Optimizing Exponential Functions in the Simplicial Taylor-Bernstein Form with Applications on Fixed-points Approximation

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Abstract

In this thesis, we study exponential functions that are given over a simplex. Proving the existence of fixed points for exponential functions in one or high dimensions suffers from the complexity issues. This required finding the minimum and maximum values through finding the critical points. We convert these functions to polynomials of finite degree. This leads to a method of optimizing the exponential functions by global bounds over simplices. The proposed method of optimization is a method that is created by Bernstein. We optimize these functions by the bounds of Bernstein expansion in the simpliciad form. First, we convert the exponent function to Taylor polynomial a bout a given point. Subsequently, we convert this polynomial to Taylor-Bernstein form over the same domain. The Taylor polynomial is contained in the minimum maximum Bernstein control points. We provide a method of computing the Bernstein bound in fast manner and test the existence of fixed points in the given domain. With these results, we can show that these functions have fixed points if the computed Bernstein control points are in the domain. Finally, we provide properties of the approach in the Taylor form with counter examples and significant comparisons.

Keywords: Exponential functions, Bernstein , Taylor polynomials, Optimization.