

CHAPTER 13

Right Triangles and Trigonometry

What You'll Learn

Key Ideas

- Multiply, divide, and simplify radical expressions. (*Lesson 13-1*)
- Use the properties of 45° - 45° - 90° and 30° - 60° - 90° triangles. (*Lessons 13-2 and 13-3*)
- Use the tangent, sine, and cosine ratios to solve problems. (*Lessons 13-4 and 13-5*)

Key Vocabulary

cosine (p. 572)

sine (p. 572)

tangent (p. 564)

radical expression (p. 549)

trigonometry (p. 564)

Why It's Important

Meteorology Meteorologists study air pressure, temperature, humidity, and wind velocity data. They then apply physical and mathematical relationships to make short-range and long-range weather forecasts. The data come from weather satellites, radars, sensors, and weather stations all over the world.

Trigonometry is used in fields like aviation, architecture, and other sciences. You will use trigonometry to determine the height of a cloud ceiling in Lesson 13-4.



Study these lessons to improve your skills.

Check Your Readiness

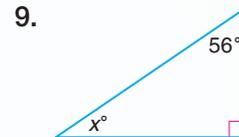
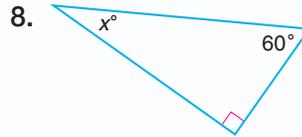
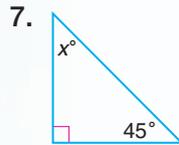
Lesson 6–6, pp. 256–261

If c is the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.

- $a = 8, b = 14, c = ?$
- $a = ?, b = 18, c = 26$
- $a = 18, b = 18, c = ?$
- $a = 6, b = ?, c = 12$
- $a = ?, b = 22, c = 28$
- $a = 31, b = 33, c = ?$

Lesson 5–2, pp. 193–197

Find the value of each variable.



Algebra Review, p. 722

Solve each equation. Check your solution.

- $x = \frac{18}{24}$
- $y = \frac{36}{24}$
- $n = \frac{36}{8}$
- $0.5 = \frac{x}{5}$
- $0.75 = \frac{y}{24}$
- $0.62 = \frac{n}{12}$
- $0.25 = \frac{x}{5}$
- $0.44 = \frac{y}{28}$
- $0.16 = \frac{n}{44}$

FOLDABLES™

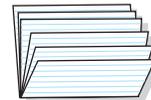
Study Organizer

Make this Foldable to help you organize your Chapter 13 notes. Begin with three sheets of notebook paper.

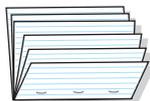
- 1 Stack the sheets of paper with edges $\frac{1}{4}$ inch apart.



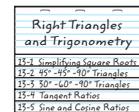
- 2 Fold up the bottom edges. All the tabs should be the same size.



- 3 Crease and staple along the fold.



- 4 Turn and label the tabs with the lesson titles.



Reading and Writing As you read and study the chapter, use each page to write main ideas, theorems, and examples for each lesson.



13-1

Simplifying Square Roots

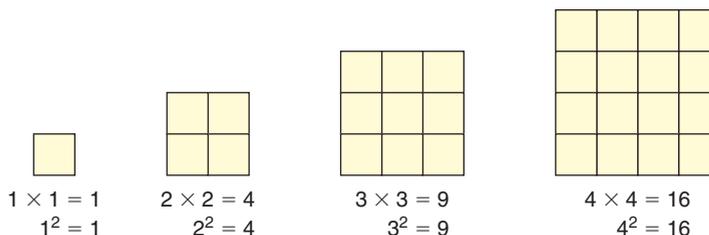
What You'll Learn

You'll learn to multiply, divide, and simplify radical expressions.

Why It's Important

Aviation Pilots use a formula with a radical expression to determine the distance to the horizon. See Example 9.

Squaring a number means multiplying a number by itself. The area of a square is found by squaring the measure of a side.



The numbers 1, 4, 9, and 16 are called **perfect squares** because $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, and $16 = 4^2$. Note that perfect squares are products of two equal factors.

The inverse (opposite) of squaring is finding a **square root**. To find a square root of 16, find two equal factors whose product is 16. The symbol $\sqrt{\quad}$, called a **radical sign**, is used to indicate the positive square root.

$$\sqrt{16} = 4 \text{ because } 4^2 = 16.$$

Read the symbol $\sqrt{16}$ as the square root of 16.

Examples

Simplify each expression.

1

$$\sqrt{49}$$

$$\sqrt{49} = 7 \text{ because } 7^2 = 49.$$

2 $\sqrt{64}$

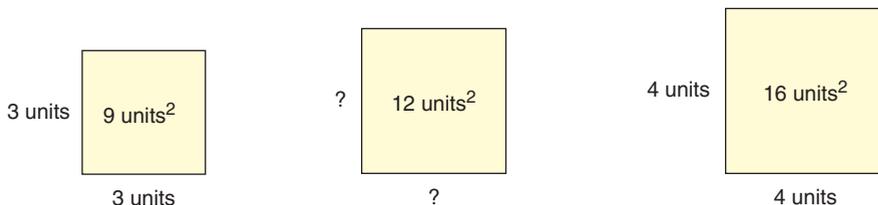
$$\sqrt{64} = 8 \text{ because } 8^2 = 64.$$

Your Turn

a. $\sqrt{25}$

b. $\sqrt{144}$

There are many squares that have area measures that are *not* perfect squares. For example, the center square has an area of 12 square units.



Look Back

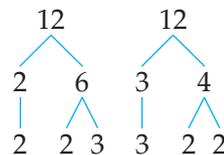
Irrational Number:
Lesson 2-1

However, there are no whole number values for $\sqrt{12}$. It is an irrational number. You can use a calculator to find an approximate value for $\sqrt{12}$.

$$\boxed{2nd} \boxed{[\sqrt{\quad}]} \boxed{12} \boxed{ENTER} \text{ 3.464101615}$$

A **radical expression** is an expression that contains a square root. The number under the radical sign is called a **radicand**. To simplify a radical expression, make sure that the radicand has no perfect square factors other than 1.

To simplify $\sqrt{12}$, use prime factorization to find a perfect square that is a factor of 12. The tree diagram shows two ways to find the prime factorization. The prime factorization of 12 is $2 \times 2 \times 3$.



To complete the simplification of $\sqrt{12}$, use the following property.

Product Property of Square Roots	Words: The square root of a product is equal to the product of each square root.
	Symbols: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ $a \geq 0, b \geq 0$
	Numbers: $\sqrt{6} = \sqrt{2} \cdot \sqrt{3}$

Example

3 Simplify $\sqrt{12}$.



Use a calculator to find the value of $2\sqrt{3}$. Compare it to the value of $\sqrt{12}$.

$$\begin{aligned} \sqrt{12} &= \sqrt{2 \cdot 2 \cdot 3} && \text{The prime factorization of 12 is } 2 \times 2 \times 3. \\ &= \sqrt{2 \cdot 2} \cdot \sqrt{3} && \text{Use the Product Property of Square Roots} \\ &= 2 \cdot \sqrt{3} && \text{to group any factors that occur in pairs.} \\ &= 2\sqrt{3} && \sqrt{2 \cdot 2} = \sqrt{4} \text{ or } 2 \\ &&& 2\sqrt{3} \text{ means 2 times } \sqrt{3}. \end{aligned}$$

Your Turn

- c. $\sqrt{8}$ d. $\sqrt{75}$ e. $\sqrt{20}$

You can also use the Product Property to multiply square roots.

Example

4 Simplify $\sqrt{3} \cdot \sqrt{6}$.

$$\begin{aligned} \sqrt{3} \cdot \sqrt{6} &= \sqrt{3 \cdot 6} && \text{Product Property of Square Roots} \\ &= \sqrt{3 \cdot 3 \cdot 2} && \text{Replace 6 with } 3 \cdot 2. \\ &= \sqrt{3 \cdot 3} \cdot \sqrt{2} && \text{Product Property of Square Roots} \\ &= 3 \cdot \sqrt{2} \text{ or } 3\sqrt{2} && \sqrt{3 \cdot 3} = 3 \end{aligned}$$

Your Turn

- f. $\sqrt{5} \cdot \sqrt{10}$ g. $\sqrt{3} \cdot \sqrt{15}$ h. $\sqrt{3} \cdot \sqrt{7}$



You can divide square roots and simplify radical expressions that involve fractions by using the following property.

Quotient Property of Square Roots

Words: The square root of a quotient is equal to the quotient of each square root.

Symbols: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $a \geq 0, b > 0$

Numbers: $\sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}}$

Examples

Simplify each expression.

5

$$\frac{\sqrt{16}}{\sqrt{8}}$$

$$\begin{aligned} \frac{\sqrt{16}}{\sqrt{8}} &= \sqrt{\frac{16}{8}} && \text{Quotient Property} \\ &= \sqrt{2} && \text{Simplify.} \end{aligned}$$

6

$$\sqrt{\frac{9}{4}}$$

$$\begin{aligned} \sqrt{\frac{9}{4}} &= \frac{\sqrt{9}}{\sqrt{4}} && \text{Quotient Property} \\ &= \frac{3}{2} && \text{Simplify.} \end{aligned}$$

Your Turn

i. $\frac{\sqrt{81}}{\sqrt{100}}$

j. $\sqrt{\frac{49}{64}}$



The process of simplifying a fraction with a radical in the denominator is called *rationalizing the denominator*.

To simplify a radical expression with fractions, make sure that it does not have a radical in the denominator. Remember that a fraction can be changed to an equivalent fraction by multiplying it by another fraction that is equivalent to 1. To simplify $\frac{\sqrt{3}}{\sqrt{5}}$, multiply it by $\frac{\sqrt{5}}{\sqrt{5}}$ because $\sqrt{5} \cdot \sqrt{5} = \sqrt{25}$ or 5, and the new denominator will not have a radical sign.

