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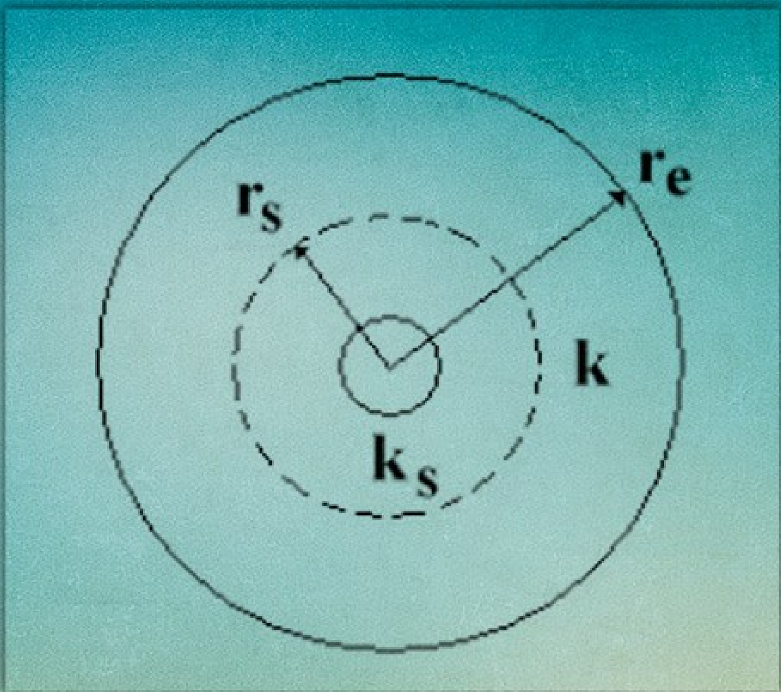
Introductory Well Testing

Tom Aage Jelmert



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Contents

1 Preface

While there are many excellent and important books on the subject of well testing, few provide an introduction to the topic at the very basic level. The objective is to provide an easy to read introduction to classical well test theory. No previous knowledge in well testing is required. The reader is expected to understand basic concepts of flow in porous media. Well test interpretation depends on mathematical models. Some calculus skill is required. Hopefully, this book will give the reader a useful introduction to a fascinating subject and stimulate further studies.

Well testing is important in many disciplines: petroleum engineering, groundwater hydrology, geology and waste water disposal. The theory is the same, but different nomenclature and units are used. The present book use consistent units and petroleum engineering nomenclature. A consistent unit system leads to dimensionless constants in all equations. Equations in a consistent unit system are dimensional transparent but inconvenient numerically. Hence a myriad of practical unit systems have evolved. Many authors present equations first in consistent units and then convert them to the practical unit system of their choice. Conversion factors are easily available in the literature.

2 Introduction

Well testing may be regarded as part of formation evaluation. The objective of formation evaluation is to provide input to a geologic model, which in turn may provide important input data for an economic model. Decisions, whether to start possible engineering projects or not, are based on economic analysis.

Classical well test interpretation depends on simplified analytical models and graphical techniques. The methodology may be described as follows: A pressure test is conducted by giving the well at least one perturbation in flow rate. The pressure response (pressure signature) is measured and matched to a mathematical model (equation or graph). Each well has a unique response which depends on the rock and fluid properties. The matched model gives rise to equations that may be solved for selected variables. There is a limit as to how many parameters that can be determined this way. Hence, well test interpretation depends on input data from other sources. For example, many interpretation techniques depend on the appearance of straight lines. These show up for specific flow periods, like radial-, linear- and pseudo steady flow. It is possible to obtain two equations from a straight line. These can be solved for two unknowns only. It is possible to solve for a few more by type curve analysis. The additional information required derives from a variety of sources, such as: geology, core analysis, well logging, seismic, etc.

An alternative to classical well testing is numerical simulation. An analytical model gives insight into the interaction of the variables that a numerical model cannot give. On the other hand, an analytical model may be too simplistic due to restrictive simplifying assumptions. Both techniques are needed. The use of numerical models also falls outside the scope of these notes.

3 Productivity of Wells

3.1 Introductory remarks

The productivity index of a well has been defined as the production rate pr. unit drawdown. This measure varies from well to well and should be as high as possible.

Usually a reservoir is produced by several wells. Each has a specific drainage area. A small reservoir may be produced by a single well. Then, the drainage area and the reservoir are the same.

If the saturation of a fluid phase becomes less than a critical value, then it will no longer exist as a continuous phase. Such a fluid phase has lost the ability to move. The relative permeability becomes zero. We refer to a flowing phase as a mobile and a non-flowing one as immobile. Suppose there is one flowing phase only. The immobile phases are compressible and will add to the system compressibility. Use of the total compressibility, c_t, will account for the immobile phases in an approximate way.

$$c_t = S_o c_o + S_w c_w + S_g c_g + c_r$$

Consider a drainage area shaped like cylinder and the well in the center. Flow is controlled by the diffusivity equation.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\varphi \mu c_t}{k}\frac{\partial p}{\partial t}$$

Pressure becomes independent of time provided the flow in and out of the drainage area are the same. Such flow is of steady state type. Then, the right hand side of the above partial differential equation is zero. The drainage area has an open external boundary as steady state flow depends on flow into the drainage area. The pressure at the outer boundary is constant, i.e. $p_e = const$. This can be achieved by a pressure maintenance project (water and or gas injection) or edge water drive.

No-flow boundaries may be due to geology or interference between production wells. Interference creates no-flow boundaries somewhere between the wells. A no-flow boundary is closed. From Darcy's law we find that a no-flow boundary may be characterized by $\frac{\partial p}{\partial n} = 0$, where n is the direction perpendicular to the boundary. For a cylindrical geometry the boundary condition becomes:

$$\frac{\partial p}{\partial r} = 0$$

The time derivative of pressure becomes constant for all values of the space variable, r. Since the rate of pressure decline is the same everywhere, the pressure difference between two points, $p(r_1) - p(r_2)$, is constant. This type of flow shares important characteristics with steady state flow and has been called pseudo-steady flow.

Both steady and pseudo-steady flow are characterized by a constant productivity index, *PI*. The important consequence is that the production rate, q, is proportional to the drawdown, Δp.

$$q = PI \cdot \Delta p$$

Both flow regimes are boundary dominated since they depend on the outer boundary condition. Also, both flow periods are preceded by an infinite-acting flow period. During the later period, the influence of the outer boundary condition is negligible.

3.2 Flow equations for boundary dominated flow

We consider steady- and pseudo steady state flow. The production rate is given by Darcy's law.

$$q = \frac{2\pi khr}{\mu B}\frac{dp}{dr}$$

Suppose the μB-product is independent of pressure and that the flow rate is independent of time. Under the assumption of constant pressure boundaries, the above equation may be integrated by separation of variables:

$$\Delta p = p_e - p_w = \frac{q\mu B}{2\pi kh}\ln\frac{r_e}{r_w}$$

Non-ideal conditions are accounted for by a skin factor, S.

$$\Delta p = p_e - p_w = \frac{q\mu B}{2\pi kh}\left(\ln\frac{r_e}{r_w} + S\right)$$

For pseudo-steady flow it may be shown that:

$$\Delta p = \overline{p} - p_w = \frac{q\mu B}{2\pi kh}\left(\ln\frac{r_e}{r_w} - \frac{3}{4} + S\right)$$

The above equation may be generalized for arbitrary geometries:

$$\Delta p = \frac{q\mu B}{2\pi kh}\left(\frac{1}{2}\ln\frac{2.25A}{C_A r_w^2} + S\right)$$

The size of drainage area is A and C_A is Dietz shape factor. Many shape factors for simple geometries have been tabulated. For example, C_A is 31.62, 30.88 and 27.6 for a circle, square and equilateral triangle respectively, and each area with the well located in the center.

It is convenient to define the drawdown in terms of average pressure for pseudo-steady flow. This is because $\overline{p}$ may be estimated by material balance calculations.

3.3 Productivity index, PI

The productivity index is a measure of the quality of a well. It has been defined as the rate of flow per unit pressure drawdown:

$$PI = \frac{q}{\Delta p}$$

A slight rearrangement yields:

$$q = PI \cdot \Delta p$$

where the productivity index assume constant values. The above equation shows up as a straight line in a q vs. Δp coordinate system with slope PI. The productivity index may be computed from the flow equations discussed in the previous section.

For steady state flow:

$$PI = \frac{q}{\Delta p} = \frac{2\pi kh}{\mu B\left(\ln\frac{r_e}{r_w} + S\right)}$$

For pseudo-steady state flow:

$$PI = \frac{2\pi kh}{\mu B\left(\ln\frac{r_e}{r_w} - \frac{3}{4} + S\right)}$$

For a drainage area of arbitrary shape, the following equation applies:

$$PI = \frac{2\pi kh}{\mu B\left(\frac{1}{2}\ln\frac{2.2458A}{C_A r_w^2} + S\right)}$$

The drawdown is $\Delta p = p_e - p_w$ for steady state and $\Delta p = \overline{p} - p_w$ for pseudo-steady flow.

Both steady state- and pseudo steady flow are ideal cases. A real case will fall somewhere in between.

A high production rate means more income. An increase in drawdown may be achieved by a change of choke. If the well is producing at full capacity, this possibility does not exist. In addition, too high drawdown may cause problems like sand-, water- and gas production, etc. Hence, the drawdown has an upper limit.

A high productivity index is advantageous (see Figure 1). Production characteristic "a" is obviously best since it gives the highest production rate for a given drawdown. The same production characteristic also gives the lowest drawdown for a given production rate.

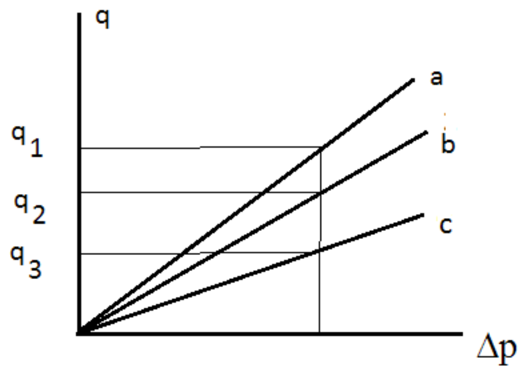

Figure 1: Effect of the productivity index on the production rate

By inspection of the above equations, we see that a well will produce at higher rate if it is possible to increase the numerator and decrease the denominator of the relevant productivity equation.

A high flow capacity, kh -product, is favorable for the production rate. It is difficult to change the flow capacity by an engineering technique.

The denominator may be decreased by:

- Reducing the skin factor, S. This can be done by a stimulation project, hydraulic fracturing or acid injection.
- Reducing the external radius, r_e , which can be achieved by infill drilling. The drainage area of each individual well is reduced since the reservoir area is shared by more wells.
- An increase in the wellbore radius, r_w , which is expensive or even impossible for deep wells.
- An increase in the effective wellbore radius, r_{we} . Fracturing.
- Decreasing the viscosity, μ. A thermal project.

3.4 Time dependency of the pseudo-steady solution

The time dependency can be explained by simple material balance model. A model may be classified according to the number of space variables. There are one-, two- and three dimensional models. Oldest is the tank model which has no space variable at all. As such, it may be classified as zero dimensional. The pressure associated with a tank model is the volumetric average pressure, $\overline{p}$. The shape of the tank is arbitrary.

Consider a slightly compressible fluid with compressibility, c_t , in a reservoir with pore volume V_p . The compressibility of the fluid is small and constant by assumption.

$$c_t = -\frac{1}{V}\frac{dV}{d\overline{p}}$$

The pore volume of the drainage area is constant for a conventional reservoir. Then, the fluid volume, V, in the reservoir and the pore volume, V_p, remains the same regardless of pressure. Suppose the average pressure is decreased from the initial pressure, p_i to, $\overline{p}$, and there is no flow into or out of the reservoir. Under these conditions, the increase in fluid volume due to expansion is produced, i.e. $dV = qBdt$. Integration yields:

$$\overline{p} = p_i - \frac{qB}{c_t V_p} t$$

Combination of the material balance equation with the generalized pseudo-steady flow equation yields:

$$p_{wf} = p_i - \frac{qB}{c_t \varphi Ah} \cdot t - \frac{q\mu B}{2\pi kh}\left(\ln\frac{A}{r_w^2} + \ln\frac{2.25}{C_A} + S \right)$$

The above equation is important for the reservoir limit test.

3.5 Flow efficiency

The productivity index depends on the unit system. Use of a normalized productivity index removes this inconvenience. In most cases, an actual well has some skin. We use the same well, but without skin, as reference condition. A well without skin has been called ideal. The Flow Efficiency, *FE*, is defined as:

$$FE = \frac{PI_{actual}}{PI_{ideal}} = \frac{\left(\frac{q}{\Delta p}\right)_{actual}}{\left(\frac{q}{\Delta p}\right)_{ideal}}$$

The actual and ideal well can be compared under the conditions of equal drawdown or equal production rate. Under the assumption equal drawdown:

$$FE = \frac{q_{actual}}{q_{ideal}}$$

Hence, the rate of an equivalent well without skin may be computed as follows.

$$q_{ideal} = \frac{q_{actual}}{FE}$$

If the flow rates are the same, Then:

$$FE = \frac{\Delta p_{ideal}}{\Delta p_{actual}} = \frac{\Delta p_{actual} - \Delta p_{skin}}{\Delta p_{actual}}$$

the drawdown without skin is:

$$\Delta p_{ideal} = FE \cdot q_{actual}$$

Suppose we have steady state radial flow, then:

$$FE = \frac{\ln \frac{r_e}{r_w}}{\ln \frac{r_w}{r_w} + S}$$

For pseudo-steady radial flow we have.

$$FE = \frac{\ln \frac{r_e}{r_w} - \frac{3}{4}}{\ln \frac{r_w}{r_w} - \frac{3}{4} + S}$$

From the above equations, we see that:

- negative skin (stimulation) leads to $FE > 1$
- no skin leads to $FE = 1$
- positive skin (damage) leads to $FE < 1$

A well with stimulation will produce better than an ideal well.

4 Skin Factor

4.1 Introductory remarks

The quality of a well may be quantified by comparison against the performance of an equivalent ideal one. A non-ideal condition may lead to either an improvement or a deterioration of the well performance. These conditions are called stimulation and damage respectively, and are characterized by a skin factor.

The skin factor accounts for all non-ideal conditions that are confined to a small region around the wellbore.

Examples are:

1. Invasion of mud filtrate (damage)
2. Partial penetration
3. Perforations
4. Fractures
5. Slant angle

Items 1 and 2 will decrease the productivity while 4 and 5 will improve it. Item 3 may decrease or improve the productivity depending on the length of the perforation density and tunnel length.

4.2 Steady state flow

If the external boundary is open such that the fluid produced is immediately replaced, then there is a possibility for steady state flow. Fluid replacement may be due to:

1. An edge water drive
2. A pressure maintenance project

It is sufficient that one boundary is of constant pressure type. For instance, the top and bottom boundaries may be of constant pressure type due to an expanding gas cap or a bottom aquifer. These conditions are not discussed in these notes.

Consider steady state flow in a reservoir shaped like a cylinder with the well in the center.

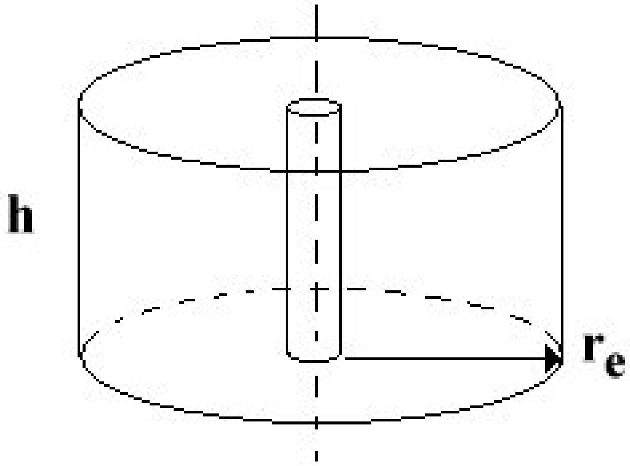

Figure 1: Schematic of a cylindrical reservoir

The pressure is constant at the external boundary and the wellbore. The top and bottom boundaries are closed. Then the flow is described by the radial flow equation.

$$p(r) = p_w + \frac{q\mu B}{2\pi kh} \ln \frac{r}{r_w}$$

The above equation will show up as a straight line on a p vs. $\ln r$ plot, see Figure. 2. The slope is given by

$$m = \frac{q\mu B}{2\pi kh}$$

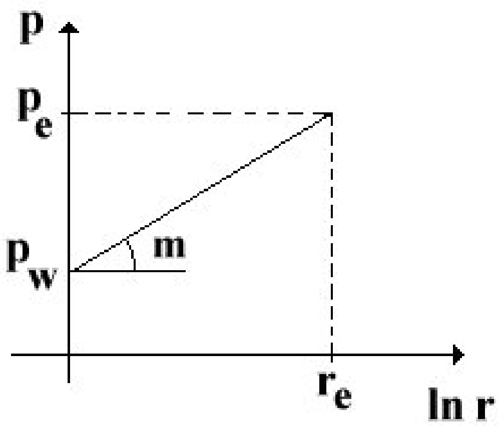

Figure 2: Pressure profile during steady state flow

Suppose there is a zone of altered permeability around the wellbore, see Figure. 3. Then the resulting pressure profile will be as shown in Figure. 4 for damage and

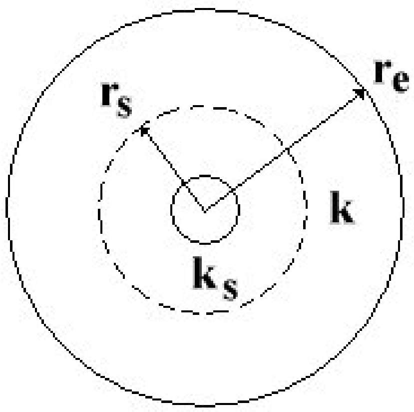

Figure 3: Schematic of a circular drainage area with an altered zone

as shown in Figure. 5 for stimulation. The word damage refers to a lower permeability in the skin zone than in the reservoir. For stimulation it is the other way around.

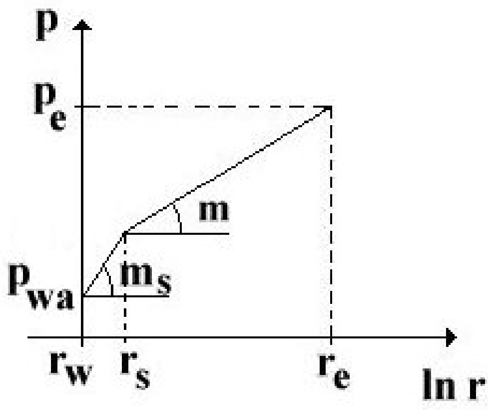

Figure 4: Pressure profile for well with damage, $k_s < k$

The radial flow equation is valid in both regions. The slopes of the straight lines are:

$$m = \frac{q\mu B}{2\pi kh} \qquad \text{and} \qquad m_s = \frac{q\mu B}{2\pi k_s h} \quad \text{respectively.}$$

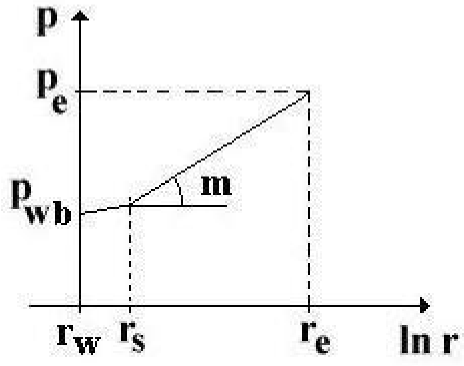

Figure 5: Schematic for stimulation, $k_s > k$

4.3 Skin factor

Usually the radius of the skin zone is negligible in comparison with the external radius of the drainage area, i.e. $r_S << r_e$. This suggests a simplification. The zone of altered permeability may be removed and replaced by a fictitious thin coating (skin) around the wellbore. Then, the radial flow equation is valid from the external boundary to the wellbore. When the fluid passes the skin, it is subject to a pressure change, Δp_{Skin}, such that the pressure at the wellbore side of the skin is equal to the actual pressure. A well without skin has been called ideal.

