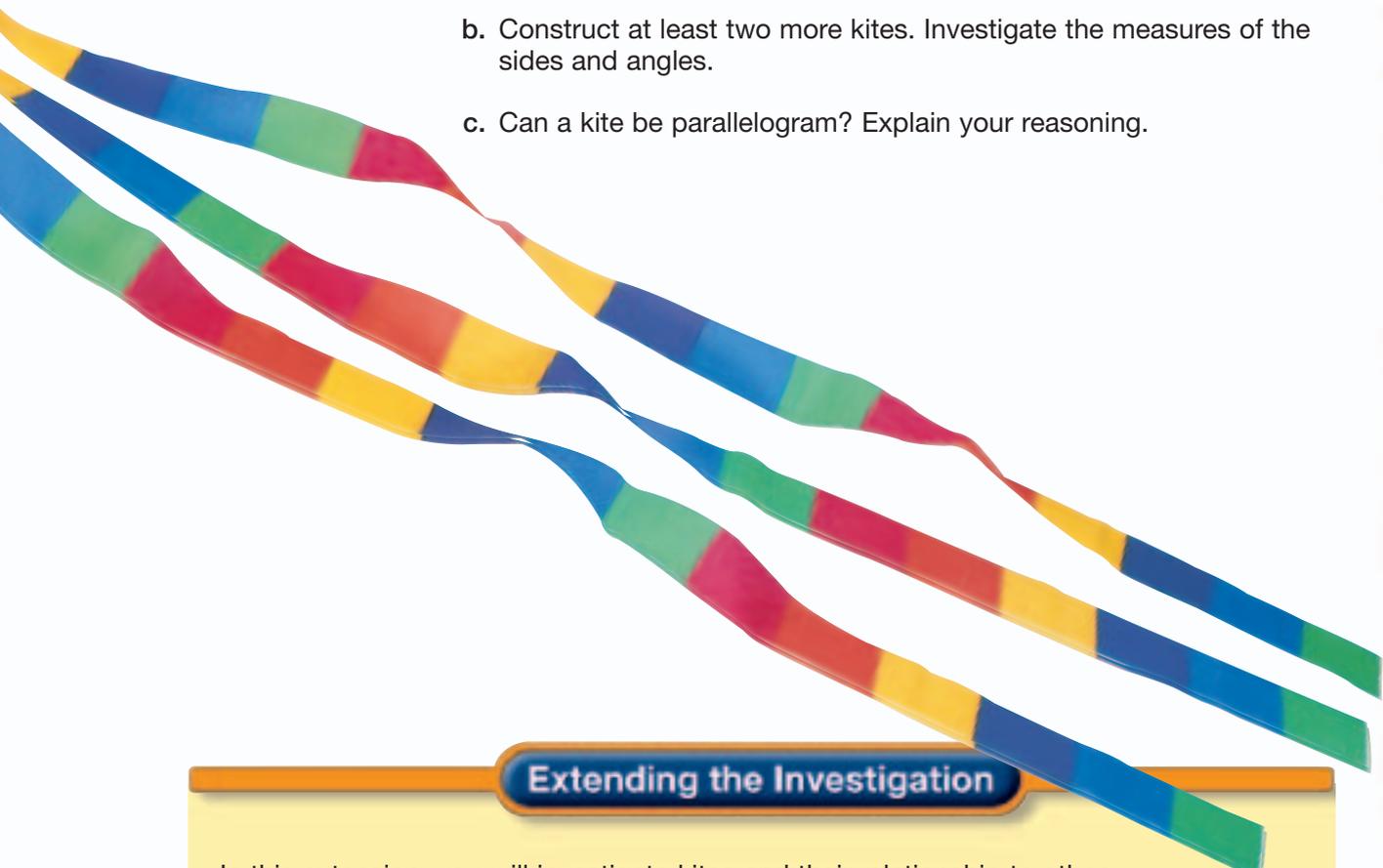
Investigate

1. Use paper, compass, and straightedge to construct a kite.
 - a. Draw a segment about six inches in length. Label the endpoints I and E . Mark a point on the segment. The point should *not* be the midpoint of \overline{IE} . Label the point X .
 - b. Construct a line that is perpendicular to \overline{IE} through X . Mark point K about two inches to the left of X on the perpendicular line. Then mark another point, T , on the right side of X so that $\overline{KX} \cong \overline{XT}$.



- c. Connect points K , I , T , and E to form a quadrilateral. $KITE$ is a kite. Use a ruler to measure the lengths of the sides of $KITE$. What do you notice?
- d. Write a definition for a kite. Compare your definition with others in the class.