Example

7

Simplify $\frac{\sqrt{3}}{\sqrt{5}}$.

$$\frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \quad \frac{\sqrt{5}}{\sqrt{5}} = 1$$

$$= \frac{\sqrt{3 \cdot 5}}{\sqrt{5 \cdot 5}} \quad \text{Product Property of Square Roots}$$

$$= \frac{\sqrt{15}}{5} \quad \sqrt{5 \cdot 5} = 5$$



Example**8** Simplify $\frac{2}{\sqrt{3}}$.

$$\begin{aligned} \frac{2}{\sqrt{3}} &= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} && \frac{\sqrt{3}}{\sqrt{3}} = 1 \\ &= \frac{2 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} && \text{Product Property of Square Roots} \\ &= \frac{2\sqrt{3}}{\sqrt{9}} && \sqrt{3} \cdot \sqrt{3} = \sqrt{9} \\ &= \frac{2\sqrt{3}}{3} && 2\sqrt{3} \text{ is in simplest form, and } \sqrt{9} = 3. \end{aligned}$$

Your Turn

k. $\frac{\sqrt{7}}{\sqrt{2}}$

l. $\frac{4}{\sqrt{3}}$

A radical expression is said to be in **simplest form** when the following conditions are met.

Rules for Simplifying Radical Expressions

1. There are no perfect square factors other than 1 in the radicand.
2. The radicand is not a fraction.
3. The denominator does not contain a radical expression.

When working with square roots of numbers that are not perfect squares, the radical form is used to show an exact number. However, decimal approximations are often used when applying radical expressions in real-life situations.

**Example****Aviation Link**

9 Pilots use the formula $d = 1.5\sqrt{h}$ to find the distance in miles that an observer can see under ideal conditions. In the formula, d is the distance in miles, and h is the height in feet of the plane. If an observer on a plane is flying at a height of 2000 feet, how far can he or she see? Round to the nearest mile.



$$\begin{aligned} d &= 1.5\sqrt{h} \\ &= 1.5 \cdot \sqrt{2000} && \text{Replace } h \text{ with } 2000. \\ 1.5 & \text{ [2nd] } [\sqrt{\quad}] 2000 \text{ [ENTER]} && 67.08203932 \end{aligned}$$

To the nearest mile, the observer can see a distance of about 67 miles.



Check for Understanding

Communicating Mathematics

- Write the symbol for the square root of 100.
- Find the next three perfect squares after 16.
- You Decide?** Talisa says that $\sqrt{30}$ is in simplest form. Robbie says $\sqrt{30}$ is *not* in simplest form. Who is correct? Explain.

Vocabulary

perfect square
square root
radical sign
radical expression
radicand
simplest form

Guided Practice



Getting Ready Simplify each expression.

Sample: $\sqrt{5} \cdot \sqrt{5}$

Solution: 5

4. $\sqrt{7} \cdot \sqrt{7}$

5. $\sqrt{2} \cdot \sqrt{2}$

6. $\sqrt{11} \cdot \sqrt{11}$

Simplify each expression.

Examples 1 & 2

7. $\sqrt{36}$

8. $\sqrt{81}$

Examples 3 & 4

9. $\sqrt{27}$

10. $\sqrt{2} \cdot \sqrt{10}$

Examples 5 & 6

11. $\frac{\sqrt{45}}{\sqrt{15}}$

12. $\sqrt{\frac{25}{36}}$

Examples 7 & 8

13. $\frac{\sqrt{7}}{\sqrt{3}}$

14. $\frac{1}{\sqrt{5}}$

Example 9

15. **Buildings** The Empire State Building is about 1500 feet tall. From that height, how far can an observer see? Use the formula in Example 9 and round to the nearest mile.

Exercises

Practice

Simplify each expression.

16. $\sqrt{100}$

17. $\sqrt{121}$

18. $\sqrt{28}$

19. $\sqrt{32}$

20. $\sqrt{50}$

21. $\sqrt{48}$

22. $\sqrt{45}$

23. $\sqrt{200}$

24. $\sqrt{2} \cdot \sqrt{6}$

25. $\sqrt{5} \cdot \sqrt{15}$

26. $\sqrt{3} \cdot \sqrt{5}$

27. $\sqrt{8} \cdot \sqrt{9}$

28. $\frac{\sqrt{9}}{\sqrt{16}}$

29. $\frac{\sqrt{30}}{\sqrt{5}}$

30. $\frac{\sqrt{21}}{\sqrt{7}}$

31. $\sqrt{\frac{16}{81}}$

32. $\frac{\sqrt{16}}{\sqrt{4}}$

33. $\frac{\sqrt{3}}{\sqrt{2}}$

34. $\frac{\sqrt{2}}{\sqrt{3}}$

35. $\frac{4}{\sqrt{7}}$

36. $\sqrt{\frac{5}{10}}$

37. $\frac{1}{\sqrt{8}}$

38. What is the square root of 400?

39. Multiply $\sqrt{19}$ and $\sqrt{2}$.

40. Write $\sqrt{14} \cdot \sqrt{2}$ in simplest form.

Homework Help

For Exercises	See Examples
16–23, 38, 41, 42	1–3
24–27, 39, 40, 44	4
28–37	5–8
43	9

Extra Practice

See page 750.

Applications and Problem Solving

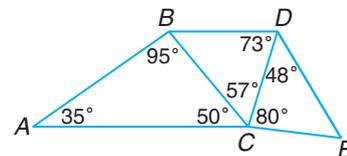
41. **Comics** Help the character from *Shoe* find the square root of 225.
42. **Measurement** The area of a square is 50 square meters. In simplest form, find the length of one of its sides.
43. **Fire Fighting** The velocity of water discharged from a nozzle is given by the formula $V = 12.14\sqrt{P}$, where V is the velocity in feet per second and P is the pressure at the nozzle in pounds per square inch. Find the velocity of water if the nozzle pressure is 64 pounds per square inch.
44. **Critical Thinking** You know that $\sqrt{5} \cdot \sqrt{5} = 5$ and $\sqrt{2} \cdot \sqrt{2} = 2$. Find the value of $\sqrt{n} \cdot \sqrt{n}$ if $n \geq 0$. Explain your reasoning.

Mixed Review

45. **Food** The world's largest cherry pie was made by the Oliver Rotary Club of Oliver, British Columbia, Canada. It measured 20 feet in diameter and was completed on July 14, 1990. Most pies are 8 inches in diameter. If the largest pie was similar to a standard pie, what is the ratio of the volume of the larger pie to the volume of the smaller pie? (*Lesson 12-7*)
46. What is the volume of a sphere with a radius of 7.3 yards? (*Lesson 12-6*)

Refer to the figure for Exercises 47–49. (*Lesson 7-3*)

47. Find the shortest segment in the figure. (The figure is not drawn to scale.)
48. Which segment is longer, \overline{AC} or \overline{CE} ? Explain.
49. Find $m\angle CBD$. Does this show that $\overline{DC} \cong \overline{AB}$?



If $\triangle PRQ \cong \triangle YXZ$, $m\angle P = 63$, $m\angle Q = 57$, $XY = 10$, and $YZ = 11$, find each measure. (*Lesson 5-4*)

50. $m\angle R$ 51. PQ

Standardized Test Practice

(A) (B) (C) (D)

52. **Multiple Choice** A basketball player made 9 out of 40 free throws last season. What percent of the free throws did she make? (*Percent Review*)
- (A) 0.225% (B) 4.4% (C) 22.5% (D) 44.4%



What You'll Learn

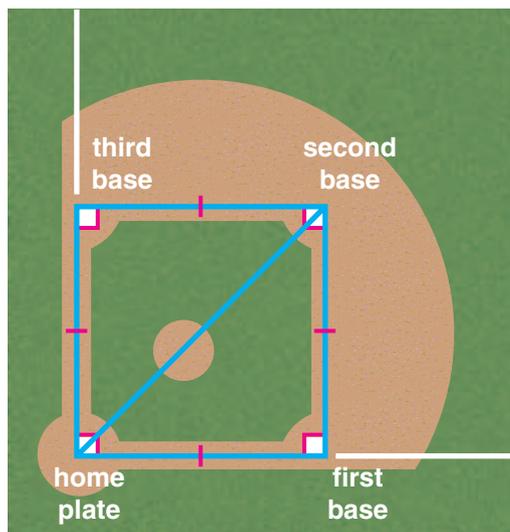
You'll learn to use the properties of 45°-45°-90° triangles.

Why It's Important**Machine Technology**

Machinists use 45°-45°-90° triangles when they cut square bolts from circular metal.

See Exercise 15.

If you're a baseball fan, you know that home plate, first base, second base, and third base form the baseball "diamond." But geometrically, a baseball diamond is actually a square.



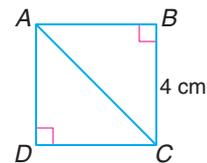
The line segment from home plate to second base is a diagonal of the square. The diagonal of a square separates the square into two 45°-45°-90° triangles.

Hands-On Geometry

Materials:  ruler  protractor

Step 1 Draw a square with sides 4 centimeters long. Label its vertices A , B , C , and D .

Step 2 Draw the diagonal \overline{AC} .

**Try These**

- Use a protractor to measure $\angle CAB$ and $\angle ACB$.
- Use the Pythagorean Theorem to find AC . Write your answer in simplest form.
- Repeat Exercise 2 for squares with sides 6 centimeters long and 8 centimeters long.
- Make a conjecture** about the length of the diagonal of a square with sides 7 inches long.

Look Back

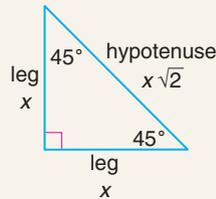
Pythagorean
Theorem:
Lesson 6-6

The results you discovered in the activity lead to Theorem 13–1.

Theorem 13–1
45°-45°-90°
Triangle
Theorem

Words: In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times the length of a leg.

Model:



A 45°-45°-90° triangle is also called an isosceles right triangle.

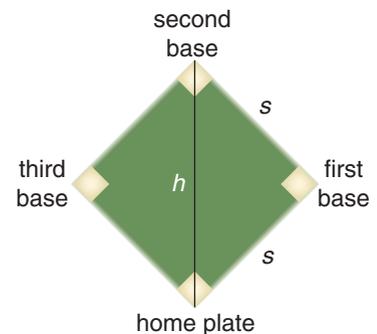
If you know the leg length of a 45°-45°-90° triangle, you can use the above theorem to find the hypotenuse length.



Example
Baseball Link

1

An official baseball diamond is a square with sides 90 feet long. How far is it from home plate to second base? Round your answer to the nearest tenth.



Explore The sides of a baseball diamond are 90 feet long. You need to find the distance from home plate to second base.

Plan The triangle formed by first base, second base, and home plate is a 45°-45°-90° triangle. The distance h from home plate to second base is the length of the hypotenuse of the triangle. Let s represent the length of the legs.