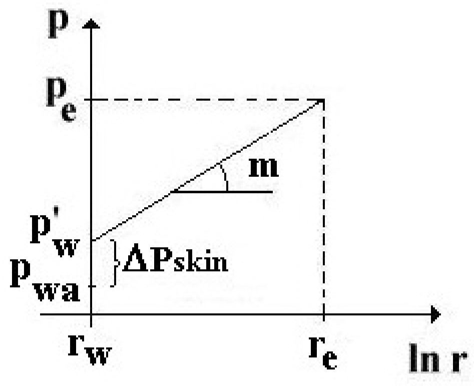

Figure 6: Pressure profile of damaged well, $k_s < k$

The pressure in the reservoir may be described by:

$$p = p'_w + \frac{q\mu B}{2\pi kh} \ln \frac{r}{r_w},$$

The pressure drop across the skin is illustrated in Figure 6 for damage and Figure 7 for stimulation.

Consider a well with damage. When passing the skin, the fluid pressure will drop to the real wellbore pressure, which is denoted by p_{wa}. There are two pressures associated with the wellbore. The pressure at the reservoir side of the skin, p'_w, is also the ideal pressure.

In the above figure, the ideal reservoir wellbore pressure is:

$$p'_w = p_{wa} + \Delta p_{skin}$$

A dimensionless skin factor, *S*, is defined such that:

$$p(r) = p_{wa} + \frac{q\mu B}{2\pi kh}\left(\ln \frac{r}{r_w} + S \right)$$

At the wellbore, $r = r_w$, the above equation will reduce to

$$p'_w = p_{wa} + \frac{q\mu B}{2\pi kh} S$$

Hence, pressure drop across the skin is given by

$$\Delta p_{skin} = \frac{q\mu B}{2\pi kh} S$$

The permeability in the skin zone may be higher than in the unaltered formation. Then the actual wellbore pressure is higher than the ideal one. The pressure profile in the one-permeability model is described by the radial flow equation.

$$p = p'_w + \frac{q\mu B}{2\pi kh} \ln \frac{r}{r_w}$$

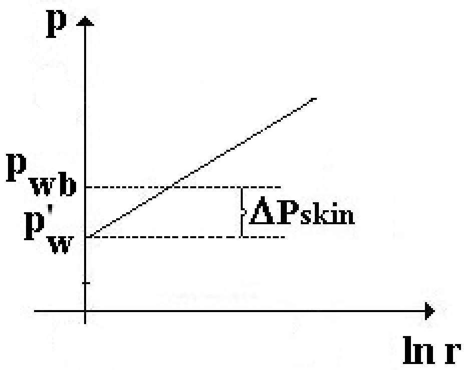

Figure 7: Pressure profile for stimulation, $k_s > k$

When passing the skin zone, the fluid pressure is increased by Δp_{skin}. This behavior may be described by a negative skin factor.

$$p(r) = p_{wb} + \frac{q\mu B}{2\pi kh}\left(\ln \frac{r}{r_w} + S\right)$$

The words damage and stimulation are used for positive and negative skin factors respectively. The inflow performance may be improved by stimulation projects. A stimulated well may have damage if the reduced skin factor ends up with a positive skin factor.

Notice that there is a lower limit on a negative skin factor. Obviously, the actual wellbore pressure, p_{wb}, cannot be higher than the pressure at the external boundary, p_e. The skin factor of a well with stimulation is rarely less than –6. It may, however, assume high positive values. If sufficiently high, the well may not produce unless it is stimulated.

4.4 Pseudo-steady flow

If the external boundary is closed, then comes a period of pseudo steady state type if the production rate is constant. During this regime, the derivative of the pressure with respect to time is constant and independent of position.

Since the rate of pressure decline is the same everywhere, the difference between the average pressure and the wellbore pressure is constant. The equation is

$$\bar{p} = p_w + \frac{q\mu B}{2\pi kh}\left(\ln\frac{r_e}{r_w} - \frac{3}{4}\right)$$

for an ideal well and

$$\bar{p} = p_w + \frac{q\mu B}{2\pi kh}\left(\ln\frac{r_e}{r_w} - \frac{3}{4} + S\right)$$

for a well with skin.

The relationship between pressure and logarithmic distance is shown in Figure 8.

Note that:

1) The initial part of curve is a straight line
2) The average pressure is located at the extension of the line at a distance $r = 0.47r_e$. Also $\ln(0.47r_e) \approx \ln r_e - 3/4$
3) The pressure will decline with increasing time, but in such a way that the difference $\bar{p} - p_w$ is constant.

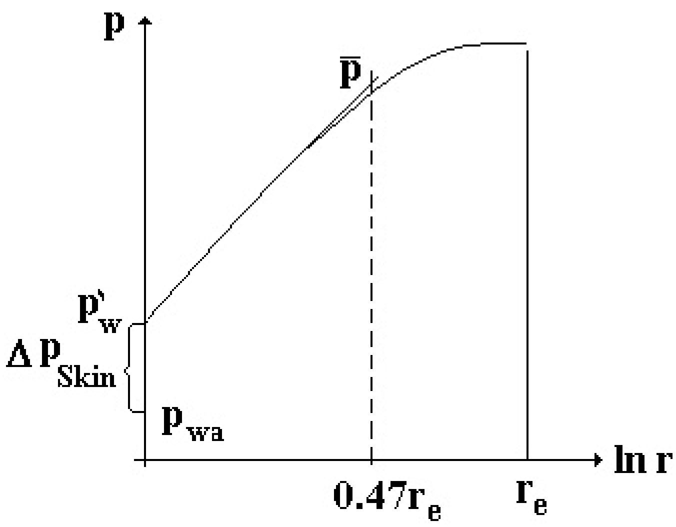

Figure 8: Instantaneous pressure profile during pseudo-steady flow

The above schematic is for a well with damage, since the pressure at the wellbore side of the skin falls below the ideal wellbore pressure. For a well with stimulation it will be the other way around.

5 Hawkins' Formula for Skin

5.1 Introductory remarks

By use of the composite (two-region) reservoir model, Hawkins clarified the concept of skin.

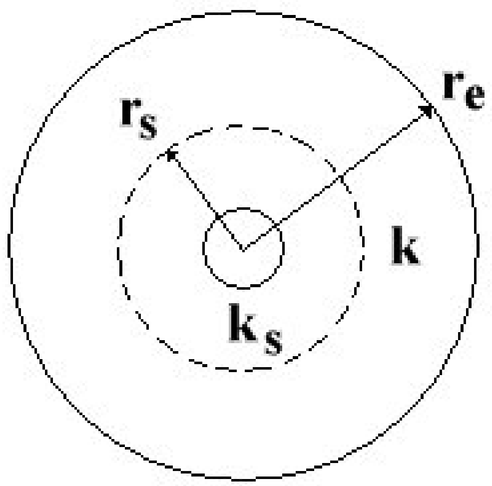

Figure 1: The two-permeability reservoir model

The model explains the effect of invasion of mud filtrate and acid injection in an intuitive way. The skin zone permeability, k_S, is reduced for the former and increased for the latter.

5.2 Hawkins' model

Consider steady state radial flow in a circular drainage area. There is an internal boundary at position r_S , see Figure 1. The pressure has to be the same on both sides of the boundary.

$$p_S^+(r_S) = p_S^-(r_S) = p_S$$

Superscripts + and – denote the outer and inner side of the internal boundary. The boundary has no storage capacity, Hence:

$$q_S^+(r_S) = q_S^-(r_S) = q$$

The relationship between the composite model (two-region) and the skin model is illustrated below.

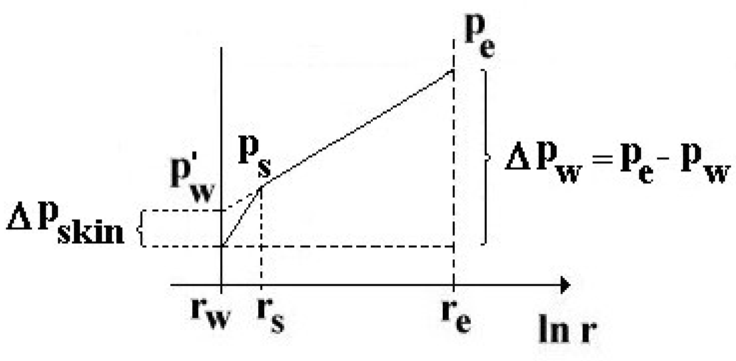

Figure 2: Pressure profiles for the two regions and the skin model

The pressure behavior of each region is given by the radial flow equation. Hence,

$$p_s - p_w = \frac{q\mu B}{2\pi k_s h}\ln\frac{r_s}{r_w}$$

and

$$p_e - p_s = \frac{q\mu B}{2\pi kh}\ln\frac{r_e}{r_s}$$

Addition of the above equations yields:

$$p_e - p_w = \frac{q\mu B}{2\pi h}\left(\frac{\ln\frac{r_s}{r_w}}{k_s} + \frac{\ln\frac{r_e}{r_s}}{k}\right)$$

According to the skin model:

$$p_e - p_w = \frac{q\mu B}{2\pi kh}\left(\ln\frac{r_e}{r_w} + S\right)$$

Elimination of the total drawdown, $p_e - p_w$, between the above equations, yields.

$$S = \left(\frac{k}{k_S} - 1\right)\ln\frac{r_s}{r_w}$$

This is the Hawkins equation. Some comments are in order:

- Damage , $k_S < k$, leads to a positive skin factor.

- Stimulation, $k_S > k$, leads to a negative skin factor.
- The skin factor and the reservoir permeability may be obtained by a well test. Hawkins equation involves two additional unknowns, r_S and k_S. If one of these is known from other sources, the other one may be determined from Hawkins equation.
- Hawkins' equation has theoretical (explanatory) rather than a practical value, since r_S and k_S is rarely known.

Hawkins' equation confirms the existence of a minimum skin factor. The maximum (theoretical) stimulation is given by: $k_S \to \infty$ and $r_S \to r_e$, then:

$$S_{\min} = -\ln \frac{r_e}{r_w}$$

6 Equivalent Wellbore Radius

6.1 Introductory remarks

For some purposes it may be more convenient to take non-ideal conditions into account by the concept equivalent radius rather than skin. It may be difficult to visualize a hydraulic fracture as a thin skin. An obvious effect of a hydraulic fracture is to capture the fluid deeper into the reservoir, see Figure 1. The length of each fracture half-wing, x_f , should at least extend beyond a possible skin zone of damage.

A fictitious well with a larger radius, r_{we}, will also capture the fluid deeper into the reservoir. Hence, this may be a more realistic model than skin which may be regarded as a film.

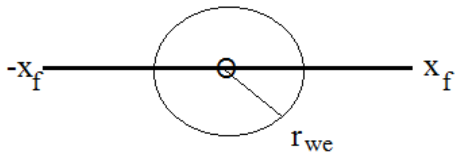

Figure 1: Well with a vertical fracture, top view

6.2 The equivalent wellbore radius

The equivalent wellbore radius may be explained as: The fictitious wellbore radius which makes the calculated pressure of an ideal well equal to that of the equivalent actual one, see Figures 2 and 3.

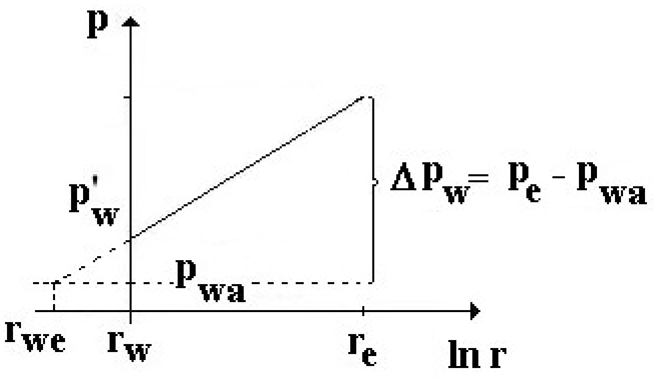

Figure 2: Equivalent wellbore radius for damage

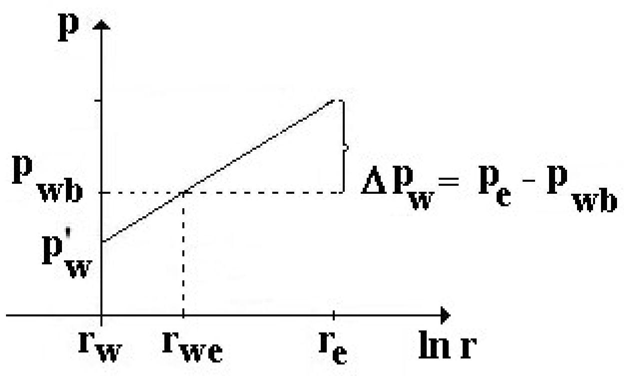

Figure 3: Equivalent wellbore radius for stimulation

Damage leads to an equivalent radius that is less than the radius of the wellbore, stimulation leads to an equivalent radius that is larger.

By use of the equivalent wellbore radius, the drawdown is given by:

$$\Delta p_w = p_e - p_w = \frac{q\mu B}{2\pi kh} \cdot \ln \frac{r_e}{r_{we}}$$

Since the equivalent radius and skin factor are designed to account for the same non-ideal conditions, these concepts are related.

The drawdown is also given by:

$$\Delta p_w = \frac{q\mu B}{2\pi kh} \left(\ln \frac{r_e}{r_w} + S \right)$$

Elimination of Δp_w between the two equations and solving for the equivalent radius yields:

$$r_{we} = r_w e^{-S} .$$

Suppose the fracture is of infinite conductivity type. The assumption implied is that fluids flow without pressure loss inside the fracture. It may be shown that the equivalent wellbore radius is:

$$r_{we} = \frac{x_f}{2}$$

The skin factor may be obtained by well testing. Once this is a known quantity, a rough estimate of the fracture half length may be obtained from:

$$x_f = 2 r_w e^{-S}$$

Suppose the wellbore radius, r_w, is 10 cm and the skin factor, S is -5, then $x_f \approx 30$ m. The accuracy of the estimate may be improved by relaxing the restrictive assumption.

7 Drawdown Test

7.1 Introductory remarks

The drawdown test is only possible for new wells or wells that have been closed in for a long period. Another disadvantage is that it may be difficult to keep the rate constant. The advantage is that one may produce while testing.

Well test interpretation depends on mathematical models. The observed pressure behavior in a test is matched to a plausible model. The target variable(s) are estimated from the matched model. If the assumed model is incorrect, the interpretation will lead to erroneous estimates. Interpretation of the drawdown test may serve as a first example of this technique. The assumptions are: Static equilibrium initially, constant production rate, homogeneous and infinite-acting reservoir.

We introduce the concept of limiting forms. These are simplified equations that are valid, either for large or small, values of time. Limiting forms are easy to use and have sufficient accuracy for engineering calculations.

7.2 Drawdown test

Procedure:

1. Increase the production rate from 0 to q at time $t = 0$ Keep the rate constant
2. Measure the pressure response, p_{wf} . Index *wf* denotes well flowing pressure.
3. Make an interpretation by matching the observed behavior to the model.

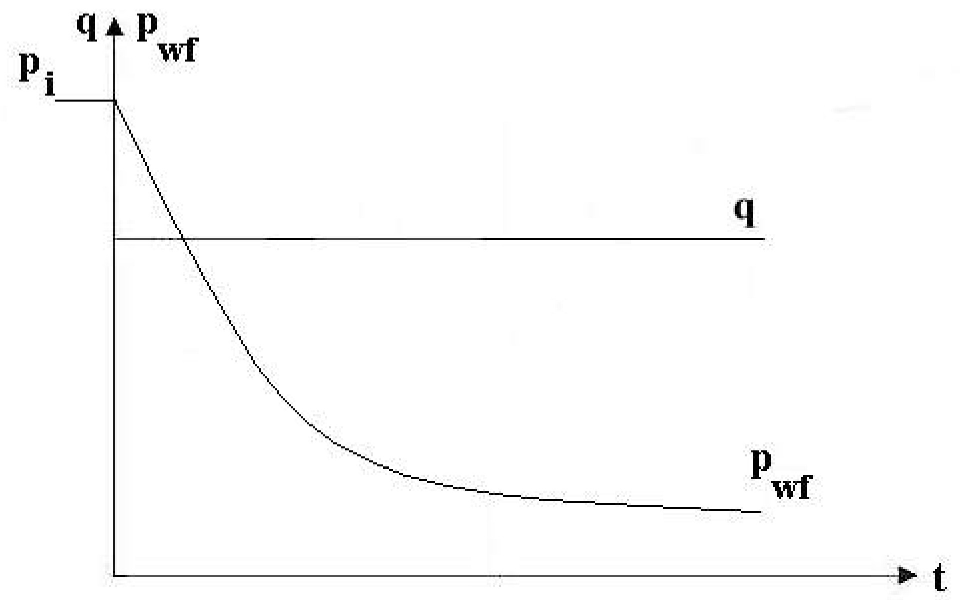

Figure 1: Schematic of production rate and well flowing pressure as a function of time

The reservoir has to be in static equilibrium initially. When the pressure vs. distance relationship assumes a constant value, then from Darcy's law there is no flow in the reservoir. Hence, the initial condition is: $p = p_i$ for $t = 0$ and $r_w \le r \le r_e$. The outer boundary condition is: $p = p_i$ when $r \to \infty$. The inner boundary condition, at the wellbore is:

$$\left(\frac{\partial p}{\partial r}\right)_{r=r_w} = \frac{q\mu B}{2\pi k h r_w} \quad , \quad t \ge 0$$

The flow in the reservoir is governed by the diffusivity equation. The solution to this problem is a complex equation. Fortunately, it has a simple limiting form:

$$p_{wf} = p_i - \frac{q\mu B \cdot 1.15}{2\pi k h}\left(\log\frac{kt}{\varphi\mu c_t r_w^2} + 0.351 + 0.87S\right)$$

which is valid for large values of time.

The above formula is for a consistent unit system. A consistent unit system is characterized by dimensionless constants. SI- and Darcy units are examples of consistent unit systems.

The variable c_t is used to denote the total compressibility

$$c_t = c_o S_o + c_w S_w + c_g S_g + c_r$$

c_r is the compressibility of the rock.

A well test typically lasts for a day or two. Hence, it is convenient to measure time in hours rather than seconds. When time is measured in hours, the following equation applies:

$$p_{\mathrm{wf}} = p_i - \frac{q\mu B \cdot 1.15}{2\pi kh}\left(\log\frac{kt}{\varphi\mu c_t r_w^2} + 3.91 + 0.87S\right)$$

The above equations are valid when:

$$\frac{3600kt}{\varphi\mu c_t r_w^2} > 25$$

Again, t is measured in hours.

The target parameters to be obtained from interpretation of a drawdown test are the permeability and skin factor. The above equation obviously involves more variables. These must be obtained from other sources like logging, core analysis, fluid analysis, production data, etc. The initial pressure is not known with sufficient precision. Hence p_i must also be obtained from another source, preferably a buildup test.