- 
2. Use compass, straightedge, protractor, and ruler to investigate kites.
 - a. Use a protractor to measure the angles of *KITE*. What do you notice about the measures of opposite and consecutive angles?
 - b. Construct at least two more kites. Investigate the measures of the sides and angles.
 - c. Can a kite be parallelogram? Explain your reasoning.

Extending the Investigation

In this extension, you will investigate kites and their relationship to other quadrilaterals. Here are some suggestions.

1. Rewrite Theorems 8–2 through 8–6 and 8–10 through 8–12 so they are true for kites.
2. Make a list of as many properties as possible for kites.
3. Build a kite using the properties you have studied.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a booklet showing the differences and similarities among the quadrilaterals you have studied. Be sure to include kites.
- Make a video about quadrilaterals. Cast your actors as the different quadrilaterals. The script should help viewers understand the properties of quadrilaterals.



Investigation For more information on kites, visit: www.geomconcepts.com



CHAPTER 8 Study Guide and Assessment

Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

Review Activities
For more review activities, visit:
www.geomconcepts.com

base angles (p. 333)
bases (p. 333)
consecutive (p. 311)
diagonals (p. 311)
isosceles trapezoid (p. 334)

kite (p. 340)
legs (p. 333)
median (p. 334)
midsegment (p. 334)
nonconsecutive (p. 311)

parallelogram (p. 316)
quadrilateral (p. 310)
rectangle (p. 327)
rhombus (p. 327)
square (p. 327)
trapezoid (p. 333)

Choose the term from the list above that best completes each statement.

- In Figure 1, $ACBD$ is best described as a(n) ____?
- In Figure 1, \overline{AB} is a(n) ____? of quadrilateral $ACBD$.
- Figure 2 is best described as a(n) ____?
- The parallel sides of a trapezoid are called ____?
- Figure 3 is best described as a(n) ____?
- Figure 4 is best described as a(n) ____?
- In Figure 4, $\angle M$ and $\angle N$ are ____?
- A(n) ____? is a quadrilateral with exactly one pair of parallel sides.
- A parallelogram with four congruent sides and four right angles is a(n) ____?
- The ____? of a trapezoid is the segment that joins the midpoints of each leg.

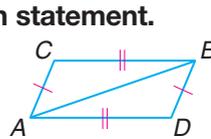


Figure 1

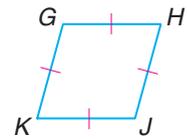


Figure 2



Figure 3

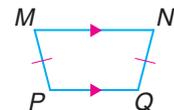


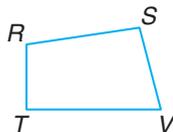
Figure 4

Skills and Concepts

Objectives and Examples

- Lesson 8-1** Identify parts of quadrilaterals and find the sum of the measures of the interior angles of a quadrilateral.

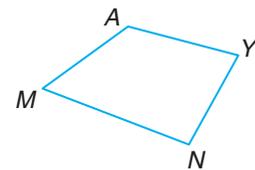
The following statements are true about quadrilateral $RSVT$.



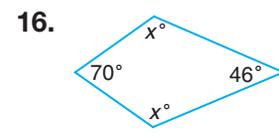
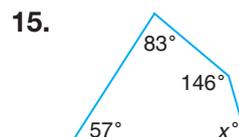
- \overline{RT} and \overline{TV} are consecutive sides.
- S and T are opposite vertices.
- The side opposite \overline{RS} is \overline{TV} .
- $\angle R$ and $\angle T$ are consecutive angles.
- $m\angle R + m\angle S + m\angle V + m\angle T = 360$

Review Exercises

- Name one pair of nonconsecutive sides.
- Name one pair of consecutive angles.
- Name the angle opposite $\angle M$.
- Name a side that is consecutive with \overline{AY} .



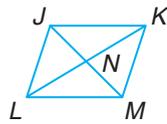
Find the missing measure(s) in each figure.



Objectives and Examples

- **Lesson 8–2** Identify and use the properties of parallelograms.

If $JKLM$ is a parallelogram, then the following statements can be made.



- | | |
|---|---|
| $\overline{JK} \parallel \overline{LM}$ | $\overline{JL} \parallel \overline{KM}$ |
| $\angle JLM \cong \angle JKM$ | $\angle LJK \cong \angle KML$ |
| $\overline{JK} \cong \overline{LM}$ | $\overline{JL} \cong \overline{KM}$ |
| $\overline{JN} \cong \overline{NM}$ | $\overline{LN} \cong \overline{NK}$ |
| $\triangle JLM \cong \triangle MKJ$ | $\triangle LJK \cong \triangle KML$ |
| $m\angle LJK + m\angle JKM = 180$ | |

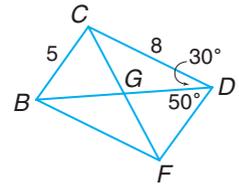
- **Lesson 8–3** Identify and use tests to show that a quadrilateral is a parallelogram.

