

CHAPTER 12

Surface Area and Volume

What You'll Learn

Key Ideas

- Identify solid figures.
(Lesson 12–1)
- Find the lateral areas, surface areas, and volumes of prisms, cylinders, regular pyramids, and cones.
(Lessons 12–2 to 12–5)
- Find the surface areas and volumes of spheres.
(Lesson 12–6)
- Identify and use the relationships between similar solid figures.
(Lesson 12–7)

Key Vocabulary

cone (p. 497)

cylinder (p. 497)

prism (p. 497)

pyramid (p. 497)

sphere (p. 528)

Why It's Important

Horticulture A greenhouse is an enclosed glass house used for growing plants in regulated temperatures, humidity, and ventilation. Greenhouses range from small rooms to large heated buildings covering many acres. Millions of dollars worth of plant products are raised in greenhouses each year.

Surface area and volume are important concepts used in fields like architecture, automotive design, and interior design. You will find the surface area of a prism to determine the amount of glass for a greenhouse in Lesson 12–2.



Study these lessons to improve your skills.

✓ **Lesson 1–6,**
pp. 35–40

✓ **Lesson 10–4,**
pp. 419–424

✓ **Lesson 10–5,**
pp. 425–430

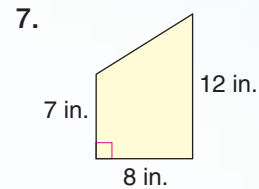
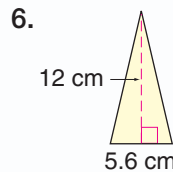
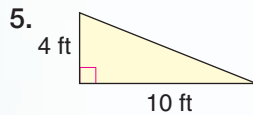
✓ **Lessons 11–5**
and 11–6,
pp. 478–487

✓ Check Your Readiness

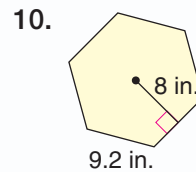
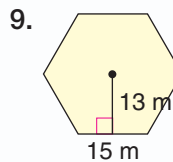
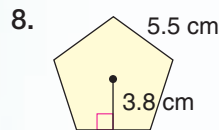
Find the area and perimeter of each rectangle described.

1. $\ell = 12$ m, $w = 4$ m
2. $\ell = 23$ in., $w = 8$ in.
3. $\ell = 13$ ft, $w = 40$ ft
4. $\ell = 6.2$ cm, $w = 1.8$ cm

Find the area of each triangle or trapezoid.



Find the area of each regular polygon.



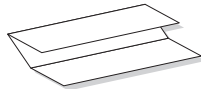
Find the circumference and area of each circle described to the nearest hundredth.

11. $r = 4$ m
12. $d = 10$ in.
13. $d = 6.4$ cm
14. $r = 8\frac{1}{2}$ ft

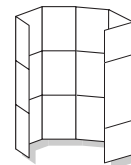
FOLDABLES™ Study Organizer

Make this Foldable to help you organize your Chapter 12 notes.
Begin with a sheet of 11" by 17" paper.

1 **Fold** in thirds lengthwise.



2 **Open** and fold a 2" tab along the short side. Then fold the rest in fifths.



3 **Draw** lines along the folds and label as shown.

Ch. 12	Prisms	Cylinders	Pyramids	Cones	Spheres
Surface Area					
Volume					

Reading and Writing As you read and study the chapter, use each page to write definitions and theorems and draw models. Also include any questions that need clarifying.



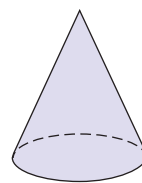
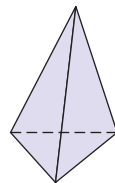
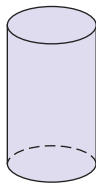
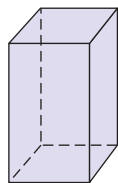
What You'll Learn

You'll learn to identify solid figures.

Why It's Important
Construction

Architects use various solid shapes to design buildings.
See Example 2.

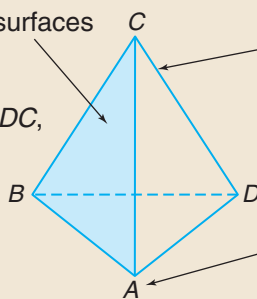
Study the figures below. How are these figures alike? How do they differ?



All of the above figures are examples of **solid figures** or **solids**. In geometry, solids enclose a part of space. Solids with flat surfaces that are polygons are called **polyhedrons** or **polyhedra**. Which of the above figures are polyhedrons?

**Parts of a Polyhedron**

The 2-dimensional surfaces are called **faces** are polygons and their interiors. $\triangle ABC$, $\triangle ADC$, $\triangle ABD$, and $\triangle BCD$ are the faces of the polyhedron.



Two faces intersect in a segment called an **edge**. \overline{CD} , \overline{CA} , \overline{CB} , \overline{AB} , \overline{AD} , and \overline{BD} are the edges of the polyhedron.

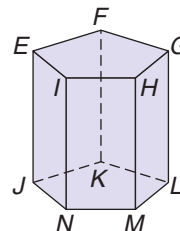
Three or more edges intersect at a point called a **vertex**. A , B , C , and D are the vertices of the polyhedron.

Example

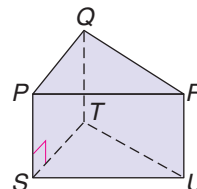
- 1 Name the faces, edges, and vertices of the polyhedron.

The faces are quadrilaterals $EFKJ$, $FGLK$, $GHML$, $HINM$, and $IEJN$ and pentagons $EFGHI$ and $JKLMN$. The edges are \overline{EJ} , \overline{FK} , \overline{GL} , \overline{HM} , \overline{IN} , \overline{EF} , \overline{FG} , \overline{GH} , \overline{HI} , \overline{IE} , \overline{JK} , \overline{KL} , \overline{LM} , \overline{MN} , and \overline{NJ} .

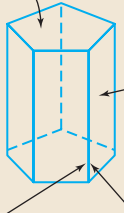
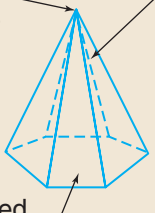
The vertices are E , F , G , H , I , J , K , L , M , and N .

**Your Turn**

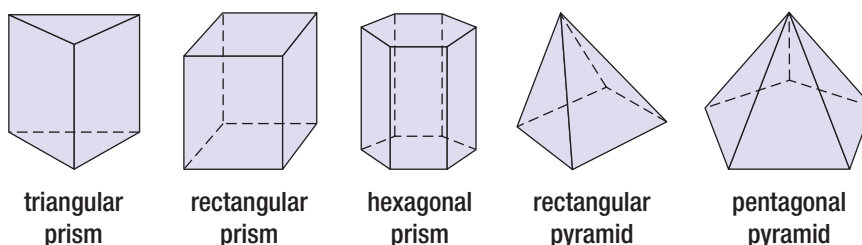
- a. Name the faces, edges, and vertices of the polyhedron.



Prisms and **pyramids** are two types of polyhedrons.

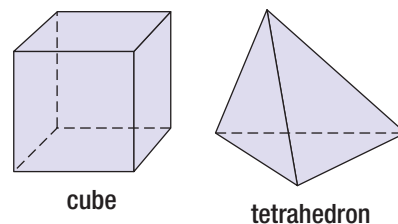
Prism	Pyramid
<p>Two parallel faces, called bases, are congruent polygons.</p>  <p>The other faces are called lateral faces. They are parallelograms.</p> <p>Each pair of adjacent lateral faces has a common lateral edge. The lateral edges are parallel segments.</p>	<p>All of the faces except one intersect at a common point called the vertex.</p>  <p>The faces that meet are called lateral faces. They are triangles.</p> <p>The other face is called the base. The base is also a polygon.</p>

Both prisms and pyramids are classified by the shape of their bases.



A **cube** is a special rectangular prism in which all of the faces are squares. A cube is one of the **Platonic solids**, which include all regular polyhedrons.

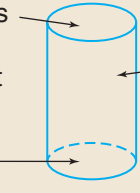
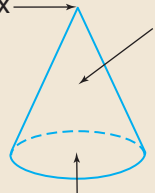
A **tetrahedron** is another name for a triangular pyramid. All of its faces are triangles.



Cylinders and **cones** are two types of solids that are not polyhedrons.

Reading Geometry

In this text, circular cylinders and circular cones will be referred to as cylinders and cones.

Cylinder	Cone
<p>The bases are two congruent circles. They are part of parallel planes.</p>  <p>The lateral surface is a curved surface.</p>	<p>The lateral surface is a curved surface.</p>  <p>The base is a circle.</p>





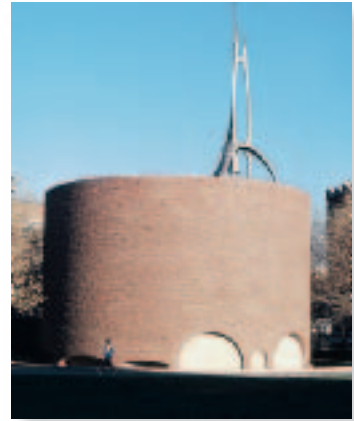
Example

Architecture Link

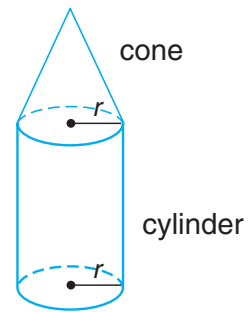
2

Eero Saarinen designed the chapel at the Massachusetts Institute of Technology (M.I.T.). Describe the basic shape of the chapel as a solid.

The chapel has two circular bases. It resembles a cylinder.



A **composite solid** is a solid that is made by combining two or more solids. The solid shown in the figure is a combination of a cylinder and a cone.



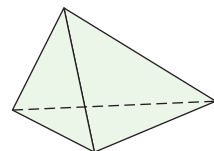
Check for Understanding

Communicating Mathematics

1. Draw an example of each solid.
 - a. cylinder
 - b. pentagonal prism
 - c. cone
 - d. rectangular pyramid
2. **Compare and contrast** each pair of solids.
 - a. prism and pyramid
 - b. cylinder and cone
 - c. prism and cylinder
 - d. pyramid and cone
3. **You Decide** Jasmine says the figure at the right is a tetrahedron. Mariam says it is a triangular pyramid. Who is correct? Explain.

Vocabulary

solid figures
polyhedrons
faces
edge
prisms
pyramids
lateral faces
lateral edge
cube
tetrahedron
cylinders
cones
composite solid
Platonic solid



Guided Practice

Getting Ready

Name the polygons that form the lateral faces of each polyhedron.

Sample 1: pentagonal pyramid

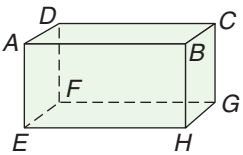
Sample 2: triangular prism

Solution: triangles

Solution: rectangles

Example 1

- 4. triangular pyramid
 - 5. cube
 - 6. hexagonal prism
 - 7. rectangular pyramid
 - 8. pentagonal prism
 - 9. tetrahedron
10. Name the faces, edges, and vertices of the polyhedron.



Example 2

Describe the basic shape of each item as a solid figure.

11.



12.



Example 2

13. **Art** David Smith creates geometric sculptures. What geometric solids did Mr. Smith use for this sculpture?

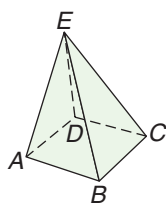
David Smith, *Cubi IX*

Exercises

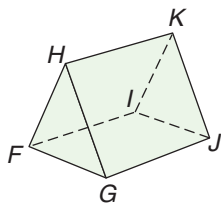
Practice

Name the faces, edges, and vertices of each polyhedron.

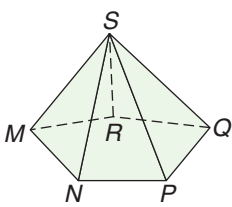
14.



15.



16.



Homework Help	
For Exercises	See Examples
14–16	1
17–22, 30, 31	2
Extra Practice	
See page 748.	

Describe the basic shape of each item as a solid.

17.



18.

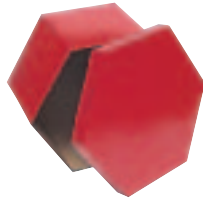


19.



Describe the basic shape of each item as a solid.

20.



21.

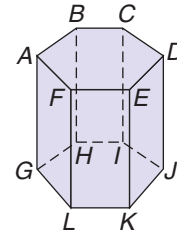


22.



Determine whether each statement is *true* or *false* for the solid.

23. The figure is a pyramid.
24. The figure is a polyhedron.
25. Hexagon $ABCDEF$ is a base.
26. Hexagon $GHIJKL$ is a lateral face.
27. There are six lateral faces.
28. The figure has 12 lateral edges.
29. $BHIC$ is a lateral face.

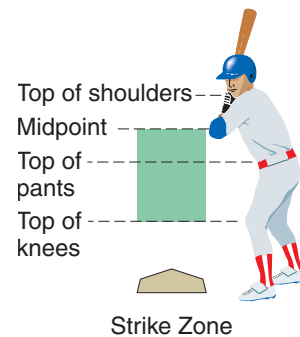


Applications and Problem Solving



The Flatiron Building

30. **Sports** In baseball, the ball is in the strike zone if the following criteria are met.
 - It is over the home plate. (Home plate is a pentagon placed on the ground.)
 - It is between the top of the batter's knees and the midpoint between the batter's shoulders and the top of his pants.



Describe the strike zone in terms of a geometric solid.

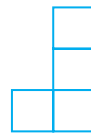
31. **History** In 1902, the Flatiron Building, New York's first skyscraper, was built. The base of the building is a polygon having three sides. Describe the basic shape of the building as a geometric solid.

32. **Orthographic Drawings** An *orthographic drawing* shows the top, the front, and the right-side views of a solid figure.

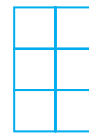
a. Use the orthographic drawing to sketch the solid it represents.



top



front



right

- b. Select a solid object in your home and draw the top, the front, and the right-side views.

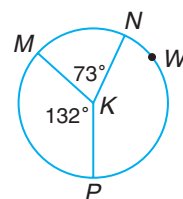
33. **Critical Thinking** The Swiss mathematician Leonhard Euler (1707–1783) was the first to discover the relationship among the number of vertices V , edges E , and faces F of a polyhedron.
- a. Copy and complete the table.

Solid	Number of Vertices (V)	Number of Faces (F)	Number of Edges (E)
triangular prism	6	5	9
triangular pyramid			
rectangular prism			
rectangular pyramid			
pentagonal prism			

- b. Look for a pattern, and write a formula using V , F , and E to show the relationship discovered by Euler.
- c. If a prism has 10 faces and 24 edges, how many vertices does it have? Use the formula you found in part b.