Solve $h = s\sqrt{2}$ *The hypotenuse is $\sqrt{2}$ times the length of a leg.*

$h = 90\sqrt{2}$ *Replace s with 90.*

90 2 127.27922

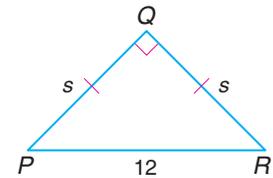
To the nearest tenth, the distance from home plate to second base is 127.3 feet.

Examine The value of $\sqrt{2}$ is about 1.5. So, the distance from home plate to second base is about $90 \cdot 1.5$ or 135. The answer seems reasonable.



Example

- 2 If $\triangle PQR$ is an isosceles right triangle and the measure of the hypotenuse is 12, find s . Write the answer in simplest form.



$$h = s\sqrt{2} \quad \text{The hypotenuse is } \sqrt{2} \text{ times the length of a leg.}$$

$$12 = s\sqrt{2} \quad \text{Replace } h \text{ with } 12.$$

$$\frac{12}{\sqrt{2}} = \frac{s\sqrt{2}}{\sqrt{2}} \quad \text{Divide each side by } \sqrt{2}.$$

$$\frac{12}{\sqrt{2}} = s \quad \text{Simplify.}$$

$$\frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = s \quad \text{Simplify the radical expression.}$$

$$\frac{12\sqrt{2}}{\sqrt{4}} = s \quad \text{Product property of square roots}$$

$$\frac{12\sqrt{2}}{2} = s \quad \text{Simplify.}$$

$$\text{Therefore, } s = \frac{12\sqrt{2}}{2} \text{ or } 6\sqrt{2}.$$

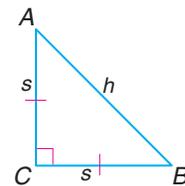
Your Turn

$\triangle ABC$ is an isosceles right triangle. Find s for each value of h .

a. 4

b. 5

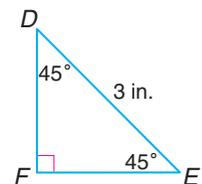
c. $3\sqrt{2}$



Check for Understanding

Communicating Mathematics

1. Draw and label a 45° - 45° - 90° triangle in which the sides are 3 centimeters, 3 centimeters, and about 4.2 centimeters.
2. Explain two different methods for finding the length of the hypotenuse of a 45° - 45° - 90° triangle.
3. **You?** **Decide** Jamie says that the length of a leg of $\triangle DEF$ is $3\sqrt{2}$ inches. Kyung says the length of a leg is $\frac{3\sqrt{2}}{2}$ inches. Who is correct? Explain your reasoning.

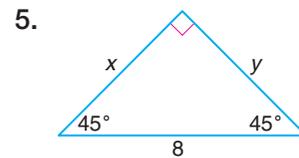
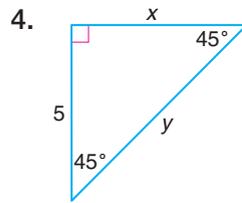


Guided Practice

Examples 1 & 2

Example 1

Find the missing measures. Write all radicals in simplest form.



6. **Maps** On the map of Milwaukee, Wisconsin, the shape formed by Route 190, Route 57, and Route 145 closely resembles an isosceles right triangle. If the distances on Routes 190 and 57 are each 2.8 miles, find the distance on Route 145 between Route 190 and Route 57. Round to the nearest tenth.

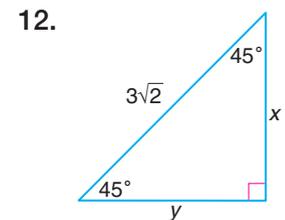
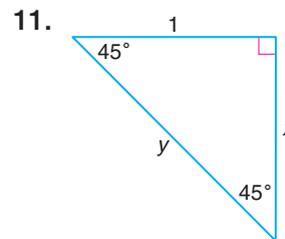
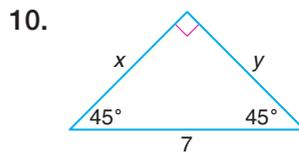
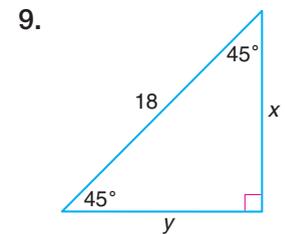
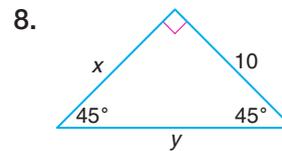
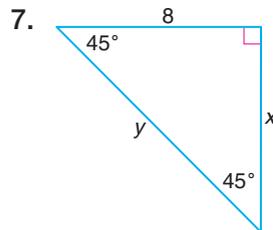


Exercises

Practice

Homework Help	
For Exercises	See Examples
7, 8, 11, 14, 15, 17	1
9, 10, 12, 13, 16	2
Extra Practice	
See page 750.	

Find the missing measures. Write all radicals in simplest form.

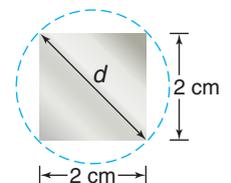


13. The length of the hypotenuse of an isosceles right triangle is $6\sqrt{2}$ feet. Find the length of a leg.

Applications and Problem Solving

14. **Measurement** The length of one side of a square is 12 meters. To the nearest tenth, find the length of a diagonal of the square.

15. **Machine Technology** A square bolt 2 centimeters on each side is to be cut from round stock. To the nearest tenth, what diameter stock is needed?

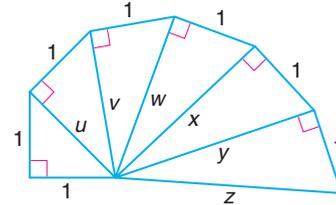


Exercise 15

16. **Logging** In order to make flat boards from a log, a miller first trims off the four sides to make a square beam. Then the beam is cut into flat boards. If the diameter of the original log was 15 inches, find the maximum width of the boards. Round your answer to the nearest tenth.



17. **Critical Thinking** Use the figure at the right to find each measure.
- | | |
|--------|--------|
| a. u | b. v |
| c. w | d. x |
| e. y | f. z |



Mixed Review

Simplify each expression. (Lesson 13-1)

18. $\sqrt{20}$

19. $\sqrt{5} \cdot \sqrt{15}$

20. $\frac{\sqrt{2}}{\sqrt{3}}$

21. **Manufacturing** The Purely Sweet Candy Company has just released its latest candy. It is a chocolate sphere filled with a candy surprise. If the sphere is 6 centimeters in diameter, what is the minimum amount of foil paper it will take to wrap the sphere? (Lesson 12-6)

Standardized Test Practice

A B C D

22. **Grid In** In a circle with a radius of 7 inches, a chord is 5 inches from the center of the circle. To the nearest tenth of an inch, what is the length of the chord? (Lesson 11-3)

23. **Short Response** Draw a semi-regular tessellation. (Lesson 10-7)

Quiz 1

Lessons 13-1 and 13-2

Simplify each expression. (Lesson 13-1)

1. $\sqrt{12}$

2. $\sqrt{6} \cdot \sqrt{3}$

3. $\frac{6}{\sqrt{2}}$

4. The measure of a leg of an isosceles right triangle is 15. Find the measure of the hypotenuse in simplest form. (Lesson 13-2)

5. **Measurement** The length of a diagonal of a square is $4\sqrt{2}$ inches. (Lesson 13-2)

- | |
|---------------------------------------------|
| a. Find the length of a side of the square. |
| b. Find the perimeter of the square. |

What You'll Learn

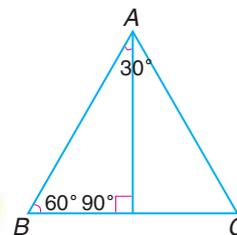
You'll learn to use the properties of 30°-60°-90° triangles.

Why It's Important

Architecture

Architects use 30°-60°-90° triangles to design certain kinds of arches. See Example 4.

An equilateral triangle has three equal sides and three equal angles. Because the sum of the measures of the angles in a triangle is 180° , the measure of each angle in an equilateral triangle is 60° . If you draw a median from vertex A to side \overline{BC} , the median bisects the angle A . The median of an equilateral triangle separates it into two **30°-60°-90° triangles**.



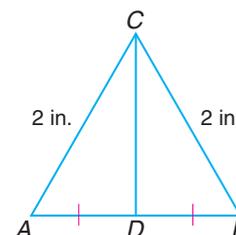
The activity explores some of the special properties of 30°-60°-90° triangles.

Hands-On Geometry

Materials: compass protractor ruler

Step 1 Construct an equilateral triangle with sides 2 in. long. Label its vertices A , B , and C .

Step 2 Find the midpoint of \overline{AB} and label it D . Draw \overline{CD} , a median.



Try These

- Use a protractor to measure $\angle ACD$, $\angle A$, and $\angle CDA$.
- Use a ruler to measure \overline{AD} .
- Copy and complete the table below. Use the Pythagorean Theorem to find CD . Write your answers in simplest form.

AC	AD	CD
2 in.		
4 in.		
3 in.		

- Suppose the length of a side of an equilateral triangle is 10 inches. What values would you expect for AC (hypotenuse), AD (shorter leg), and CD (longer leg)?

Reading Geometry

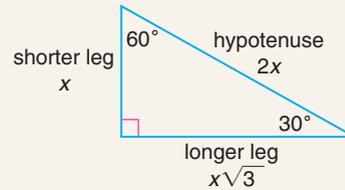
Remember that the shorter leg is always opposite the 30° angle, and the longer leg is opposite the 60° angle.

The results you discovered in the activity lead to Theorem 13–2.

Theorem 13–2 30° - 60° - 90° Triangle Theorem

Words: In a 30° - 60° - 90° triangle, the hypotenuse is twice the length of the shorter leg, and the longer leg is $\sqrt{3}$ times the length of the shorter leg.

Model:



Examples

- 1 In $\triangle ABC$, $b = 7$. Find a and c . Write in simplest form.

Since b is opposite the 30° angle, b is the measure of the shorter leg. Therefore, a is the measure of the longer leg, and c is the measure of the hypotenuse.

