The Existence of Fixed Points for Trigonometric Functions in Lagrange-Bernstein Polynomial Form

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Abstract

In this thesis, we provide a method of proving the existence of fixed points for trigonometric Functions over intervals. This method based on converting the trigonometric function to Lagrange Interpolating polynomials of finite degree. Subsequently, we expand Lagrange to Bernstein polynomial of higher degree. The Bernstein basis is considered in this work over boxes. By computing the coefficients of Bernstein, the trigonometric function is contained in the minimum and maximum Bernstein coefficients. For this reason, we study important properties for Lagrange and Bernstein polynomials. First, Lagrange can be expanded to Bernstein form of finite degree and the error bound can be optimized by the maximum range. Second, Lagrange can be optimized by the minimum and maximum Bernstein coefficients of higher degree. Last, if the minimum and maximum Bernstein coefficients are contained in the given interval, then the trigonometric has fixed points in the same domain. Finally, applications of our results on positivity analysis of continuous functions are considered.

Keywords: Trigonometric Functions, Lagrange Interpolating Polynomials, Bernstein Polynomial, Fixed Point.