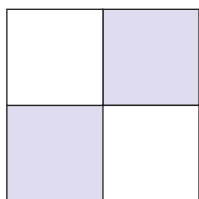
Mixed Review

34. **Recreation** The area of a circular pool is approximately 707 square feet. The owner wishes to purchase a new cover for the pool. What is the diameter of the cover? (*Lesson 11–6*)
35. **Transportation** Suppose the wheels of a car have 29-inch diameters. How many full revolutions will each front wheel make when the car travels 1 mile? (*Lesson 11–5*)
36. In $\odot K$, find $m\widehat{MN}$, $m\widehat{MWP}$, and $m\widehat{NWP}$. (*Lesson 11–2*)

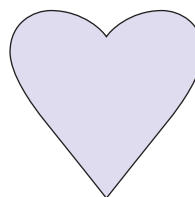


Determine whether each figure has line symmetry. If it does, copy the figure, and draw all lines of symmetry. If not, write *no*. (*Lesson 10–6*)

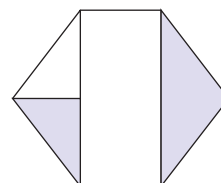
37.



38.



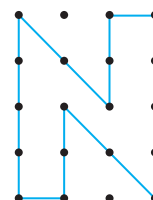
39.



Standardized Test Practice

(A) (B) (C) (D)

40. **Multiple Choice** What is the area of the polygonal region in square units? (*Lesson 10–3*)
- (A) 6 square units (B) 8 square units
- (C) 10 square units (D) 12 square units



Take a Slice

Materials



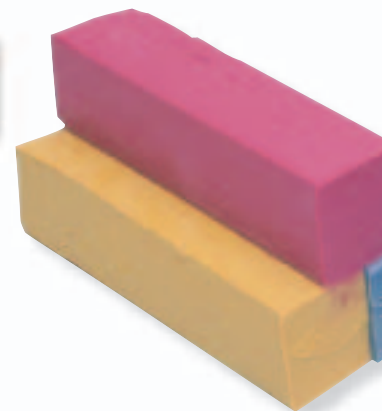
modeling clay



dental floss

Cross Sections of Solids

What would happen if you sliced through the center of a basketball? The shape you would see is a circle. If you sliced through the ball away from the center, you would see a smaller circle. Let's find out what happens when you slice through solid figures.



Investigate

1. Use modeling clay to investigate slicing a right circular cylinder.

- Roll a piece of modeling clay on your desk to form a thick tube. Then use dental floss to cut off the ends. Make your cuts perpendicular to the sides of the tube. You have created a cylinder.

- Use dental floss to slice through the cylinder horizontally as shown at the right. Place the cut surface on a piece of paper and trace around it. What shape do you get?



- Remake the cylinder. Use dental floss to slice through the cylinder on an angle as shown at the right. Place the cut surface on a piece of paper and trace around it. What shape do you get?

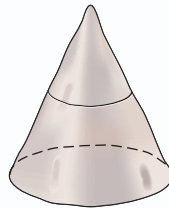


- Remake the cylinder. Use dental floss to slice through the cylinder vertically as shown at the right. Place the cut surface on a piece of paper and trace around it. What shape do you get?

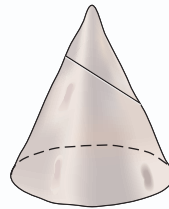




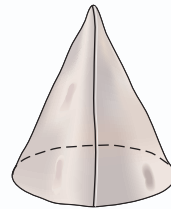
2. Use modeling clay to investigate slicing a cone.
 - a. Use modeling clay to form a cone.
 - b. Use dental floss to slice through the cone as shown in each diagram. Place the cut surface on a piece of paper and trace around it. Identify the shape of each cross section. Remember to remake your cone after each slice.



horizontal slice



angled slice



vertical slice

Extending the Investigation

In this extension, you will use modeling clay to identify the cross sections of solid figures. Make a horizontal slice, an angled slice, and a vertical slice of each solid figure.

1. rectangular prism or cube
2. triangular prism
3. square pyramid
4. a solid of your choice

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a poster with drawings of your solids and the tracings of the shapes of the cross sections.
- Write a paragraph about each solid and its various cross sections.
- Consider a solid that you have not studied in this Investigation. Draw and make the solid. Identify the cross sections that would result from this solid.



Investigation For more information on cross sections of solids, visit: www.geomconcepts.com

Surface Areas of Prisms and Cylinders

What You'll Learn

You'll learn to find the lateral areas and surface areas of prisms and cylinders.

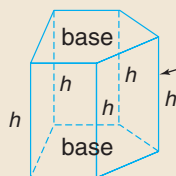
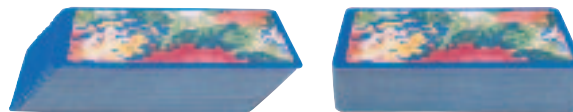
Why It's Important

Home Improvement

Plumbers use lateral area to determine the amount of insulation needed to cover the sides of a water heater.

See Example 4.

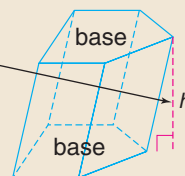
The heights of the two decks of cards are the same, but the shapes are different. One is oblique and the other is right.



In a **right prism**, a lateral edge is also an altitude.

An *altitude* of a prism is a segment perpendicular to the two planes that contain the bases.

The length of an altitude is also called the height of the prism. Its measure is represented by h .



In an **oblique prism**, a lateral edge is *not* an altitude.



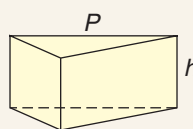
The **lateral area** of a solid figure is the sum of the areas of its lateral faces. The **surface area** of a solid figure is the sum of the areas of all its surfaces.

The activity shows how the formula for the lateral area and surface area are derived.

Theorem 12-1 Lateral Area of a Prism

Words: If a prism has a lateral area of L square units and a height of h units and each base has a perimeter of P units, then $L = Ph$.

Model:



Symbols: $L = Ph$



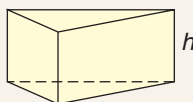
In this text, you can assume that all prisms are right unless noted otherwise.

To find the surface area of a prism, the areas of the two bases must be included. Remember that the two bases are congruent.

Theorem 12-2 Surface Area of a Prism

Words: If a prism has a surface area of S square units and a height of h units and each base has a perimeter of P units and an area of B square units, then $S = Ph + 2B$.

Model:



Symbols: $S = Ph + 2B$

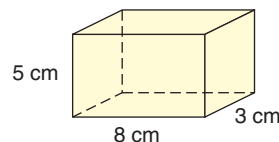
The formula for surface area can also be written in terms of lateral area, $S = L + 2B$.

Examples

1

Find the lateral area and the surface area of the rectangular prism.

First, find the perimeter of the base, P .
Then, find the area of the base, B .



Perimeter of Base

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2(8) + 2(3) \\ &= 16 + 6 \text{ or } 22 \end{aligned}$$

Area of Base

$$\begin{aligned} B &= \ell w \\ &= 8(3) \text{ or } 24 \end{aligned}$$

Use this information to find the lateral area and the surface area.

$$L = Ph$$

$$\begin{aligned} &= 22(5) \\ &= 110 \end{aligned}$$

$$S = L + 2B$$

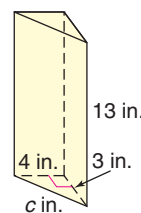
$$\begin{aligned} &= 110 + 2(24) \\ &= 110 + 48 \text{ or } 158 \end{aligned}$$

The lateral area is 110 square centimeters, and the surface area is 158 square centimeters.

2

Find the lateral area and the surface area of the triangular prism.

First, use the Pythagorean Theorem to find c , the measure of the hypotenuse. Use the value of c to find the perimeter of the base. Then, find the area.



Perimeter of Base

$$\begin{aligned} P &= 4 + 3 + c \\ &= 4 + 3 + 5 \\ &= 12 \end{aligned}$$

Area of Base

$$\begin{aligned} B &= \frac{1}{2}bh \\ &= \frac{1}{2}(4)(3) \\ &= 6 \end{aligned}$$

$$c^2 = 3^2 + 4^2$$

$$c^2 = 9 + 16$$

$$c^2 = 25$$

$$c = \sqrt{25} \text{ or } 5$$

Use this information to find the lateral area and surface area.

$$L = Ph$$

$$\begin{aligned} &= 12(13) \\ &= 156 \end{aligned}$$

$$S = L + 2B$$

$$\begin{aligned} &= 156 + 2(6) \\ &= 156 + 12 \text{ or } 168 \end{aligned}$$

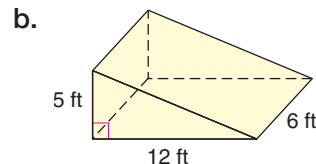
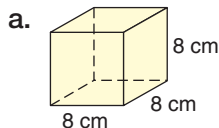
The lateral area is 156 square inches, and the surface area is 168 square inches.

Look Back

Pythagorean
Theorem:
Lesson 6–6

Your Turn

Find the lateral area and the surface area of each prism.



You can use the CellSheet™ App on a TI-83 Plus/TI-84 Plus graphing calculator to investigate how changing a dimension of a rectangular prism changes the surface area of the prism.





Graphing Calculator Exploration

To open a CellSheet session, press **APPS**, select CellSheet and press **ENTER**. Press any key twice to continue. A blank spreadsheet is opened and cell A1 is selected.

Step 1 Enter the labels LENGTH, WIDTH, HEIGHT, and SA in cells A1, B1, C1, and D1, respectively. To enter text press **2nd** **ALPHA** ["], then press each letter.

	A	B	C	D
1	LENGTH	WIDTH	HEIGHT	SA
2	8	5	3	
3	16	5	3	
4	8	10	3	
5	8	5	6	
6	24	15	9	

Step 2 Enter the dimensions of a rectangular prism in cells A2, B2, and C2.

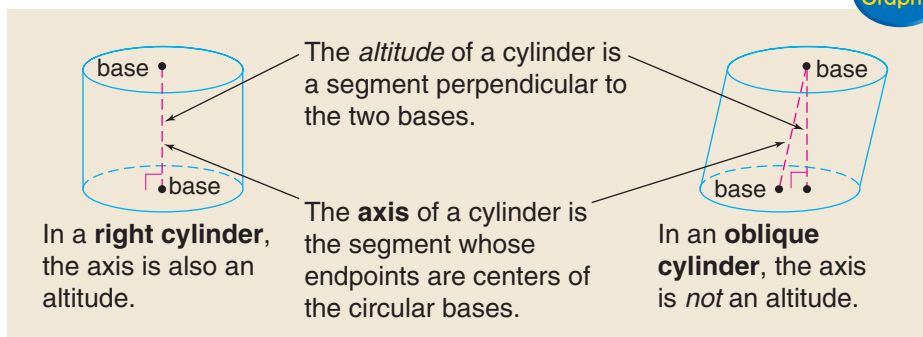
Step 3 Enter the formula for the surface area of a prism in cell D2. To enter the formula, press **STO** **▸** **(** **2** **×** **ALPHA** **[A]** **2** **+** **2** **×** **ALPHA** **[B]** **2** **+** **2** **×** **ALPHA** **[C]** **2** **+** **2** **×** **ALPHA** **[A]** **2** **+** **2** **×** **ALPHA** **[B]** **2** **ENTER**. The formula uses the information in cells A2, B2, and C2 to automatically calculate the surface area.

Try These

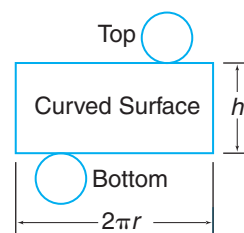
1. Use CellSheet to find the surface area of a prism. Then double one dimension of the prism. How is the surface area affected?
2. How is the surface area of a prism affected if all of the dimensions are doubled?
3. How do you think the surface area of a prism is affected if all of the dimensions are tripled? Use CellSheet to verify your conjecture.

Reading Geometry

In this text, you can assume that all cylinders are right circular cylinders unless noted otherwise.



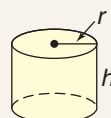
The lateral area of a cylinder is the area of the curved surface. If a cylinder were cut across the lateral side and unfolded, it would resemble a rectangle. Its **net** is shown at the right. A net is a two-dimensional pattern that folds to form a solid. The width of the rectangle is the height h of the cylinder. The length of the rectangle is the distance around the circular base, or the circumference, $2\pi r$. Since $\ell = 2\pi r$ and $w = h$, $L = \ell w$ becomes $L = (2\pi r)h$.



Theorem 12-3 Lateral Area of a Cylinder

Words: If a cylinder has a lateral area of L square units and a height of h units and the bases have radii of r units, then $L = 2\pi rh$.

Model:



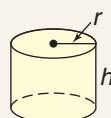
Symbols: $L = 2\pi rh$

The surface area of a cylinder is still found by using $S = L + 2B$. However, L can be replaced with $2\pi rh$, and B can be replaced with πr^2 .

Theorem 12-4 Surface Area of a Cylinder

Words: If a cylinder has a surface area of S square units and a height of h units and the bases have radii of r units, then $S = 2\pi rh + 2\pi r^2$.

Model:



Symbols: $S = 2\pi rh + 2\pi r^2$

Example

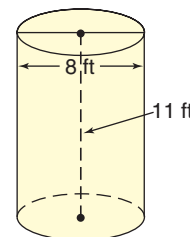
- 3** Find the lateral area and surface area of the cylinder to the nearest hundredth.

Lateral Area

$$\begin{aligned} L &= 2\pi rh & d &= 8, \\ &= 2\pi(4)(11) & r &= 4 \\ &\approx 276.46 \end{aligned}$$

Surface Area

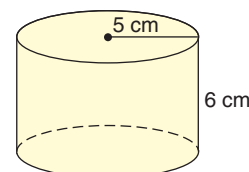
$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 \\ &= 2\pi(4)(11) + 2\pi(4)^2 \\ &\approx 376.99 \end{aligned}$$



To the nearest hundredth, the lateral area is about 276.46 square feet, and the surface area is about 376.99 square feet.

Your Turn

- c. Find the lateral area and the surface area of the cylinder to the nearest hundredth.





Example

Plumbing Link

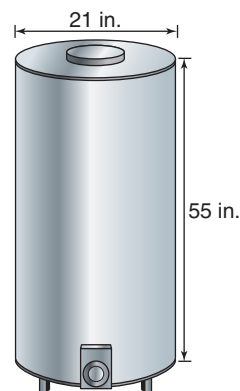
4

Find the amount of insulation needed to cover the sides of the 50-gallon hot water heater to the nearest hundredth.