Find c .

$$c = 2b \quad \text{The hypotenuse is twice the shorter leg.}$$

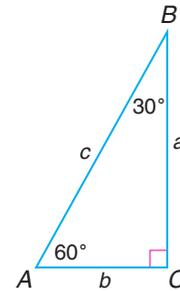
$$c = 2(7) \quad \text{Replace } b \text{ with } 7.$$

$$c = 14 \quad \text{Multiply.}$$

Find a .

$$a = b\sqrt{3} \quad \text{The longer leg is } \sqrt{3} \text{ times the length of the shorter leg.}$$

$$a = 7\sqrt{3} \quad \text{Replace } b \text{ with } 7.$$



- 2 In $\triangle ABC$, $c = 18$. Find a and b . Write in simplest form.

Since b is opposite the 30° angle, b is the measure of the shorter leg.

Find b .

$$c = 2b \quad \text{The hypotenuse is twice the shorter leg.}$$

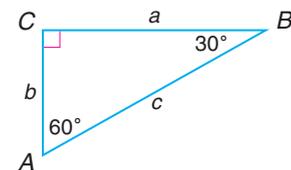
$$18 = 2b \quad \text{Replace } c \text{ with } 18.$$

$$9 = b \quad \text{Divide each side by } 2.$$

Now, find a .

$$a = b\sqrt{3} \quad \text{The longer leg is } \sqrt{3} \text{ times the shorter leg.}$$

$$a = 9\sqrt{3} \quad \text{Replace } b \text{ with } 9.$$

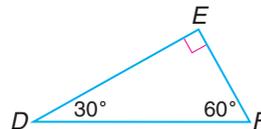


Your Turn

- Refer to $\triangle ABC$ above. If $b = 8$, find a and c .
- Refer to $\triangle ABC$ above. If $c = 10$, find a and b .

Example

3 In $\triangle DEF$, $DE = 12$. Find EF and DF . Write in simplest form.



First, find EF .

$$DE = (EF)\sqrt{3} \quad \text{The longer leg is } \sqrt{3} \text{ times the shorter leg.}$$

$$12 = (EF)\sqrt{3} \quad \text{Replace } DE \text{ with } 12.$$

$$\frac{12}{\sqrt{3}} = \frac{(EF)\sqrt{3}}{\sqrt{3}} \quad \text{Divide each side by } \sqrt{3}.$$

$$EF = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \quad \frac{\sqrt{3}}{\sqrt{3}} = 1$$

$$EF = \frac{12\sqrt{3}}{3} \quad \sqrt{3} \cdot \sqrt{3} = 3$$

$$EF = 4\sqrt{3} \quad \text{Simplify.}$$

Now, find DF .

$$DF = 2(EF) \quad \text{The hypotenuse is twice the shorter leg.}$$

$$= 2(4\sqrt{3}) \quad \text{Replace } EF \text{ with } 4\sqrt{3}.$$

$$= (2 \cdot 4)\sqrt{3} \quad \text{Associative Property}$$

$$= 8\sqrt{3} \quad \text{Multiply.}$$

Your Turn

c. Refer to $\triangle DEF$ above. If $DE = 8$, find EF and DF .

You can use the properties of a 30° - 60° - 90° triangle to solve real-world problems that involve equilateral triangles.



Example

Architecture Link

4 The Gothic arch is based on an equilateral triangle. Find the height of the arch to the nearest tenth.

The height h separates the triangle into two 30° - 60° - 90° triangles. The hypotenuse is 12 feet, and the side opposite the 30° angle is 6 feet. The height of the arch is the length of the side opposite the 60° angle.

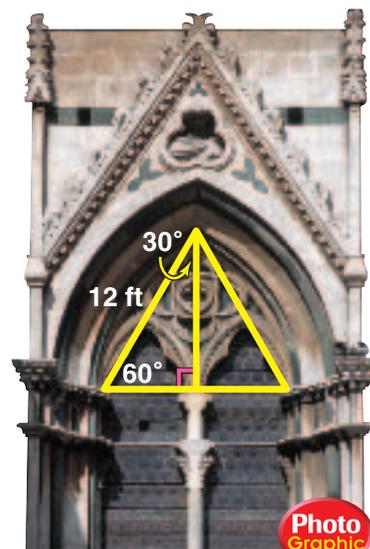


Photo Graphic



Data Update

For more information on Gothic architecture, visit: www.geomconcepts.com

$$h = 6\sqrt{3} \quad \text{Theorem 13-2}$$

$$6 \quad \boxed{2nd} \quad [\sqrt{ }] \quad \boxed{3} \quad \boxed{ENTER} \quad 10.39230485$$

To the nearest tenth, the height of the arch is 10.4 feet.



Check for Understanding

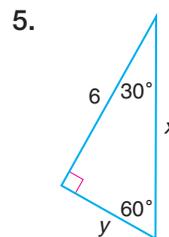
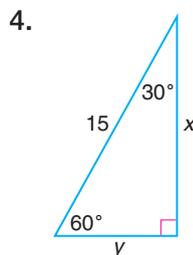
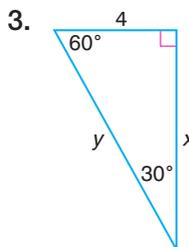
Communicating Mathematics

1. Draw and label a 30° - 60° - 90° triangle in which the sides are 5 inches, 10 inches, and $5\sqrt{3}$ inches.
2. **Writing Math** Compare and contrast the 30° - 60° - 90° Triangle Theorem and the 45° - 45° - 90° Triangle Theorem.

Guided Practice

Examples 1–3

Find the missing measures. Write all radicals in simplest form.



Example 4

6. **Design** The hexagons in the stained-glass window are made of equilateral triangles. If the length of a side of a triangle is 14 centimeters, what is the height of the triangle? Round to the nearest tenth.

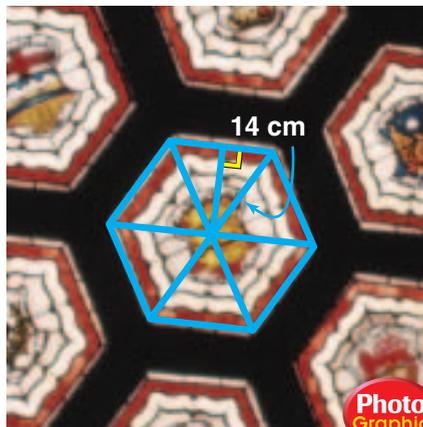
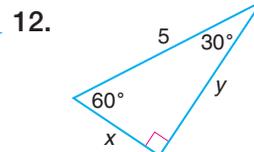
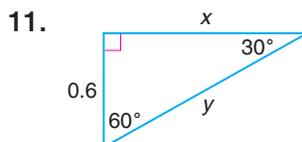
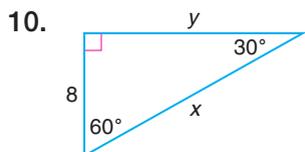
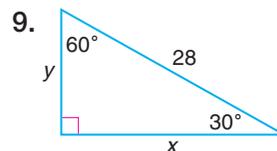
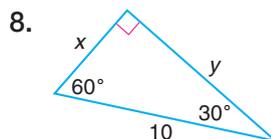
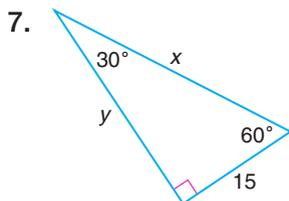


Photo Graphic

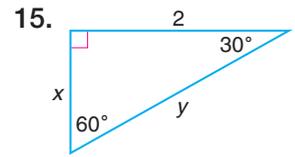
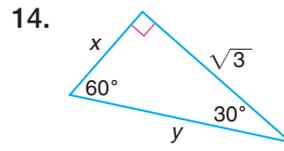
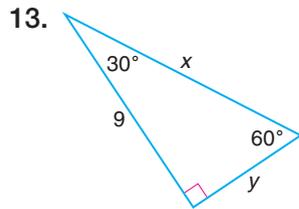
Exercises

Practice

Find the missing measures. Write all radicals in simplest form.



Homework Help	
For Exercises	See Examples
7, 9–11, 13–17	1, 3
8, 12	2, 4
Extra Practice	
See page 751.	



16. The length of the shorter leg of a 30° - 60° - 90° triangle is 24 meters. Find the length of the hypotenuse.

Applications and Problem Solving

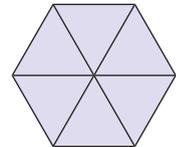
17. **Measurement** At the same time that the sun's rays make a 60° angle with the ground, the shadow cast by a flagpole is 24 feet. To the nearest foot, find the height of the flagpole.



Exercise 17

18. **Measurement** The length of one side of an equilateral triangle is 10 meters.
 a. Find the length of an altitude.
 b. Find the area of the triangle.

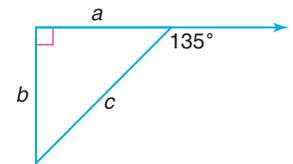
19. **Critical Thinking** A regular hexagon is made up of six congruent equilateral triangles. Find the area of a regular hexagon whose perimeter is 24 feet.



Mixed Review

The measure of one side of the triangle is given. Find the missing measures to complete the table. (Lesson 13-2)

	<i>a</i>	<i>b</i>	<i>c</i>
20.	18		
21.		$5\sqrt{2}$	



22. **Electricity** The current that can be generated in a circuit is given by the formula $I = \sqrt{\frac{P}{R}}$, where *I* is the current in amperes, *P* is the power in watts, and *R* is the resistance in ohms. Find the current when *P* = 25 watts and *R* = 400 ohms. (Lesson 13-1)

Standardized Test Practice



23. **Grid In** Find the distance between *A*(0, 0) and *B*(3, 4). (Lesson 6-7)

24. **Multiple Choice** Line *ℓ* has a slope of $\frac{1}{2}$ and a *y*-intercept of -7 . Which of the following is the equation of *ℓ* written in slope-intercept form? (Algebra Review)

(A) $y = \frac{1}{2}x - 7$

(B) $y = \frac{1}{2}x + 7$

(C) $2x + y = 7$

(D) $x - 2y = 14$



13-4 Tangent Ratio

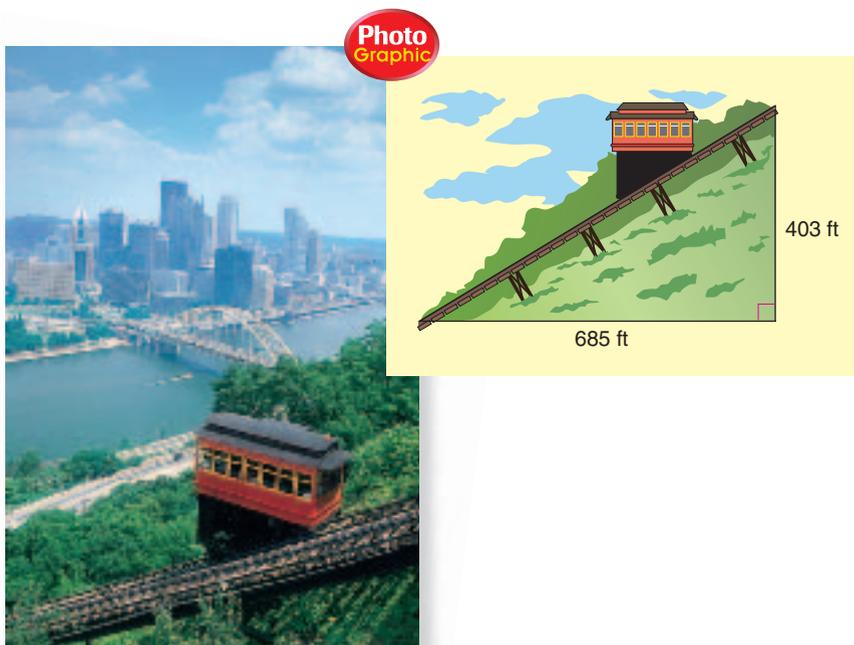
What You'll Learn

You'll learn to use the tangent ratio to solve problems.

