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VERIFYING THE VALIDITY OF IMPLICATIONS THAT INVOLVE QUANTIFIERS USING THE SIMPLIFICATION AND LOGICAL INFERENCE METHODS

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Abstract. *A computer scientist aims to develop an algorithm that if followed will solve the related problem. Algorithms are finite processes that contain a step by step list of instructions in solving a problem. In the same context, discrete mathematics is the background behind many computer operations, and it is concerned with structures which take on a discrete value often infinite. Besides, it analyzes data whose values are separated (such as integers) and provide the needed understanding of important topics such as logical inferences and mathematical proof. Moreover, it shows that algorithms always produce the correct results. This paper proposes a new approach to prove the validity of implications involving quantifiers and logical operators. The new approach is based on simplification and logical inference methods which are used for the first time in solving such problems. Therefore, the proposed methods were used to verify the validity of different implications. The results for all previously known to be valid formulas using the simplification method were valid. And when using the logical inference method, the conclusions were true in all cases; accordingly, the arguments were valid.*
Keywords: Universal quantifier, Existential quantifier, Logical operators, Implication, Logical inference, Logical simplification

1. Introduction.

Discrete mathematics is one of the core components of the mathematics of modern computer science, in particular, combinatorics and graph theory. It gives the opportunity to explore and analyze real-world problems challenges and teaches mathematical reasoning along with essential proof techniques by applying some fundamental concepts in many different ways. The difference between discrete mathematics and other disciplines is the basic foundation on proof as its modus operandi for determining truth, whereas science, for example, relies on carefully analyzed experience. According to [1], a proof is any reasoned argument accepted as such by other mathematicians.

Moreover, mathematical discrete has important applications in computer science, for instance, to verify that computer programs produce the correct output for all possible input values and to show algorithms always produce the correct results [2,3]. Also, it deals with discrete objects which are separated from each other. Examples of discrete objects include integers and rational numbers.

Many researchers have expressed that logic is an essential topic in the field of computer science [4-6]. It is generally agreed that computer scientist should have a strong basis in discrete mathematics [7-11]. Mathematical logic is frequently utilized for logical proofs, where proofs are valid arguments that decide the truth values of mathematical statements [12], while the argument is a group of statements; all its first statements are called premises except the last statement called the conclusion. The symbol "$\therefore$" denotes "therefore", which is placed before the conclusion.

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A valid argument is one where the conclusion must follow from the truth of the preceding statements. Usually the truth table is used to show the validity of an argument; however, this method is considered to be a repetitive approach. Alternatively, the validity of some relatively simple argument forms can be established by using the rules of inference which give the guidelines for constructing valid arguments from the statements [13,14].

Some quantifiers are used to formulate complex mathematical statements, such as "There exists (written $\exists$)" and "For all (written $\forall$)". The universe of a variable is the collection of values it is allowed to take. The universal quantifier $\forall$ confirms that the given assertion is true for all allowed replacements for a variable. The existential quantifier $\exists$ confirms that the given assertion is true for at least one allowed replacements for a variable [15,16]. The validity of the previous implications was proved using the method of truth table [17]. However, this can be considered as time-consuming due to the time it takes. In [18], a method to verify the validity of implications that involve quantifiers distribution over logical operators was proposed. This method is based on the fact that whenever the premise of the implication is true, the conclusion of that implication cannot be false (must be true), which results in a valid implication.

This work aims to introduce a new technique to prove the validity of some implications, in order to overcome the limitations of the currently used methods. The simplification and logical inference methods will be used for the first time to assure the validity of five well-known implications that have been proved valid using fundamental methods. The rest of this paper is organized as follows: in the following section, the work methodology will be presented, the results of the research instruments will be discussed in Section three, while Section four will conclude the paper.