The hot water heater is in the shape of a cylinder. Since the insulation covers only the sides of the hot water heater, find the lateral area. The diameter is 21 inches. So, the radius is 10.5 inches.

$$\begin{aligned}
 L &= 2\pi rh && \text{Theorem 12-3} \\
 &= 2\pi(10.5)(55) && \text{Replace } r \text{ with } 10.5 \text{ and } h \text{ with } 55. \\
 &\approx 3628.54 && \text{Use a calculator.}
 \end{aligned}$$

The amount of insulation needed is 3628.54 square inches.



Check for Understanding

Communicating Mathematics

1. Explain the difference between lateral area and surface area.
2. Draw an oblique cylinder and a right cylinder. Write a sentence or two explaining the difference between the two kinds of cylinders.

Vocabulary

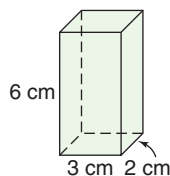
right prism
 oblique prism
 lateral area
 surface area
 net
 right cylinder
 axis
 oblique cylinder

Guided Practice

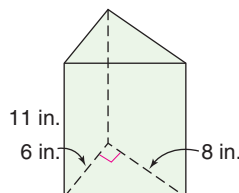
Examples 1–3

Find the lateral area and the surface area for each solid. Round to the nearest hundredth, if necessary.

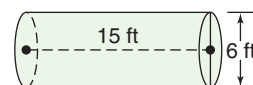
3.



4.



5.



Examples 3 & 4

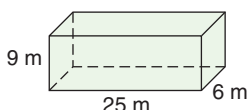
6. **Manufacturing** A soup can has a height of 10 centimeters and a diameter of 6.5 centimeters.
 - a. Find the amount of paper needed to cover the can.
 - b. Find the amount of steel needed to make the can.

Exercises

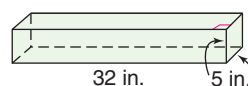
Practice

Find the lateral area and the surface area for each solid. Round to the nearest hundredth, if necessary.

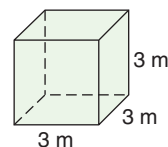
7.



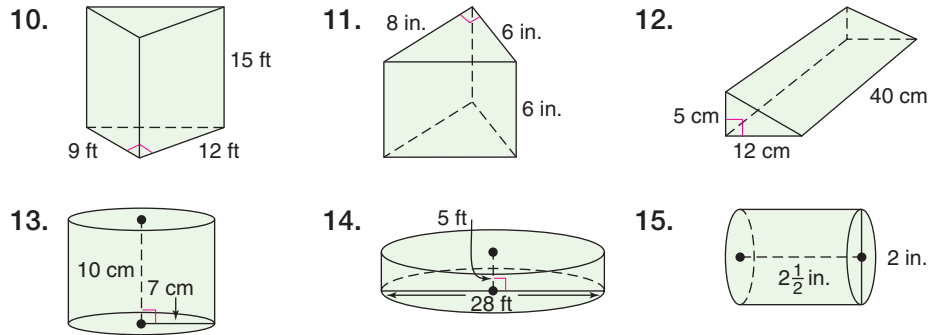
8.



9.



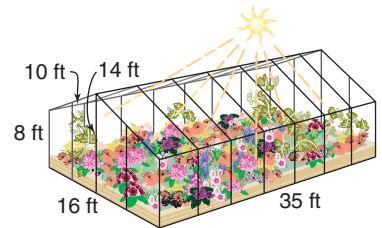
Homework Help	
For Exercises	See Examples
7–19, 16, 19	1
10–12	2
13–15, 17	3, 4
18	1, 2
Extra Practice	
See page 748.	



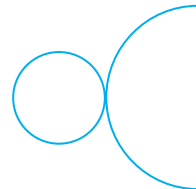
16. Draw a rectangular prism that is 4 centimeters by 5 centimeters by 8 centimeters. Find the surface area of the prism.
17. A cylinder has a diameter of 32 feet and a height of 20 feet.
 - a. Find the lateral area of the cylinder.
 - b. Find the surface area of the cylinder.

Applications and Problem Solving

18. **Architecture** Find the amount of glass needed to build the greenhouse.



19. **Painting** A rectangular room is 12 feet by 21 feet. The walls are 8 feet tall. Paint is sold in one-gallon containers. If a gallon of paint covers 450 square feet, how many gallons of paint should Poloma buy to paint the walls of the room?
20. **Critical Thinking** Identify the solid by its net shown at the right.



Mixed Review

21. Draw an example of a pentagonal pyramid. (*Lesson 12–1*)
22. Find the area of a 30° sector of a circle if the radius of the circle is 36 meters. Round to the nearest hundredth. (*Lesson 11–6*)
23. **Nature** What is the circumference of a bird's nest if its diameter is 7 inches? (*Lesson 11–5*)
24. **Grid In** Find the perimeter in centimeters of a regular nonagon whose sides are 3.9 centimeters long. (*Lesson 10–1*)
25. **Multiple Choice** Candace bought a cordless screwdriver on sale for \$22.50. The regular price was \$30. What was the percent of discount? (*Percent Review*)

(A) 15%	(B) 20%	(C) 25%	(D) 30%
---------	---------	---------	---------

Standardized Test Practice

(A) (B) (C) (D)



What You'll Learn

You'll learn to find the volumes of prisms and cylinders.

Why It's Important

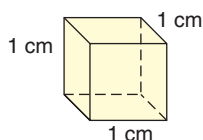
Automotive Design

Engineers calculate the volume of cylinders to determine the displacement of an engine.

See Example 4.

The amount of water a fish tank can hold, the amount of grain a silo can hold, or the amount of concrete needed for a patio floor are all examples of volume.

Volume measures the space contained within a solid. Volume is measured in cubic units. The cube below has a volume of 1 cubic centimeter or 1 cm^3 . Each of its sides is 1 centimeter long.



Hands-On Geometry

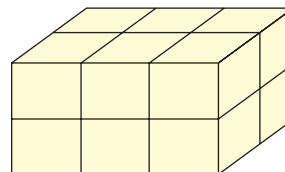
Materials:



cubes/blocks

Step 1 Make a prism like the one shown at the right.

Step 2 Make at least three different rectangular prisms.



Try These

1. Assume that the edge of each cube represents 1 unit. Then, the area of each surface of each cube is 1 square unit, and the volume of each cube is 1 cubic unit. Copy and complete the table for your prisms.

Prism	Area of Base	Height	Volume
1	6	2	12
2			
3			
4			

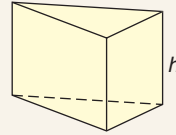
2. Describe how the area of the base and the height of a prism are related to its volume.
3. The corner view of the prism is shown. Draw the top, front, and side views of the prism with one cube removed.

The activity above suggests the following postulate.

Theorem 12-5 Volume of a Prism

Words: If a prism has a volume of V cubic units, a base with an area of B square units, and a height of h units, then $V = Bh$.

Model:



Symbols: $V = Bh$

Examples

1

Find the volume of the triangular prism.

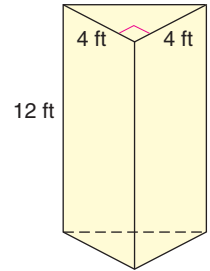
The area of the triangular base can be calculated using the measures of the two legs.

Therefore, $B = \frac{1}{2}(4)(4)$ or 8.

$V = Bh$ *Theorem 12-5*

$= 8(12)$ *Replace B with 8 and h with 12.*

$= 96$ The volume of the triangular prism is 96 cubic feet.



2

The base of the prism is a regular pentagon with sides of 4 centimeters and an apothem of 2.75 centimeters. Find the volume of the prism.

The perimeter of the base is $5(4)$ or 20.

$B = \frac{1}{2}Pa$

$V = Bh$

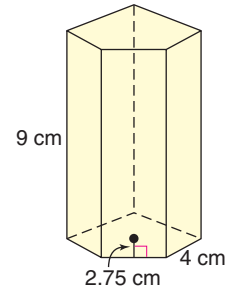
$= \frac{1}{2}(20)(2.75)$

$= 27.5(9)$

$= 27.5$

$= 247.5$

The volume of the pentagonal prism is 247.5 cubic centimeters.



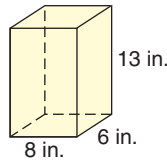
Look Back

Area of Regular
Polygons:
Lesson 10-5

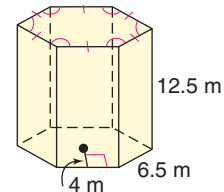
Your Turn

Find the volume of each prism.

a.



b.



A stack of coins is shaped like a cylinder. The volume of a cylinder is given by the same general formula as for a prism. The volume is the product of the cylinder's base and height. The base of this cylinder is a circle with area πr^2 , and its height is h .

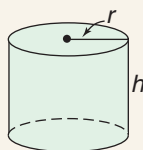
$$\begin{aligned} V &= Bh \\ &= (\pi r^2)h \text{ or } \pi r^2 h \end{aligned}$$



Theorem 12-6 Volume of a Cylinder

Words: If a cylinder has a volume of V cubic units, a radius of r units, and a height of h units, then $V = \pi r^2 h$.

Model:



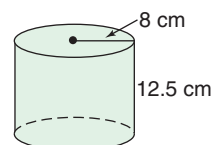
Symbols: $V = \pi r^2 h$

Example

3

Find the volume of the cylinder to the nearest hundredth.

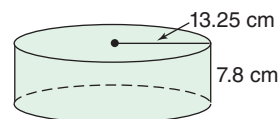
$$\begin{aligned} V &= \pi r^2 h && \text{Theorem 12-6} \\ &= \pi(8^2)(12.5) && \text{Replace } r \text{ with } 8 \text{ and } h \text{ with } 12.5. \\ &\approx 2513.27 && \text{Use a calculator.} \end{aligned}$$



The volume of the cylinder is 2513.27 cubic centimeters.

Your Turn

c. Find the volume of the cylinder to the nearest hundredth.

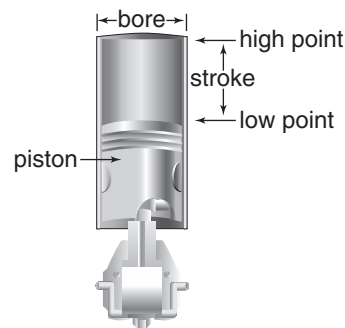


Example

Automotive Link

4

In an automotive engine, the piston moves up and down in a cylinder. The volume of space through which the piston moves is called the *displacement*. A certain automobile has a stroke of 3.54 inches and a bore of 3.23 inches. The bore is the diameter of the cylinder.



A. Find the displacement of each cylinder.

The diameter d of the cylinder is 3.23 inches. So, the radius is 1.615 inches.

$$\begin{aligned} V &= \pi r^2 h && \text{Theorem 12-6} \\ &= \pi(1.615)^2(3.54) && \text{Replace } r \text{ with } 1.615 \text{ and } h \text{ with } 3.54. \\ &\approx 29 && \text{Use a calculator.} \end{aligned}$$

The displacement of each cylinder is about 29 cubic inches.

B. Find the total displacement of the car's four cylinders.

Since the car has four cylinders, the total displacement is about 4×29 or about 116 cubic inches.



Check for Understanding

Communicating Mathematics

Vocabulary

volume

1. Draw two right rectangular prisms with a volume of 24 cubic inches, but with different dimensions.
2. Compare and contrast surface area and volume. Be sure to include the type of unit used for each measurement.
3. **You Decide?** A fish tank is 18 inches by 12 inches by 10 inches. Rebecca says that if she were to double the dimensions of the fish tank, she would need twice as much water to fill the tank. Caitlin disagrees. Who is correct? Explain.



Guided Practice



Getting Ready

Find the area of each figure to nearest hundredth.

Sample: rectangle: length, 4.9 inches; width, 5 inches

Solution: $A = \ell w$

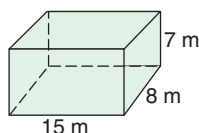
$A = 4.9 \times 5$ or 24.5 The area is 24.5 square inches.

4. triangle: base, $3\frac{3}{4}$ feet; height, 4 feet
5. circle: diameter, 7 meters
6. regular pentagon: side, 6 yd; apothem, 4.1 yd

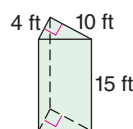
Examples 1–3

Find the volume of each solid. Round to the nearest hundredth, if necessary.

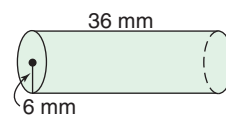
7.



8.



9.



Example 4

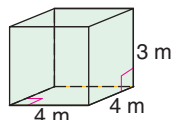
10. **Environmental Engineering** A classroom is 30 feet long, 24 feet wide, and 10 feet high. If each person in the room needs 300 cubic feet of air, find the maximum capacity of the room.

Exercises

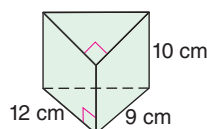
Practice

Find the volume of each solid. Round to the nearest hundredth, if necessary.

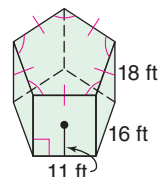
11.



12.

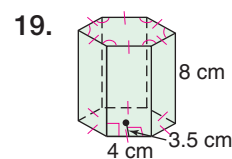
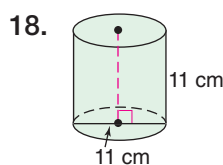
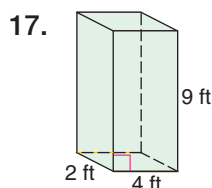
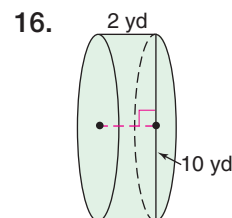
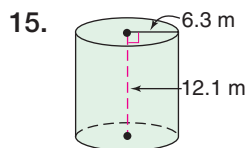
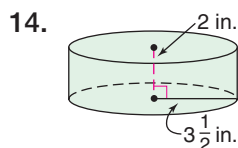


13.



Homework Help	
For Exercises	See Examples
11–13, 17, 19–21, 25	1, 2
14–16, 18, 22, 23	3, 4
24	1–3
Extra Practice	
See page 748.	

Find the volume of each solid. Round to the nearest hundredth, if necessary.



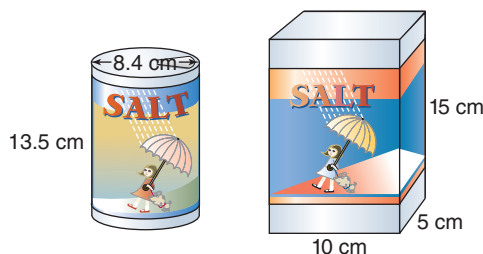
20. What is the volume of a cube that has a 5-inch edge?
21. Draw a rectangular prism that is 4 centimeters by 8 centimeters by 3 centimeters. Find the volume of the prism.
22. A cylinder has a base diameter of 14 inches and a height of 18 inches. What is the volume of the cylinder?