The well flowing pressure is predicted by the above equation (mathematical model). The same pressure, p_{wf} , may also be measured in an actual test. Once measured, the latter becomes a known function of time.

7.3 Determination of permeability

The drawdown equation shows up as a straight line segment in a semi-logarithmic coordinate system. The early part of the curve is influenced by wellbore storage and skin, the late part by the external boundary. The drawdown equation is valid for the middle part straight line segment only. A schematic is shown in Figure. 2.

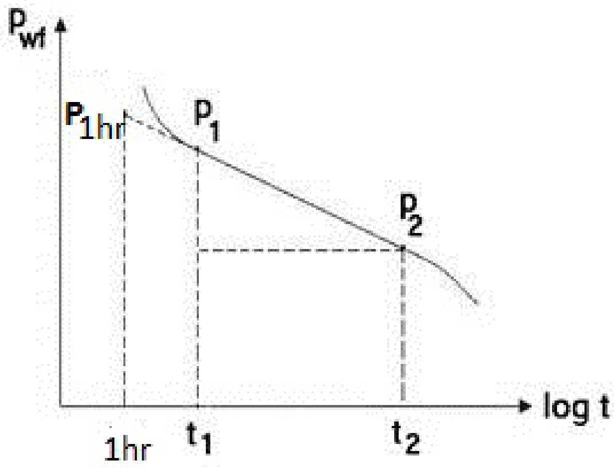

Figure 2: Schematic of semi-log plot for a drawdown test

The pressure, p_{wf}, is a decreasing function of time. For convenience, we characterize the slope by a positive number. The pressure decrease is accounted for by the minus sign in the drawdown equation.

The slope, m, is given by the group:

$$m = \frac{q\mu B \cdot 1.15}{2\pi kh}$$

Hence, the *kh*-product may be obtained from the slope. In most cases, the reservoir thickness, *h*, may be obtained by logs. Then, one may obtain the reservoir permeability:

$$k = \frac{q\mu B \cdot 1.15}{2\pi mh}$$

The test is matched to the model by the slope, *m*, which is calculated from measured pressure values. Then the variable, *m*, has a known value and the above equation may be solved for one unknown, usually the permeability.

The slope may be obtained from two points on the straight line.

$$m = \frac{p_1 - p_2}{\log t_2 - \log t_1}$$

which simplifies to:

$$m = p_1 - p_2$$

when t_2 and t_1 are one decade apart. Note that the unit used for time, *t*, has no influence on the calculated slope. In case of noisy data, the best fit to the logarithmic straight line may be obtained by linear regression.

7.4 Determination of skin factor

A straight line is uniquely defined by the slope and its position in the coordinate system. The position is defined once the coordinates of a point on the line are known. Hence, a point on the straight line gives information that may be used to solve the model equation for an additional unknown.

The traditional approach is to use the point given by the pressure at 1 hour, p_{1hr}.

This point may not be on the straight line. This is because the pressure response may still be influenced by wellbore storage and skin. In such cases, the straight line is extrapolated until it intersects the vertical $t = 1$ hr line. The technique is illustrated in Figure. 2.

Once the pressure at 1 hour has been obtained from the test, the drawdown equation becomes:

$$p_{1hr} = p_i - \frac{q\mu B \cdot 1.15}{2\pi kh}\left(\log\frac{k}{\varphi\mu c_t r_w^2} + 3.91 + 0.87S\right)$$

With known slope and permeability, the above equation may be solved for the skin factor. The result is:

$$S = 1.15\left(\frac{p_i - p_{1hr}}{m} - \log\frac{k}{\phi \mu c_t r_w^2} - 3.91\right)$$

The skin factor is dimensionless.

Note that the initial pressure, p_i, has to be obtained from another source, preferably a buildup test. The permeability, k, was obtained from the slope calculated from the pressure signature. For consistency, use the same slope, m, that was obtained from the actual test.

Note that only a small part of the available data has been used in the analysis. Use of the early and late time data in well test interpretation by straight lines in Cartesian coordinate system will be discussed in the chapters on Wellbore Storage and Reservoir Limit Test, respectively. We refer to straight line analysis of specific flow periods as special analysis. It is valid for a particular flow period only. The alternative is the type curve matching. By use of this technique, several flow periods may be analyzed simultaneously, see the chapter on Pressure Derivative.

8 Reservoir Limit Test

8.1 Introductory remarks

The reservoir limit test is a long drawdown test. The objective is to quantify the pore volume, V_p, which is of economic importance. For instance, the initial oil in place, N_{oi}, is given by:

$$N_{oi} = \frac{\overline{\varphi} \cdot \overline{S}_o V_B}{\overline{B}_o}$$

In the above equation, the formation volume factor, B_o and the bulk volume, V_B, may be obtained by PVT-analysis and seismic studies respectively. The average porosity, $\overline{\varphi}$, and the oil saturation, $\overline{S}_o$, may be estimated by core analysis or well logging.

The length scale of core analysis and logging is much smaller than the length scale of reservoir analysis while the length scale of seismic is larger. Upscaling and downscaling of measurements to the required length scale is uncertain. It is advantageous to avoid length scale problems as far as possible. An additional problem with core analysis is that only a tiny part of the reservoir is sampled. These problems can be mitigated by the reservoir limit test.

The pore volume, V_p, is given by:

$$V_p = \overline{\varphi} \cdot V_B$$

With known pore volume, we can solve for either the average porosity or bulk volume if the other variable has a reliable estimate.

8.2 Reservoir limit test

Test procedure

1. Produce the well for a long period at a constant rate
2. Measure the well flowing pressure
3. Make interpretation from the matched model

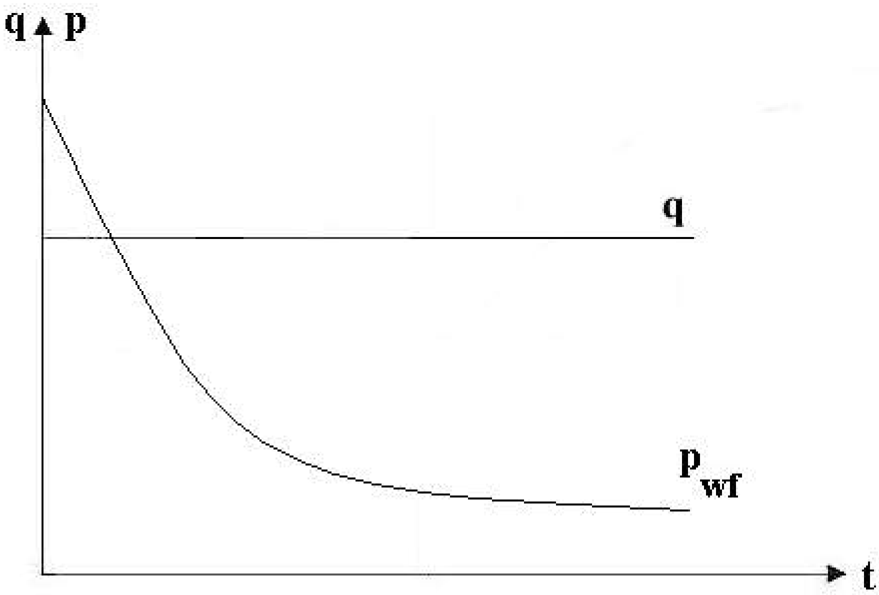

Figure 1: Schematic of a reservoir limit test

The basic assumptions are as for the drawdown test, except for a change in the outer boundary condition. The outer boundary is of no-flow type, i.e.:

$$\left(\frac{\partial p}{\partial r}\right)_{r=r_e} = 0$$

for a cylindrical drainage area. Again, the well test interpretation relies on a limiting equation.

8.3 Determination of pore volume, circular drainage area

During pseudo-steady flow, the drawdown equation for a circular reservoir is given by:

$$p_{wf} = p_i - \frac{qB}{\phi c_t \pi r_e^2 h} \cdot t - \frac{q\mu B}{2\pi kh}\left(\ln\frac{r_e}{r_w} - \frac{3}{4} + S\right)$$

The above equation will show up as a straight line in a Cartesian coordinate system.

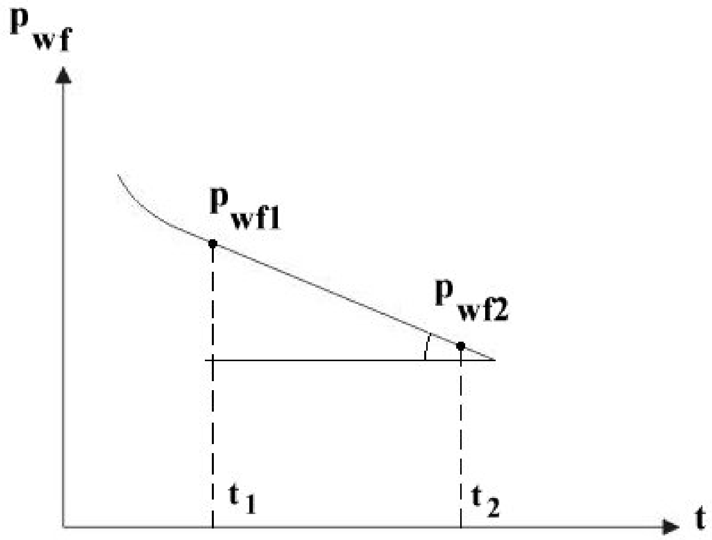

Figure 2: Pressure response during pseudo-steady flow

The slope is given by

$$m = \frac{qB}{c_t \varphi \pi r_e^2 h}$$

The bulk volume of the reservoir is:

$$V_B = \pi r_e^2 h$$

and the pore volume is:

$$V_p = \varphi V_B = \varphi A h$$

Hence, the equation for the slope will reduce to:

$$m = \frac{qB}{c_t V_p}$$

The above equation is valid regardless of the reservoir shape. Hence, the pore volume may be computed from:

$$V_p = \frac{qB}{c_t m}$$

From the actual test we have:

$$m = \frac{p_{wf1} - p_{wf2}}{(t_2 - t_1) \cdot 3600}$$

The time is measured in hours.

Reservoir of non-cylindrical shape

Suppose several production wells share the same reservoir area. Then, interference between wells will set up no-flow boundaries somewhere in between them. This is because the pressure is a continuous function of distance and that the pressure will increase away from each production well. The no-flow boundary will be at the maximum point, see Figure. 3. Note that the closed boundary is pushed towards the well with the lowest rate. Hence the better producer will drain a larger area.

It is common practice to allocate to each well a drainage area that is proportional to the production rate and in such a way that the sum of the areas adds up to the total area. Suppose we have two wells, well a and well b in a small reservoir, and that the latter produce with twice the rate of the former. Then the drainage area of well b will be twice as large as for well a, see Figure. 4.

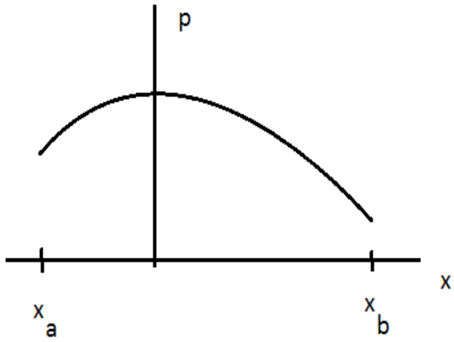

Figure 3. Pressure vs. distance profile

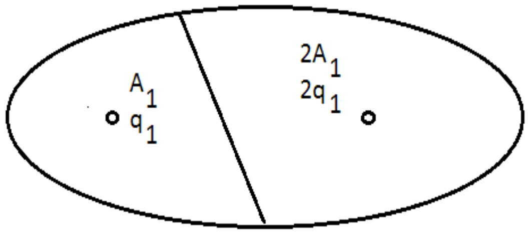

Figure 4: Rate allocation according to production rate

Dietz showed that a reservoir of area A and simple arbitrary shape obeys the following equation:

$$p_{wf} = p_i - \frac{qB}{c_t V_p} \cdot t - \frac{q\mu B}{4\pi kh}\left(\ln\frac{A}{r_w^2} + \ln\frac{2.25}{C_A} + 2S \right)$$

The constant C_A has been called the Dietz shape factor. It depends on the shape of the drainage area and the location of the well within the area. The shape factor has been tabulated for areas of simple shapes. For example, C_A=31.6 for a circle, 30.9 for a quadrate and 27.6 for an equilateral triangle, all with a well in the centre. Many more are listed in the original work, Dietz, D.N., 1965. "Determination of reservoir average pressure from buildup surveys", JPT, Aug. 955–959.

The above equation will show up as a straight line in a Cartesian coordinate system. The slope of the line is given by:

$$m = \frac{qB}{c_t V_p} = \frac{qB}{c_t \overline{\varphi} Ah}$$

The line may be extrapolated back to the intercept with the vertical axis, which is given by:

$$p_{\text{int}} = p_i - \frac{q\mu B}{4\pi kh}\left(\ln\frac{A}{r_w^2} + \ln\frac{2.25}{C_A} + 2S \right)$$

The corresponding straight line segment obtained by the actual test may also be extrapolated back to the intercept pressure. Then the left hand side of the above equation has a known value. The above equation may be solved for an additional unknown, either area or shape factor.

The number of unknowns, V_p, $\bar{\varphi}$, A and C_A are problematic. All cannot be determined from the straight line. Assuming the other variables are known, there are 3 equations and 4 unknowns. Assuming that the drainage areas may be allocated according to the production rates, a crude estimate of C_A may be obtained. The value obtained may be compared against the values listed in the table provided by Dietz, to get some idea of the shape of the drainage area and the location of the well.

The above theory implies the assumption of constant production rate. This condition is difficult to achieve in practice. Hence, it may be necessary to resort to Horner time, t_p, to account for previous rate changes. There is a brief discussion of Horner time in the chapter on buildup testing. Before the test, the rate is stabilized at q_{last} and N_p is the volume produced before the start of the test.

$$t_p = \frac{N_p}{q_{last}}$$

Then, use a synthetic value for the time of interest, t.

$$t = t_p + \Delta t$$

Δt is the production time since the nominal start of the test.

9 Interference Test – Type Curve Matching

9.1 Introductory remarks

An interference test involves more than one well. It is conducted by producing at least one well and measuring the pressure response in at least one observation well. An observation well has a pressure gauge to record pressure. The flow rate is zero.

We discuss interpretation by traditional type curve matching. Today, this can easily be done on a computer, but the principle remains unchanged. The theory can best be explained by the original approach. Interpretation of a sequence of flow periods is possible. A classic example is the wellbore storage transition into radial flow. Type curve analysis can be based on more information than semi-log analysis, which is valid for constant rate radial flow only. The method depends on some curvature in the curves to obtain a unique match.

A type curve is a log-log plot of the mathematical solution, i.e. the p_D-function for the test. If the correct mathematical model is applied, then the type curve may be matched to the field curve, log Δp vs. log t plot, to obtain the correct interpretation. Choice of a wrong model will lead to erroneous results. Unfortunately, well test responses are not unique. More than one plausible model may be available.

9.2 Interference test

The objectives of an interference test are:

- To determine whether two or more wells are in pressure communication. If communication exists, some wells may be turned into injectors to displace hydrocarbons towards the production wells.
- When communication exists, to provide estimates of the average permeability and storage capacity, k and φc_t-product, between the wells.
- To investigate the directional properties of the permeability (anisotropy). More than one observation well are necessary.

We use a simplified model. Assume two wells, one observation and one production well in a homogeneous infinite reservoir. The observation well does not produce and the wells are separated by a distance r.

The effect of the producing well in the observation well may be approximated by the line-source solution. A possible skin around the production well has negligible effect on the pressure measured in the observation well, unless the distance between the wells is short.

9.3 The line source solution

The assumptions are: Static equilibrium initially, constant production, homogeneous- and infinite-acting reservoir. These are the same as for a drawdown test. The diffusivity equation has simple solution when the well assumes the shape of a line, i.e. $r_w \rightarrow 0$. Then, the modified wellbore condition becomes:

$$\lim_{r\rightarrow 0}\left(r\frac{\partial p}{\partial r}\right)=\frac{q\mu B}{2\pi kh}$$

The solution to the diffusivity equation is:

$$\Delta p(r,t) \;=\; p_i - p(r,t) \;=\; \frac{q\mu B}{4\pi kh}\;\; Ei\left(-\;\frac{\varphi\mu c_t r^2}{4kt}\right)$$

The above equation is the exact solution for a line source well and an approximate solution to a cylindrical one. There is negligible difference between cylindrical- and line source solutions for large values of time.

The *Ei*-function is defined as an integral.

$$-E(-x)=\int_x^{\infty}\frac{e^{-u}}{u}du$$

The integral has to be evaluated by numerical methods.

The *Ei*-function has simple limiting forms. The long time approximation is:

$$E(-x)\approx \ln x+0.577\,,\; x<0.01$$

and the short time one is:

$$E(-x)\approx 0\,,\; x>10$$

Use of the long time approximation yields a semi-log equation:

$$\Delta p(r,t) \;=\; p_i - p(r,t) \;=\; \frac{q\mu B\cdot 1.15}{2\pi kh}\left(\log\frac{kt}{\varphi\mu c_t r^2}+0.351\right)$$

Hence, there is a possibility for semi-log analysis for interference tests. One may obtain the permeability-thickness product from the slope of the semi-log straight line and the φc_t -product from the position of the straight line. This is rarely done since it will take a long time before the semi-log approximation has sufficient accuracy. Type curve analysis is a practical alternative.

9.4 Type curve matching

Type curve matching depends on dimensionless variables. Conventional definitions are:

$$p_D = \frac{2\pi kh}{q\mu B}(p_i - p(r,t))$$

$$r_D = \frac{r}{r_w}$$

$$\frac{kt}{\varphi\mu c_t r_w}$$

Substitution into the line-source solution yields:

$$p_D = -\frac{1}{2} Ei\left(-\frac{r_D^2}{4t_D}\right) = -\frac{1}{2} Ei\left(-\frac{1}{\frac{4t_D}{r_D^2}}\right)$$

Observe that the numerical value of the argument in the *Ei*-function is the same in dimensionless and dimensional form. Hence, the *Ei*-functions come out with the same numerical values in *t*- and t_D coordinates.

Dimensionless variables are not unique. Several definitions are possible. An alternative dimensionless time is:

$$t'_D = \frac{kt}{\varphi\mu c_t r^2}$$

Multiplication and division with r_w^2 on the right hand side yields.

$$t'_D = \frac{kt}{\varphi\mu c_t r_w^2} \cdot \frac{r_w^2}{r^2}$$

$$t'_D = \frac{t_D}{r_D^2}$$

The group $\frac{t_D}{r_D^2}$ may be regarded as a dimensionless time with basis in r^2.

The type curve for the *Ei*-solution is usually presented as a log p_D vs. $\log\frac{t_D}{r_D^2}$ graph. Then, the line-source solution (for any distance) will show up as a single curve.

The principle of type curve analysis may be explained as follows: Three dimensionless variables have been defined. Each is proportional to the corresponding real one. Hence:

$$p_D(r_D, t_D) = \alpha\left(p_i - p(r,t)\right)$$

$$t'_D = \beta t$$

$$r_D = \gamma r$$

Taking the logarithm yields:

$$\log p_D = \log \Delta p + \log \alpha$$

and

$$\log t'_D = \log t + \log \beta$$

The important consequence of the above equations is that plots of $\log p_D$ vs. $\log t'_D$ and $\log \Delta p$ vs. $\log t$ in the same log-log coordinate system are identical in shape, but are shifted relative to each other. Since the curves are of identical shape, one may put one on top of the other. Then, we have the opposite condition. The curves have the same position, but the axes are shifted, see Figure 1. We say that the curves are matched when they overlap. The observed behavior has been matched to the model. To determine the displacement, we choose an arbitrary match-point, *m*.