2. **Methodology.** This section proposes a new approach to prove the validity of some implications involving quantifiers and logical operators. In order to utilize the logical content in discrete mathematics with greater emphasis on problem solving and symbolic reasoning, essential equivalences (logical identities) and rules of inference should be included in each step of simplifying and logical inferencing. The proposed approach is based on using simplification and logical inference methods. In the beginning, the following valid implications will be proved using each of our suggested methods.

Implication 1. $\forall x P(x) \rightarrow \exists x P(x)$

Implication 2. $[\forall x P(x) \vee \forall x Q(x)] \rightarrow \forall x[P(x) \vee Q(x)]$

Implication 3. $\exists x[P(x) \wedge Q(x)] \rightarrow [\exists x P(x) \wedge \exists x Q(x)]$

Implication 4. $\forall x[P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)]$

Implication 5. $[\exists x P(x) \rightarrow \exists x Q(x)] \rightarrow \exists x[P(x) \rightarrow Q(x)]$

Proving the above implications begins with three important steps. The first step is to assume that the universe is the set of $\{0, 1\}$, the next step is to express each quantifier by equivalent logical operator as the following: $\forall x P(x)$ can be expressed by conjunction as $P(0) \wedge P(1)$ and $\exists x P(x)$ can be expressed by disjunction as $P(0) \vee P(1)$. Finally, let $P(0)$ be denoted by A, $P(1)$ be denoted by B, $Q(0)$ be denoted by C, and $Q(1)$ be denoted by D.

2.1. **Simplification method.** The validity of the previous implications will be verified using the simplification method which can be defined as the process of replacing a mathematical expression by an equivalent one, which is simpler and shorter. This method is somewhat long, but it is considered to be a novel technique.

Formula 1.

$$
\begin{aligned}
& \forall x P(x) \rightarrow \exists x P(x) \\
= {} & [P(0) \wedge P(1)] \rightarrow [P(0) \vee P(1)] \\
= {} & (A \wedge B) \rightarrow (A \vee B)
\end{aligned}
$$

$$
\begin{aligned}
&= (\neg A \vee \neg B) \vee (A \vee B) \\
&= (\neg A \vee A) \vee (\neg B \vee B) \\
&= 1 \vee 1 = 1 \text{ (that means it is a valid implication)}
\end{aligned}
$$

Formula 2.

$$
\begin{aligned}
&[\forall x P(x) \vee \forall x Q(x)] \to \forall x[P(x) \vee Q(x)] \\
&= \{[P(0) \wedge P(1)] \vee [Q(0) \wedge Q(1)]\} \to \{[P(0) \vee Q(0)] \wedge [P(1) \vee Q(1)]\} \\
&= [(A \wedge B) \vee (C \wedge D)] \to [(A \vee C) \wedge (B \vee D)] \\
&= [\neg(A \wedge B) \wedge \neg(C \wedge D)] \vee [(A \vee C) \wedge (B \vee D)] \\
&= [(\neg A \vee \neg B) \wedge (\neg C \vee \neg D)] \vee [(A \vee C) \wedge (B \vee D)] \\
&= \{[(\neg A \vee \neg B) \wedge (\neg C \vee \neg D)] \vee (A \vee C)\} \wedge \{[(\neg A \vee \neg B) \wedge (\neg C \vee \neg D)] \vee (B \vee D)\} \\
&= \{[(\neg A \vee \neg B) \vee (A \vee C)] \wedge [(\neg C \vee \neg D) \vee (A \vee C)]\} \wedge \{[(\neg A \vee \neg B) \vee (B \vee D)] \\
&\quad \wedge [(\neg C \vee \neg D) \vee (B \vee D)]\} \\
&= \{[(\neg A \vee A) \vee \neg B \vee C] \wedge [(\neg C \vee C) \vee \neg D \vee A]\} \wedge \{[\neg A \vee (\neg B \vee B) \vee D] \\
&\quad \wedge [(\neg C \vee (\neg D \vee D) \vee B)]\} \\
&= [1] \wedge [1] \wedge [1] \wedge [1] \\
&= 1 \text{ (that means it is a valid implication)}
\end{aligned}
$$