Applications and Problem Solving

23. **Weather** Rain enters the rain gauge through a funnel-shaped top. It is measured in the cylindrical collector. Find the volume of the cylindrical collector of the rain gauge.



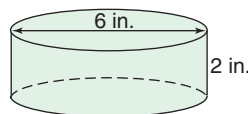
24. **Packaging** Salt is usually packaged in cylindrical boxes. In 1976, the Leslie Salt Company in Newark, California, tried to package salt in rectangular boxes to save space on supermarket shelves.
- Compare the volumes of the two salt boxes.
 - Which design is better for storing, stacking, and shipping? Explain your reasoning.



25. **Critical Thinking** The areas of the faces of a rectangular prism are 12 square inches, 32 square inches, and 24 square inches. The lengths of the edges are represented by whole numbers. Find the volume of the prism. Explain how you solved the problem.

Mixed Review

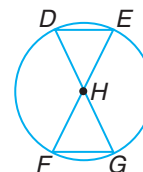
26. Find the lateral area and the surface area of the cylinder to the nearest hundredth.
(Lesson 12-2)



27. Describe the basic shape of a package of computer disks as a geometric solid. (Lesson 12-1)



28. **Algebra** In $\odot H$, $\widehat{DE} \cong \widehat{FG}$.
If $DE = 3x$ and $FG = 4x - 6$,
what is the value of x ? (Lesson 11-3)



Standardized Test Practice

A B C D

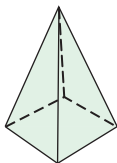
29. **Short Response** Find the area of a regular octagon whose perimeter is 49.6 feet and whose apothem is 7.5 feet long. (Lesson 10-5)
30. **Multiple Choice** Which ordered pair is a solution to $3x + 4y \geq 17$? (Algebra Review)
- (A) (1, 1) (B) (-3, 7) (C) (4, 0) (D) (2, -3)

Quiz 1

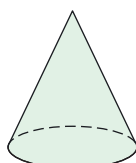
Lessons 12-1 through 12-3

Identify each solid. (Lesson 12-1)

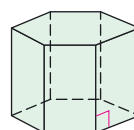
1.



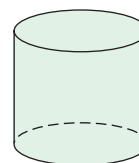
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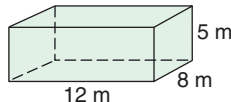


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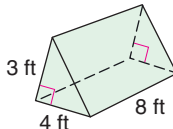


Find the lateral area and the surface area for each solid. Round to the nearest hundredth, if necessary. (Lesson 12-2)

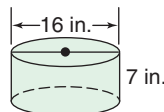
5.



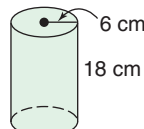
6.



7.



8.



9. Find the volume of a cylinder with a base diameter of 24 meters and a height of 28 meters. (Lesson 12-3)

10. **Construction** How many cubic yards of concrete will be needed for a driveway that is 40 feet long, 18 feet wide, and 4 inches deep? Round the answer to the nearest tenth. (Hint: $27 \text{ ft}^3 = 1 \text{ yd}^3$) (Lesson 12-3)



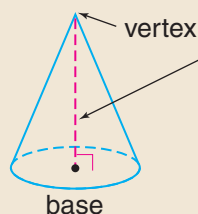
What You'll Learn

You'll learn to find the lateral areas and surface areas of regular pyramids and cones.

Why It's Important

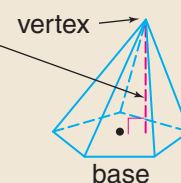
Building Architects can determine the area of the outside walls of a building in the shape of a pyramid. See Example 2.

Just as there are right and oblique prisms and cylinders, there are also right and oblique pyramids and cones.



In a **right pyramid** or **right cone**, the altitude is perpendicular to the base at its center.

The *altitude* of a pyramid or cone is the segment from the vertex perpendicular to the plane containing the base.



In an **oblique pyramid** or **oblique cone**, the altitude is perpendicular to the base at a point other than its center.

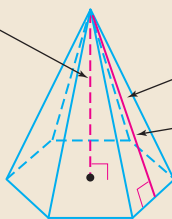
Info Graphic

The Great American Pyramid is an arena in Memphis, Tennessee. It is an example of a regular pyramid.

Reading Geometry

In this text, you can assume that all pyramids are right unless noted otherwise.

A pyramid is a **regular pyramid** if and only if it is a right pyramid and its base is a regular polygon.



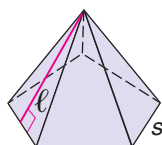
The lateral faces of a regular pyramid form congruent isosceles triangles.

The height of each lateral face is called the **slant height** of the pyramid. The measure of the slant height is represented by ℓ .

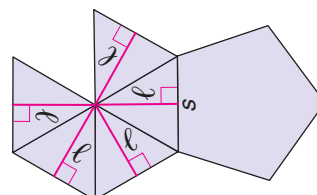
Info Graphic

Consider the regular pentagonal pyramid below. Its lateral area L can be found by adding the areas of all its congruent triangular faces. Because the base is a regular polygon, the length of each edge along the base has equal measure s .

Solid



Net

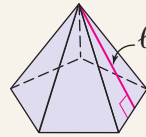


$$\begin{aligned}
 L &= \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell + \frac{1}{2}s\ell \\
 &= \frac{1}{2}(s + s + s + s + s)\ell && \text{Distributive Property} \\
 &= \frac{1}{2}P\ell && P = s + s + s + s + s
 \end{aligned}$$

Theorem 12-7 Lateral Area of a Regular Pyramid

Words: If a regular pyramid has a lateral area of L square units, a base with a perimeter of P units, and a slant height of ℓ units, then $L = \frac{1}{2}P\ell$.

Model:



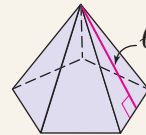
Symbols: $L = \frac{1}{2}P\ell$

With prisms and cylinders, the formula for the surface area is $S = L + 2B$. Since a pyramid has only one base, the formula for its surface area is $S = L + B$, where $L = \frac{1}{2}P\ell$.

Theorem 12-8 Surface Area of a Regular Pyramid

Words: If a regular pyramid has a surface area of S square units, a slant height of ℓ units, and a base with perimeter of P units and area of B square units, then $S = \frac{1}{2}P\ell + B$.

Model:



Symbols: $S = \frac{1}{2}P\ell + B$

The formula for surface area can also be written in terms of lateral area, $S = L + B$.

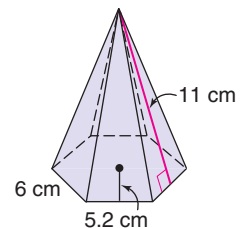
Examples

1

Find the lateral area and the surface area of the regular hexagonal pyramid.

First, find the perimeter and area of the hexagon. For a regular hexagon, the perimeter is 6 times the length of one side. The area is one-half the perimeter times the apothem.

$$\begin{aligned}
 P &= 6s && B = \frac{1}{2}Pa \\
 &= 6(6) \text{ or } 36 && = \frac{1}{2}(36)(5.2) \text{ or } 93.6
 \end{aligned}$$



(continued on the next page)



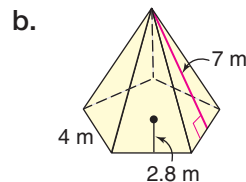
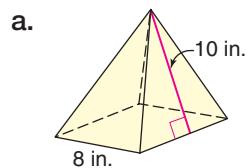
Now use this information to find the lateral area and surface area.

$$\begin{aligned} L &= \frac{1}{2}P\ell & S &= L + B \\ &= \frac{1}{2}(36)(11) & &= 198 + 93.6 \\ &= 198 & &= 291.6 \end{aligned}$$

The lateral area is 198 square centimeters, and the surface area is 291.6 square centimeters.

Your Turn

Find the lateral area and the surface area of each regular pyramid.

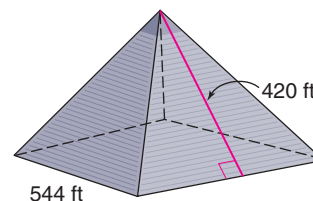


Architecture Link



2

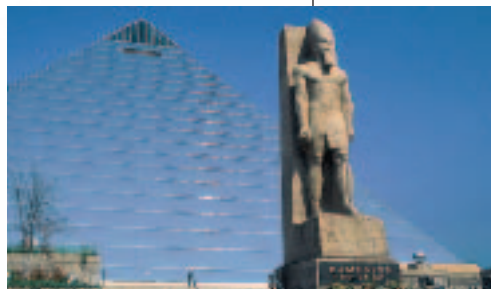
The Great American Pyramid in Memphis, Tennessee, is a regular pyramid with a square base. Each side of the base is about 544 feet long. The slant height is about 420 feet. Find the area of the outside walls of this structure.



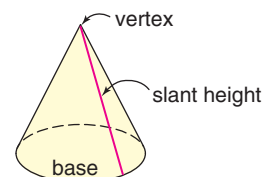
The area of the outside walls is the lateral area of the pyramid. The perimeter of the base is $4(544)$ or 2176 feet.

$$\begin{aligned} L &= \frac{1}{2}P\ell && \text{Theorem 12-7} \\ &= \frac{1}{2}(2176)(420) && \text{Replace } P \text{ with 2176 and } \ell \text{ with 420.} \\ &= 456,960 \end{aligned}$$

The area of the outside walls of the Great American Pyramid is about 456,960 square feet.



The slant height of a cone is the length of any segment whose endpoints are the vertex of the cone and a point on the circle that forms the base.



The formulas for finding the lateral area and surface area of a cone are similar to those for a regular pyramid. However, since the base is a circle, the perimeter becomes the circumference, and the area of the base is πr^2 square units.



Theorem 12–9 Lateral Area of a Cone

Words: If a cone has a lateral area of L square units, a slant height of ℓ units, and a base with a radius of r units, then $L = \frac{1}{2} \cdot 2\pi r \cdot \ell$ or $\pi r \ell$.

Model:



Symbols: $L = \pi r \ell$

To find the surface area of a cone, add its lateral area and the area of its base.

Theorem 12–10 Surface Area of a Cone

Words: If a cone has a surface area of S square units, a slant height of ℓ units, and a base with a radius of r units, then $S = \pi r \ell + \pi r^2$.

Symbols: $S = \pi r \ell + \pi r^2$

Example

- 3** Find the lateral area and the surface area of the cone to the nearest hundredth.

Since the diameter of the base is 10 feet, the radius is 5 feet. Use the Pythagorean Theorem to find ℓ .

$$\ell^2 = 5^2 + 12^2$$

Use the Pythagorean Theorem.

$$\ell^2 = 25 + 144$$

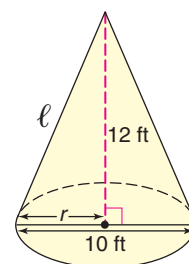
$$5^2 = 25; 12^2 = 144$$

$$\ell^2 = 169$$

Add 25 and 144.

$$\ell = \sqrt{169} \text{ or } 13$$

Take the square root of each side.



Use this value to find the lateral area and the surface area.

$$\begin{aligned} L &= \pi r \ell \\ &= \pi(5)(13) \\ &\approx 204.20 \end{aligned}$$

$$\begin{aligned} S &= \pi r \ell + \pi r^2 \\ &\approx 204.20 + \pi(5^2) \\ &\approx 282.74 \end{aligned}$$

Theorems 12–9 and 12–10

Substitution

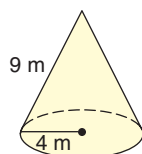
Use a calculator.

The lateral area is 204.20 square feet, and the surface area is 282.74 square feet.

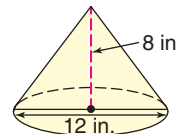
Your Turn

Find the lateral area and the surface area of each cone. Round to the nearest hundredth.

c.



d.



Check for Understanding

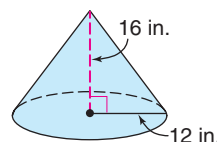
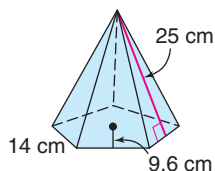
Communicating Mathematics

1. Explain the difference between the slant height and the altitude of a regular pyramid.
2. Draw a cone in which the altitude is not perpendicular to the base at its center. What is the name of this solid?
3. **Writing Math** Write a paragraph comparing the methods for finding the surface areas of prisms, cylinders, pyramids, and cones.

Vocabulary

right pyramid
right cone
oblique pyramid
oblique cone
regular pyramid
slant height

Guided Practice Examples 1 & 3



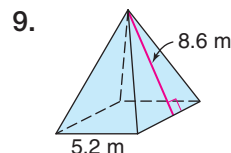
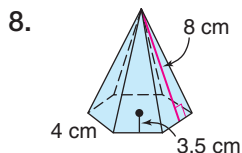
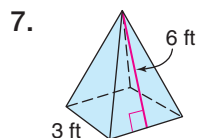
Example 3

6. **Manufacturing** The Party Palace makes cone-shaped party hats out of cardboard. If the diameter of the hat is $6\frac{1}{2}$ inches and the slant height is 7 inches, find the amount of cardboard needed for each hat.

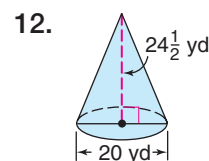
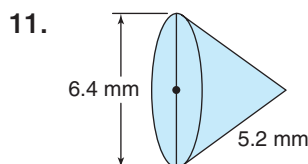
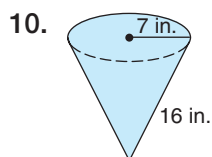
Exercises

Practice

Find the lateral area and the surface area of each regular pyramid.

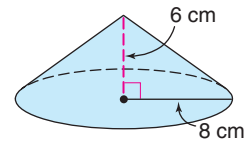
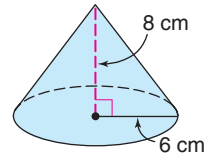


Find the lateral area and the surface area of each cone. Round to the nearest hundredth.



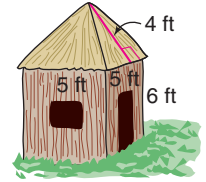
13. A regular pyramid has an altitude of 4 feet. The base is a square with sides 6 feet long. What is the surface area of the pyramid?

14. Determine which cone has the greater lateral area.



Applications and Problem Solving

15. **Tourism** A tropical resort maintains cabanas on the beach for their guests. Find the amount of canvas needed to cover one cabana.