There is a difference between the solutions of line-source and cylindrical wells for small values of time. The line-source solution is valid when:

$$\frac{kt}{\varphi \mu c_t r^2} = t'_D > 25$$

As a result, it may not be possible to match the field curve to the entire type curve. In such cases, disregard the few earliest points.

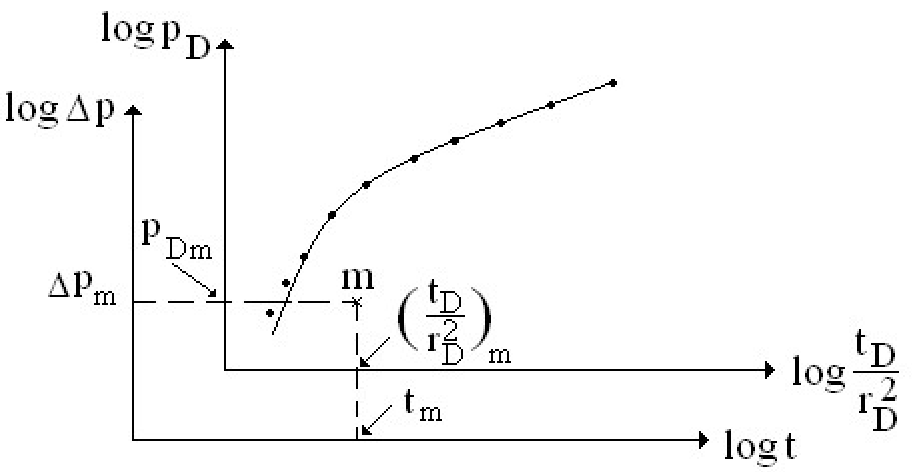

Figure 1: Type curve matching

Index m denotes match-point. The displacement in the vertical and horizontal direction depends on α and β, respectively. The displacement can be calculated from the coordinates of the match-point.

$$\Delta y = \log \alpha = \log p_{Dm} - \log \Delta p_m$$

$$\Delta x = \log \beta = \log t'_{Dm} - \log t_m$$

A different match-point has different coordinates, but their differences remain unchanged.

A log-scale coordinate system has the same distance between decades. The values on the log-axis are the anti-log or inverse, i.e. $\log^{-1} u = u$. A log-difference corresponds to division in the argument of $\log u$. The values of the translation factors can be computed from the coordinates of the match points. These are available on the graph. With known values for α and β, their definitions provide two equations that can be solved for two unknowns.

Hence:

$$\alpha = \frac{2\pi kh}{q\mu B} = \frac{p_{Dm}}{\Delta p_m}$$

and

$$\beta = \frac{k}{\varphi\mu c_t r^2} = \frac{t'_{Dm}}{t_m}$$

Procedure:

a) Locate a log-log dimensionless plot of the line source-solution. This graph is plausible type curve for the simplified interference test.
b) Plot the field curve, $\Delta p = p_i - p(r,t)$ vs. t, on log-log transparent paper. Use the same scale as for the type curve.
c) Slide the field curve horizontally and vertically on top of the type curve until a match is found. The curves are matched when they overlap. The grid axes must be parallel while sliding.
d) Pick a convenient match point. Mark both graphs, use a needle. Read the value (coordinates) of the match points on both graphs.
e) Calculate the permeability, *k*, from the pressure match:

$$k = \frac{q\mu B}{2\pi h}\frac{p_{Dm}}{\Delta p_m}$$

f) Calculate the storage capacity, φc_t -product, from the time match:

$$\varphi c_t = \frac{k}{\mu r^2}\left(\frac{t}{\frac{t_D}{r_D^2}}\right)_m$$

10 Pressure Buildup Test

10.1 Introductory remarks

A pressure drawdown test has to start from a no-flow condition. Hence, a long flow period prior to shutin is required. In addition, it is difficult to keep the rate constant in a producing well. The latter problem is mitigated in a buildup test. The production rate is essentially zero (constant) once afterflow becomes negligible. In addition, the buildup test may be conducted any time. The disadvantage of the method is that the well has to be closed and will not generate income. From an economic perspective, the shutin time should be as short as possible. Unfortunately, shorter shutin period leads to less information. Hence, a trade-off between technical and economic objectives is necessary.

10.2 Pressure buildup test

Procedure:

1. Produce the well at a constant (stabilized) rate. At time t_p close the well.
2. Measure the last flowing pressure, $p_{wf}\left(t_p\right)$, and the shutin pressure, p_{ws}.
3. Interprete by use of the matched model.

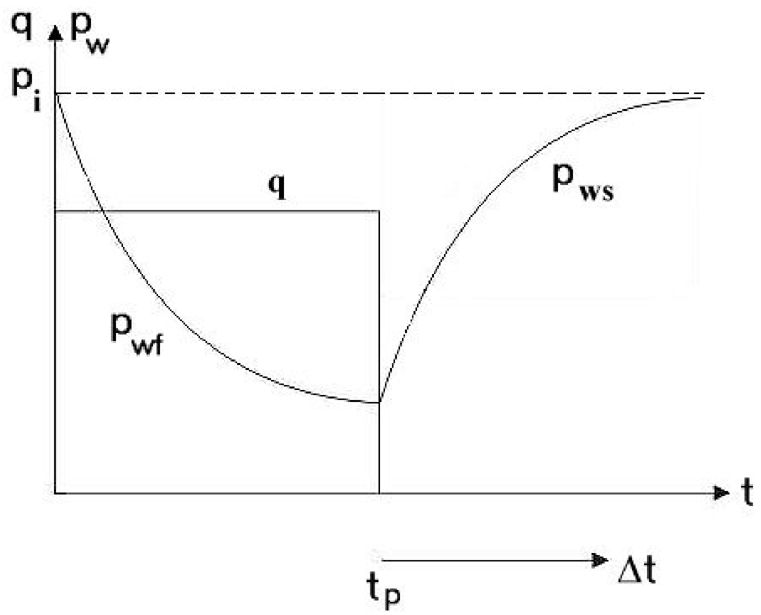

Figure 1: Schematic of an ideal buildup test

t_p and Δt denote production time and shutin time, respectively.

10.3 Infinite-acting reservoir

For a new well, the pressure wave associated with the flow period may not have reached the outer boundary. Then the following equation applies:

$$p_{ws} = p_i - \frac{q\mu B \cdot 1.15}{2\pi kh} \log \frac{t_p + \Delta t}{\Delta t}$$

This is the Horner equation. It may be derived as a straight forward application of superposition in time.

The above equation shows up as a straight line on a p_{ws} vs. log $\frac{t_p + \Delta t}{\Delta t}$ plot.

10.4 Determination of permeability

Note that the rate profile of Figure 1 is idealized. An instantaneous rate-change is not possible. There will be some flow into the well immediately after shutin. This phenomenon has been called afterflow (see: Wellbore Storage). As a consequence, the measured pressure will not obey the Horner equation initially. Note that the early part of the buildup curve falls on the right hand side of the Horner plot. This is because the shutin time, Δt, increases to the left (see Figure 2).

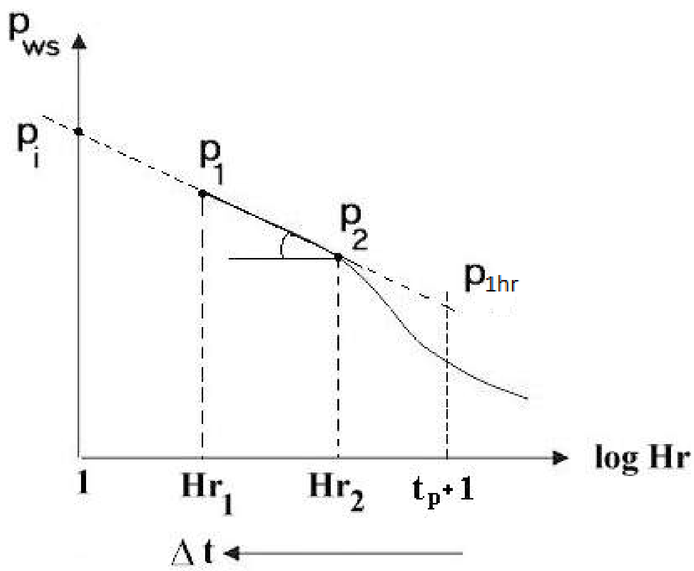

Figure 2: Schematic of a Horner plot of a well with afterflow and skin.

The symbol *Hr* is used to denote the Horner ratio: $Hr = \frac{t_p + \Delta t}{\Delta t}$.

The slope of the straight line in Figure 2 is given by:

$$m = \frac{q\mu B \cdot 1.15}{2\pi kh}$$

Hence, the permeability may be determined from the following equation:

$$k = \frac{q\mu B \cdot 1.15}{2\pi mh}$$

The slope is also defined by two points on the straight line

$$m = \frac{p_1 - p_2}{\log Hr_1 - \log Hr_2}$$

which simplifies to:

$$m = p_1 - p_2$$

when Hr_1 and Hr_2 are one decade apart.

10.5 Determination of the initial reservoir pressure

The Horner equation may be written as:

$$p_{ws} = p_i - m \log Hr$$

Note that:

$$p_{ws} = p_i \qquad \text{for} \qquad Hr = 1$$

The Horner ratio will approach 1 for infinite shutin time, Δt. Hence, the initial reservoir pressure may be obtained by extrapolating the straight line to Hr = 1. The technique is illustrated in Figure 2.

10.6 Determination of the skin factor

The skin is not included in the Horner equation. To involve this parameter, the last flowing pressure, p_{wf} at Δt = 0 is subtracted from both sides of the Horner equation. On the left hand side, we subtract the observed pressure, and on the right hand side the mathematical model (the drawdown equation).

The result is:

$$p_{ws}\left(t_p+\Delta t\right)-p_{wf}\left(t_p\right)=m\left(\log\frac{t_p+\Delta t}{\Delta t}-\log\frac{kt_p}{\phi\mu c_t r_w^2}-0.351-0.87S\right)$$

Both pressures on the left hand side are obtained from the test. The above equation may be solved for an additional unknown, the skin factor, *S*.

Usually the shutin time is small in comparison with the production time. Hence:

$$t_p+\Delta t\approx t_p$$

Then the above equation will simplify since the production time, t_p, is cancelled from the above equation.

The extended Horner equation may be solved for the skin factor, once the shutin time is specified. The traditional choice is Δt = 1 hr. This choice leads to:

$$S=1.15\left(\frac{p_{1hr}-p_{wf}\left(t_p\right)}{m}-\log\frac{k}{\phi\mu c_t r_w^2}-3.91\right)$$

Observations:

1. The skin factor is controlled by the difference Δ:

$$\Delta=p_{1hr}-p_{wf}\left(t_p\right)$$

2. The skin factor S will increase with increasing value of Δ.

Due to afterflow, the measured wellbore pressure at 1 hour may not be on the straight Horner line. The formula for the skin factor implies the assumption that p_{1hr} is on the straight line. it is on the straight line. This condition can be satisfied by extrapolating the straight line to where it intersects the $Hr_{\Delta t=1hr}=t_p+1$ vertical line. The technique is illustrated in Figure 3.

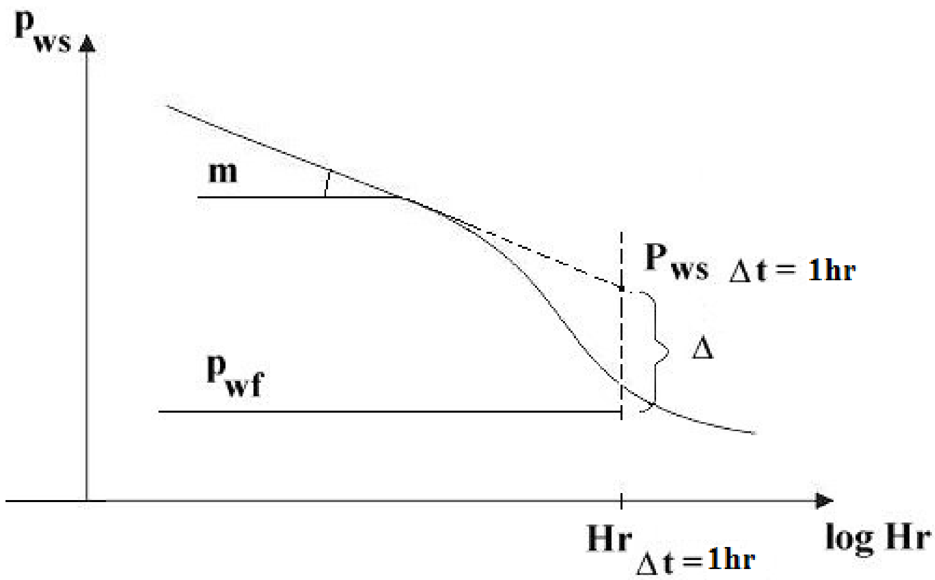

Figure 3: The skin depends on the distance Δ

10.7 Bounded reservoir

Sooner or later the pressure wave associated with the flow period will hit the outer boundary. Under this condition, the Horner equation is not valid. Suppose the external boundary is of no-flow type. If the well is closed during boundary dominated flow, then the pressure will build up towards the average pressure rather than the initial pressure. This is illustrated in Figure 4. An intuitive explanation may go as follows: Suppose the pressure vs. distance profile at the instant of shutin is as shown in the schematic. Then, flow towards the well will continue due to the existing pressure gradient. As a result, the pressure will increase in the region that receives fluid (near wellbore region) and decrease where it leaves (outer region). In theory, the end result is a uniform pressure. In reality, the pressure in the reservoir may start to decrease after an initial buildup period. This is due to interference from surrounding production wells. The pressure decrease is normally not seen in a buildup test, since the shutin time is too short.

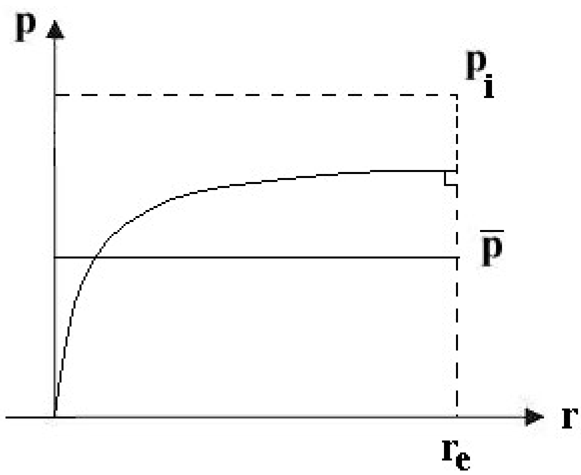

Figure 4: Pressure versus distance, pseudo-steady flow

Experience has shown that the effect of the external boundary appears at the late part of the Horner plot only. The middle straight line- and early part remains practically unchanged. The boundary effect will show up as a break off from the straight line.

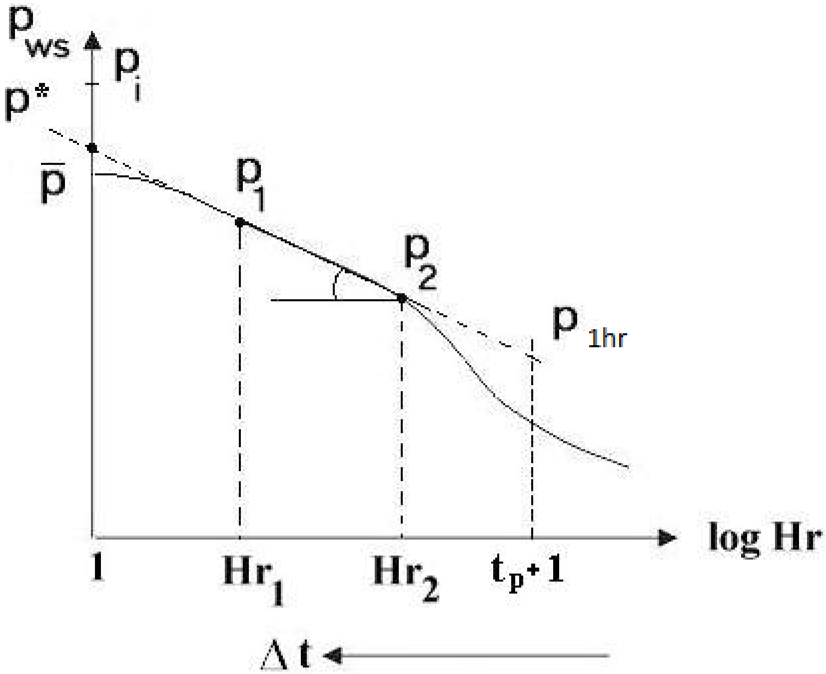

Figure 5: Horner plot, bounded reservoir

The Horner equation for the straight line section may be written:

$$p_{ws} = p^* - \frac{q\mu B \cdot 1.15}{2\pi kh} \log \frac{t + \Delta t}{\Delta t}$$

where p^* is the intersection with the $Hr = 1$ axis. The pressure, p^*, has been called the false pressure. It has no physical interpretation, but is related to the average pressure, $\bar{p}$. The straight line on the Horner plot may be used to determine the permeability and skin factor as discussed previously. The procedure will not be repeated here.

10.8 Determination of the average pressure

Matthews, Brons and Hazebroek presented charts that relate the false pressure to the average pressure for various geometries. The theory was based on the method of images, see the chapter on superposition.

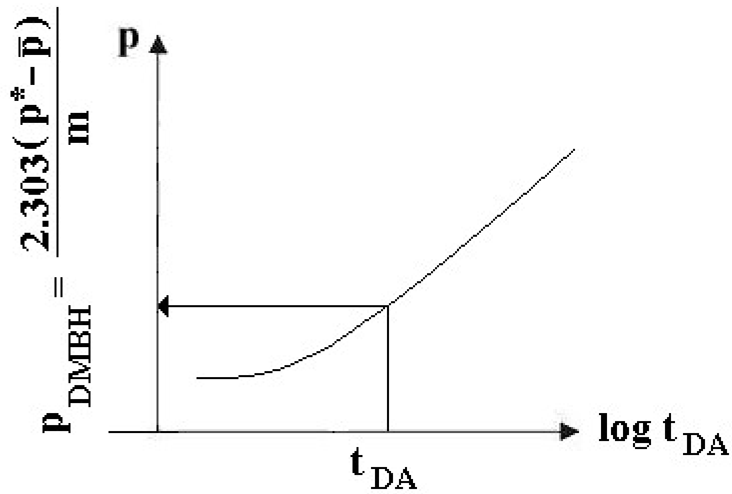

Figure 6: Schematic of a Mattews,Brons and Hazebroek plot

We use index D to denote dimensionless variables.

The average pressure may be calculated as follows:

1. Obtain the slope m and the false pressure, p^*, from the Horner plot.
2. Estimate the shape and size of the drainage area. The traditional assumption is that the drainage area of each well is proportional to the flow rate and that the sum adds up to the total area.
3. Compute the dimensionless production time from the formula:

$$t_{DA} = \frac{kt_p}{\varphi \mu c_t A}$$

4. Look up the MBH-curve that best approximates the estimated shape of the drainage area, and find p_{DMBH} .
5. Calculate the average pressure from the formula:

$$\overline{p} = p^* - \frac{m \cdot p_{DMBH}}{2.303}$$

The difficult part in this calculation procedure is point 2. Estimation of the size and shape of the drainage area is beyond the scope of these notes.