You can use the following tests to show that a quadrilateral is a parallelogram.

- Theorem 8–7* Both pairs of opposite sides are congruent.
- Theorem 8–8* One pair of opposite sides is parallel and congruent.
- Theorem 8–9* The diagonals bisect each other.

Review Exercises

In the parallelogram, $CG = 4.5$ and $BD = 12$. Find each measure.

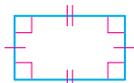


17. FD
18. BF
19. $m\angle CBF$
20. $m\angle BCD$
21. BG
22. GF

Exercises 17–22

23. In a parallelogram, the measure of one angle is 28. Determine the measures of the other angles.

- **Lesson 8–4** Identify and use the properties of rectangles, rhombi, and squares.



rectangle



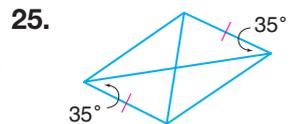
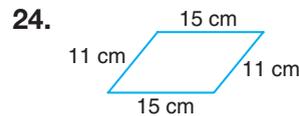
rhombus



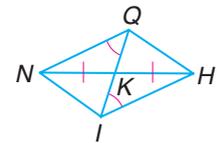
square

- Theorem 8–10* The diagonals of a rectangle are congruent.
- Theorem 8–11* The diagonals of a rhombus are perpendicular.
- Theorem 8–12* Each diagonal of a rhombus bisects a pair of opposite angles.

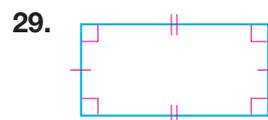
Determine whether each quadrilateral is a parallelogram. Write *yes* or *no*. If *yes*, give a reason for your answer.



26. In quadrilateral $QNIH$, $\angle NQI \cong \angle QIH$ and $\overline{NK} \cong \overline{KH}$. Explain why quadrilateral $QNIH$ is a parallelogram. Support your explanation with reasons.



Identify each parallelogram as a *rectangle*, *rhombus*, *square*, or *none of these*.



Objectives and Examples

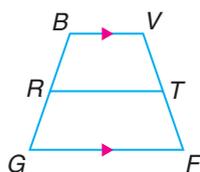
- **Lesson 8–5** Identify and use the properties of trapezoids and isosceles trapezoids.

If quadrilateral $BVFG$ is an isosceles trapezoid, and \overline{RT} is the median, then each is true.

$$\overline{BV} \parallel \overline{GF} \quad \overline{BG} \cong \overline{VF}$$

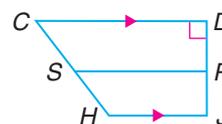
$$\angle G \cong \angle F \quad \angle B \cong \angle V$$

$$RT = \frac{1}{2}(BV + GF)$$



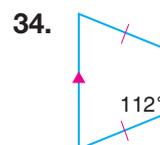
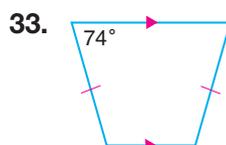
Review Exercises

- 31. Name the bases, legs, and base angles of trapezoid $CDJH$ where \overline{SP} is the median.
- 32. If $CD = 27$ yards and $HJ = 15$ yards, find SP .



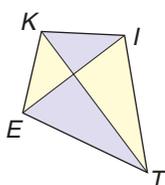
Exercises 33–34

Find the missing angle measures in each isosceles trapezoid.

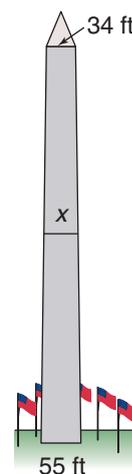


Applications and Problem Solving

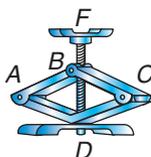
- 35. **Recreation** Diamond kites are one of the most popular kites to fly and to make because of their simple design. In the diamond kite, $m\angle K = 135$ and $m\angle T = 65$. The measure of the remaining two angles must be equal in order to ensure a diamond shape. Find $m\angle I$ and $m\angle E$. (Lesson 8–1)