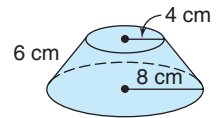


16. **Aircraft Design** An important design factor of aircraft is known as *wing loading*. Wing loading is the total weight of the aircraft and its load on take-off divided by the surface area of its wings.

Aircraft	Maximum Takeoff Weight (lb)	Surface Area of Wings (ft ²)
Wright brothers' plane	750	532
Concorde	408,000	3856
Nighthawk	52,500	913
Tomcat	70,280	565

Source: *Aircraft of the World*

- a. Determine the wing loading for each aircraft to the nearest hundredth.
- b. What do the resulting wing loading numbers mean?
17. **Critical Thinking** A **truncated solid** is the remaining part of a solid after one or more vertices have been cut off.



- a. Find the surface area of the truncated cone to the nearest hundredth. (*Hint: Sketch the entire cone.*)
- b. A lampshade resembles a truncated cone without a solid top or bottom. Find the lateral area of a lampshade in your home.

Mixed Review

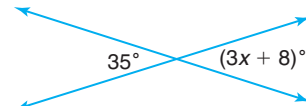
18. **Construction** The town of West Mountfort recently built a new cylindrical water tower. If the tower is 275 feet tall and has a diameter of 87 feet, how many cubic feet of water can the tank hold? Round the answer to the nearest cubic foot. (*Lesson 12-3*)
19. Draw and label a rectangular prism with a length of 3 centimeters, a width of 2 centimeters, and a height of 4 centimeters. Then find the surface area of the prism. (*Lesson 12-2*)
20. Find the area of a trapezoid whose height measures 8 inches and whose bases are 7 inches and 12 inches. (*Lesson 10-4*)

Standardized Test Practice

(A) (B) (C) (D)

21. **Short Response** Given $L(2, 1)$, $M(4, 3)$, $P(0, 2)$, and $Q(3, -1)$, determine if \overline{LM} and \overline{PQ} are parallel, perpendicular, or neither. (*Lesson 4-5*)
22. **Multiple Choice** Find the value of x in the figure. (*Lesson 3-6*)

- (A) 3 (B) 5
(C) 7 (D) 9



What You'll Learn

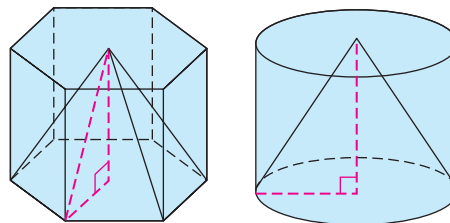
You'll learn to find the volumes of pyramids and cones.

Why It's Important

Entertainment

Theater managers can determine the amount of popcorn in a container. See Example 3.

The pyramid and prism at the right have the same base and height. The cone and cylinder have the same base and height. If you consider these figures in terms of volume, you can see that the volume of the pyramid is less than the volume of the prism. Likewise, the volume of the cone is less than the volume of the cylinder.

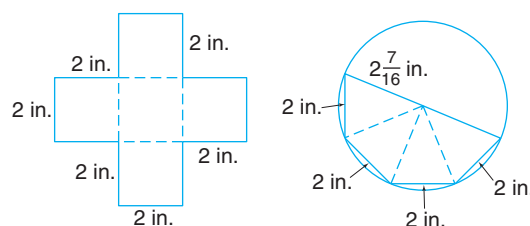


Let's investigate the relationship between the volumes of a prism and a pyramid with the same base and height.

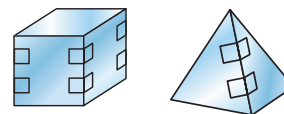
Hands-On Geometry

Materials: card stock ruler compass
 scissors tape rice

Step 1 Draw nets for a cube and a pyramid on card stock using a ruler and a compass.



Step 2 Cut out the nets. Fold along the dashed lines to form the cube and the pyramid with open bases. Tape the edges together to form the solids as shown.



Step 3 Fill the pyramid with rice. Then pour the rice into the cube. Repeat the process until the prism (cube) is full.

Try These

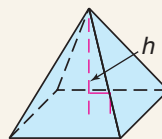
1. Set the cube and pyramid on your desk with the base of each figure on the desk. Compare the heights of the cube and pyramid.
2. Place the base of the pyramid on top of the cube. Compare their bases.
3. How many times did you fill the pyramid in order to fill the cube?
4. In general, if a pyramid and a prism have the same height and base, how would you expect their volumes to compare?

This activity suggests Theorem 12–11. Recall that Bh is the volume of a cube with height h and area of its base B .

Theorem 12–11 Volume of a Pyramid

Words: If a pyramid has a volume of V cubic units and a height of h units and the area of the base is B square units, then $V = \frac{1}{3}Bh$.

Model:



Symbols: $V = \frac{1}{3}Bh$

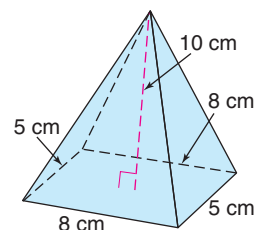
Example

- 1** Find the volume of the rectangular pyramid to the nearest hundredth.

In a rectangle, $A = \ell w$. Therefore, $B = 8(5)$ or 40.

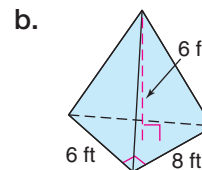
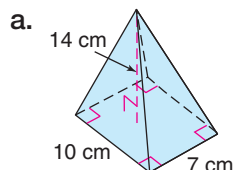
$$\begin{aligned} V &= \frac{1}{3}Bh && \text{Theorem 12–11} \\ &= \frac{1}{3}(40)(10) && \text{Replace } B \text{ with } 40 \text{ and } h \text{ with } 10. \\ &\approx 133.33 && \text{Simplify.} \end{aligned}$$

The volume of the pyramid is 133.33 cubic centimeters.



Your Turn

Find the volume of each pyramid. Round to the nearest hundredth.

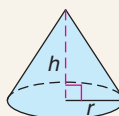


The relationship between the volumes of a cone and a cylinder is similar to the relationship between the volumes of a pyramid and a prism. The volume of a cone is $\frac{1}{3}$ the volume of cylinder with the same base and height. The volume of a cylinder is $\pi r^2 h$, so the volume of a cone is $\frac{1}{3}\pi r^2 h$.

Theorem 12–12 Volume of a Cone

Words: If a cone has a volume of V cubic units, a radius of r units, and a height of h units, then $V = \frac{1}{3}\pi r^2 h$.

Model:



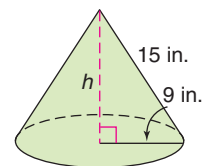
Symbols: $V = \frac{1}{3}\pi r^2 h$



Example**2**

Find the volume of the cone to the nearest hundredth.

The triangle formed by the height, radius, and slant height is a right triangle. So, you can use the Pythagorean Theorem to find the measure of the height h .



$$a^2 + b^2 = c^2$$

Pythagorean Theorem

$$h^2 + 9^2 = 15^2$$

Replace a with h , b with 9, and c with 15.

$$h^2 + 81 = 225$$

$$9^2 = 81; 15^2 = 225$$

$$h^2 + 81 - 81 = 225 - 81$$

Subtract 81 from each side.

$$h^2 = 144$$

Simplify.

$$h = \sqrt{144} \text{ or } 12$$

Take the square root of each side.

Use this value of h to find the volume of the cone.

$$V = \frac{1}{3}\pi r^2 h$$

Theorem 12-12

$$= \frac{1}{3}\pi(9)^2(12)$$

Replace r with 9 and h with 12.

$$\approx 1017.88$$

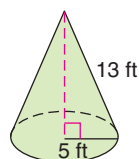
Use a calculator.

The volume of the cone is 1017.88 cubic inches.

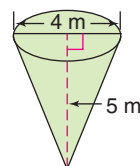
Your Turn

Find the volume of each cone to the nearest hundredth.

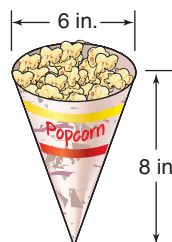
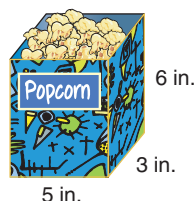
c.



d.

**Example****3****Entertainment Link**

Fernando Soto is the manager of a theater. He can purchase one of two containers to hold a small order of popcorn. How much more does the box hold than the cone?



First, find the volume of each container.

Volume of Prism

$$\begin{aligned} V &= Bh && \text{Theorem 12-5} \\ &= 15(6) && B = 5(3) \text{ or } 15 \\ &= 90 && \text{Simplify.} \end{aligned}$$

Volume of Cone

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h && \text{Theorem 12-12} \\ &= \frac{1}{3}\pi(3)^2(8) && \text{If } d = 6, r = 3. \\ &\approx 75.40 && \text{Use a calculator.} \end{aligned}$$

The volume of the box is 90 cubic inches. The volume of the cone is about 75.40 cubic inches. Thus, the box holds $90 - 75.40$ or 14.6 cubic inches more than the cone.

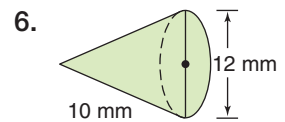
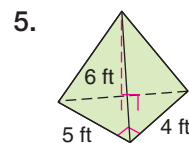
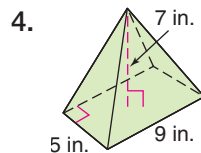
Check for Understanding

Communicating Mathematics

1. Compare and contrast the formulas for the volume of a prism and the volume of a pyramid.
2. Explain why both $V = \frac{1}{3}Bh$ and $V = \frac{1}{3}\pi r^2 h$ can be used to find the volume of a cone.
3. **You Decide?** Darnell believes that doubling the radius of a cone increases the volume more than doubling the height. Nicole disagrees. Who is correct? Explain.

Guided Practice Examples 1 & 2

Find the volume of each solid. Round to the nearest hundredth, if necessary.



Example 3

7. **Architecture** The Muttart Conservatory in Edmonton, Alberta consists of four greenhouses in the shape of pyramids. Each of the two largest pyramids has a height of about 24 meters and a base with an area of about 625 square meters. Find the total volume of the two pyramids.

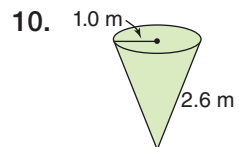
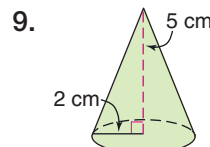
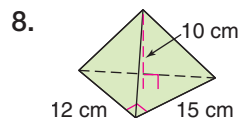


The Muttart Conservatory

Exercises

Practice

Find the volume of each solid. Round to the nearest hundredth, if necessary.



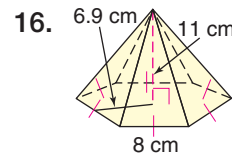
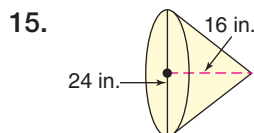
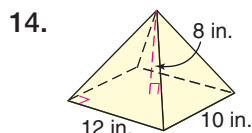
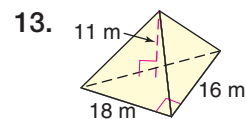
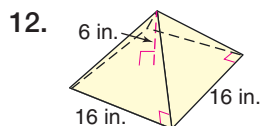
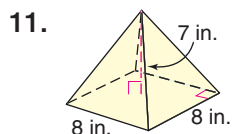
Homework Help

For Exercises	See Examples
8, 11–14, 16, 17, 20	1
9, 10, 15, 18, 19, 21	2

Extra Practice

See page 749.

Find the volume of each solid. Round to the nearest hundredth, if necessary.



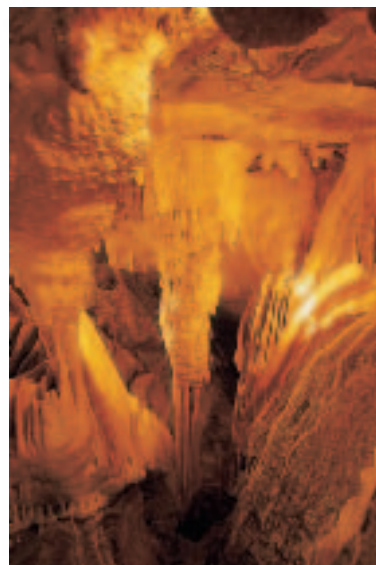
17. A pyramid has a height of 5 centimeters and a base with area of 18 square centimeters. What is its volume?
18. A cone has a height of 10 meters and a base with a radius of 3 meters. Find the volume of the cone.
19. The diameter of the base of a cone is 18 feet. The height of the cone is 12 feet. What is the volume of the cone?
20. The base of a pyramid is a triangle with a base of 12 inches and a height of 8 inches. The height of the pyramid is 10 inches. Find the volume of the pyramid.

Applications and Problem Solving

21. **Geology** A stalactite in Endless Caverns in Virginia is shaped like a cone. It is 4 feet tall and has a diameter at the roof of $1\frac{1}{2}$ feet. Find the volume of the stalactite.

22. **History** The Great Pyramid at Giza was built about 2500 B.C. It is a square pyramid.

- a. Originally the Great Pyramid was about 481 feet tall. Each side of the base was about 755 feet long. What was the original volume of the pyramid?
- b. Today the Great Pyramid is about 450 feet tall. Each side of the base is still about 755 feet long. What is the current volume of the pyramid?
- c. What is the difference between the volume of the original pyramid and the current pyramid?
- d. What was the yearly average (mean) loss in the volume of the pyramid from 2500 B.C. to 2000 A.D.?

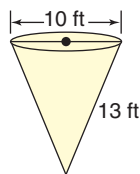


Stalactite formation

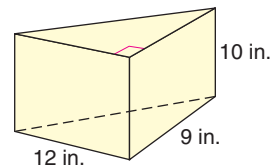
23. **Critical Thinking** A cone and a cylinder have the same volume, and their radii have the same measure. What is true about these two solids?