The average pressure is important for material balance calculations. It may also be used to calculate the flow efficiency, FE.

$$FE = \frac{\overline{p} - p_{wf} - \Delta p_S}{\overline{p} - p_{wf}}$$

p_{wf} is the last flowing pressure. For pseudo-steady flow, the difference $\overline{p} - p_{wf}$ is independent of time. This condition leads to a constant value of the flow efficiency. The same is true for steady state flow. Otherwise it will depend on time.

Sometimes the flow efficiency is approximated by: $FE = \dfrac{p^* - p_{wf} - \Delta p_S}{(p^* - p_{wf})}$.

The latter result may not be as accurate, but is easier to obtain.

10.9 Average reservoir pressure

The Matthews, Brons and Hazebroek methodology leads to calculation of the average pressure of a drainage area. After obtaining the average pressure for each well, the next step is to weight these to obtain the reservoir average pressure. Suppose the total production rate from the reservoir is q_t and that each well, *j*, drains a volume proportional to the rate, q_j , during pseudo steady flow. Then:

$$\frac{V_j}{V_t} = \frac{q_j}{q_t}$$

The average reservoir pressure may be approximated by:

$$\overline{p}_{res} = \overline{p}_1 \frac{q_1}{q_t} + \overline{p}_2 \frac{q_2}{q_t} + ...$$

10.10 Horner time

As mentioned previously, it is difficult to keep the flow rate constant. The rate may have fluctuated significantly prior to shutin. A real buildup test therefore involves three steps: stabilization of the production rate, a production period and shutin period.

The variable rate condition contradicts the assumption of constant production rate which is implied in the buildup equation. Horner proposed the following correction to account for previous rate changes in an approximate way:

$$t_p = \frac{N_p}{q_{LAST}}$$

N_p is the cumulative production since the last major shutin period. This has usually a known value. The last stabilized rate is denoted by q_{LAST}. Hence, the Horner time, t_p, is a synthetic production time. The Horner correction is not rigorous. One may, however, put forward intuitive arguments: It is consistent with material balance. It has dimension like time. The last production rate is usually more important than previous. Finally, the concept has been around since the fifties and still in use.

If the well is new, then one may calculate the variable rate buildup response by superposition in time. For an old well, superposition is impractical. If the cumulative production, N_p, is not known, one may use the Miller-Dyes –Hutchinson method. The latter method is not discussed here.

11 Pressure Derivative

11.1 Introductory remarks

The logarithmic pressure derivative has the advantage of being dimensionally consistent with the pressure difference. Hence the two curves may be displayed in the same coordinate system. The field curves may be matched to their respective type curves simultaneously rather than sequentially. This is because the field curves for the pressure and pressure derivatives are related to their respective type curves in the same way:

$$p_{Dwf} = \frac{2\pi kh}{q\mu B}\Delta p_{wf}$$

and

$$p'_{Dwf\ln} = \frac{2\pi kh}{q\mu B}\Delta p'_{wf\ln}$$

Hence, the two field curves are shifted the same distance relative to their respective type curves. This improves the reliability of type curve analysis. The field curves may be matched against two type curves rather than one.

The pressure derivative has important diagnostic properties and may be used as an aid in identifying a plausible reservoir model. Specific flow periods show up as straight lines of known slopes on a log-log plot. For example, the following sequence of slopes: 1, 0.5, 0, suggests: wellbore storage, linear flow and radial flow. A plausible model may be a well with a vertical fracture in a large drainage area. A change in the the order of the sequence gives rise to other interpretations. For example, the sequence: 0, 0.5, 1, leads to the following interpretation: radial flow, linear and pseudo steady state flow. A plausible model could be: A vertical well in channel sand with no-flow external boundaries.

11.2 Drawdown

During radial flow the following equation applies:

$$\Delta p_{wf} = \frac{q\mu B}{4\pi kh}\left(\ln\frac{kt}{\varphi\mu c_t r_w^2} + 0.809 + 2S\right)$$

The above equation will show up as a straight line on a Δp_{wf} versus $\ln t$ -plot. The slope of the straight line is given by the coefficient of the logarithmic term. This implies that the logarithmic pressure derivative is independent of time.

$$\frac{d\Delta p_{wf}}{d\ln t} = \frac{q\mu B}{4\pi kh}$$

A constant shows up as a horizontal line in any coordinate system. Use of the chain rule leads to:

$$\frac{d\Delta p_{wf}}{d\ln t} = t\frac{d\Delta p_{wf}}{dt}$$

During the wellbore storage period, the following equation applies:

$$\Delta p_{wf} = \frac{qB}{C_s}\cdot t$$

The logarithmic derivative becomes:

$$\frac{d\Delta p_{wf}}{d\ln t} = \frac{qB}{C_s}\cdot t$$

which is the same as the right hand side of the pressure difference, Δp_{wf}. Hence, the two curves will overlap initially. C_S is the wellbore storage constant.

Flow period diagnostics is best done in a log-log coordinate system. Taking the logarithm on both sides of the above equation yields:

$$\log \Delta p'_{wf\ln} = \log\frac{qB}{C_s} + \log t$$

The above equation shows up as a unit slope straight line since the logarithmic term has an implied coefficient of value 1.

A wellbore storage transition into radial flow has a characteristic shape on diagnostic (log – log) plot which is shown below.

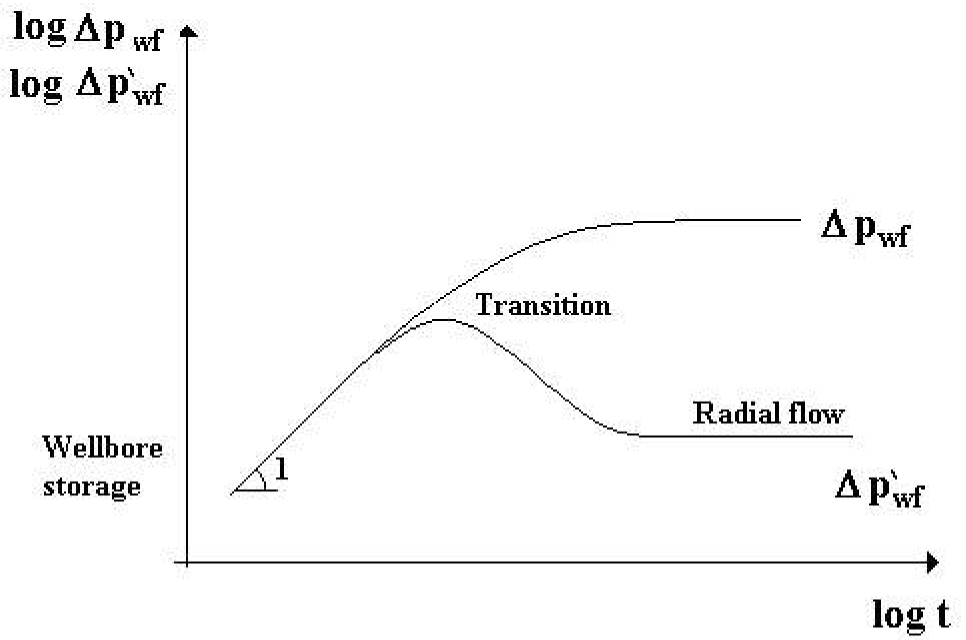

Figure 1: Pressure and pressure derivative diagnostic plot for drawdown

This example illustrates the diagnostic properties of the pressure derivative. Wellbore storage may be recognized as an early unit slope straight line and the radial flow period shows up as a horizontal line. The hump in the derivative indicates a positive skin factor. With a negative skin factor, the pressure derivative curve will approach the horizontal line from below.

Other specific flow periods are of power-law type. A power-law flow period obeys the equation:

$$\Delta p_{wf} = At^n + B$$

A and B are constants and the exponent n is a real number, $n \leq 1$. Classic examples of this type of behavior are linear-, $n = 1/2$, and spherical flow, $n = -1/2$.

The constant B will disappear in the derivative operation and the logarithmic derivative becomes:

$$t\frac{d\Delta p_{wf}}{dt} = nAt^n$$

The above equation will plot as a straight line with slope n in a log-log coordinate system since:

$$\log \Delta p'_{wf\ln} = \log nA + n \log t$$

For example, a linear flow period will show up as a"half slope" straight line, i.e. $n = 1/2$.

$$\log \Delta p'_{wf\ln} = \log 0.5A + 0.5 \log t$$

For large values of time the constant B may be neglected in comparison with the A-t-term. Then:

$$\log \Delta p_{wf} \approx \log A + 0.5 \log t$$

Hence, a linear flow period will show up as two parallel half slope straight lines. The logarithmic derivative, $\Delta p'_{wf\ln}$, is displaced downwards in comparison to the drawdown, Δp_{wf}

11.3 Buildup

Drawdown type curves may be used to analyze buildup tests under two conditions:

- $\Delta t << t_p$
- Two concurrent flow periods of the same type.

Concurrent flow periods may be explained as follows: The effect of a well with a piecewise constant rate schedule may be computed by superposition in time. The technique may be thought about as replacing the actual well with constant rate fictitious wells, one for each rate change. Two wells are required for an ideal buildup, one production- and one injection well. For a well with after-flow, an additional well is required. The latter has time dependent rate to account for fluid flow into the well after shutin.

Suppose we have radial flow towards the well and that there is no afterflow. If both the imaginary production – and the injection well are in the radial infinite-acting period, then there are two flow periods of the same type. As a consequence, the concept of equivalent time is valid. The concept, which is explained below, works equally well for flow periods of power-law type. The equations for equivalent time, however, will be different.

Suppose there are two simultaneous radial flow periods. Then, the buildup may be described by the extended Horner equation

$$\Delta p_{BU} = \frac{q\mu B}{4\pi kh}(\ln \frac{kt_p \Delta t}{\varphi\mu c_t r_w^2 (t_p + \Delta t)} + 0.809 + 2S)$$

where $\Delta p_{BU} = p_{ws} - p_{wf}(t_p)$. The pressure difference, Δp_{BU}, is illustrated in Figure 2. Note that $p_{wf}(t_p)$ is the last recorded flowing pressure.

The basic idea of equivalent time is to equalize the buildup and drawdown equation in the infinite-acting flow period. The drawdown equation is:

$$\Delta p_{wf} = \frac{q\mu B}{4\pi kh}\left(\ln \frac{kt}{\varphi\mu c_t r_w^2} + 0.809 + 2S \right)$$

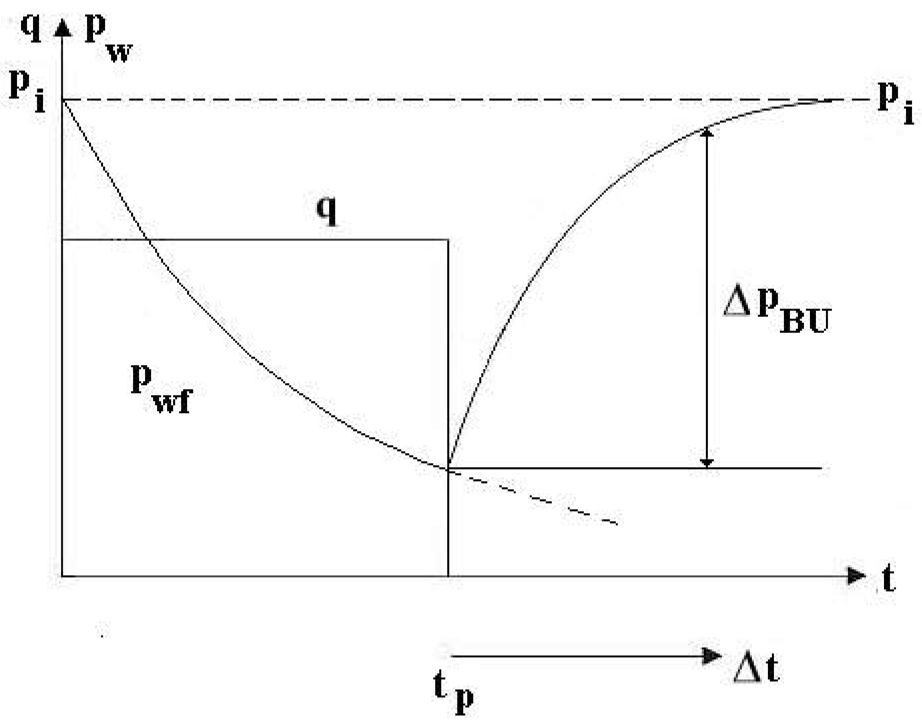

Figure 2: The development of Δp_{BU} with time.

One may define an equivalent time:

$$\Delta t_e = \frac{t_p \Delta t}{t_p + \Delta t}$$

Substitution of the equivalent time into the buildup equation yields:

$$\Delta p_{BU} = \frac{q\mu B}{4\pi kh}(\ln \frac{k\Delta t_e}{\varphi\mu c_t r_w^2} + 0.809 + 2S)$$

The above equation has the same appearance as the corresponding drawdown equation and the pressure derivative becomes:

$$\frac{d\Delta P_{Bu}}{d \ln \Delta t_e} = \frac{q\mu B}{4\pi kh}$$

Immediately after the well has been closed, fluid will continue to flow into the well. This phenomenon has been called afterflow. From the definition of equivalent time, we conclude that for small values of shutin time, there is essentially no difference between the shutin time, Δt, and equivalent time, Δt_e. Hence:

$$\Delta t = \Delta t_e$$

The dimensionless pressure change for build up is:

$$p_{DBU} = p_{Dw}(\Delta t_D) - p_{Dw}(t_{Dp} + \Delta t_D) + p_{Dw}(t_{Dp})$$

The two last terms on the right hand side tends to cancel when the shutin time is negligible compared to the production time.

$$p_{DBU} \approx p_{Dw}(\Delta t_D)$$

If there is a unit slope straight line in the pressure difference and pressure derivative, then the pressure response is dominated by afterflow.

$$\Delta p_{BU} = \frac{qB}{C_S}\Delta t$$

The above equation will show up as a straight line with unit slope on a log-log plot.

The equivalent dimensionless equation becomes:

$$_{DBU} \qquad \frac{\quad}{_{SD}}$$

The diagnostic plot for a transition from after-flow to radial flow is shown below.

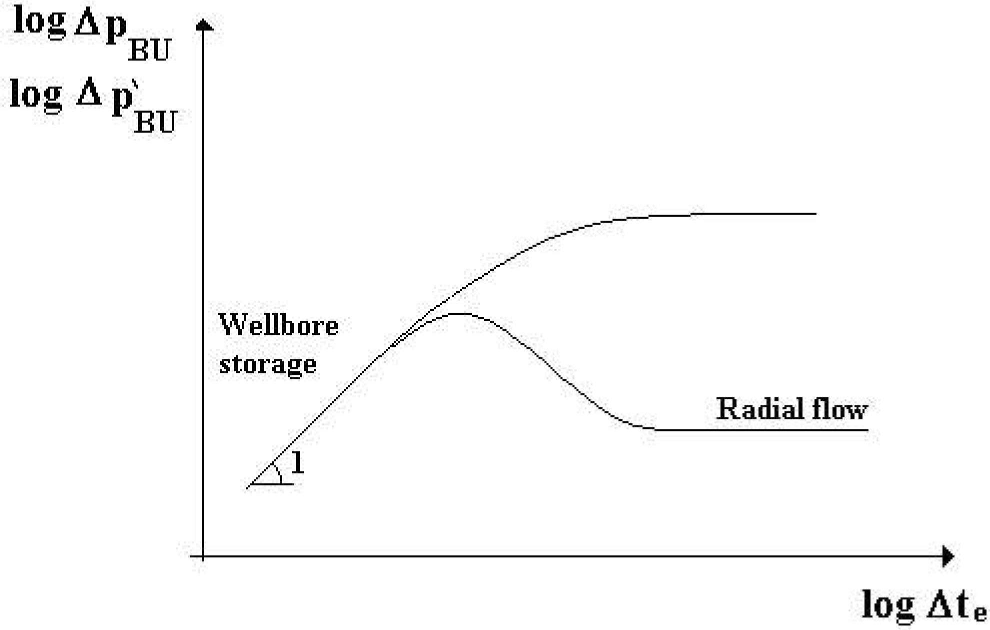

Figure 3: The buildup diagnostic plot has the same appearance as for drawdown.

For an old well, the condition that shutin time is small in comparison with the production time is valid during the entire test, then the buildup equation will simplify to:

$$\Delta p_{BU} = \frac{q\mu B}{4\pi kh}\left(\ln \frac{k\Delta t}{\phi \mu c_t r_w^2} + 0.809 + 2S \right)$$

The transition between wellbore storage period, defined by the unit slope straight line (see Chapter 12), and radial flow will show up as the previous curves when plotted against the shutin time, Δ*t*.

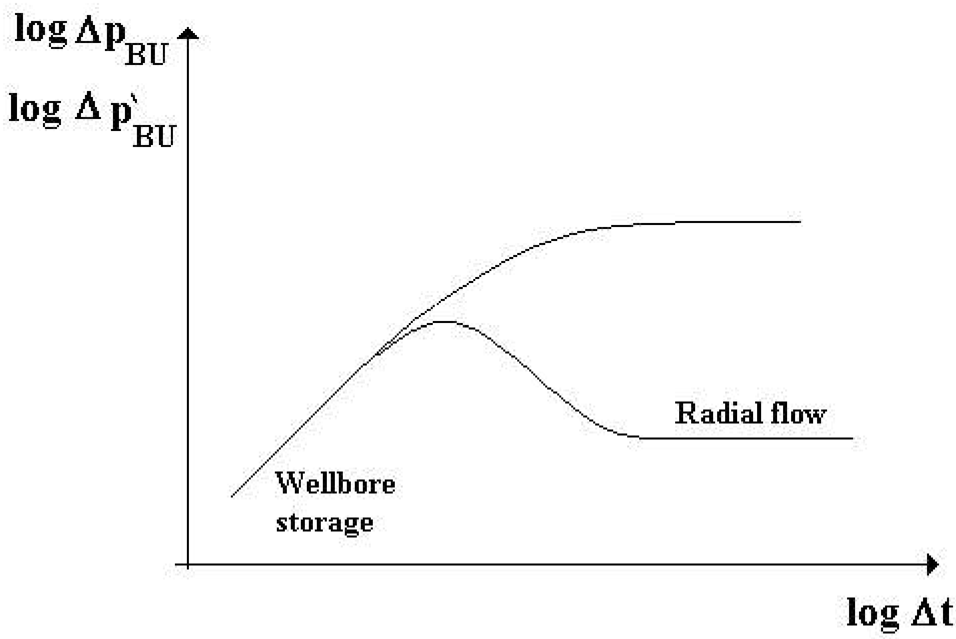

Figure 4: Pressure and pressure derivative for buildup in an old well, $t >> \Delta t$

11.4 Derivation algorithm

A derivative is usually approximated by a finite difference quotient. As such it is sensitive to noise. Some smoothing may be necessary to filter out high frequency noise.

Most numerical filters depend on an even sampling rate. This is rarely the case in well testing. For a non-even spacing between sampled the points, the derivative may be approximated by the slope of a least square line through neighboring points. All points in the interval contribute to the computed derivative.

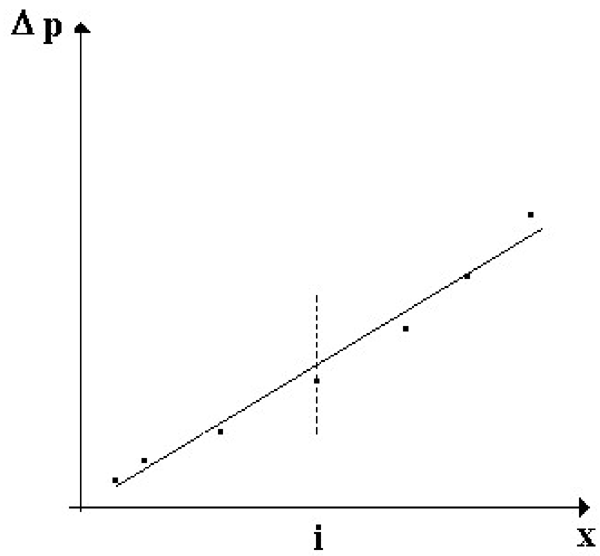

Figure 5: Schematic of a 7 point least square filter

An alternative algorithm uses three points only, the point of interest, one to the left and one to the right.