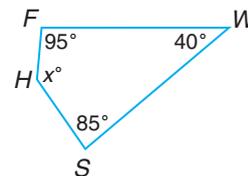
- 36. **Architecture** The Washington Monument is an *obelisk*, a large stone pillar that gradually tapers as it rises, ending with a pyramid on top. Each face of the monument under the pyramid is a trapezoid. The monument's base is about 55 feet wide, and the width at the top, just below the pyramid, is about 34 feet. How wide is the monument at its median? (Lesson 8–5)



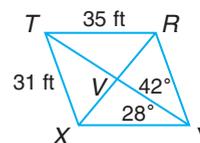
- 37. **Car Repair** To change a flat tire, a driver needs to use a device called a jack to raise the corner of the car. In the jack, $AB = BC = CD = DA$. Each of these metal pieces is attached by a hinge that allows it to pivot. Explain why nonconsecutive sides of the jack remain parallel as the tool is raised to point F . (Lesson 8–3)



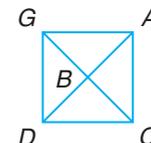
1. Name a diagonal in quadrilateral $FHSW$.
2. Name a side consecutive with \overline{SW} .
3. Find the measure of the missing angle in quadrilateral $FHSW$.
4. In $\square XTRY$, find XY and RY .
5. Name the angle that is opposite $\angle XYR$.
6. Find $m\angle XTR$.
7. Find $m\angle TRY$.
8. If $TV = 32$, find TY .
9. In square $GACD$, if $DA = 14$, find BC .
10. Find $m\angle DBC$.



Exercises 1–3

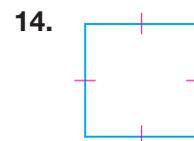
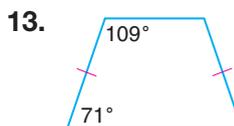
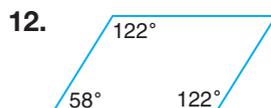
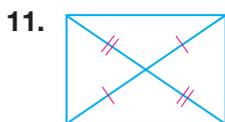


Exercises 4–8

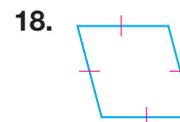
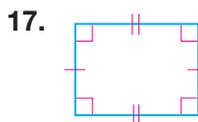
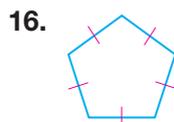
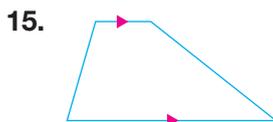


Exercises 9–10

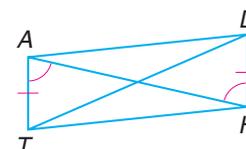
Determine whether each quadrilateral is a parallelogram. Write *yes* or *no*. If *yes*, give a reason for your answer.



Identify each figure as a *quadrilateral*, *parallelogram*, *rhombus*, *rectangle*, *square*, *trapezoid*, or *none of these*.



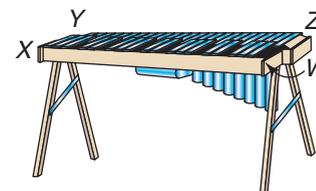
19. Determine whether quadrilateral $ADHT$ is a parallelogram. Support your answer with reasons.
20. In rhombus $WQTZ$, the measure of one side is 18 yards, and the measure of one angle is 57. Determine the measures of the other three sides and angles.
21. NP is the median of isosceles trapezoid $JKML$. If \overline{JK} and \overline{LM} are the bases, $JK = 24$, and $LM = 44$, find NP .



Exercise 19

Identify each statement as *true* or *false*.

22. All squares are rectangles.
23. All rhombi are squares.
24. **Music** A series of wooden bars of varying lengths are arranged in the shape of a quadrilateral to form an instrument called a xylophone. In the figure, $\overline{XY} \parallel \overline{WZ}$, but $\overline{XW} \not\parallel \overline{YZ}$. What is the best description of quadrilateral $WXYZ$?
25. **Algebra** Two sides of a rhombus measure $5x$ and $2x + 18$. Find x .



Coordinate Geometry Problems

Standardized tests often include problems that involve points on a coordinate grid. You'll need to identify the coordinates of points, calculate midpoints of segments, find the distance between points, and identify intercepts of lines and axes.

Be sure you understand these concepts.

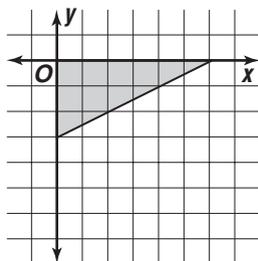
axis	coordinates	distance	intercept
line	midpoint	ordered pair	

Test-Taking Tip

If no drawing is provided, draw one to help you understand the problem. Label the drawing with the information given in the problem.