Mixed Review

24. Find the lateral area of the cone to the nearest hundredth.
(Lesson 12-4)



25. Find the volume of the triangular prism. (Lesson 12-3)

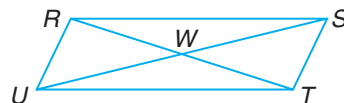


Standardized Test Practice

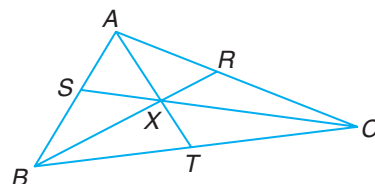
A B C D

26. **Construction** In a blueprint, 1 inch represents an actual length of 13 feet. If the dimensions of a kitchen on the blueprint are 1.25 inches by 0.75 inch, what are the actual dimensions? (Lesson 9-2)

27. **Grid In** In $\square RSTU$, find RW if $RT = 47.4$ units.
(Lesson 8-2)



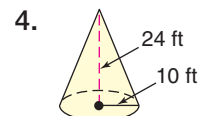
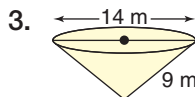
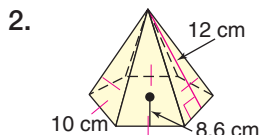
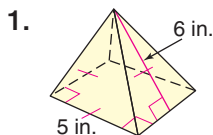
28. **Short Response** In $\triangle ABC$, \overline{AT} , \overline{BR} , and \overline{CS} are medians. What is the measure of \overline{XR} if $BR = 18$?
(Lesson 6-1)



Quiz 2

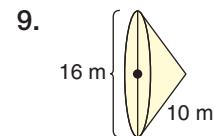
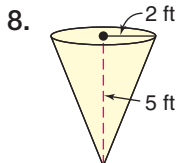
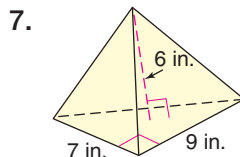
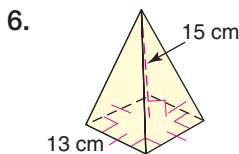
Lessons 12-4 and 12-5

Find the lateral area and the surface area of each solid. Round to the nearest hundredth, if necessary. (Lesson 12-4)



5. **Architecture** The base of the glass pyramid that serves as the entrance to the Louvre in Paris is a square with sides measuring 115 feet. The slant height of the pyramid is about 92 feet. To the nearest square foot, find the area of the glass covering the outside of the pyramid. (Lesson 12-4)

Find the volume of each solid. Round to the nearest hundredth, if necessary. (Lesson 12-5)



10. **History** Monk's Mound in Illinois is an earthen pyramid built around A.D. 600. It is 30.5 meters high. The base of this pyramid is a 216.6-meter by 329.4-meter rectangle. Find the volume of the soil used to build this mound to the nearest cubic meter. (Lesson 12-5)



What You'll Learn

You'll learn to find the surface areas and volumes of spheres.

Why It's Important
Sports

Manufacturers need to determine the amount of leather needed to cover a baseball. See Exercise 6.

A circle is the set of all points in a plane that are a fixed distance from a point in the plane, called the center. Suppose you were not limited to a plane. From a center, all points in space at a fixed distance form a hollow shell called a **sphere**. The hollow shell of the sphere is its surface.

Definition of a Sphere

Words: A sphere is the set of all points that are a fixed distance from a given point called the center.

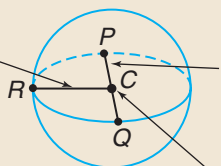
Model:



sphere with center at point C

A sphere has many properties like those of a circle.

A **radius** of a sphere is a segment whose endpoints are the center and a point on the sphere. \overline{CR} is a radius.

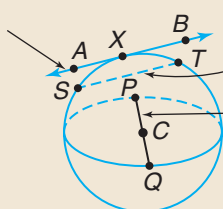


A **diameter** of a sphere is a segment that joins two points on the sphere and passes through its center. \overline{PQ} is a diameter.

Point C is the center of the sphere.

Info
Graphic

A **tangent** to a sphere is a line that intersects the sphere at exactly one point. \overline{AB} is tangent to the sphere at point X .




A **chord** of a sphere is a segment whose endpoints are points on the sphere. \overline{ST} and \overline{PQ} are chords.

The diameter \overline{PQ} is a special type of chord.

Info
Graphic

The formulas for finding the surface area and the volume of a sphere are given on the next page.

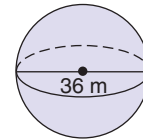
Theorem	Words	Model and Symbols
12–13 Surface Area of a Sphere	If a sphere has a surface area of S square units and a radius of r units, then $S = 4\pi r^2$.	 $S = 4\pi r^2$
12–14 Volume of a Sphere	If a sphere has a volume of V cubic units and a radius of r units, then $V = \frac{4}{3}\pi r^3$.	$V = \frac{4}{3}\pi r^3$

Example

1

Find the surface area and volume of the sphere.

Since the diameter is 36 meters, the radius is 18 meters.



Surface Area

$$\begin{aligned}
 S &= 4\pi r^2 && \text{Theorem 12–13} \\
 &= 4\pi(18)^2 && \text{Replace } r \text{ with } 18. \\
 &\approx 4071.50 && \text{Use a calculator.}
 \end{aligned}$$

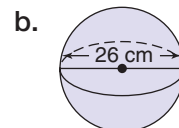
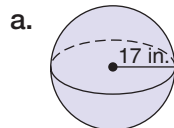
Volume

$$\begin{aligned}
 V &= \frac{4}{3}\pi r^3 && \text{Theorem 12–14} \\
 &= \frac{4}{3}\pi(18)^3 && \text{Replace } r \text{ with } 18. \\
 &\approx 24,429.02 && \text{Use a calculator.}
 \end{aligned}$$

The surface area is about 4071.50 square meters. The volume is about 24,429.02 cubic meters.

Your Turn

Find the surface area and volume of each sphere. Round to the nearest hundredth.



When a plane intersects a sphere so that congruent halves are formed, each half is called a hemisphere.

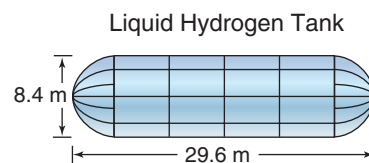


Example

Aerospace Link

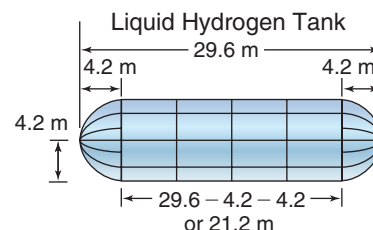
2

The large external tank attached to the space shuttle at the time of launch contains the propellants for takeoff. It holds three tanks, including the liquid hydrogen tank. If the ends of the liquid hydrogen tank are hemispheres, find the volume of this tank to the nearest hundredth.



Explore You know the length and the diameter of the tank. You need to find the volume of the tank.

Plan Think of the tank as two hemispheres and a cylinder. The radius of each hemisphere and the cylinder is $\frac{1}{2}(8.4)$ or 4.2 meters. Draw a diagram showing the measurements, and calculate the sum of the volumes of each shape. You can think of the two hemispheres as one sphere.



InterNET CONNECT

Data Update For the latest information on space shuttles, visit: www.geomconcepts.com

Solve

Volume of Sphere

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi(4.2)^3 \\ &\approx 310.34 \end{aligned}$$

$$310.34 + 1174.86 = 1485.2$$

The volume of the liquid hydrogen tank is about 1485.2 cubic meters.

Examine

Find the volume of a cylindrical tank with a radius of 4.2 meters and a height of 29.6 meters. Its volume should be slightly more than the liquid hydrogen tank.

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(4.2)^2(29.6) \\ &\approx 1640.36 \end{aligned}$$

The answer seems reasonable.

Volume of Cylinder

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi(4.2)^2(21.2) \\ &\approx 1174.86 \end{aligned}$$



Check for Understanding

Communicating Mathematics

1. Compare and contrast circles and spheres.
2. Draw and label a sphere with center at point P and chord \overline{MN} that is not a diameter.
3. **Writing Math** Describe real-world examples of five different solids. Write the formula used to find the volume of each type of solid.

Vocabulary

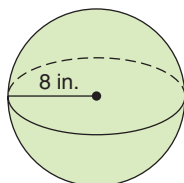
sphere

Guided Practice

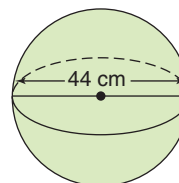
Example 1

Find the surface area and volume of each sphere. Round to the nearest hundredth.

4.

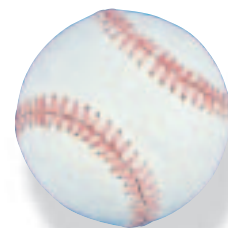


5.



Example 2

6. **Sports** Find the amount of leather needed to cover an official major league baseball if its diameter is 7.4 centimeters.



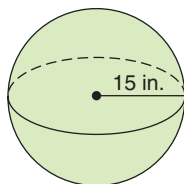
Exercises

Practice

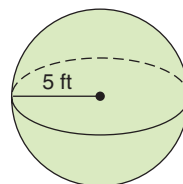
Find the surface area and volume of each sphere. Round to the nearest hundredth.

Homework Help	
For Exercises	See Examples
7–18, 20, 21	1
19	2
Extra Practice	
See page 749.	

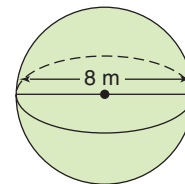
7.



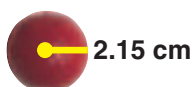
8.



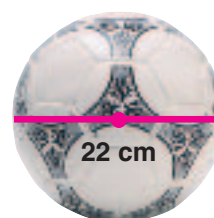
9.



10.



11.



12.



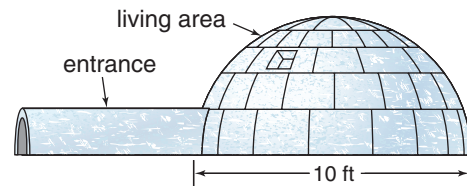
13. Find the surface area of a sphere with a diameter of 24 meters. Round to the nearest hundredth.
14. Find the volume of a sphere with a radius of 10 inches. Round to the nearest hundredth.
15. What is the volume of a sphere with a radius of 7 feet? Express the answer in cubic yards. Round to the nearest hundredth.
16. What is the surface area of a sphere with a diameter of 18 centimeters? Express the answer in square meters. Round to the nearest hundredth.

Applications and Problem Solving

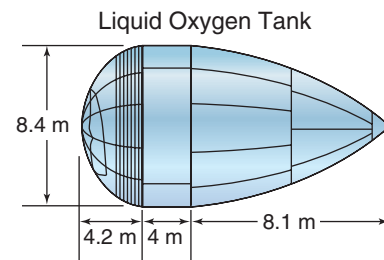
17. **Food** An ice cream cone is 10 centimeters deep and has a diameter of 4 centimeters. A spherical scoop of ice cream that is 4 centimeters in diameter rests on top of the cone. If all the ice cream melts into the cone, will the cone overflow? Explain.

18. **Housing** The Algonquin people live in northern Canada, Greenland, Alaska, and eastern Siberia. Their traditional homes are igloos that resemble hemispheres.

- a. Find the square footage of the living area of the igloo to the nearest hundredth.
- b. Find the volume of the living area of the igloo to the nearest hundredth.

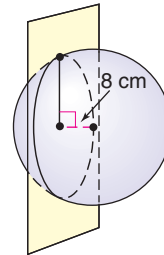


19. **Aerospace** The liquid oxygen tank in the external tank of the space shuttle resembles a combination of a hemisphere, a cylinder, and a cone. Find the volume of the liquid oxygen tank to the nearest hundredth.



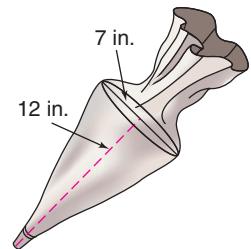
20. **Sales** The Boy Scouts of America recently made a giant popcorn ball to promote their popcorn sales. Scouts and other helpers added to the popcorn ball as it traveled around the country.
 - a. What was the volume of the popcorn ball when it had a diameter of 6 feet? Round to the nearest hundredth.
 - b. Suppose the diameter of the popcorn ball was enlarged to 9 feet. What would be its volume to the nearest hundredth?
 - c. What was the volume of the popcorn needed to increase the popcorn ball from a diameter of 6 feet to a diameter of 9 feet?

21. **Science** The diameter of Earth is about 7900 miles.
- Find the surface area of Earth to the nearest hundred square miles.
 - Find the volume of Earth to the nearest hundred cubic miles.
 - Most of Earth's atmosphere is less than 50 miles above the surface. Find the volume of Earth's atmosphere to the nearest hundred cubic miles.
22. **Critical Thinking** A plane slices a sphere as shown. If the radius of the sphere is 10 centimeters, find the area of the circle formed by the intersection of the sphere and the plane to the nearest hundredth.



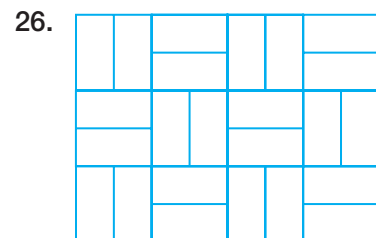
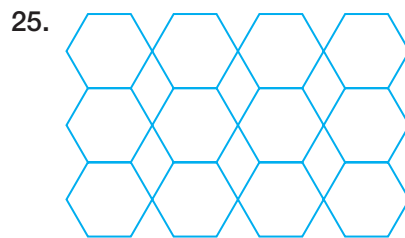
Mixed Review

23. **Cooking** A cake-decorating bag is in the shape of a cone. To the nearest hundredth, how much frosting will fit into a cake-decorating bag that has a diameter of 7 inches and a height of 12 inches? (*Lesson 12-5*)



24. A square pyramid has a base with sides 3.6 meters long and with a slant height of 6.5 meters. (*Lesson 12-4*)
- Draw a figure to represent this situation.
 - Find the surface area of the pyramid to the nearest hundredth.

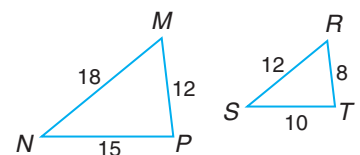
Identify the figures used to create each tessellation. Then identify the tessellation as **regular**, **semi-regular**, or **neither**. (*Lesson 10-7*)



Standardized Test Practice

(A) (B) (C) (D)

27. **Short Response** Determine the scale factor of $\triangle MNP$ to $\triangle RST$. (*Lesson 9-7*)



28. **Multiple Choice** Which of the following represents the distance between the points with coordinates $(a, 0)$ and $(0, b)$? (*Lesson 6-7*)

(A) $\sqrt{2a + 2b}$ (B) $\sqrt{a^2 + b^2}$ (C) $\sqrt{a + b}$ (D) $\sqrt{a^2 - b^2}$



What You'll Learn

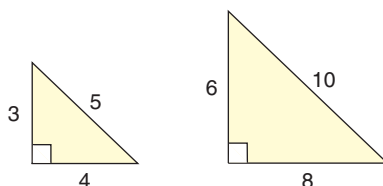
You'll learn to identify and use the relationships between similar solid figures.

Why It's Important

Automotive Design

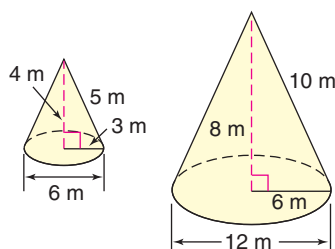
Car designers make models that are similar to proposed new cars. See Exercise 6.