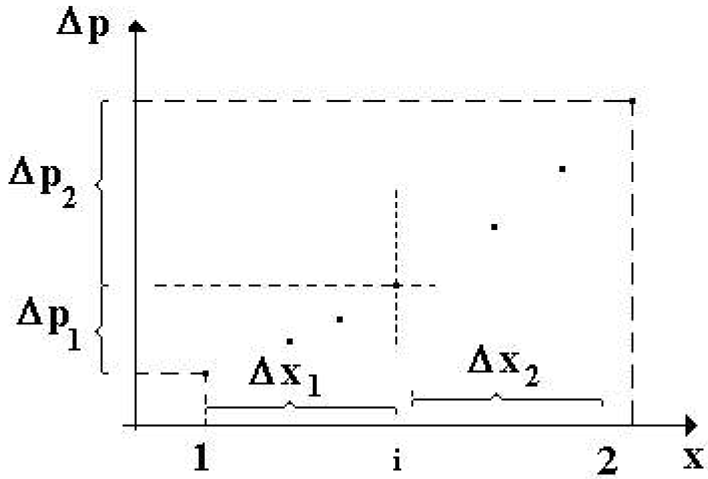

Figure 6: Schematic of a derivation algorithm

The derivative is approximated as the weighted average between the left and right hand slopes.

$$\left[\frac{d\Delta p}{dx}\right]_i = \frac{\frac{\Delta p_1}{\Delta x_1}\Delta x_2 + \frac{\Delta p_2}{\Delta x_2}\Delta x_1}{\Delta x_1 + \Delta x_2}$$

A smaller difference, Δx, on the horizontal axis leads to a better approximation of the derivative. Hence, the highest weight factor is applied to the finite difference quotient with the smallest difference Δx.

It is recommended to start with consecutive points. If the degree of noise is unacceptable, then the distances Δx_1 and Δx_2 have to be increased. The points 1 and 2 are selected as the first points such that

$$\Delta x_1 , \Delta x_2 \geq L$$

If $x = \ln t_e$, then

$$\ln \frac{t_2}{t_i} \geq L$$

and

$$\ln \frac{t_i}{t_1} \geq L$$

L has been called the smoothing parameter. The degree of smoothing will increase with increasing smoothing parameter. On the other hand, the accuracy of the derivative approximation will decrease. Hence, L should be kept as small as possible. Experience has shown that $L \approx 0.2$ is often a natural choice.

Any derivation algorithm has a problem near the ends of the interval. The calculation scheme is then running out of points. This phenomenon has been called the end effect of the pressure derivative. These regions have to be treated separately. One solution is to reduce the smoothing parameter close to the ends of the intervals.

12 Wellbore Storage

12.1 Introductory remarks

Immediately after a rate change, the pressure response may be influenced by wellbore storage. In case of a decreased rate, more fluid will flow into the well than out. For an increase it will be the other way around.

We discuss the storage effect for drawdown and buildup tests by use of simplified models. These may or may not be realistic. If the pressure signature is inconsistent with the predicted behavior, one should resort to a more complex model. Such models will not be discussed here.

A wellbore storage transition into constant rate infinite-acting behavior may be separated into three periods: the wellbore storage period, transition and finally infinite-acting. The wellbore storage period shows up as an early unit slope straight line on a log-log plot, Δp_w vs. t. This period may or may not appear in a given test. It may have ended before the first pressure reading. If it does show up, type curve matching becomes more reliable since the interpretation is based on more information. The wellbore storage effect has died out at the end of the transition period. Then, constant rate infinite-acting behavior will exist until the pressure signal is influenced by the external boundary condition. For a small drainage area, the infinite-acting period may be short, even non-existing.

Elementary well test interpretation techniques rely on the assumption of a constant rate. Suppose it is possible to keep the surface flow rate, q, constant. Then the first production derives from the wellbore rather than the reservoir. This phenomenon has been called well unloading. Unloading could be the result of fluid expansion or a moving gas liquid interface. The reservoir flow rate, q_{sf}, will build up gradually towards the surface flow rate. A schematic of the rate vs. time relationship is shown in Figure 1.

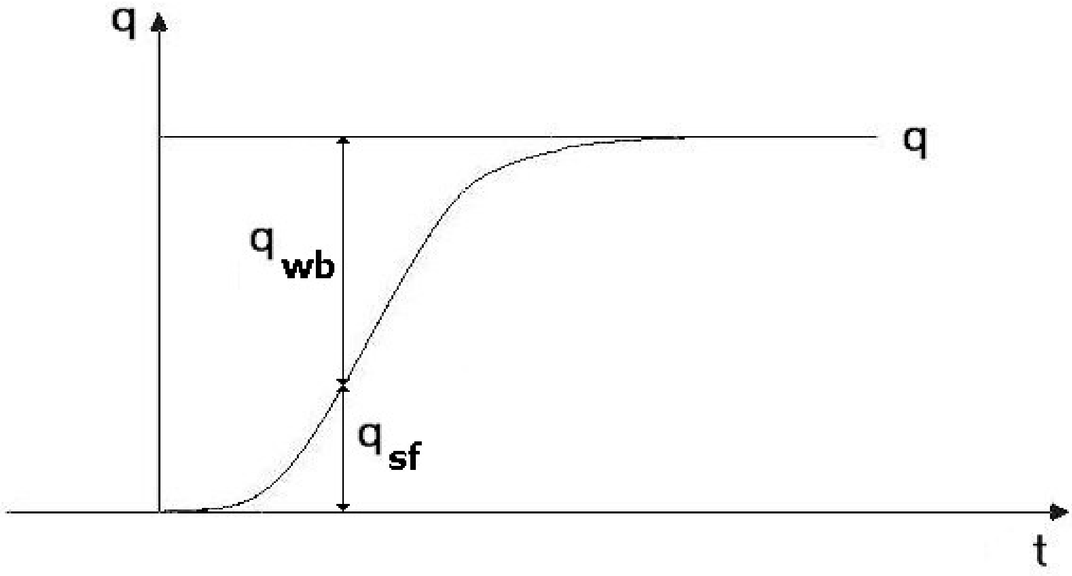

Figure 1: Surface and sandface production rate as a function of time

Semi-log analysis is possible when $q_{wb} \approx 0$. Then, the wellbore storage effect has died out.

12.2 Drawdown

We consider two simplified models.

1. One phase in the wellbore

Suppose the well is filled with a high pressure reservoir fluid (oil, gas or water) as shown in Figure 2. Production starts when the choke at the top is opened and the well is exposed to a lower pressure. As a result, the fluid in the wellbore will expand.

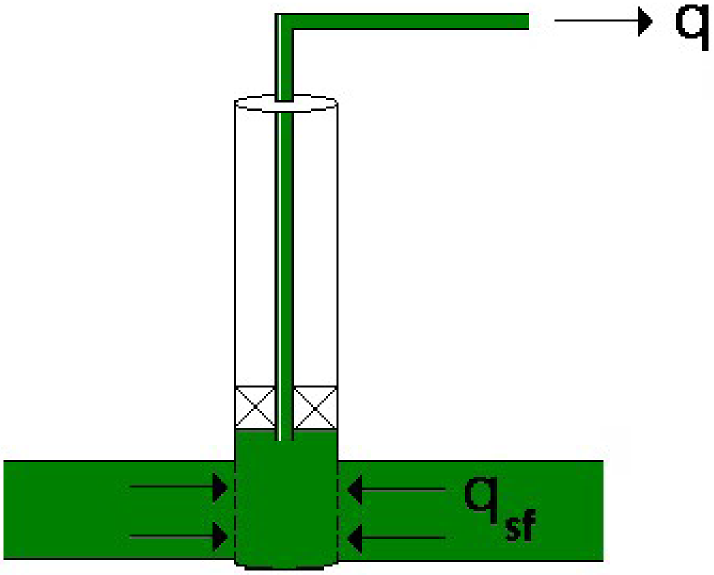

Figure 2: Fluid of high pressure in the wellbore

A simple material balance of the fluid in the wellbore gives the relationship between density and pressure. The principle of conservation of mass states that:

Mass rate in – mass rate out = rate of accumulation

$$q_{sf}\rho - q\rho = V_{wb}\frac{d\rho}{dt}$$

If more fluid is flowing out of the well that in, the time derivative of density becomes negative. This corresponds to a decrease in pressure. The wellbore storage effect has died out when the time derivative of the density becomes negligible. A change of density will have influence on both constant and multi-rate tests.

Division by the density, ρ, and use of the chain rule yields:

$$q_{wb} = q_{sf} - q = C_S\frac{dp_w}{dt}$$

The wellbore storage constant is defined as:

$$C_S = c_{wb}V_{wb}$$

where $c_{wb} = \frac{1}{\rho}\frac{d\rho}{dp_w}$ is the compressibility of the fluid in the wellbore. The compressibility is essentially constant for a slightly compressible fluid. For a gas, the compressibility may depend on the pressure. V_{wb} is the volume of the compressed fluid.

Note that all the variables are measured close to reservoir conditions. These may be converted to standard conditions by division by the formation volume factor:

$$B = \frac{V_{res}}{V_{sc}}$$

$$q_{wb} = q_{sf} - q = \frac{C_S}{B}\frac{dp_w}{dt}$$

The above production rates are referred to standard conditions.

In case of negligible sandface production, i.e. $q_{sf} \approx 0$, the fluid produced at the surface is the result of fluid expansion. This has been called well unloading. Since the surface rate, q, is constant, the above differential equation may be solved by separation of variables:

$$\int_{p_i}^{p_{wf}} dp_w = -\frac{qB}{C_S}\int_0^t dt$$

which yields:

$$p_{wf} = p_i - \frac{qBt}{C_S}$$

The above equation describes a straight line with slope $m = \frac{qB}{C_s}$, and intercept p_i with the vertical axis.

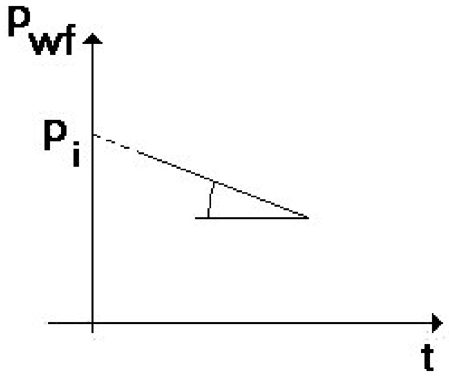

Figure 3: Well flowing pressure as a function of time

Because the reservoir flow rate is negligible initially, the pressure response does not reflect reservoir properties. Only wellbore properties have influence. If the wellbore storage constant determined from the slope is too large, this may indicate a leak. Since V_{wb} is known from well data, the wellbore storage constant may be estimated independently. Ideally, these values should agree.

Dimensionless variables are important for type curve analysis. For constant rate problems, these may be defined as:

$$q_{Dsf} = \frac{q_{sf}}{q}.$$

$$t_D = \frac{kt}{\varphi \mu c_t r_w^2}$$

and

$$p_{Dw} = \frac{2\pi kh}{q\mu B} \Delta p_w$$

For a drawdown test we use:

$$\Delta p_{wf} = p_i - p_{wf}$$

Substitution of the dimensionless variables into:

$$q_{wb} = q_{sf} - q = \frac{C_S}{B} \frac{dp_{wf}}{dt}$$

yields:

$$q_{Dwb} = 1 - q_{Dsf} = C_{SD}\frac{dp_{Dwf}}{dt_D}$$

Under the condition of negligible sandface production, $q_{Dsf} \approx 0$, the above equation may be integrated by separation of variables:

$$p_{Dwf} = \frac{t_D}{C_{SD}}$$

where:

$$C_{SD} = \frac{C_S}{2\pi\varphi c_t h r_w^2}$$

2. Production by a moving gas-liquid interface

Consider the well in Figure 4, which has been closed. The pressure is a decreasing function of the vertical distance above the reservoir. At some position it may fall below the bubble point. Then gas comes out of solution and flow to the top. Suppose there is no packer and that the tubing is filled with a liquid (oil or water) all the way to the choke. The first fluid produced will be due to a falling liquid level in the annular space, which is the space between the casing and tubing.

The model depends on simplifying assumptions.

- The pressure at the top of the well, p_t, is constant:
- The density of the gas is negligible in comparison with the liquid density.
- The effects of density changes are negligible.
- The effect of friction is negligible. (roughness of tubing surface)

Let z denote the position of the gas-liquid interface. Under these conditions, the wellbore pressure may be approximated by the static equation.

$$p_{wf} \approx p_w = p_t + \rho g z$$

Derivation with respect to time yields the relationship between position and pressure.

$$\frac{dz}{dt} = \frac{1}{\rho g}\frac{dp_{wf}}{dt}$$

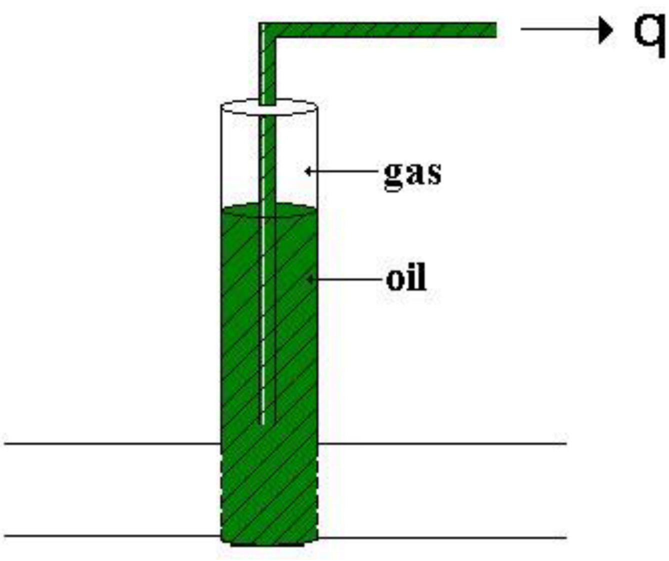

Figure 4: Oil production by a falling gas liquid interface

The area of annular space is:

$$A_{wb} = \pi(r_w^2 - r_{tubing}^2)$$

The difference in inflow and outflow has to be compensated by the falling liquid level:

$$q_{wb}\left(=q_{sf}-q\right)B=A_{wb}\frac{dz}{dt}$$

Substitution of the relationship between pressure and position yields:

$$\left(q_{sf}-q\right)B=\frac{A_{wb}}{\rho g}\frac{dp_{wf}}{dt}$$

Note that the above equation has the same form as for unloading by fluid expansion,

$$\left(q_{sf}-q\right)=\frac{C_S}{B}\frac{dp_{wf}}{dt}$$

The definition of the wellbore storage constant, however, is different.

$$C_s=\frac{A_{wb}}{\rho g}$$

Since $q_{wb}\approx q$ initially, the above differential equation may be integrated to yield:

$$p_{wf}=p_i-\frac{qBt}{C_S}$$

While the wellbore storage equation is the same for fluid expansion and moving gas-liquid column, the numerical value of the wellbore storage constant assumes a much larger value for a moving gas-liquid interface.

12.3 Buildup

As shown in the chapter on buildup testing, the pressure in the near wellbore region is lower than farther away after shutin. Fluid will continue to flow towards the near-wellbore region. It will also flow into the well. This is called afterflow. The fluid in the wellbore is compressed since fluid cannot flow out of the well. The material balance equation for the well is the same as for drawdown, but this time more is flowing into the well than out.

$$q_{wb}=q_{sf}-q=\frac{C_S}{B}\frac{dp_w}{dt}$$

During the wellbore storage period (afterflow) we have: $q = 0$, and $q_{sf}(t_p) = q$. Then:

$$q_{wb} = q(t_p) = V_{wb} c_{wb} \frac{dp_w}{dt}$$

$$\int_{p_{wf}(t_p)}^{p_{ws}} dp_w = -\frac{qB}{C_S} \int_0^{\Delta t} dt$$

The above equation may be integrated to yield:

$$p_{ws} = p_{wf}(t_p) + \frac{qB\Delta t}{C_S}$$

The dimensionless equation becomes:

$$p_{DBU} = \frac{\Delta t_D}{C_{SD}}$$

where:

$$p_{DBU} = \frac{2\pi kh}{q\mu B} \Delta p_{BU}$$

and

$$\Delta t_D = \frac{k\Delta t}{\varphi \mu c_t r_w^2}$$

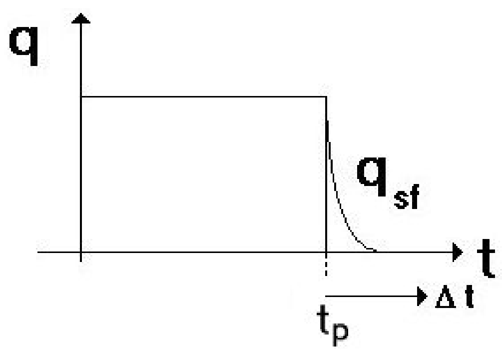

Figure 5: Surface and sandface production rates as function of time

Observations:

1. The pressure drop across the skin is constant when $q_{sf} \approx q(t_p) = q$ since:

$$\Delta p_{skin} = \frac{q_{sf} B\mu}{2\pi kh} S$$

2. Semi-log (Horner) analysis is possible when ${}_{wb} \approx$ and $q_{sf} \approx 0$. Then, there is no pressure drop associated with the skin.
3. The skin pressure drop is decreasing function of time during afterflow period since q_{sf} is decreasing from $q(t_p) = q$ to $q_{sf} = 0$.

The effect of afterflow may be minimized by downhole shutin. If there is one phase flowing into the wellbore, the storage constant is given by:

$$C_S = c_{wb} V_{wb}$$

Downhole shutin will reduce the volume of the compressed fluid dramatically. This is illustrated in Fig. 6. The compressed volume for surface shutin is shown in Fig. 4.

Downhole shutin is rarely done for production wells, but it is common practice in drillstem tests.

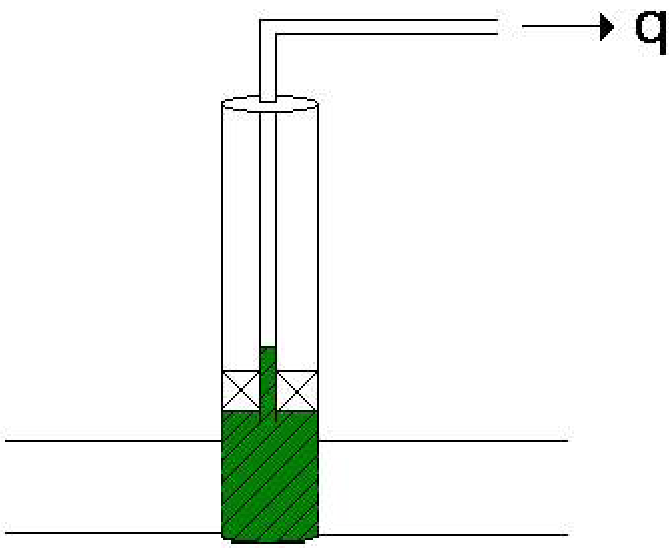

Figure 6: Volume of compressed fluid, downhole shutin

13 Principle of Superposition

13.1 Introductory remarks

The present notes deals with the principle of superposition. Three applications will be discussed: Several wells in an infinite reservoir, the method of images and superposition in time.