Why It's Important

Surveying Surveyors use the tangent ratio to find distances that cannot be measured directly. See Example 2.

In Pittsburgh, Pennsylvania, the Duquesne Incline transports passengers from the river valley up to Mount Washington. The Incline has a 403-foot rise and a 685-foot run. What angle is made by the track and the run? *This problem will be solved in Example 4.*



The Duquesne Incline

Trigonometry is the study of the properties of triangles. The word *trigonometry* comes from two Greek words meaning *angle measurement*. A **trigonometric ratio** is a ratio of the measures of two sides of a right triangle. Using trigonometric ratios, you can find the unknown measures of angles and sides of right triangles.

One of the most common trigonometric ratios is the **tangent** ratio.

Reading Geometry

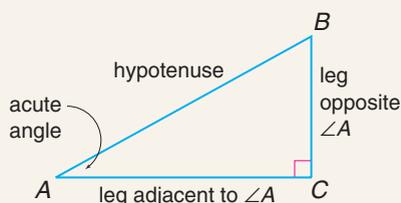
The symbol $\tan A$ is read *the tangent of angle A*.

Definition of Tangent

Words: If A is an acute angle of a right triangle,

$$\tan A = \frac{\text{measure of the leg opposite } \angle A}{\text{measure of the leg adjacent to } \angle A}$$

Model:



Symbols:

$$\tan A = \frac{BC}{AC}$$

Example**1**Find $\tan A$ and $\tan B$.

$$\tan A = \frac{BC}{AC}$$

*opposite
adjacent*

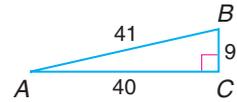
$$= \frac{9}{40} \text{ or } 0.225$$

Replace BC with 9 and AC with 40.

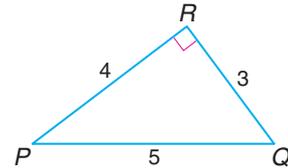
$$\tan B = \frac{AC}{BC}$$

*AC is the leg opposite $\angle B$.**BC is the leg adjacent to $\angle B$.*

$$= \frac{40}{9} \text{ or } 4.4444$$

Replace AC with 40 and BC with 9.**Reading
Geometry**

When trigonometric ratios are expressed as decimals, they are usually rounded to four decimal places.

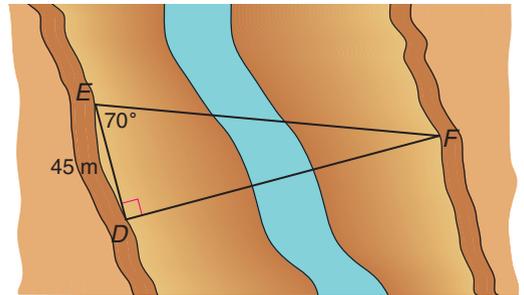
Your Turna. Find $\tan P$ and $\tan Q$.

The tangent of an angle in a right triangle depends on the measure of the angle, not on the size of the triangle. So, an expression like $\tan 70^\circ$ has a unique value. You can use the TAN function on your calculator to find the value of $\tan 70^\circ$, or you can find the value in a trigonometric table.

If you know the measure of an angle and one leg, you can use the tangent ratio to find distances that cannot be measured directly.

**Example****Surveying Link****2**

A surveyor standing at the edge of a canyon made the measurements shown at the right. Find the distance across the canyon from D to F .



$$\tan 70^\circ = \frac{DF}{DE}$$

*opposite
adjacent*

$$\tan 70^\circ = \frac{DF}{45}$$

Replace DE with 45.

$$45 \cdot \tan 70^\circ = 45 \cdot \frac{DF}{45}$$

Multiply each side by 45.

$$45 \cdot \tan 70^\circ = DF$$

Use a calculator. Be sure it is in degree mode.

$$45 \quad \times \quad \boxed{\text{TAN}} \quad 70 \quad \boxed{\text{ENTER}} \quad 123.6364839$$

Therefore, the distance across the canyon is about 123.6 meters.



Some applications of trigonometry use an angle of elevation or angle of depression. In the photo below, the angle made by the line of sight from the boat and a horizontal line is called an **angle of elevation**. The angle made by the line of sight from the parasail and a horizontal line is called an **angle of depression**.

You can use parallel lines and alternate interior angles to show that the angle of elevation and the angle of depression are congruent.

Look Back

Alternate Interior Angles: Lesson 4-2

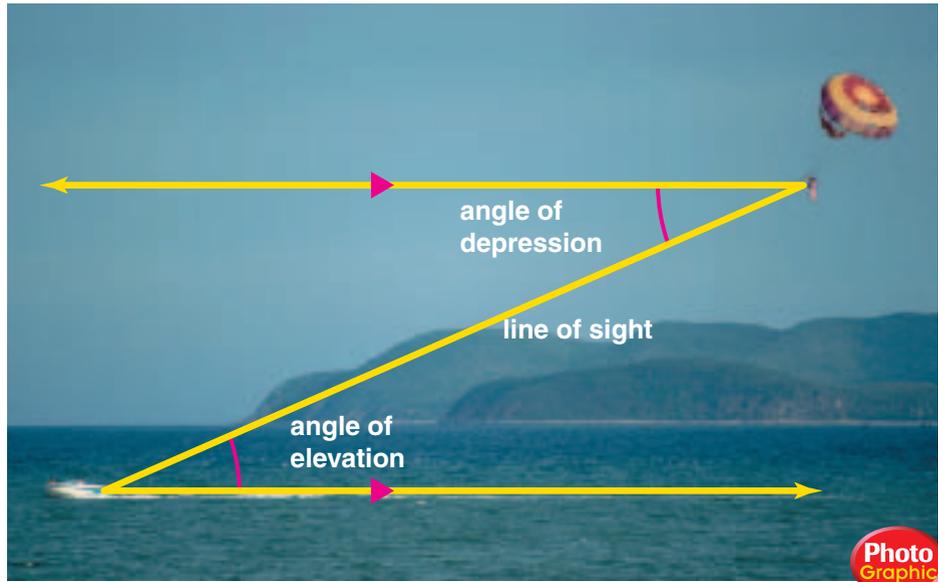


Photo Graphic

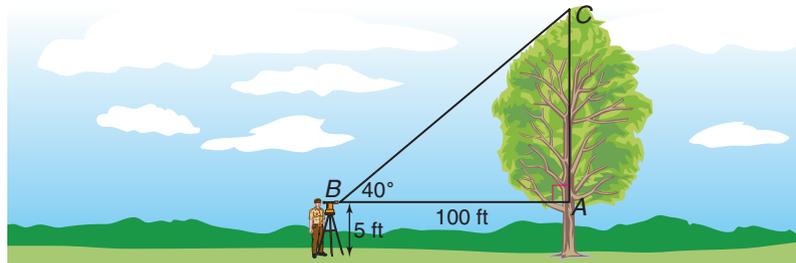


Example

Forestry Link

3

A ranger standing 100 feet from a tree sights the top of the tree at a 40° angle of elevation. Find the height of the tree.



$$\tan 40^\circ = \frac{AC}{AB} \quad \text{opposite over adjacent}$$

$$\tan 40^\circ = \frac{AC}{100} \quad \text{Replace } AB \text{ with } 100.$$

$$100 \cdot \tan 40^\circ = 100 \cdot \frac{AC}{100} \quad \text{Multiply each side by } 100.$$

$$100 \cdot \tan 40^\circ = AC \quad \text{Use a calculator.}$$

$$100 \quad \times \quad \text{TAN} \quad 40 \quad \text{ENTER} \quad 83.90996312$$

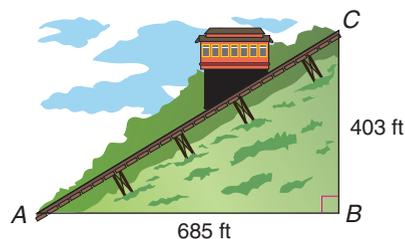
The height of the ranger is 5 feet. Therefore, the height of the tree is about $83.9 + 5$ or 88.9 feet.



Example Engineering Link

You can use the TAN^{-1} function on your calculator to find the measure of an acute angle of a right triangle when you know the measures of the legs. The TAN^{-1} function is called the *inverse tangent*. *The inverse tangent is also called the arctangent.*

- 4 Refer to the beginning of the lesson. Find the angle between the track and the run of the Duquesne Incline to the nearest tenth.



$$\begin{aligned}\tan A &= \frac{BC}{AB} \quad \begin{array}{l} \textit{opposite} \\ \textit{adjacent} \end{array} \\ &= \frac{403}{685} \quad \textit{Replace BC with 403 and AB with 685.}\end{aligned}$$

Now, use a calculator to find the measure of $\angle A$, an angle whose tangent ratio is $\frac{403}{685}$. $m\angle A = \tan^{-1}\left(\frac{403}{685}\right)$

`2nd` `[TAN-1]` `403` `÷` `685` `ENTER` *30.46920015*

To the nearest tenth, the measure of $\angle A$ is about 30.5° .