Similar solids are solids that have the same shape but are not necessarily the same size. Just as with similar polygons, corresponding linear measures have equivalent ratios. For example, two triangles are similar if the ratio of their corresponding sides are equal. In other words, the ratio of their corresponding sides are proportional.



$$\frac{8}{4} = \frac{6}{3} = \frac{10}{5}$$

The two cones shown below are similar because the ratios of the diameters to the bases are proportional, that is, $\frac{6}{12} = \frac{4}{8}$.

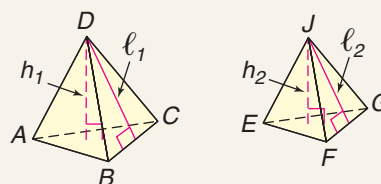


Recall that the ratio of measures is called a *scale factor*.

Characteristics of Similar Solids

Words: For similar solids, the corresponding lengths are proportional, and the corresponding faces are similar.

Model:

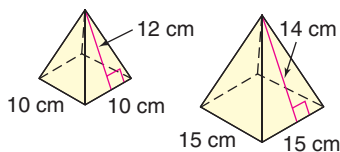


Symbols: $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CA}{GE} = \frac{AD}{EJ} = \frac{BD}{FJ} = \frac{CD}{GJ} = \frac{h_1}{h_2} = \frac{\ell_1}{\ell_2}$,
 $\triangle ABC \sim \triangle EFG$, $\triangle ABD \sim \triangle EFJ$,
 $\triangle BCD \sim \triangle FGJ$, $\triangle ACD \sim \triangle EGJ$

Examples

Determine whether each pair of solids is similar.

1



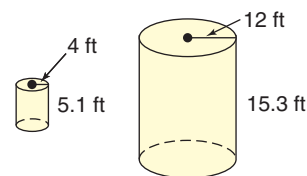
$$\frac{10}{15} \stackrel{?}{=} \frac{12}{14}$$

$$10(14) \stackrel{?}{=} 15(12)$$

$$140 \neq 180$$

The pyramids are not similar.

2



$$\frac{4}{12} \stackrel{?}{=} \frac{5.1}{15.3}$$

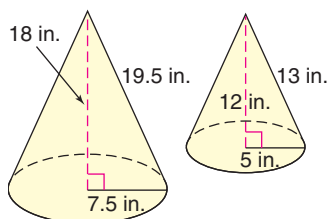
$$4(15.3) \stackrel{?}{=} 12(5.1)$$

$$61.2 = 61.2 \quad \checkmark$$

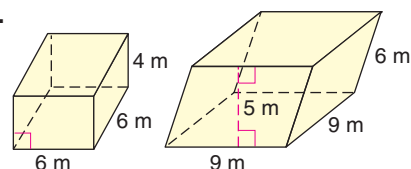
The cylinders are similar.

Your Turn

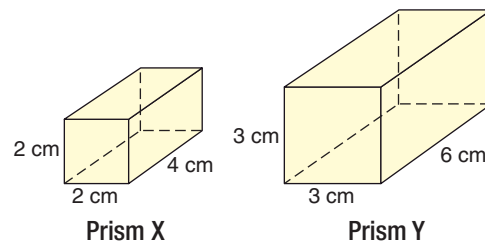
a.



b.



Study the two similar prisms at the right. The scale factor of prism X to prism Y is $\frac{2}{3}$. What is the ratio of the surface area of prism X to the surface area of prism Y?



Surface Area of Prism X

$$S = Ph + 2B$$

$$= 12(2) + 2(8)$$

$$= 40$$

Surface Area of Prism Y

$$S = Ph + 2B$$

$$= 18(3) + 2(18)$$

$$= 90$$

The ratio of the surface area of prism X to the surface area of prism Y is $\frac{40}{90}$ or $\frac{4}{9}$. Notice that $\frac{4}{9} = \frac{2^2}{3^2}$.

Now, compare the ratio of the volume of prism X to the volume of prism Y.

Volume of Prism X

$$V = Bh$$

$$= 8(2)$$

$$= 16$$

Volume of Prism Y

$$V = Bh$$

$$= 18(3)$$

$$= 54$$



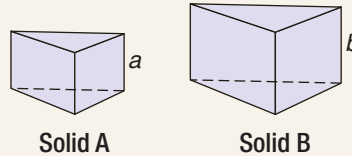
The ratio of the volume of prism X to the volume of prism Y is $\frac{16}{54}$ or $\frac{8}{27}$.
 Notice that $\frac{8}{27} = \frac{2^3}{3^3}$.

The relationships between prism X and prism Y suggest Theorem 12–15.

Theorem 12–15

Words: If two solids are similar with a scale factor of $a:b$, then the surface areas have a ratio of $a^2:b^2$ and the volumes have a ratio of $a^3:b^3$.

Model:



Solid A

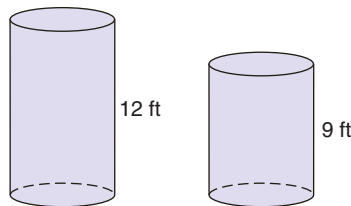
Solid B

Symbols: scale factor of solid A to solid B = $\frac{a}{b}$
 $\frac{\text{surface area of solid A}}{\text{surface area of solid B}} = \frac{a^2}{b^2}$
 $\frac{\text{volume of solid A}}{\text{volume of solid B}} = \frac{a^3}{b^3}$

Example

3

For the similar cylinders, find the scale factor of the cylinder on the left to the cylinder on the right. Then find the ratios of the surface areas and the volumes.



The scale factor is $\frac{12}{9}$ or $\frac{4}{3}$.

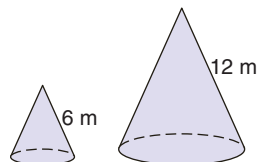
The ratio of the surface areas is $\frac{4^2}{3^2}$ or $\frac{16}{9}$.

The ratio of the volumes is $\frac{4^3}{3^3}$ or $\frac{64}{27}$.

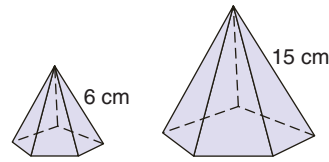
Your Turn

For each pair of similar solids, find the scale factor of the solid on the left to the solid on the right. Then find the ratios of the surface areas and the volumes.

c.



d.





Example

Social Studies Link

4

Mrs. Gomez's social studies class is using cardboard to build a scale model of the Great Pyramid of Khufu in Egypt. The surface area of this pyramid is about 1,496,510 square feet. If the scale factor of the model to the original is 1:100, how much cardboard will the class need to make the model?

Let S represent the surface area of the model.

$$\frac{\text{surface area of model}}{\text{surface area of Great Pyramid}} = \frac{1^2}{100^2} \quad \text{Theorem 12-15}$$

$$\frac{S}{1,496,510} = \frac{1}{10,000} \quad \text{Substitution}$$

$$S(10,000) = 1,496,510(1) \quad \text{Cross Products}$$

$$10,000S = 1,496,510 \quad \text{Multiply.}$$

$$\frac{10,000S}{10,000} = \frac{1,496,510}{10,000} \quad \text{Divide each side by 10,000.}$$

$$S = 149.651 \quad \text{Simplify.}$$

The class will need about 150 square feet of cardboard.



Great Pyramid of Khufu

Check for Understanding

Communicating Mathematics

1. **Explain** the meaning of similar solids. Can two solids which have the same size and shape be similar? Explain.
2. **Draw** two spheres such that the ratio of their volumes is 1:64.

Vocabulary

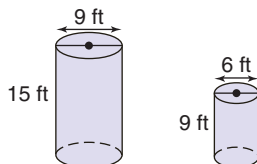
similar solids

Guided Practice

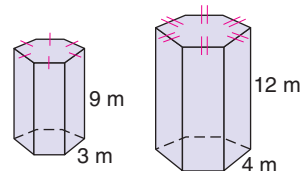
Examples 1 & 2

Determine whether each pair of solids is similar.

3.

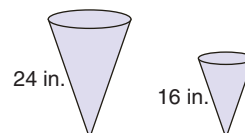


4.



Example 3

5. For the similar cones, find the scale factor of the cone on the left to the cone on the right. Then find the ratios of the surface areas and the volumes.



Example 4

6. **Automotive Design** Car designers often build clay models of the concept car they are creating.
 - a. If the 30-inch model represents a 15-foot car, what is the scale factor of the model to the actual car? (*Hint: Change feet to inches.*)
 - b. What is the ratio of the surface areas of the model to the actual car?

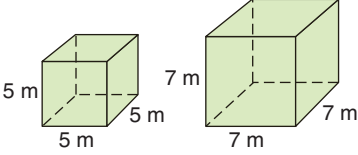
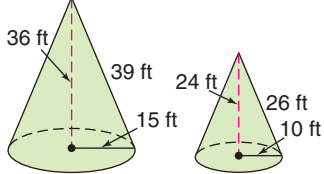
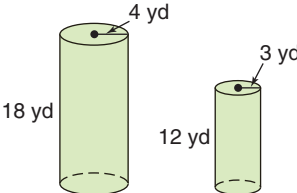
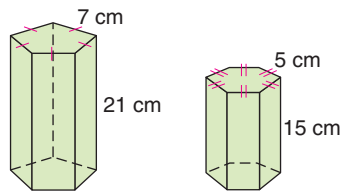
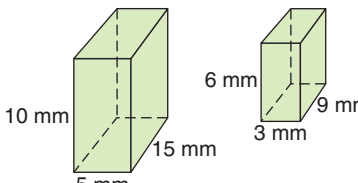
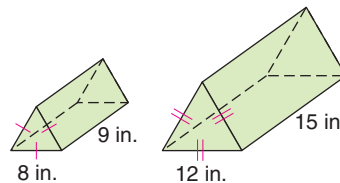


Exercises

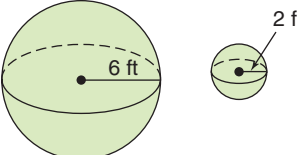
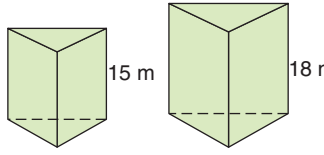
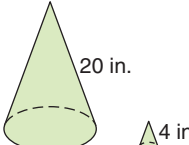
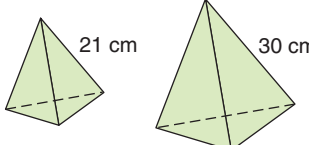
Practice

Homework Help	
For Exercises	See Examples
7–12	1, 2
13–23	3
Extra Practice	
See page 750.	

Determine whether each pair of solids is similar.

7. 
8. 
9. 
10. 
11. 
12. 

For each pair of similar solids, find the scale factor of the solid on the left to the solid on the right. Then find the ratios of the surface areas and the volumes.

13. 
14. 
15. 
16. 

17. The dimensions of a prism are doubled.
 - a. How does the surface area change?
 - b. How does the volume change?
18. The ratio of the heights of two similar prisms is 5:3.
 - a. Find the ratio of their surface areas.
 - b. Find the ratio of their volumes.
19. The ratio of the surface areas of two similar cones is 9:16.
 - a. What is the scale factor of the cones?
 - b. What is the ratio of the volumes of the cones?

20. The ratio of the volumes of two similar pyramids is 27:1000.
- Find the scale factor of the pyramids.
 - Find the ratio of the surface areas of the pyramids.

Applications and Problem Solving



21. **Baking** In 1989, a very large pecan pie was created for the Pecan Festival in Okmulgee, Oklahoma. The pie was 40 feet in diameter. If the pie was similar to a normal pie with an 8-inch diameter, find the ratio of volume of the large pie to the volume of the normal pie.

22. **Miniatures** The Carole & Barry Kaye Museum of Miniatures in Los Angeles displays a tiny desk made from a thousand pieces of wood.

- If the 3-inch tall desk represents a real desk that is 30 inches tall, what is the scale factor of the miniature to the real desk?
- What is the ratio of the surface areas of the miniature to the real desk?
- What is the ratio of the volumes of the miniature to the real desk?

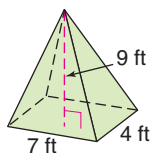


23. **Critical Thinking** Explain why all cubes are similar to each other. Name another type of solid that is always similar to others in the category.

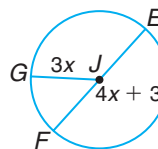
Mixed Review

24. **Sports** What is the surface area and volume of a racquetball if its diameter is 2.25 inches? Round to the nearest hundredth.
(Lesson 12-6)

25. Find the volume of the rectangular pyramid to the nearest hundredth.
(Lesson 12-5)



26. **Algebra** Find the measure of radius \overline{JG} if $JG = 3x$ and $EF = 4x + 3$. (Lesson 11-1)



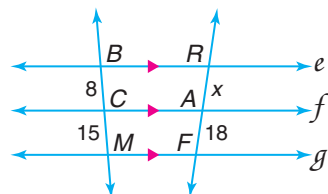
Standardized Test Practice

(A) (B) (C) (D)

27. **Grid In** Find the measure of one exterior angle of a regular hexagon.
(Lesson 10-2)

28. **Multiple Choice** In the figure at the right, $e \parallel f \parallel g$. Find the value of x .
(Lesson 9-6)

- (A) $8\frac{1}{4}$ (B) $8\frac{2}{3}$
(C) $9\frac{1}{2}$ (D) $9\frac{3}{5}$



CHAPTER 12 Study Guide and Assessment

Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

axis (p. 506)
 composite solid (p. 498)
 cone (p. 497)
 cube (p. 497)
 cylinder (p. 497)
 edge (p. 496)
 face (p. 496)
 lateral area (p. 504)
 lateral edge (p. 497)
 lateral face (p. 497)
 net (p. 507)

oblique cone (p. 516)
 oblique cylinder (p. 506)
 oblique prism (p. 504)
 oblique pyramid (p. 516)
 Platonic solid (p. 497)
 polyhedron (p. 496)
 prism (p. 496)
 pyramid (p. 496)
 regular pyramid (p. 516)
 right cone (p. 516)
 right cylinder (p. 506)



right prism (p. 504)
 right pyramid (p. 516)
 similar solids (p. 534)
 slant height (p. 516)
 solid figures (p. 496)
 sphere (p. 528)
 surface area (p. 504)
 tetrahedron (p. 497)
 truncated solid (p. 521)
 volume (p. 510)

Choose the letter of the term that best matches each phrase.