Formula 3.

$$
\begin{aligned}
&\exists x[P(x) \wedge Q(x)] \to [\exists x P(x) \wedge \exists x Q(x)] \\
&= \{[P(0) \wedge Q(0)] \vee [P(1) \wedge Q(1)]\} \to \{[P(0) \vee P(1)] \wedge [Q(0) \vee Q(1)]\} \\
&= [(A \wedge C) \vee (B \wedge D)] \to [(A \vee B) \wedge (C \vee D)] \\
&= [\neg(A \wedge C) \wedge \neg(B \wedge D)] \vee [(A \vee B) \wedge (C \vee D)] \\
&= [(\neg A \vee \neg C) \wedge (\neg B \vee \neg D)] \vee [(A \vee B) \wedge (C \vee D)] \\
&= \{[(\neg A \vee \neg C) \wedge (\neg B \vee \neg D)] \vee (A \vee B)\} \wedge \{[(\neg A \vee \neg C) \wedge (\neg B \vee \neg D)] \vee (C \vee D)\} \\
&= \{[(\neg A \vee \neg C) \vee (A \vee B)] \wedge [(\neg B \vee \neg D) \vee (A \vee B)]\} \wedge \{[(\neg A \vee \neg C) \vee (C \vee D)] \\
&\quad \wedge [(\neg B \vee \neg D) \vee (C \vee D)]\} \\
&= [(\neg A \vee A) \vee \neg C \vee B] \wedge [(\neg B \vee B) \vee \neg D \vee A] \wedge [\neg A \vee (\neg C \vee C) \vee D] \\
&\quad \wedge [\neg B \vee (\neg D \vee D) \vee C] \\
&= [1] \wedge [1] \wedge [1] \wedge [1] \\
&= 1 \text{ (that means it is valid implication)}
\end{aligned}
$$

Formula 4.

$$
\begin{aligned}
&\forall x[P(x) \to Q(x)] \to [\forall x P(x) \to \forall x Q(x)] \\
&= \forall x[\neg P(x) \vee Q(x)] \to [\neg(\forall x P(x)) \vee \forall x Q(x)] \\
&= \forall x[\neg P(x) \vee Q(x)] \to [\exists x \neg P(x) \vee \forall x Q(x)] \\
&= \{[\neg P(0) \vee Q(0)] \wedge [\neg P(1) \vee Q(1)]\} \to \{[\neg P(0) \vee \neg P(1)] \vee [Q(0) \wedge Q(1)]\} \\
&= \{[P(0) \wedge \neg Q(0)] \vee [P(1) \wedge \neg Q(1)]\} \vee \{[\neg P(0) \vee \neg P(1)] \vee [Q(0) \wedge Q(1)]\} \\
&= [(A \wedge \neg C) \vee (B \wedge \neg D)] \vee [(\neg A \vee \neg B) \vee (C \wedge D)] \\
&= \{[(A \vee \neg A) \wedge (\neg C \vee \neg A)] \vee [(B \vee \neg B) \wedge (\neg D \vee \neg B)]\} \vee (C \wedge D) \\
&= \{[1 \wedge (\neg C \vee \neg A)] \vee [1 \wedge (\neg D \vee \neg B)]\} \vee (C \wedge D) \\
&= [\neg C \vee \neg A \vee \neg B \vee \neg D] \vee (C \wedge D) \\
&= [\neg C \vee \neg A \vee \neg B] \vee [(\neg D \vee D) \wedge (\neg D \vee C)] \\
&= [\neg C \vee \neg A \vee \neg B] \vee [1 \wedge (\neg D \vee C)]
\end{aligned}
$$

$$
\begin{aligned}
&= [\neg C \vee \neg A \vee \neg B \vee \neg D \vee C] \\
&= [(\neg C \vee C) \vee \neg B \vee \neg D] \\
&= [1 \vee \neg B \vee \neg D] \\
&= 1 \text{ (that means it is a valid implication)}
\end{aligned}
$$