Your Turn

- b. The access ramp to a parking lot has a rise of 10 feet and a run of 48 feet. To the nearest degree, find the angle between the ramp and the run.

Check for Understanding

Communicating Mathematics

1. Write a definition of *tangent*.
2. Draw right triangle LMN in which $\angle N$ is an acute angle. Label the leg opposite $\angle N$ and the leg adjacent to $\angle N$.

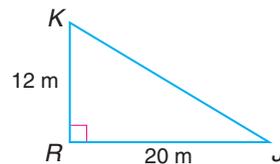
Vocabulary

trigonometry
trigonometric ratio
tangent
angle of elevation
angle of depression

Guided Practice Example 1

Find each tangent. Round to four decimal places, if necessary.

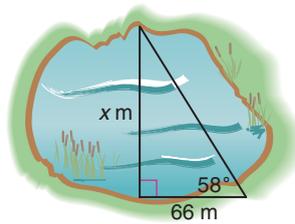
3. $\tan J$
4. $\tan K$



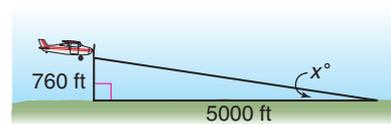
Examples 2 & 3

Find each missing measure. Round to the nearest tenth.

5.



6.



Example 4

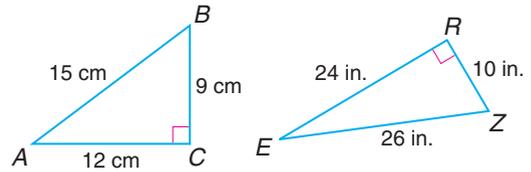
7. **Travel** The distance from a boat to a bridge is 200 meters. A person aboard measures the angle of elevation to the bridge as 12° . To the nearest tenth, how far above the water is the bridge?

Exercises

Practice

Find each tangent. Round to four decimal places, if necessary.

8. $\tan A$ 9. $\tan B$
 10. $\tan E$ 11. $\tan Z$



Homework Help	
For Exercises	See Examples
8–11	1
12, 14, 17, 20	2, 3
13, 15, 16, 21	4
Extra Practice See page 751.	

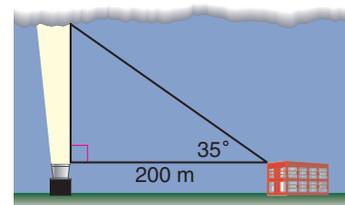
Find each missing measure. Round to the nearest tenth.

12. 13. 14.
 15. 16. 17.

18. If the leg adjacent to a 29° angle in a right triangle is 9 feet long, what is the measure of the other leg to the nearest tenth?

Applications and Problem Solving

19. **Meteorology** A searchlight located 200 meters from a weather office is shined directly overhead. If the angle of elevation to the spot of light on the clouds is 35° , what is the altitude of the cloud ceiling? Round to the nearest tenth.

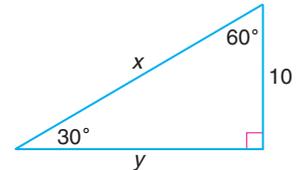


20. **Farming** A pile of corn makes an angle of 27.5° with the ground. If the distance from the center of the pile to the outside edge is 25 feet, how high is the pile of corn?
21. **Critical Thinking** In a right triangle, the tangent of one of the acute angles is 1. How are the measures of the two legs related?



Mixed Review

22. Find the missing measures. Write all radicals in simplest form. (Lesson 13-3)



23. **Sports** Many younger children like to play a game similar to baseball called tee-ball. Instead of trying to hit a ball thrown by a pitcher, the batter hits the ball off a tee. To accommodate younger children, the bases are only 40 feet apart. Find the distance between home plate and second base. (Lesson 13-2)

Determine whether it is possible for a trapezoid to have the following conditions. Write yes or no. If yes, draw the trapezoid. (Lesson 8-5)

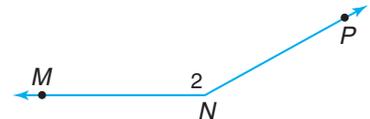
24. congruent diagonals 25. three obtuse angles

Standardized Test Practice

- (A) (B) (C) (D)

26. **Multiple Choice** Which is *not* a name for this angle? (Lesson 3-1)

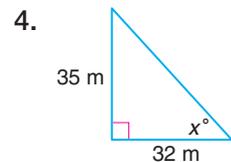
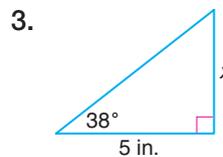
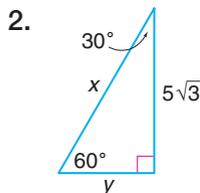
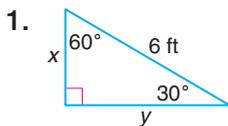
- (A) $\angle MNP$ (B) $\angle 2$
 (C) $\angle NPM$ (D) $\angle PNM$



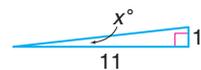
Quiz 2

Lessons 13-3 and 13-4

Find each missing measure. Write radicals in simplest form. Round decimals to the nearest tenth. (Lessons 13-3 & 13-4)



5. **Transportation** The steepest grade of any standard railway system in the world is in France between Chedde and Servoz. The track rises 1 foot for every 11 feet of run. Find the measure of the angle formed by the track and the run. (Lesson 13-4)





Materials

-  protractor
-  index card
-  straws
-  paper clips
-  string
-  tape

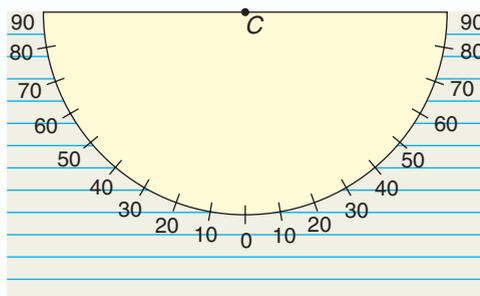
Indirect Measurement Using a Hypsometer

The General Sherman giant sequoia in Sequoia National Park, California, has been measured at 275 feet tall. But it's unlikely that someone actually measured the height with a tape measure. It's more likely that the height was calculated using trigonometry.

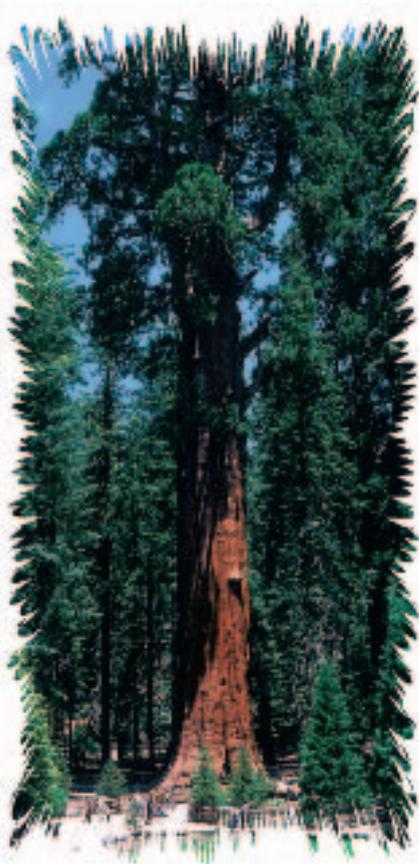
A **hypsometer** is an instrument that measures angles of elevation. You can use a hypsometer and what you know about the tangent ratio to find the heights of objects that are difficult to measure directly. *A hypsometer is sometimes called a clinometer.*

Investigate

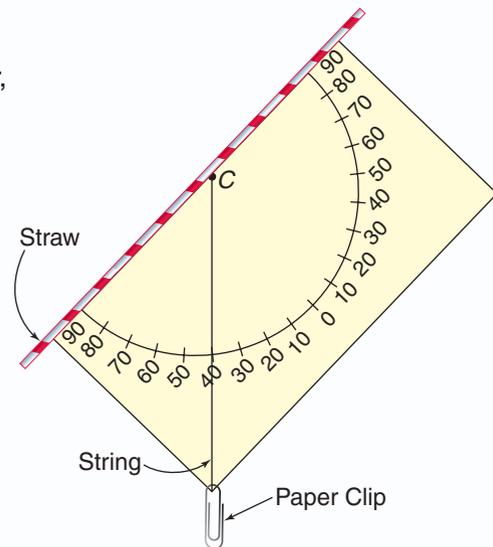
1. Make a hypsometer by following these steps.
 - a. Tape a protractor on an index card so that both zero points align with the edge of the card. Mark the center of the protractor on the card and label it C . Then mark and label every 10° on the card like the one shown below.



- b. Tie a piece of string to a large paper clip. Attach the other end of the string to the index card at C .
 - c. Tape a straw to the edge of the index card that contains C .



- To use the hypsometer, look through the straw at an object like the top of your classroom door. Ask another student to read the angle from the scale. This is the angle of elevation.



- Use your hypsometer to find the angle of elevation to the top of a tree or flagpole on your school property. Also, measure the distance from the hypsometer to the ground and from your foot to the base of the tree.
- Make a sketch that you can use to find the height of the tree. Use your measurements from Exercise 3.
- Explain how you can use trigonometry to find the height of the tree. Then find the height of the tree.

Extending the Investigation

In this project, you will use your hypsometer to find the height of three objects on your school property. Here are some suggestions.

- tree
- flagpole
- basketball hoop
- goal post
- school building

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make scale drawings that show the angle of elevation, distance to the ground, and distance from the object.
- Research *indirect measurement*. Write a paragraph that explains how indirect measurement was used in ancient times.



Investigation For more information on indirect measurement, visit: www.geomconcepts.com

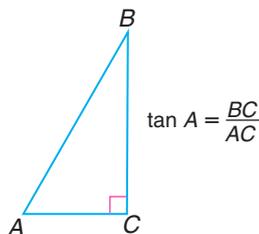
What You'll Learn

You'll learn to use the sine and cosine ratios to solve problems.

Why It's Important Navigation

Flight navigators use trigonometry to determine flight paths. See Exercise 16.

In Lesson 13-4, you learned about the tangent ratio. In a right triangle, the tangent ratio $\frac{BC}{AC}$ is related to the angle A .