Example 1

In the figure at the right, which of the following points lies within the shaded region?



- Ⓐ $(-1, 1)$ Ⓑ $(1, -2)$
 Ⓒ $(4, 3)$ Ⓓ $(5, -4)$
 Ⓔ $(7, 0)$

Hint Try to eliminate impossible choices in multiple-choice questions.

Solution Notice that the shaded region lies in the quadrant where x is positive and y is negative. Look at the answer choices. Since x must be positive and y must be negative for a point within the region, you can eliminate choices A, C, and E.

Plot the remaining choices, B and D, on the grid. You will see that $(1, -2)$ is inside the region and $(5, -4)$ is not. So, the answer is B.

Example 2

A segment has endpoints at $P(-2, 6)$ and $Q(6, 2)$.

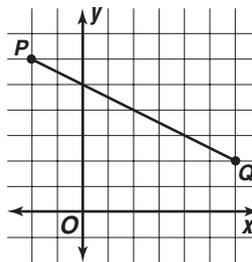
Part A Draw segment PQ .

Part B Explain how you know whether the midpoint of segment PQ is the same as the y -intercept of segment PQ .

Hint You may be asked to draw points or segments on a grid. Be sure to use labels.

Solution

Part A



Part B Use the Midpoint Formula.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

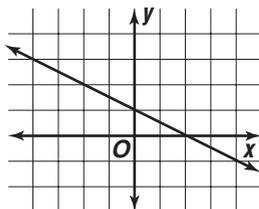
The midpoint of \overline{PQ} is $\left(\frac{-2 + 6}{2}, \frac{6 + 2}{2} \right)$

or $(2, 4)$. The y -intercept is $(0, 5)$. So they are not the same point.

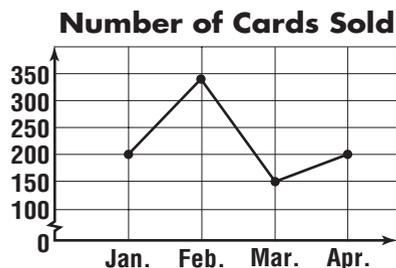
After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

- The graph of $y = -\frac{1}{2}x + 1$ is shown. What is the x -intercept?
(Algebra Review)
 A 2 B 1
 C 0 D -1
- A team has 8 seniors, 7 juniors, 3 sophomores, and 2 freshmen. What is the probability that a player selected at random is *not* a junior or a freshman?
(Statistics Review)
 A $\frac{9}{20}$ B $\frac{11}{20}$ C $\frac{13}{20}$ D $\frac{9}{11}$
- A cubic inch is about 0.000579 cubic feet. How is this expressed in scientific notation?
(Algebra Review)
 A 5.79×10^{-4} B 57.9×10^{-6}
 C 57.9×10^{-4} D 579×10^{-6}
- Joey has at least one quarter, one dime, one nickel, and one penny. If he has twice as many pennies as nickels, twice as many nickels as dimes, and twice as many dimes as quarters, what is the least amount of money he could have?
(Algebra Review)
 A \$0.41 B \$0.64 C \$0.71
 D \$0.73 E \$2.51
- On a floor plan, two consecutive corners of a room are at (3, 15) and (18, 2). The architect places a window in the center of the wall containing these two points. What are the coordinates of the center of the window?
(Lesson 2–6)
 A (8.5, 10.5) B (10.5, 8.5)
 C (17, 21) D (21, 17)
- Find the distance between (-2, 1) and (1, -3).
(Lesson 6–7)
 A 3 B 4 C 5
 D 6 E 7



- The graph shows a store's sales of greeting cards. The average price of a greeting card was \$2. Which is the best estimate of the total sales during the 4-month period?
(Algebra Review)



- A less than \$1000
 B between \$1000 and \$2000
 C between \$2000 and \$3000
 D between \$3000 and \$4000
- At a music store, the price of a CD is three times the price of a cassette tape. If 40 CDs were sold for a total of \$480, and the combined sales of CDs and cassette tapes totaled \$600, how many cassette tapes were sold?
(Algebra Review)
 A 4 B 12 C 30 D 120

Short Response

- Two segments with lengths 3 feet and 5 feet form two sides of a triangle. Draw a number line that shows possible lengths for the third side.
(Lesson 7–4)

Extended Response

- Make a bar graph for the data below.
(Statistics Review)

Destination	Frequency
Circle Center shopping district	
Indianapolis Children's Museum	
RCA Dome	
Indianapolis 500	
Indianapolis Art Museum	