Formula 5.

$$
\begin{aligned}
&\quad [\exists x P(x) \to \exists x Q(x)] \to \exists x[P(x) \to Q(x)] \\
&= \{[P(0) \vee P(1)] \to [Q(0) \vee Q(1)]\} \to \{[P(0) \to Q(0)] \vee [P(1) \to Q(1)]\} \\
&= [(A \vee B) \to (C \vee D)] \to [(A \to C) \vee (B \to D)] \\
&= [(\neg A \wedge \neg B) \vee (C \vee D)] \to [(\neg A \vee C) \vee (\neg B \vee D)] \\
&= [(A \vee B) \wedge (\neg C \wedge \neg D)] \vee [(\neg A \vee C) \vee (\neg B \vee D)] \\
&= [(A \vee B) \vee (\neg A \vee C)] \wedge [(\neg C \wedge \neg D) \vee (\neg A \vee C)] \vee (\neg B \vee D) \\
&= [(A \vee \neg A \vee B \vee C)] \wedge \{[\neg C \vee (\neg A \vee C)] \wedge [\neg D \vee (\neg A \vee C)]\} \vee (\neg B \vee D) \\
&= [1] \wedge \{1 \wedge [\neg D \vee (\neg A \vee C)]\} \vee (\neg B \vee D) \\
&= [\neg D \vee (\neg A \vee C)] \vee (\neg B \vee D) \\
&= [\neg D \vee D \vee \neg A \vee C \vee \neg B] \\
&= 1 \text{ (that means it is a valid implication)}
\end{aligned}
$$

2.2. **Logical inference method.** In this part the validity of the previous implications will be verified using the logical inference method. This method is considered to be short, easy, and efficient. Rules of Inference aim to provide the guidelines for constructing valid arguments from the statements. As mentioned in Section 1, an argument is a sequence of statements. The last statement is the conclusion and all its preceding statements are called premises (or hypothesis). The symbol "$\therefore$" (read therefore) is placed before the conclusion. An argument is said to be valid if whenever all premises are true, the conclusion is true.

Formula 1.

$$
\begin{aligned}
&\quad \forall x P(x) \to \exists x P(x) \\
&= [P(0) \wedge P(1)] \to [P(0) \vee P(1)] \\
&= (A \wedge B) \to (A \vee B)
\end{aligned}
$$

In the form of an argument, we can write this formula as the following:

$$
\frac{(A \wedge B)}{\therefore (A \vee B)}
$$

The premise statement $(A \wedge B)$ has only one case which makes it true as follows.
The case is: A is true and B is true; therefore, the conclusion $(A \vee B)$ is true.
The conclusion is true in this case (unique). Therefore, the argument is valid.

Formula 2.

$$
\begin{aligned}
&\quad [\forall x P(x) \vee \forall x Q(x)] \to \forall x[P(x) \vee Q(x)] \\
&= \{[P(0) \wedge P(1)] \vee [Q(0) \wedge Q(1)]\} \to \{[P(0) \vee Q(0)] \wedge [P(1) \vee Q(1)]\} \\
&= [(A \wedge B) \vee (C \wedge D)] \to [(A \vee C) \wedge (B \vee D)]
\end{aligned}
$$

In the form of an argument, we can write this formula as follows:

$$
\frac{(A \wedge B) \vee (C \wedge D)}{\therefore (A \vee C) \wedge (B \vee D)}
$$

In the above argument the premise statement $(A \wedge B) \vee (C \wedge D)$ has three cases which make it true as follows.