The principle of superposition may be summarized as follows:

- The addition of solutions to a linear differential equation gives a new solution but for different boundary conditions.
- The total response of several wells may be computed by adding the response of each well as if this one was acting alone in infinite reservoir. Adding fictitious wells is a powerful technique to obtain solutions to the diffusivity equation.
- The diffusivity equation has a unique solution. If a solution can be found that satisfies the differential equation and the boundary conditions, then this is the only solution. It may be possible to find an alternative solution to the same problem by use of another technique. If the resulting equation has a different appearance, it may be shown that they are equivalent by proving that the difference is zero.

The criteria for linearity, both for the diffusivity equation and the boundary conditions, are:

- All derivatives (of pressure) with respect to the space and time variable in the first power.
- No product of derivatives.
- All coefficients to derivatives independent of the solution, which in pressure transient analysis is pressure.

The line source solution is often used in well test interpretation. It is the solution to the diffusivity equation for a line source well. The drawdown at the wellbore is:

$$\Delta p_{wf} = p_i - p_{wf} = -\frac{q\mu B}{4\pi kh}\left(Ei\left(-\frac{\varphi\mu c_t r_w^2}{4kt} \right) - 2S \right)$$

It is usually assumed that the effect of a skin at long distance from the well is negligible. Hence, the drawdown at some distance, *r*, is given by:

$$\Delta p = p_i - p(r,t) = -\frac{q\mu B}{4\pi kh} Ei\left(-\frac{\varphi\mu c_t r^2}{4kt} \right)$$

The line source solution is an approximate solution for a cylindrical well.

The line source solution implies the following assumptions:

- Infinite reservoir
- One well
- Constant production rate
- The well is shaped like a line, $r_w \to 0$.

The objective of this chapter is to generalize the utility of the line source solution by relaxing the first three assumptions.

13.2 Several wells in an infinite reservoir

Consider three wells A, B and C in an infinite reservoir. Each well is produced at a constant rate. The wells are put on production at different times and are produced with different rates.

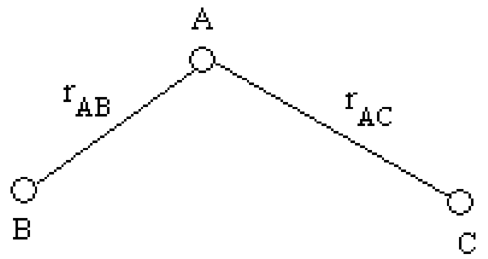

Figure 1: Three wells in an infinite reservoir

According to the principle of superposition, the total response of several wells may be found by computing the pressure change resulting from each well separately and then adding the pressure changes.

Suppose we are interested in computing the pressure in well A. According to the principle of superposition, the total pressure change may be separated into three components.

$$\Delta p_{A,tot} = \Delta p_{A,A} + \Delta p_{A,B} + \Delta p_{A,C}$$

Index A,B denotes the pressure change in well A due to the production of well B. In the same way index A,A denotes the pressure change in well due to well A.

Then,

$$\Delta p_{A,tot} = -\frac{q_A \mu B}{4\pi kh}\left(Ei\left(-\frac{\varphi \mu c_t r_w^2}{4k\left(t-\tau_A\right)}\right) - 2S\right)$$

$$-\frac{q_B \mu B}{4\pi kh} Ei\left(-\frac{\varphi \mu c_t r_{AB}^2}{4k\left(t-\tau_B\right)}\right)$$

$$-\frac{q_C \mu B}{4\pi kh} Ei\left(-\frac{\varphi \mu c_t r_{AC}^2}{4k\left(t-\tau_C\right)}\right)$$

The indices A, B and C denote wells and τ denote start of production. Hence, the difference $t-\tau$ is the production time.

The response of each well is controlled by the production time. If a production time is negative, the corresponding *Ei*-function is zero since a well cannot create a pressure drop before it has been put on production.

13.3 Method of images

This discussion is limited to the application of the method of images to generate no-flow and constant pressure boundaries. So far, only infinite reservoirs have been discussed. Physical boundaries may be simulated by the method of images. We discuss the method in an intuitive way

The method of images may be thought of as a two step procedure.

1. Remove all boundaries.
2. Add imaginary wells until the boundary conditions are satisfied.

Addition of solutions to a linear equation gives a new solution, but for different boundary conditions. The boundary condition is satisfied when perfect symmetry with respect to all boundaries has been achieved.

Consider an infinite reservoir with a no-flow fault. It is known that the behavior of a well near a sealing fault in an infinite reservoir may be modeled by addition of an identical well at a symmetrical position with respect to the fault plane, just like a mirror image. The fictitious well has been called an image well. This is illustrated in Figure. 2. The position of a fault, after removal, is indicated by the broken line.

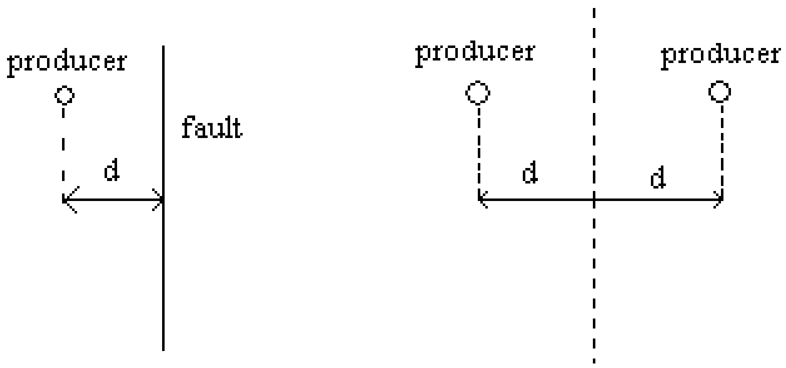

Figure 2: Use of the method of images to generate a no-flow boundary.

The image well interacts with the real well so that no flow occurs across the broken line. This is readily verified by showing that $\frac{\partial p}{\partial x} = 0$ at this position. The same conclusion may be reached by symmetry considerations. A fluid particle at the fault position is equally attracted towards the real and the image well. Hence, no fluid can flow across this boundary.

A constant pressure boundary may be generated by the same procedure. In the latter case, there must be one injection well and one production well. The physical argument for assuming that interference between the actual well and the image well creates a constant pressure boundary is that the fluid produced is immediately replaced by flow across the constant pressure boundary. It is reasonable to expect that a gas cap or a large aquifer will approximately behave this way for a short well test. Such a boundary may also be created by a pressure maintenance scheme, i.e. gas and/or water injection.

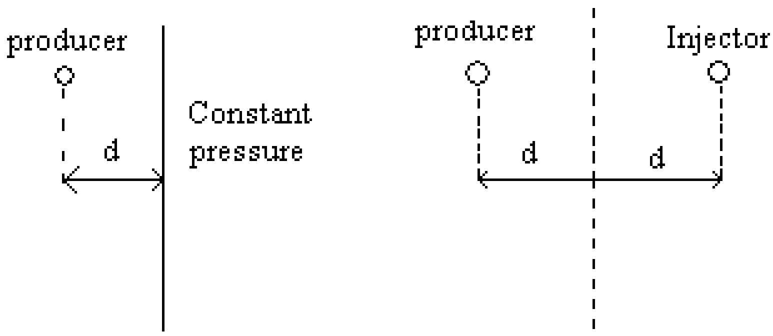

Figure 3: Use of the method of images to generate a constant pressure boundary.

The proposition that interference between wells generates a constant pressure boundary may be verified as follows. Choose a coordinate system as shown in Figure 4 and compute the total pressure change, Δp, at an arbitrary position y_0 along the y-axis.

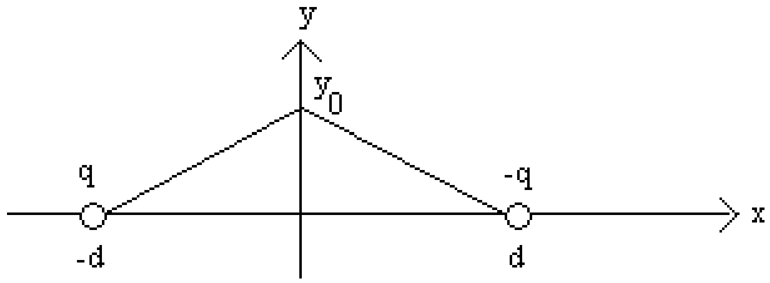

Figure 4: The fault plane (the y-axis) is a constant pressure boundary.

Application of the principle of superposition yields:

$$\Delta p(x=0,y_0) = -\frac{q\mu B}{4\pi kh} Ei\left(-\frac{\phi\mu c_t\left(y_0^2+(-d)^2\right)}{4kt}\right)$$

$$+\frac{q\mu B}{4\pi kh} Ei\left(-\frac{\phi\mu c_t\left(y_0^2+d^2\right)}{4kt}\right)$$

The result becomes:

$$\Delta p\left(x=0,y_0\right)=0$$

Clearly, the method of images extends to systems with more than one boundary. Remember that the boundary conditions are satisfied when perfect symmetry has been achieved.

A well in an infinite reservoir between two parallel faults has been selected as an example. Consider a well "A" in an infinite reservoir between two no-flow boundaries. Such reservoirs have been called channel sands.

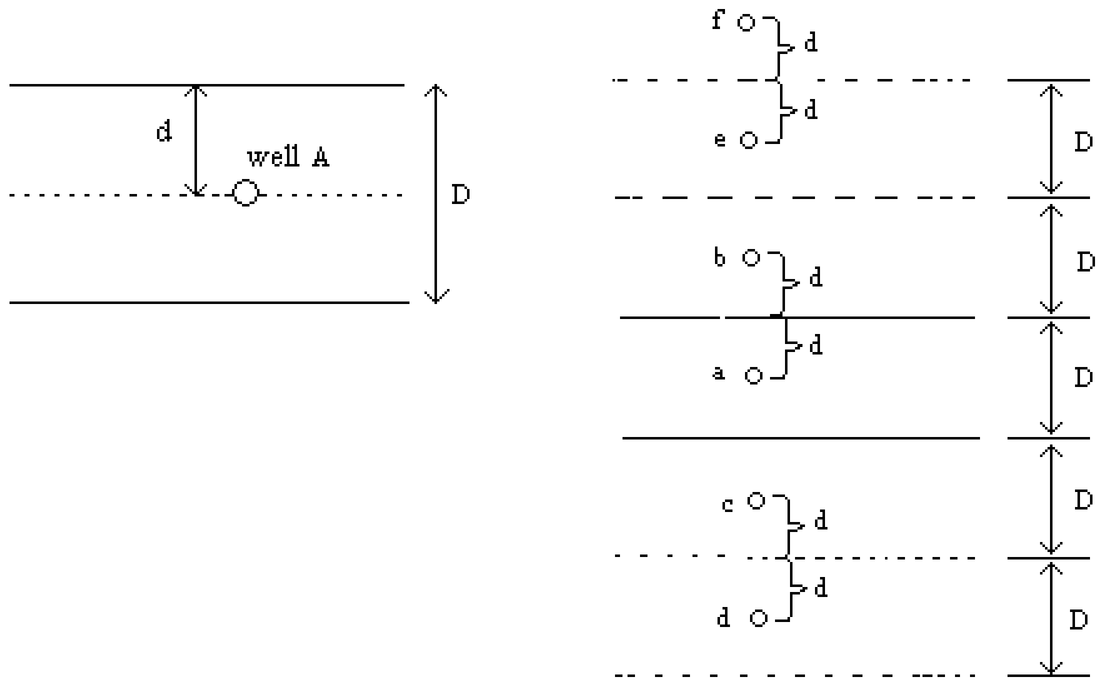

Figure 5: A well between two parallel no-flow boundaries

The upper boundary will be closed by adding image well b in Figure 5. In order to close the lower boundary, image wells c and d has to be added. This in turn will destroy the symmetry around the upper boundary. To restore the symmetry, image wells e and f must be added and the symmetry around the lower boundary is destroyed. This procedure may be continued until we have an infinite series of image wells. Then, the symmetry is perfect around both the upper and lower boundaries.

The combined effect of all wells may be obtained by adding the pressure drop created by each well. The pressure drop created by any well, may be computed as if this particular well was acting alone in an infinite reservoir.

In practice, sufficient accuracy is achieved by adding a comparatively small number of image wells. Wells located at large distances from the physical boundaries will contribute less to the reservoir pressure drop than close ones. Hence, the series may be truncated when the contribution from a distant image well becomes insignificant.

When summing up the series, we use the infinite reservoir solution, which is the line source solution.

13.4 Superposition in time

Consider a well with a piecewise constant rate schedule. The actual well may be replaced by fictitious wells located at the same position as the real one. The production rate and start-up time of an imaginary well is determined from the condition that the total (sum) rate of all fictitious wells is equal to the production rate of the actual one. This technique is the basis for traditional buildup – and multi-rate test analysis.

As an illustration of the technique, consider the rate sequence of Figure 6.a (the upper one).

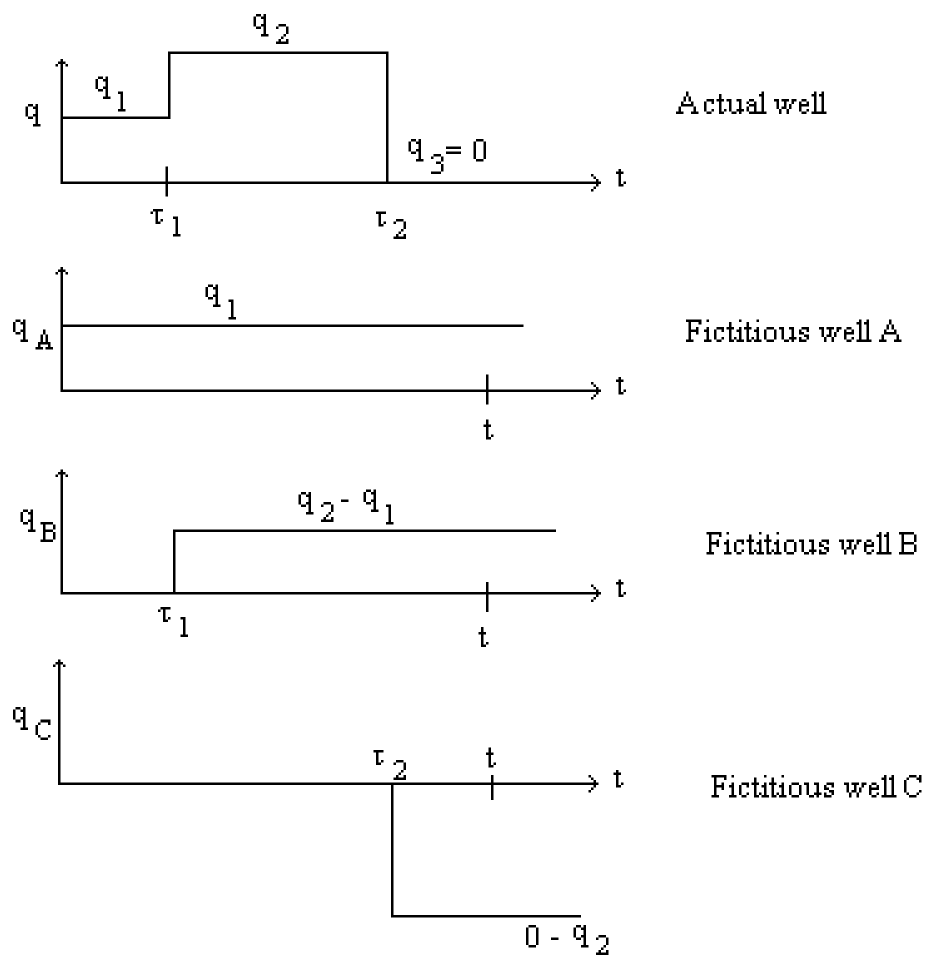

Figure 6a, g, c and d: Simulation of a piecewise constant rate schedule by fictitious wells

The same rate schedule may be obtained by adding the rates of Figure 6.b to Figure 6.d. For instance, suppose the time of interest, *t*, is larger than τ_1 but smaller than τ_2. Then, the net rate is: $q_1 + q_2 - q_1 = q_2$. Well C has no effect since $t - \tau_2 < 0$.

From the principle of superposition, the total pressure drop from wells A, B and C at the time of interest, *t*, may be obtained by addition of the resulting pressure changes.

$$\Delta p_w = \Delta p_{wA} + \Delta p_{wB} + \Delta p_{wC}$$

The pressure drop due to a well producing at a constant rate in an infinite reservoir is:

$$\Delta p_w = -\frac{q\mu B}{4\pi kh}\left(Ei\left(-\frac{\varphi\mu c_t r_w^2}{4kt} \right) - 2S \right)$$

Note that while the first well has produced the entire time period, *t*, the producing time of the last two wells are $t-\tau_1$ and $t-\tau_2$ respectively.

From the principle of superposition, the total pressure drop during is:

$$\Delta p_w = -\frac{q_1 \mu B}{4\pi kh}\left(Ei\left(-\frac{\varphi \mu c_t r_w^2}{4kt} \right) - 2S \right)$$

$$-\frac{\left(q_2 - q_1\right) \mu B}{4\pi kh}\left(Ei\left(-\frac{\varphi \mu c_t r_w^2}{4k\left(t-\tau_1\right)} \right) - 2S \right)$$

$$-\frac{\left(0 - q_2\right) \mu B}{4\pi kh}\left(Ei\left(-\frac{\varphi \mu c_t r_w^2}{4k\left(t-\tau_2\right)} \right) - 2S \right)$$

During the shutin period, i.e. $t-\tau_2 > 0$, all skin terms will cancel sine each term occurs twice with opposite sign. This is what we expect since:

$$\Delta p_{skin} = \frac{q_3 \mu B}{2\pi kh} S$$

The rate q_3 is zero.

When the time of interest falls in between τ_1 and τ_2, the skin pressure change is:

$$\Delta p_{skin} = \frac{q_2 \mu B}{2\pi kh} S$$

The latter is different from zero unless the actual well behaves like an ideal one.

14 Appendix A: Core Analysis

14.1 Introduction

The most direct way of obtaining rock properties is by core analysis. Unfortunately the method is expensive and tedious. Hence cheaper indirect methods are preferred. A series of logging techniques are available. Results from indirect methods need to be calibrated against results obtained by core analysis. Also, results obtained by well testing need to be checked against results from core analysis. Hence, it is common practice to select a few wells for coring. The alternative is to use core analysis from a similar formation elsewhere. The traditional procedure is to take one sample for every foot drilled. Each sample is assumed to be representative for the one foot interval.

These notes will be focused on the comparison of well testing and core analysis. An example of results from core analysis is shown in Table 1. The formation is conspicuously heterogeneous on the length scale of core analysis. A length scale of a few centimeters is impractical for reservoir flow studies. For instance, the length scale of a computational cell in reservoir simulation may be a few meters in the vertical direction and around 100 m in the horizontal directions. Average values for the rock properties are required for reservoir studies. Unfortunately it is unclear what the best average is.