Two other useful trigonometric ratios are the **sine** ratio and the **cosine** ratio. Both sine and cosine relate an angle measure to the ratio of the measures of a triangle's leg to its hypotenuse.

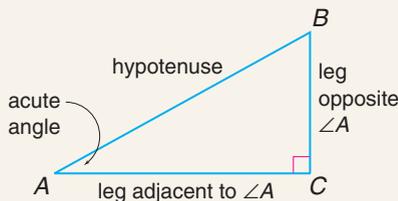
Reading Geometry

The symbol $\sin A$ is read as *the sine of angle A*. The symbol $\cos A$ is read as *the cosine of angle A*.

Definition of Sine and Cosine

Words: If A is an acute angle of a right triangle,
 $\sin A = \frac{\text{measure of the leg opposite } \angle A}{\text{measure of the hypotenuse}}$, and
 $\cos A = \frac{\text{measure of the leg adjacent to } \angle A}{\text{measure of the hypotenuse}}$.

Model:



Symbols: $\sin A = \frac{BC}{AB}$ $\cos A = \frac{AC}{AB}$

Example

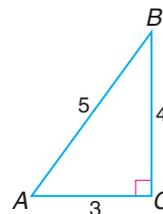
1 Find $\sin A$, $\cos A$, $\sin B$, and $\cos B$.

$$\begin{aligned} \sin A &= \frac{BC}{AB} && \frac{\text{opposite}}{\text{hypotenuse}} \\ &= \frac{4}{5} \text{ or } 0.8 \end{aligned}$$

$$\begin{aligned} \cos A &= \frac{AC}{AB} && \frac{\text{adjacent}}{\text{hypotenuse}} \\ &= \frac{3}{5} \text{ or } 0.6 \end{aligned}$$

$$\begin{aligned} \sin B &= \frac{AC}{AB} \\ &= \frac{3}{5} \text{ or } 0.6 \end{aligned}$$

$$\begin{aligned} \cos B &= \frac{BC}{AB} \\ &= \frac{4}{5} \text{ or } 0.8 \end{aligned}$$

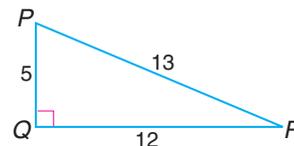


Your Turn

Find each value.

- a. $\sin P$
c. $\cos P$

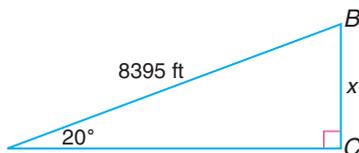
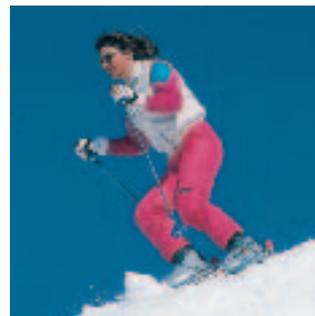
- b. $\sin R$
d. $\cos R$



Example
Recreation Link

2

The Aerial Ski Run in Snowbird, Utah, is 8395 feet long and, on average, has a 20° angle of elevation. What is the vertical drop? Round to the nearest tenth.



You know the length of the hypotenuse and the measure of $\angle A$. You need to find the measure of the leg opposite $\angle A$. Use the sine ratio.

$$\sin A = \frac{BC}{AB} \quad \begin{array}{l} \textit{opposite} \\ \textit{hypotenuse} \end{array}$$

$$\sin 20^\circ = \frac{x}{8395} \quad \begin{array}{l} \textit{Replace } AB \textit{ with } 8395, \\ \textit{BC with } x, \textit{ and } A \textit{ with } 20^\circ. \end{array}$$

$$8395 \cdot \sin 20^\circ = 8395 \cdot \frac{x}{8395} \quad \textit{Multiply each side by } 8395.$$

$$8395 \cdot \sin 20^\circ = x \quad \textit{Simplify.}$$

$$8395 \times \text{[SIN] } 20 \text{ [ENTER] } 2871.259103$$

Therefore, the vertical drop in the Aerial Ski Run is about 2871.3 feet.

**Graphing
Calculator Tutorial**

See pp. 782–785.

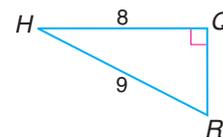
You can use the SIN^{-1} or COS^{-1} function on your calculator to find the measure of an acute angle of a right triangle when you know the measures of a leg and the measure of the hypotenuse. The SIN^{-1} function is called the *inverse sine*, and the COS^{-1} function is called the *inverse cosine*. *The inverse sine and cosine are also called the arcsine and arccosine, respectively.*

Example

3

Find the measure of $\angle H$ to the nearest degree.

You know the lengths of the side adjacent to $\angle H$ and the hypotenuse. You can use the cosine ratio.



$$\cos H = \frac{HQ}{HR} \quad \begin{array}{l} \textit{adjacent} \\ \textit{hypotenuse} \end{array}$$

$$\cos H = \frac{8}{9} \quad \textit{Replace } HQ \textit{ with } 8 \textit{ and } HR \textit{ with } 9.$$

Now, use a calculator to find the measure of $\angle H$, an angle whose tangent ratio is $\frac{8}{9}$. $m\angle H = \cos^{-1}\left(\frac{8}{9}\right)$

$$\text{[2nd] [COS }^{-1} \text{] } 8 \text{ [} \div \text{] } 9 \text{ [ENTER] } 27.26604445$$

To the nearest degree, the measure of $\angle H$ is about 27° .



Your Turn

- e. Find the measure of $\angle R$ to the nearest degree if $HQ = 35$ and $HR = 37$.

There are many relationships that can be proven among trigonometric functions. Such relationships are called **trigonometric identities**. In the following theorem, the tangent function is defined as the ratio $\frac{\sin x}{\cos x}$. As long as $\cos x \neq 0$, this relationship is true and defines the tangent function. You can use a TI-92 to show that Theorem 13-3 is a trigonometric identity.

Theorem 13-3

If x is the measure of an acute angle of a right triangle, then

$$\frac{\sin x}{\cos x} = \tan x.$$



Graphing Calculator Exploration

To find $\sin 37^\circ$, type $\boxed{\text{SIN}} \ 37 \ \boxed{)} \ \boxed{\text{ENTER}}$. To four decimal places, the result is 0.6018. *Make sure your calculator is in degree mode.*

Try These

1. Copy and complete the table below. Round to four decimal places, if necessary.

x	0°	15°	30°	45°	60°	75°
$\frac{\sin x}{\cos x}$						
$\tan x$						

2. Compare your results in rows 2 and 3.
3. What happens when you find both results for $x = 90^\circ$? Why?
4. You can also use graphing to verify Theorem 13-3. Change the viewing window by entering $\boxed{\text{WINDOW}} \ -90 \ \boxed{\text{ENTER}} \ 720$
 $\boxed{\text{ENTER}} \ 90 \ \boxed{\text{ENTER}} \ -4 \ \boxed{\text{ENTER}} \ 4 \ \boxed{\text{ENTER}} \ 1 \ \boxed{\text{ENTER}} \ \boxed{Y=}$
 $\boxed{\text{SIN}} \ \boxed{X,T,\theta,n} \ \boxed{)} \ \boxed{\div} \ \boxed{\text{COS}} \ \boxed{X,T,\theta,n} \ \boxed{)} \ \boxed{\text{ENTER}} \ \boxed{\text{GRAPH}}$.
 Repeat with $y = \tan x$. What do you notice about the graphs?

Check for Understanding

Communicating Mathematics

1. Compare and contrast the sine and cosine ratios.
2. Draw a right triangle DEF for which $\sin D = \frac{4}{5}$, $\cos D = \frac{3}{5}$, and $\tan D = \frac{4}{3}$.
3. **Writing Math** Some Old Horse—Caught A Horse—Taking Oats Away is a helpful mnemonic device for remembering the trigonometric ratios. S, C, and T represent sine, cosine, and tangent, respectively, while O, H, and A represent opposite, hypotenuse, and adjacent, respectively. Make up your own mnemonic device for remembering the ratios.

Vocabulary

sine
cosine
trigonometric identities

Guided Practice



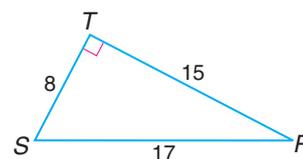
Getting Ready

Identify each segment in the figure below.

Sample: leg opposite $\angle S$

Solution: \overline{RT}

4. leg adjacent to $\angle S$
5. leg opposite $\angle R$
6. leg adjacent to $\angle R$



Exercises 4–8

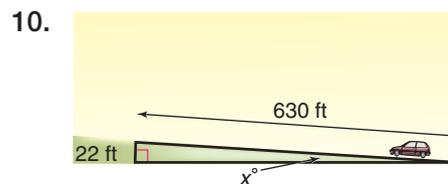
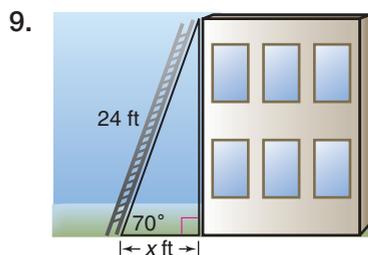
Example 1

Find each sine or cosine. Round to four decimal places, if necessary.

7. $\sin R$
8. $\cos R$

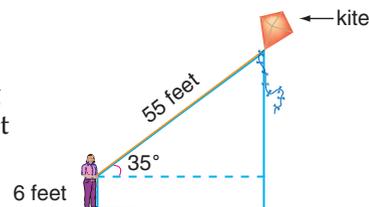
Examples 2 & 3

Find each missing measure. Round to the nearest tenth.



Example 3

11. **Recreation** Sierra is flying a kite. She has let out 55 feet of string. If the angle of elevation is 35° and the hand holding the string is 6 feet from the ground, what is the altitude of the kite? Round to the nearest tenth.



