The Existence of Fixed Points for Exponential Functions in Taylor Bernstein Polynomial Form

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Abstract

In this thesis, we provide a method of proving the existence of fixed points for Exponential Functions over intervals. This method based on converting the exponential function to Taylor polynomials of finite degree. Subsequently, we expand Taylor to Bernstein polynomial of higher degree. The Bernstein basis is considered in this work over boxes. By computing the coefficients of Bernstein, the exponential function is contained in the minimum and maximum Bernstein coefficients. For this reason, we study important properties for Taylor and Bernstein polynomials. First, Taylor can be expanded to Bernstein form of finite degree and the error bound can be optimized by a maximum value. Second, Taylor can be optimized by the minimum and maximum Bernstein coefficients of higher degree. Last, if the minimum and maximum Bernstein coefficients are in interval, then the exponential has fixed points in the same domain. Finally, application of our results on positivity analysis of continuous functions is considered.

Keywords: Exponential Functions, Taylor's Polynomials, Bernstein polynomial, Control Design, Control Function in Lyapunov.