- First case: $(A \wedge B)$ is true, then A is true and B is true, and therefore, the conclusion $(A \vee C) \wedge (B \vee D)$ is true because $(A \vee C)$ is true (A is true) and $(B \vee D)$ is true (B is true).
- Second case: $(C \wedge D)$ is true, then C is true and D is true, and therefore, the conclusion $(A \vee C) \wedge (B \vee D)$ is true because $(A \vee C)$ is true (C is true) and $(B \vee D)$ is true (D is true).
- Third case: $(A \wedge B)$ is true and $(C \wedge D)$ is true, then A, B, C and D are all true, and therefore, the conclusion $(A \vee C) \wedge (B \vee D)$ is true.

The conclusion is true in the three cases. Therefore, the argument is valid.

Formula 3.

$$\begin{aligned}
&\exists x[P(x) \wedge Q(x)] \rightarrow [\exists x P(x) \wedge \exists x Q(x)] \\
=& \{[P(0) \wedge Q(0)] \vee [P(1) \wedge Q(1)]\} \rightarrow \{[P(0) \vee P(1)] \wedge [Q(0) \vee Q(1)]\} \\
=& [(A \wedge C) \vee (B \wedge D)] \rightarrow [(A \vee B) \wedge (C \vee D)]
\end{aligned}$$

In the form of an argument, this formula can be written as follows:

$$\frac{(A \wedge C) \vee (B \wedge D)}{\therefore (A \vee B) \wedge (C \vee D)}$$

The premise statement $(A \wedge C) \vee (B \wedge D)$ has three cases which make it true as follows.

- First case: $(A \wedge C)$ is true, then A is true and C is true, and therefore, the conclusion $(A \vee B) \wedge (C \vee D)$ is true because $(A \vee B)$ is true (A is true) and $(C \vee D)$ is true (C is true).
- Second case: $(B \wedge D)$ is true, then B is true and D is true, and therefore, the conclusion $(A \vee B) \wedge (C \vee D)$ is true because $(A \vee B)$ is true (B is true) and $(C \vee D)$ is true (D is true).
- Third case: $(A \wedge C)$ is true and $(B \wedge D)$ is true, then A, B, C and D are all true, and therefore, the conclusion $(A \vee B) \wedge (C \vee D)$ is true.

The conclusion is true in the three cases. Therefore, the argument is valid.

Formula 4.

$$\begin{aligned}
&\forall x[P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)] \\
=& \forall x[\neg P(x) \vee Q(x)] \rightarrow [\neg \forall x P(x) \vee \forall x Q(x)] \\
=& \forall x[\neg P(x) \vee Q(x)] \rightarrow [\exists x \neg P(x) \vee \forall x Q(x)] \\
=& \{[\neg P(0) \vee Q(0)] \wedge [\neg P(1) \vee Q(1)]\} \rightarrow \{[\neg P(0) \vee \neg P(1)] \vee [Q(0) \wedge Q(1)]\} \\
=& [(\neg A \vee C) \wedge (\neg B \vee D)] \rightarrow [(\neg A \vee \neg B) \vee (C \wedge D)]
\end{aligned}$$

In the form of an argument, we can write this formula as the following:

$$\frac{(\neg A \vee C) \wedge (\neg B \vee D)}{\therefore (\neg A \vee \neg B) \vee (C \wedge D)}$$

The premise statement $(\neg A \vee C) \wedge (\neg B \vee D)$ has four cases which make it true as follows.

- First case: $(\neg A)$ is true and $(\neg B)$ is true, and therefore, the conclusion $(\neg A \vee \neg B) \vee (C \wedge D)$ is true.
- Second case: $(\neg A)$ is true and (D) is true, and therefore, the conclusion $(\neg A \vee \neg B) \vee (C \wedge D)$ is true.
- Third case: (C) is true and $(\neg B)$ is true, and therefore, the conclusion $(\neg A \vee \neg B) \vee (C \wedge D)$ is true.
- Fourth case: (C) is true and (D) is true, and therefore, the conclusion $(\neg A \vee \neg B) \vee (C \wedge D)$ is true.

The conclusion is true in the four cases. Therefore, the argument is valid.