			Air Perm	Log Perm	Porosity
Sample	top Depth	end (ft)	(mD)	Log(mD)	(%)
1	7352	52,3	0,59	-0,229	12,7
2	7352,3	53,4	0,22	-0,658	10,8
3	7353,4	54,5	0,43	-0,367	12,6
4	7354,5	55,2	0,53	-0,276	12,8
5	7355,2	56,4	0,79	-0,102	13,8
6	7356,4	57,3	0,3	-0,523	10,3
7	7357,3	58,3	0,14	-0,854	11,3
8	7358,3	59,4	0,08	-1,097	10,2
9	7359,4	60,4	0,13	-0,886	10,8
10	7360,4	61,3	0,09	-1,046	10,2
11	7362	62,4	0,08	-1,097	8,6
12	7362,4	63,3	0,14	-0,854	9,8
13	7363,3	64,3	0,96	-0,018	13,2
14	7364,3	65,3	0,99	-0,004	13,8
15	7365,3	66,4	0,16	-0,796	11,9
16	7366,4	67,4	0,07	-1,155	7,8

Table 1: Thor Field. Well A5

The problem of obtaining a reliable average to be used on a larger length scale (upscaling) remains a challenge. We will choose the traditional approach which is based on simplified analytical models.

Core analysis is fraught with sources of errors. For example, coring and handling may alter the rock properties. Exposure to a radically different stress condition may have the same effect. It may be impossible to obtain an unbiased set of samples. Some intervals may be to weak for coring, others may be too thin. It may be impractical to analyze tight cores, etc.

14.2 Well testing

A well test operates on a much larger length scale (10–100m in the horizontal direction) than core analysis. The pressure signature obtained, by well test, will in many cases behave as for an equivalent homogeneous reservoir. The effect of heterogeneities will show up only for small values of time. Observe that the length scale of well testing is comparable to that of reservoir simulation. A well test is expensive and may not be available for all wells.

14.3 Average porosity obtained by core analysis

The arithmetic average may be best for the porosity, since it is consistent with material balance. Also the variation in the porosity values is moderate. This is illustrated by the schematic below.

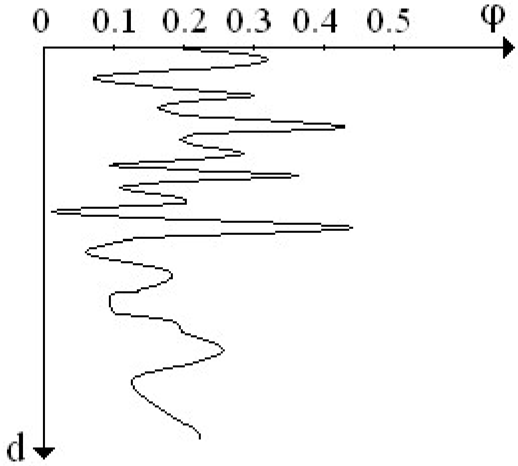

Figure 1: Porosity as a function of depth

The average porosity may be defined as

$$\bar{\varphi} = \frac{\sum_{i=1}^{N} \varphi_i h_i}{\sum_{i=1}^{N} h_i}$$

where N is the number of cores and h_i is the interval between the samples. The porosity of sample i is supposed to be representative for the length h_i.

For a uniform sample interval, $h_i = \Delta h$, the above equation will simplify to:

$$\bar{\varphi} = \frac{\Delta h \sum_{i=1}^{N} \varphi_i}{h} = \frac{1}{N} \sum_{i=1}^{N} \varphi_i$$

The arithmetic average may be regarded as the expected value:

$$\bar{\varphi} = \frac{\sum \varphi_i h_i}{\sum h_i} = \sum_{i=1}^{N} \varphi_i P_i = E(\varphi)$$

where P_i is the probability of φ_i and E(φ) is the expected value.

14.4 Average permeability obtained by core analysis

The degree of variability in the permeability is much higher than for porosity. This is illustrated below.

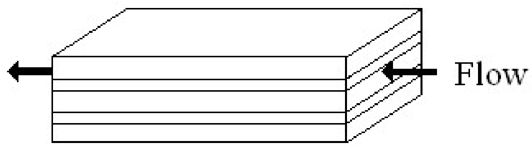

Figure 2: Permeability as a function of depth

The average permeability is a problematic concept since the values can be different by several orders of magnitudes. Also the average permeability depends on how the permeability is distributed in the spatial domain.

The average permeability may be established by the use of analytical flow models in a few simple cases. Steady state flow may be described by Darcy's law. An analysis based on integration of Darcy's law leads to important observations.

14.5 Arithmetic average

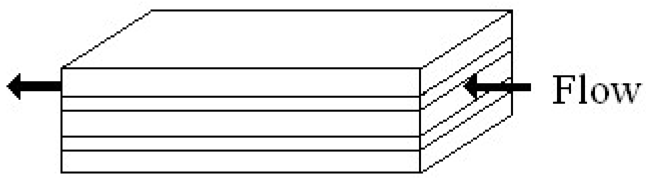

Figure 3: Flow parallell to the layers

Suppose the flow is parallel to the layers (no cross flow), then:

$$\bar{k} = \frac{\sum_{i=1}^{N} k_i h_i}{\sum h_i} = \sum k_i P_i = E(k)$$

P_i denotes the probability of k_i.

In the case of a uniform sample interval, $h_i = \Delta h$, the above equation will simplify to:

$$\bar{k} = \frac{1}{N}\sum_{i=1}^{N} k_i$$

Observe that the largest permeability values have more influence on the arithmetic average than the smaller one.

14.6 Harmonic average

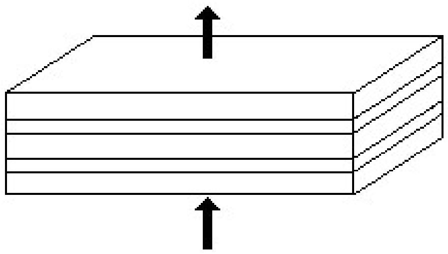

Figure 4: Flow perpendicular to the layers

Suppose the flow is perpendicular to the layers, then:

$$\bar{k}^{-1} = \frac{\sum k_i^{-1} h_i}{\sum h_i} = \sum k^{-1} P_i = E\left(k^{-1}\right)$$

which is the harmonic average. Note that P_i denotes the probability k_i^{-1}.

In case of a uniform sampling interval, the above formula will simplify to:

$$\overline{k}_H^{-1} = \frac{1}{N}\sum_{i=1}^{N} k^{-1} = E\left(k^{-1}\right)$$

or

$$\overline{k}_H = N\left(\sum_{i=1}^{N} k^{-1}\right)^{-1}$$

Zero permeability cores have to be excluded.

14.7 Probability distribution function

The probability distribution function for the permeability is in many cases of log-normal type. This means that the logarithm of the permeability is normally distributed.

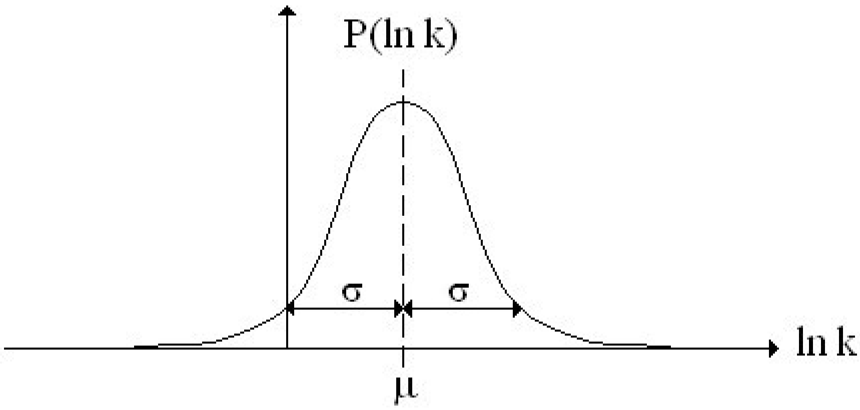

Figure 5: The log-normal distribution function

The equation of the log-normal (or log-Gaussian) distribution function is:

$$P\left(\ln k\right) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left[\frac{\ln k - \mu}{\sigma}\right]^2\right)$$

where μ is the expected value of ln k and
σ is the standard deviation of ln k.

14.8 Geometric average

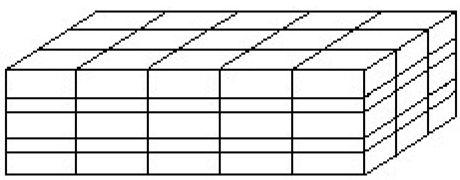

Figure 6: The permeability of the cells are randomly distributed

Matheron (1967) found that the geometric average is appropriate under the following conditions:

1. Small variation
2. The permeability has a log – normal probability distribution function.

Then,

$$\overline{k}_g = e^{\frac{1}{N}\sum_1^N \ln k_i} = e^{E(\ln k)}$$

or

$$\overline{k}_g = \sqrt[N]{k_1 \cdot k_2 \cdots \cdot k_N}$$

Note that tight cores have to be excluded since only one core of zero permeability will reduce the geometric average to zero.

A numerical study, by Warren and Piece suggests, that the geometric average is valid also under relaxed assumptions. Other analytical formulas are not known.

14.9 Powerlaw average

There is some evidence that the correct average is of powerlaw type, Desbarats 1987.

$$\overline{k}_p^p = \frac{1}{N}\sum_{i=1}^{N} k_i^p$$

The above expression is arithmetic average of k^p.

Solving for k_p yields:

$$\overline{k}_p = \left(\frac{1}{N}\sum_1^N k_i^p\right)^{\frac{1}{p}}$$

The geometrical average is used for p=0, i.e.:

$$\overline{k}_0 = e^{\frac{1}{N}\sum_{i=1}^{N} \ln k_i}$$

The correct value of the powerlaw exponent, p, is unclear. It is reasonable to assume that powerlaw exponent is bounded as follows:

$$-1 \leq p \leq 1$$

Note that the powerlaw average includes all the elementary averages as follows:

1. The harmonic average, p = -1
2. The geometric average, p = 0
3. The arithmetic average, p = 1
4. All possible powerlaw averages in between the averages listed above.

Observations:

1. All powerlaw averages are the same for a homogeneous reservoir of permeability k. Then, $\overline{k}_p = k$.
2. An increase in the degree of heterogeneity leads to a larger difference between the p-averages.
3. $\overline{k}_H \leq \overline{k}_G \leq \overline{k}_A$.

The arithmetic average will emphasize high values, while the hyperbolic average emphasizes low values.

14.10 Commingled reservoir

A layered reservoir without crossflow is called a commingled reservoir. Flow from one layer into another one has been called crossflow. This possibility is ruled out by the existence of impervious layers between the permeable ones. For such cases, one may use the geometric average within each layer and the estimate the total permeability by use of the arithmetic average of the layer permeabilities.

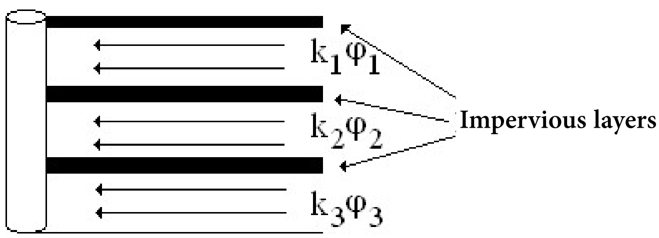

Figure 7: Commingled reservoir

15 Appendix B: A Note on Unit Systems

15.1 Introductory remarks

So far, all equations have been expressed in a consistent unit system. The SI unit system is consistent. A consistent unit system respects the dimensions of all variables such that all constants are dimensionless. For example, the constant 2π is dimensionless. Dimensional analysis shows that the dimension of permeability is L^2 (like area). The corresponding SI unit is m^2. Consistent SI units are listed in Table 1. The advantage is transparency. It is easy to check whether an equation is dimensional consistent or not.

No unit system can be flexible enough to suit all purposes. A consistent unit system may lead to key variables with inconvenient numerical values. For example, pressure may assume the value of 3000 psi or $2.1 \cdot 10^8$ Pa depending on the unit system. In the same unit systems, the permeability could assume either the value of 1 mD or $1 \cdot 10^{-16}$ m^2. The magnitudes of the first values are considered more "practical" than the latter. The reason is that numerical values fall within a range that most people are familiar with. A practical unit system is designed to give convenient numerical values. This may be achieved by use of dimensional constants in the equations.

Use of dimensionless variables is important for many purposes. We consider two cases, consistent SI units and American Field Units.

	Variable	Darcy	American	SI
Formation Factor	B	dimless	dimless	dimless
Compressibilty	c	1/atm	1/psi	1/Pa
Thickness	h	cm	ft	m
Permeability	k	D	mD	m^2
Pressure	p	atm	psi	Pa
Porosity	ϕ	dimless	dimless	dimless
Rate	q	cc/s	STB/Day	m^3/s
Radius	r	cm	ft	m
Time	t	s	h	s
Viscocity	µ	cP	cP	Pa s

Table 1: Darcy, SI and American Field Units

The traditional way of obtaining permeability is by core analysis. Hence, the unit for permeability (Darcy denoted by D) is designed for small scale laboratory experiments. The resulting numerical values are convenient for laboratory work. This unit system has been called Darcy units. It is a "mixed unit system" since pressure (atm) and viscosity (cP) depends on different unit systems. The Darcy unit system is inconvenient for oilfield calculations since the basic units are too small. A practical unit system, for field behavior, can be based on a metric system, American Field Units or a combination.

15.2 Consistent SI Units

To illustrate the advantage of a consistent unit system, let us check the dimensional consistency of the drawdown equation for radial flow:

$$p_i - p_{wf} = \frac{1.15 \cdot q\mu B}{2\pi kh}\left\{\log\frac{kt}{\varphi\mu c_t r_w^2} + 0.351 + 0.87S\right\}$$

For dimensional consistency (dimensionless constants only), the terms within bracket are required to be dimensionless. This is because the skin factor is dimensionless. The left hand side of the equation has the unit for pressure, which is Pa (Pascal). Hence, dimensional consistency dictates that the group of variables in front of the bracket also has unit Pa.

The above proposition may be tested by including the units in the equation. These are enclosed within square brackets.

$$(p_i - p_{wf})\left[Pa\right] = \frac{1.15 \cdot q\left[\frac{m^3}{s}\right]\mu\left[Pa \cdot s\right]B}{2\pi k\left[m^2\right]h\left[m\right]}\left\{\log \frac{k\left[m^2\right]t\left[s\right]}{\varphi\mu\left[Pa \cdot s\right]c_t\left[Pa^{-1}\right]r_w^2\left[m^2\right]} + 0.351 + 0.87S\right\}$$

By cancellation of common units, it is clear that the argument in the logarithmic expression is dimensionless. Again (by cancellation of units) the coefficient to the parenthesis has the unit Pa, which is the unit of pressure. The last requirement for dimensional consistency is that the left and right hand side of the equation have the same unit, Pa. It is clear that all requirements are satisfied. Thus, the dimensional consistency of the equation has been verified.

The argument of the logarithmic term is a dimensionless function of time. The group has been called dimensionless time.

$$t_D = \frac{kt}{\varphi\mu c_t r_w^2}$$

A dimensionless group for pressure may be defined in the same way. The ratio of the pressure drawdown**,** Δp_{wf}, to the pressure group on the right hand side of the drawdown equation is obviously dimensionless. Hence a dimensionless pressure, p_D-function, may be defined as:

$$p_D = \frac{kh}{q \quad B}\Delta p_{wf}$$

Substitution of the dimensionless variables into the original drawdown equation yields:

$$p_D = 1.15\left\{\log t_D + 0.351 + 0.87S\right\}$$

A dimensionless equation is independent of any unit system. The definition of dimensionless variables, however, may involve a constant that depends on the unit system.

Note that the number of variables in the dimensionless drawdown equation is dramatically reduced when compared against the corresponding dimensional one. This is because dimensionless variables are defined by grouping variables. Hence, dimensionless solutions are useful for graphs and tables.

15.3 American Field Units

We introduce the American Field Units in the dimensionless time. With each variable, associate a conversion factor to convert the practical unit back to the corresponding SI unit, which is characterized by dimensionless constants.

From any table of conversion factors, the following relations may be obtained:

$$1\left[psi\right] = 6.894757\cdot 10^{3}\left[Pa\right] \qquad 1\ \left[STB\right] = 1.589873\cdot 10^{-1}\left[m^{3}\right]$$

$$1\ \left[mD\right] = 9.8886923\cdot 10^{-16}\left[m^{2}\right] \qquad 1\ \left[ft\right] = 0.3048\ \left[m\right]$$

$$t_D = \frac{k\left[mD\right]\cdot 9.869\cdot 10^{-16}\left[\frac{m^2}{mD}\right]t\left[h\right]3600\left[\frac{s}{h}\right]}{\varphi\mu\left[cP\right]\cdot 10^{-3}\left[\frac{Pas}{cP}\right]c_t\left[\frac{1}{psi}\right]\cdot 1.450^{-4}\left[\frac{psi}{Pa}\right]r_w^2\left[ft^2\right]\cdot 0.305^2\left[\frac{m^2}{ft^2}\right]}$$

which simplifies to:

$$t_D = \frac{0.000264\cdot k\left[mD\right]t\left[h\right]}{\varphi\mu\left[cP\right]c_t\left[psi^{-1}\right]r_w^2\left[ft^2\right]}$$

It may seem like a paradox that dimensional time involves all the units listed above. The reason is that the constant, 0.000264, involves a complicated expression of units that makes the group dimensionless. The equation for dimensional time is, correctly (since it is dimensionless), presented without units:

$$p_{Dw} = \frac{2\pi k}{q\mu B}\Delta p_w$$

A dimensionless variable is always proportional to the real one. Hence:

$$p_D = \alpha \Delta p_{wf}$$

α has unit Pa^{-1}. The value of the constant may obtained by the same procedure:

$$\alpha = \frac{2\pi k\left[mD\right]\cdot 9.896\cdot 10^{-16}\left[\frac{m^2}{mD}\right]\cdot h\left[ft\right]\cdot 0.305\left[\frac{m}{ft}\right]}{q\left[\frac{STB}{Day}\right]\cdot 0.159\left[\frac{m^3}{STB}\right]\cdot\frac{1}{3600\cdot 24}\left[\frac{Day}{s}\right]\mu\left[cP\right]\cdot\left[\frac{Pas}{cP}\right]B}$$

$$\alpha = 1.018\cdot 10^{-6}\left[Pa^{-1}\right]$$

As the last step:

$$p_D = \frac{1.018\cdot 10^{-6}kh}{q\mu B}\left[Pa^{-1}\right]\cdot\Delta p_{wf}\left[psi\right]6.894\cdot 10^3\left[\frac{Pa}{psi}\right]$$

which yields:

$$p_D = \frac{0.0071kh}{q\mu B}\Delta p_{wf}$$

The dimensionless drawdown equation remains unchanged:

$$p_D = 1.15\left\{\log t_D + 0.351 + 0.87S\right\}$$

Substitution of the dimensionless variable into the above equation yields:

$$p_i - p_{wf} = \frac{162.6\cdot q\mu B}{kh}\left\{\log\frac{kt}{\varphi\mu c_t r_w^2} - 3.23 + 0.87S\right\}$$

15.4 Conclusion

The SI unit system is consistent. As such, it has the advantage of dimensional transparency and dimensionless constants. Key variables may assume inconvenient numerical values.

A practical system has the advantage of convenient numerical values. This objective may be obtained at the expense of reduced dimensional transparency.

Dimensionless equations remain unchanged regardless of unit system. This property has important consequences:

- The number of variables in a dimensionless equation is significantly reduced when compare against the corresponding dimensional one. Hence, dimensionless solutions are convenient for tables and graphs.
- Type curve matching can be performed with the same dimensionless graphs regardless of unit system. The translation factors, however, depend on the unit system.
- Dimensionless equations can easily be converted back to any unit system.
- The diffusivity equation is important in many disciplines. All share the same solutions. Notable non-petroleum examples are: heat flow and groundwater hydrology. Use of dimensionless equations may facilitate technology transfer between disciplines.