- the sum of the areas of a solid's surfaces
- a special rectangular prism in which all of the faces are squares
- the measurement of the space occupied by a solid region
- the height of each lateral face of a regular pyramid
- solids that have the same shape, but not necessarily the same size
- solid with flat surfaces that are polygons
- figure that encloses a part of space
- another name for a special kind of triangular pyramid
- the set of all points that are a given distance from the center
- intersection of two faces of a polyhedron

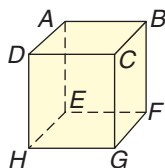
- polyhedron
- volume
- cube
- surface area
- slant height
- sphere
- edge
- tetrahedron
- similar solids
- solid

Skills and Concepts

Objectives and Examples

- Lesson 12-1** Identify solid figures.

The polyhedron has 6 faces, 12 edges, and 8 vertices. It is a rectangular prism.



Review Exercises

- Name the faces, edges, and vertices of the polyhedron at the left.

Refer to the figure at the left to determine whether each statement is **true** or **false**.

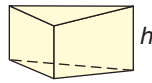
- $EFGH$ is a lateral face.
- $ABCD$ and $EFGH$ are bases.
- CG is a lateral edge.



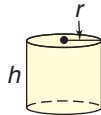
Objectives and Examples

- **Lesson 12-2** Find the lateral areas and surface areas of prisms and cylinders.

Prism
 $L = Ph$
 $S = Ph + 2B$

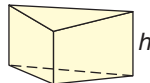


Cylinder
 $L = 2\pi rh$
 $S = 2\pi rh + 2\pi r^2$

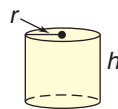


- **Lesson 12-3** Find the volumes of prisms and cylinders.

Volume of a Prism
 $V = Bh$

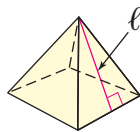


Volume of a Cylinder
 $V = \pi r^2 h$



- **Lesson 12-4** Find the lateral areas and surface areas of regular pyramids and cones.

Regular Pyramid
 $L = \frac{1}{2}P\ell$
 $S = \frac{1}{2}P\ell + B$

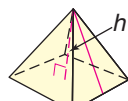


Cone
 $L = \pi r\ell$
 $S = \pi r\ell + \pi r^2$

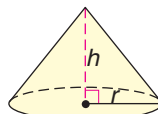


- **Lesson 12-5** Find the volumes of pyramids and cones.

Volume of a Pyramid
 $V = \frac{1}{3}Bh$



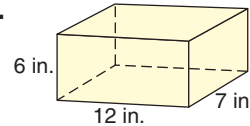
Volume of a Cone
 $V = \frac{1}{3}\pi r^2 h$



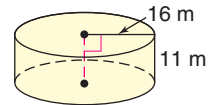
Review Exercises

Find the lateral area and the surface area for each solid. Round to the nearest hundredth, if necessary.

15.



16.



Find the volume of each solid described. Round to the nearest hundredth, if necessary.

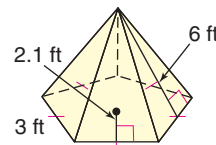
17. rectangular prism that is 5 in. by 9 in. by 9 in.

18. cylinder with a base diameter of 6 cm and a height of 5 cm

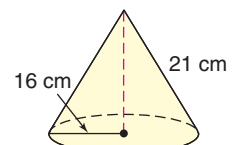
19. triangular prism with a base area of 4 m² and a height of 8 m

Find the lateral area and the surface area for each solid. Round to the nearest hundredth, if necessary.

20.

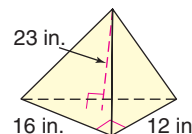


21.

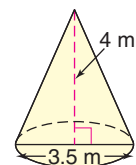


Find the volume of each solid. Round to the nearest hundredth, if necessary.

22.



23.



Objectives and Examples

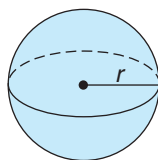
- Lesson 12-6** Find the surface areas and volumes of spheres.

Surface Area of a Sphere

$$S = 4\pi r^2$$

Volume of a Sphere

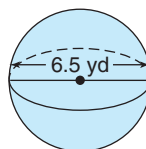
$$V = \frac{4}{3}\pi r^3$$



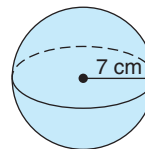
Review Exercises

Find the surface area and volume of each sphere. Round to the nearest hundredth.

24.



25.



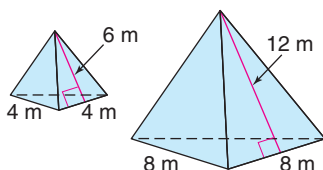
- Lesson 12-7** Identify and use the relationships between similar solid figures.

The pyramids are similar.

$$\frac{4}{6} = \frac{8}{12}$$

$$4(12) = 6(8)$$

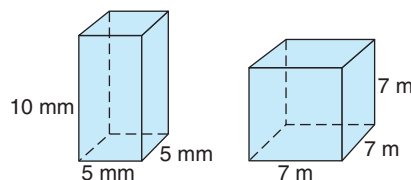
$$48 = 48$$



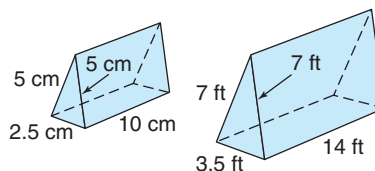
The scale factor of the pyramid on the left to the pyramid on the right is $\frac{1}{2}$. The ratio of the surface area is $\frac{1^2}{2^2}$ or $\frac{1}{4}$. The ratio of the volumes is $\frac{1^3}{2^3}$ or $\frac{1}{8}$.

Determine whether each pair of solids is similar.

26.



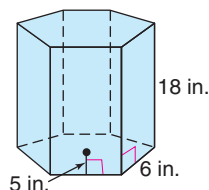
27.



28. The ratio of the heights of two similar right cylinders is 3:4. Find the ratio of their surface areas and the ratio of their volumes.

Applications and Problem Solving

29. **Hobbies** Jeanette's aquarium is a regular hexagonal prism. Find the volume of water the aquarium holds when it is completely full to the nearest cubic inch. (Lesson 12-3)



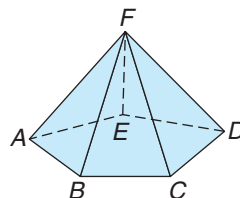
30. **Landscaping** A truckload of fill dirt is dumped in front of a newly-built home. The pile of dirt is cone-shaped. It has a height of 7 feet and a diameter of 15 feet. Find the volume of the dirt to the nearest hundredth. (Lesson 12-5)
31. **Astronomy** Find the surface area and volume of the moon if its diameter is approximately 2160 miles. (Lesson 12-6)

CHAPTER 12 Test

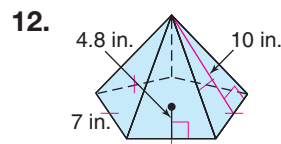
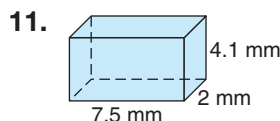
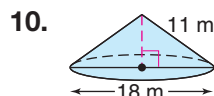
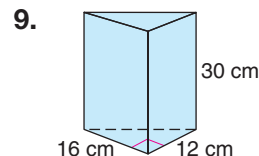
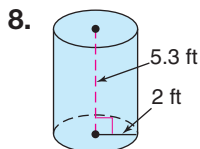
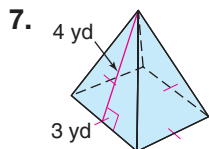
1. Compare and contrast surface area and volume.
2. Define the term *sphere*, and name three common items that are shaped like spheres.

Determine whether each statement is *true* or *false* for the geometric solid.

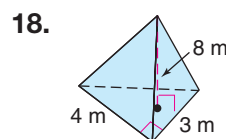
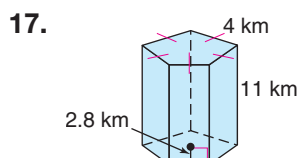
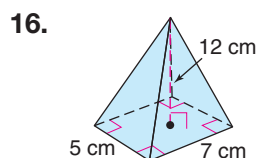
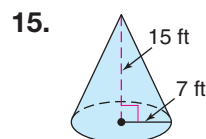
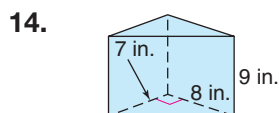
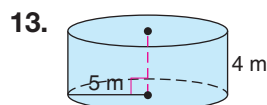
3. The figure has 10 edges.
4. The figure is a polyhedron.
5. The figure is a circular cone.
6. The figure is a tetrahedron.



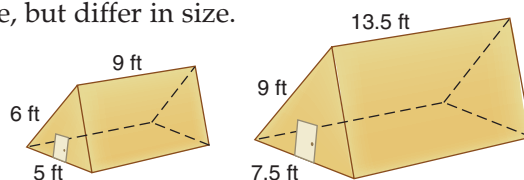
Find the lateral area and the surface area of each geometric solid. Round to the nearest hundredth, if necessary.



Find the volume of each solid. Round to the nearest hundredth, if necessary.



19. **Recreation** A beach ball has a diameter of 24 inches. Find the surface area and volume of the beach ball to the nearest hundredth.
20. **Storage** ABC Lumber Company sells plans and materials for several storage sheds. The two designs shown have a similar shape, but differ in size.
 - a. Find the scale factor of the shed on the left to the shed on the right.
 - b. Find the ratios of the surface areas and the volumes.



CHAPTER 12 Preparing for Standardized Tests

Angle, Line, and Arc Problems

Geometry problems on standardized tests often involve parallel lines and circles.

Review these concepts.

Angles: vertical angles, supplementary angles, complementary angles

Parallel lines: transversals, alternate interior angles

Circles: inscribed angles, central angles, arc length, tangent line

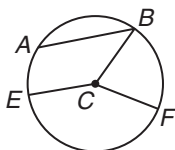
Test-Taking Tip

Use the square corner of a sheet of paper to estimate angle measure.

Example 1

Name each of the following in the figure below.

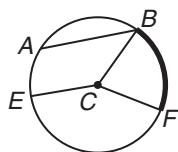
- an arc
- a sector
- a chord



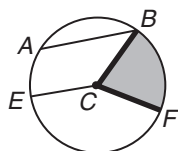
Hint Before taking a standardized test, it is a good idea to review vocabulary words.

Solution

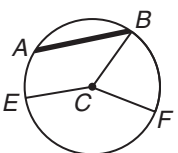
- BF is an arc.



- BCF is a sector.



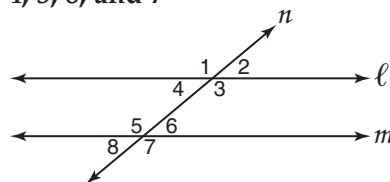
- AB is a chord.



Example 2

In the figure below, line ℓ is parallel to line m . Line n intersects both ℓ and m , with angles 1, 2, 3, 4, 5, 6, 7, and 8 as shown. Which of the following lists includes all of the angles that are supplementary to $\angle 1$?

- angles 2, 4, 6, and 8
- angles 3, 5, and 7
- angles 2, 4, and 3
- angles 5, 6, 7, and 8
- angles 4, 3, 8, and 7



Hint Look for words like *supplementary*.

Solution Look carefully at the figure.

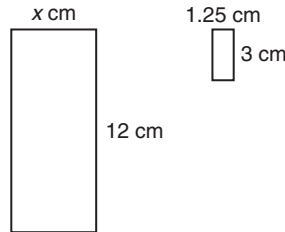
- Find $\angle 1$. Notice that $\angle 1$ and $\angle 2$ form a linear pair, so $\angle 2$ is supplementary to $\angle 1$.
- Since $\angle 2$ and $\angle 4$ are vertical angles, they are equal in measure. So $\angle 4$ is also supplementary to $\angle 1$.
- Since $\angle 4$ and $\angle 6$ are alternate interior angles, they are equal. So $\angle 6$ is supplementary to $\angle 1$.
- And since $\angle 6$ and $\angle 8$ are vertical angles, $\angle 8$ is supplementary to $\angle 1$.

The angles supplementary to $\angle 1$ are angles 2, 4, 6, and 8. The answer is A.

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

1. The figures at the right are similar. Find the value of x . (Lesson 9–2)
- (A) 5.0 (B) 28.8
(C) 3.2 (D) 45.0

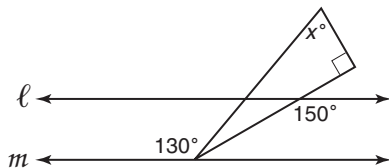


2. Which expression could be used to find the value of y ? (Algebra Review)

- (A) $2x + 1$
(B) $1 - 3x$
(C) $3x - 1$
(D) $3x + 1$

x	y
1	2
2	5
3	8
4	11

3. If line ℓ is parallel to line m in the figure below, what is the value of x ? (Lesson 5–2)



- (A) 20 (B) 50 (C) 70
(D) 80 (E) 90

4. $5\frac{1}{3} - 6\frac{1}{4} = ?$ (Algebra Review)

- (A) $-\frac{11}{12}$ (B) $-\frac{1}{2}$ (C) $-\frac{2}{7}$
(D) $\frac{1}{2}$ (E) $\frac{9}{12}$

5. Two number cubes are rolled. What is the probability that both number cubes will show a number less than 4? (Statistics Review)

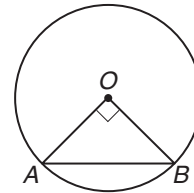
- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{4}{9}$ (D) $\frac{1}{2}$

6. What is the slope of a line perpendicular to the line represented by the equation $3x - 6y = 12$? (Lesson 4–5)

- (A) -2 (B) $-\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

7. What is the height of a triangle with an area of 36 square centimeters and a base of 4 centimeters? (Lesson 10–4)
- (A) 9 cm (B) 18 cm (C) 36 cm (D) 72 cm

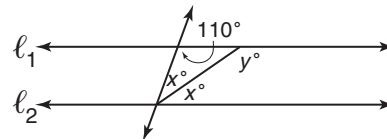
8. Which statement about $\angle OAB$ is true? (Lesson 11–2)



- (A) $\angle OAB$ measures more than 90° .
(B) $\angle OAB$ measures less than 90° but more than 45° .
(C) $\angle OAB$ measures exactly 45° .
(D) $\angle OAB$ measures less than 45° .

Grid In

9. If ℓ_1 is parallel to ℓ_2 in the figure below, what is the value of y ? (Lesson 5–2)



Extended Response

10. The height h , in feet, of a ball t seconds after being hit in the air from a height of 4 feet is $h = 4 + vt - 16t^2$, where v is the initial upward velocity. (Algebra Review)

Part A Make a table of values showing the height of a baseball hit with an initial upward velocity of 128 feet per second. Track the height for every half second for the first four seconds.

Part B How long will it take for the ball to reach a height of 256 feet on its way up?