Formula 5.

$$\begin{aligned}
&[\exists x P(x) \to \exists x Q(x)] \to \exists x[P(x) \to Q(x)]\\
&= \{[P(0) \vee P(1)] \to [Q(0) \vee Q(1)]\} \to \{[P(0) \to Q(0)] \vee [P(1) \to Q(1)]\}\\
&= [(A \vee B) \to (C \vee D)] \to [(A \to C) \vee (B \to D)]\\
&= [(\neg A \wedge \neg B) \vee (C \vee D)] \to [(\neg A \vee C) \vee (\neg B \vee D)]
\end{aligned}$$

In the form of an argument, we can write this formula as the following:

$$\frac{(\neg A \wedge \neg B) \vee (C \vee D)}{\therefore (\neg A \vee C) \vee (\neg B \vee D)}$$

The premise statement $(\neg A \wedge \neg B) \vee (C \vee D)$ has three cases which make it true as follows.

- First case: $(\neg A \wedge \neg B)$ is true, then $(\neg A)$ is true and $(\neg B)$ is true, and therefore, the conclusion $(\neg A \vee C) \vee (\neg B \vee D)$ is true.
- Second case: (C) is true, and therefore, the conclusion $(\neg A \vee C) \vee (\neg B \vee D)$ is true.
- Third case: (D) is true, and therefore, the conclusion $(\neg A \vee C) \vee (\neg B \vee D)$ is true.

The conclusion is true in the three cases. Therefore, the argument is valid.

3. **Discussion.** In the previous section a new approach that relies on simplification and logical inference methods was used to test the validity of several implications, and the results for all formulas using the simplification method are "1" which means all implications are valid. And when using the logical inference method, the conclusions are true in all cases; therefore, the argument is valid. In this section some examples are used to verify the validity of implications using the proposed approach and it will be proved using the both discussed methods.

Example 3.1. *Prove or disprove the validity of the following implication:*

$$[\forall x P(x) \vee \forall x Q(x)] \to [\exists x P(x) \vee \exists x Q(x)]$$

Proving the implication using the simplification method:

$$\begin{aligned}
&[\forall x P(x) \vee \forall x Q(x)] \to [\exists x P(x) \vee \exists x Q(x)]\\
&= \{[P(0) \wedge P(1)] \vee [Q(0) \wedge Q(1)]\} \to \{[P(0) \vee P(1)] \vee [Q(0) \vee Q(1)]\}\\
&= [(A \wedge B) \vee (C \wedge D)] \to [(A \vee B) \vee (C \vee D)]\\
&= [\neg(A \wedge B) \wedge \neg(C \wedge D)] \vee [(A \vee B) \vee (C \vee D)]\\
&= [(\neg A \vee \neg B) \wedge (\neg C \vee \neg D)] \vee [(A \vee B) \vee (C \vee D)]\\
&= \{[(\neg A \vee \neg B) \vee (A \vee B)] \wedge [(\neg C \vee \neg D) \vee (A \vee B)]\} \vee (C \vee D)\\
&= \{[1] \wedge [(\neg C \vee \neg D) \vee (A \vee B)]\} \vee (C \vee D)\\
&= [(\neg C \vee \neg D) \vee (A \vee B)] \vee (C \vee D)\\
&= (\neg C \vee \neg D) \vee (C \vee D) \vee (A \vee B)\\
&= (\neg C \vee C) \vee (\neg D \vee D) \vee (A \vee B)\\
&= 1 \vee 1 \vee (A \vee B) = 1 \ (\textit{that means it is valid implication})
\end{aligned}$$

Proving the implication using the logical inference:

$$\begin{aligned}
&[\forall x P(x) \vee \forall x Q(x)] \to [\exists x P(x) \vee \exists x Q(x)]\\
&= \{[P(0) \wedge P(1)] \vee [Q(0) \wedge Q(1)]\} \to \{[P(0) \vee P(1)] \vee [Q(0) \vee Q(1)]\}\\
&= [(A \wedge B) \vee (C \wedge D)] \to [(A \vee B) \vee (C \vee D)]
\end{aligned}$$

$$\frac{(A \wedge B) \vee (C \wedge D)}{\therefore (A \vee B) \vee (C \vee D)}$$

The premise statement $(A \wedge B) \vee (C \wedge D)$ has three cases which make it true as follows.

- *Case 1:* $(A \wedge B)$ *is true, then* A *is true and* B *is true, and therefore, the conclusion* $(A \vee B) \vee (C \vee D)$ *is true.*
- *Case 2:* $(C \wedge D)$ *is true, then* C *is true and* D *is true, and therefore, the conclusion* $(A \vee B) \vee (C \vee D)$ *is true.*
- *Case 3:* $(A \wedge B)$ *is true and* $(C \wedge D)$ *is true, and therefore, the conclusion* $(A \vee B) \vee (C \vee D)$ *is true.*

The conclusion is true in the three cases. Therefore, the argument is valid.

Example 3.2. *Prove or disprove the validity of the following implication:*

$$\exists x P(x) \rightarrow \forall x P(x)$$

Proving the implication using the simplification method:

$$\begin{aligned} & \exists x P(x) \rightarrow \forall x P(x) \\ = & [P(0) \vee P(1)] \rightarrow [P(0) \wedge P(1)] \\ = & (A \vee B) \rightarrow (A \wedge B) \\ = & (\neg A \wedge \neg B) \vee (A \wedge B) \\ = & A \leftrightarrow B \end{aligned}$$

It is clear that it is not equivalent to 1. Therefore, it is not valid.
Proving the implication using the logical inference:

$$\begin{aligned} & \exists x P(x) \rightarrow \forall x P(x) \\ = & [P(0) \vee P(1)] \rightarrow [P(0) \wedge P(1)] \\ = & (A \vee B) \rightarrow (A \wedge B) \end{aligned}$$

$$\frac{(A \vee B)}{\therefore (A \wedge B)}$$

The premise statement $(A \vee B)$ *has three cases which make it true as follows.*

- *Case 1:* A *is true and* B *is false, and as a result the conclusion is false.*
- *Case 2:* B *is true and* A *is false, and as a result the conclusion is false.*
- *Case 3:* A *is true and* B *is true, and as a result the conclusion is true in this case.*

So, not in all cases the conclusion is true, which means the implication (argument) is not valid.

4. **Conclusions.** This research discussed the importance of discrete mathematics in computer science and its significance as a skill for the aspiring computer scientist. Moreover, this work proposed a new approach to verify the validity of implications that involves quantifiers and logical operators that can be used instead of currently known methods. The new approach is based on simplification and logical inference methods which were used for the first time in solving such problems. Accordingly, the proposed methods were used to verify the validity of the following five implications: $\forall x P(x) \rightarrow \exists x P(x)$; $[\forall x P(x) \vee \forall x Q(x)] \rightarrow \forall x[P(x) \vee Q(x)]$; $\exists x[P(x) \wedge Q(x)] \rightarrow [\exists x P(x) \wedge \exists x Q(x)]$; $\forall x[P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)]$; $[\exists x P(x) \rightarrow \exists x Q(x)] \rightarrow \exists x[P(x) \rightarrow Q(x)]$. The results for all formulas using the simplification method were valid. And when using the logical inference method, the conclusions were true in all cases; therefore, the arguments are valid. Furthermore, two problems were verified using the proposed approach. The tested problems were: $[\forall x P(x) \vee \forall x Q(x)] \rightarrow [\exists x P(x) \vee \exists x Q(x)]$ and $\exists x P(x) \rightarrow \forall x P(x)$. The verifications of validity results for those problems were valid and not valid, correspondingly. In future work, more research is needed to apply and test other methods that can be used to prove relations between quantifiers and logical operators.

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