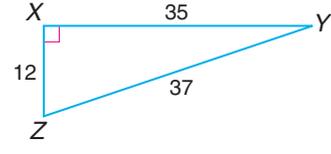
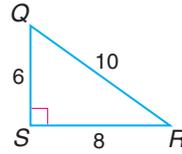
Exercises

Practice

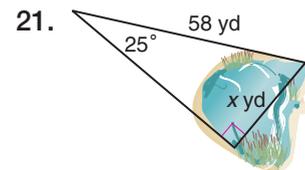
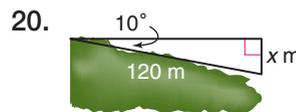
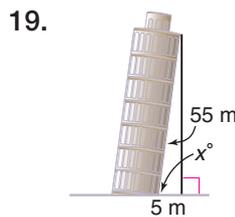
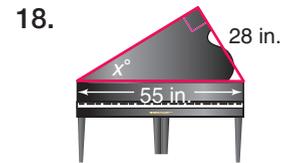
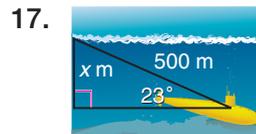
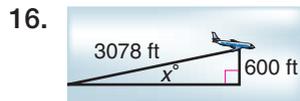
Homework Help	
For Exercises	See Examples
12–15, 22–29, 35	1
16, 18, 19	3
17, 20, 21, 31–34	2
Extra Practice	
See page 751.	

Find each sine or cosine. Round to four decimal places, if necessary.

12. $\sin Q$ 13. $\sin R$
 14. $\cos Z$ 15. $\cos Y$

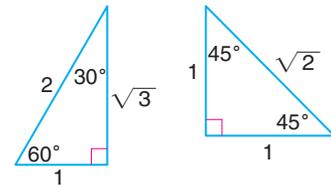


Find each measure. Round to the nearest tenth.



Use the 30° - 60° - 90° and 45° - 45° - 90° triangles to find each value. Round to four decimal places, if necessary.

22. $\sin 30^\circ$ 23. $\sin 60^\circ$
 24. $\sin 45^\circ$ 25. $\cos 30^\circ$
 26. $\cos 60^\circ$ 27. $\cos 45^\circ$
 28. $\tan 30^\circ$ 29. $\tan 60^\circ$
 30. $\tan 45^\circ$



31. If the hypotenuse of a right triangle is 5 feet and $m\angle A = 68$, find the measure of the leg adjacent to $\angle A$.
 32. In a right triangle, the hypotenuse is 24 centimeters and $m\angle D = 16$. What is the measure of a leg opposite $\angle D$?

Applications and Problem Solving

33. **Safety** To guard against a fall, a ladder should make an angle of 75° or less with the ground. What is the maximum height that a 20-foot ladder can reach safely?
 34. **Engineering** According to the Parking Standards in Santa Clarita, California, an access ramp to a parking lot cannot have a slope exceeding 11° . Suppose a parking lot is 10 feet above the road. If the length of the ramp is 60 feet, does this access ramp meet the requirements of the code? Explain your reasoning.

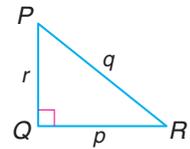


35. **Critical Thinking** Verify each step in parts a through e. Then solve parts f and g.

Theorem 13-4

If x is the measure of an acute angle of a right triangle, then $\sin^2 x + \cos^2 x = 1$.

- a. $\sin P = \frac{p}{q}$ and $\cos P = \frac{r}{q}$
 b. $\sin^2 P = \frac{p^2}{q^2}$ and $\cos^2 P = \frac{r^2}{q^2}$
 c. $\sin^2 P + \cos^2 P = \frac{p^2}{q^2} + \frac{r^2}{q^2}$ or $\frac{p^2 + r^2}{q^2}$
 d. $p^2 + r^2 = q^2$
 e. $\sin^2 P + \cos^2 P = \frac{q^2}{q^2}$ or 1
 f. Find $\sin x$ if $\cos x = \frac{3}{5}$.
 g. Find $\cos x$ if $\sin x = \frac{5}{13}$.



Mixed Review

The heights of several tourist attractions are given in the table. Find the angle of elevation from a point 100 feet from the base of each attraction to its top. (Lesson 13-4)

36. Chief Crazy Horse Statue
 37. Washington Monument
 38. Empire State Building

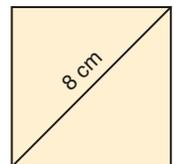
Heights of Tourist Attractions

Empire State Building, New York City	1250 ft
Gateway to the West Arch, St. Louis	.630 ft
Chief Crazy Horse Statue, South Dakota	.563 ft
Washington Monument, Washington, D.C.	.555 ft
Statue of Liberty, New York City	.305 ft



Chief Crazy Horse Statue, South Dakota

39. What is the area of the square? (Lesson 13-2)



Exercise 39

Standardized Test Practice

(A) (B) (C) (D)

40. **Grid In** If the radius of a circle is 15 inches and the diameter is $(3x + 7)$ inches, what is the value of x ? Round to the nearest hundredth. (Lesson 11-1)

41. **Multiple Choice** $\triangle PIG \sim \triangle COW$. If $PI = 6$, $IG = 4$, $CO = x + 3$, and $OW = x$, find the value of x . (Lesson 9-2)
 (A) 1.5 (B) 6 (C) 7.5 (D) 9



Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

30°-60°-90° triangle (p. 559)
 45°-45°-90° triangle (p. 554)
 angle of depression (p. 566)
 angle of elevation (p. 566)
 cosine (p. 572)
 hypsometer (p. 570)

perfect square (p. 548)
 radical expression (p. 549)
 radical sign (p. 548)
 radicand (p. 549)
 simplest form (p. 551)
 sine (p. 572)

INTERNET
CONTENTS **Review Activities**
 For more review activities, visit:
www.geomconcepts.com

square root (p. 548)
 tangent (p. 564)
 trigonometric identity (p. 574)
 trigonometric ratio (p. 564)
 trigonometry (p. 564)

Choose the correct term to complete each sentence.

1. A (perfect square, trigonometric ratio) is a ratio of the measures of two sides of a right triangle.
2. The symbol used to indicate a square root is called a (radical sign, tangent).
3. (Square roots, Trigonometry) can be used to find the measures of unknown angles as well as unknown sides of a right triangle.
4. The number 25 is an example of a (perfect square, radical expression).
5. The opposite of squaring is finding a(n) (angle of elevation, square root).
6. Sine, tangent, and (radical sign, cosine) are three common trigonometric ratios.
7. Parallel lines and alternate interior angles can be used to show that the angle of elevation and the (tangent, angle of depression) are congruent.
8. A simplified definition of (tangent, sine) is opposite over adjacent.
9. The (angle of depression, angle of elevation) is below the line of sight.
10. Cosine and (sine, tangent) are both trigonometric ratios that use the hypotenuse.

Skills and Concepts

Objectives and Examples

- **Lesson 13–1** Multiply, divide, and simplify radical expressions.

$$\begin{aligned} \text{Simplify } \sqrt{6} \cdot \sqrt{10}. \\ \sqrt{6} \cdot \sqrt{10} &= \sqrt{6 \cdot 10} && \text{Product Property} \\ &= \sqrt{2 \cdot 3 \cdot 2 \cdot 5} \\ &= \sqrt{2 \cdot 2 \cdot 3 \cdot 5} && \text{Product Property} \\ &= \sqrt{2 \cdot 2} \cdot \sqrt{3 \cdot 5} \\ &= 2\sqrt{15} && \sqrt{2 \cdot 2} = 2 \end{aligned}$$

Review Exercises

Simplify each expression.

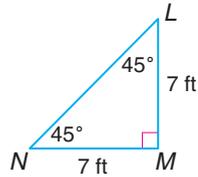
- | | |
|-----------------------------------|--------------------------------|
| 11. $\sqrt{36}$ | 12. $\sqrt{56}$ |
| 13. $\frac{\sqrt{25}}{\sqrt{10}}$ | 14. $\frac{2}{\sqrt{6}}$ |
| 15. $\sqrt{6} \cdot \sqrt{8}$ | 16. $\sqrt{3} \cdot \sqrt{18}$ |



Objectives and Examples

- **Lesson 13–2** Use the properties of 45°-45°-90° triangles.

$\triangle LMN$ is an isosceles right triangle. Find the length of the hypotenuse.



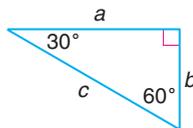
$$h = s\sqrt{2}$$

$$h = 7\sqrt{2}$$

The length of the hypotenuse is $7\sqrt{2}$ feet.

- **Lesson 13–3** Use the properties of 30°-60°-90° triangles.

In the triangle, $b = 15$. Find a and c .



$a = b\sqrt{3}$ *The longer leg is $\sqrt{3}$ times as long as the shorter leg.*

$a = 15\sqrt{3}$ *Replace b with 15.*

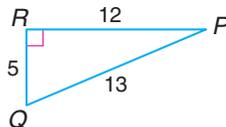
$c = 2b$ *The hypotenuse is twice the shorter leg.*

$c = 2(15)$ *Replace b with 15.*

$c = 30$ *Simplify.*

- **Lesson 13–4** Use the tangent ratio to solve problems.

Find $\tan P$ rounded to four decimal places.



$\tan P = \frac{QR}{PR}$ *opposite adjacent*

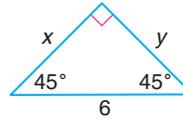
$\tan P = \frac{5}{12}$ *Replace QR with 5 and PR with 12.*

$\tan P = 0.4167$ *Use a calculator.*

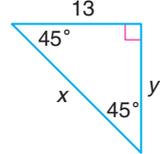
Review Exercises

Find the missing measures. Write all radicals in simplest form.

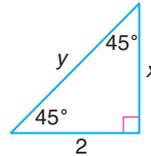
17.



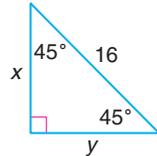
18.



19.

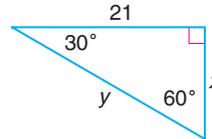


20.

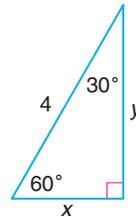


Find the missing measures. Write all radicals in simplest form.

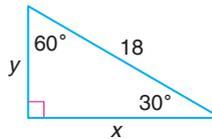
21.



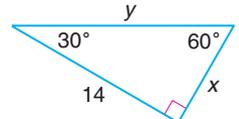
22.



23.

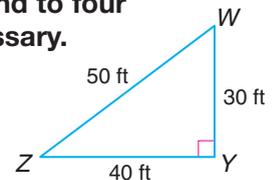


24.

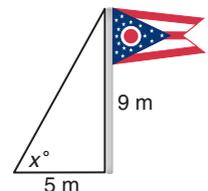


Find each tangent. Round to four decimal places, if necessary.

- 25. $\tan W$
- 26. $\tan Z$



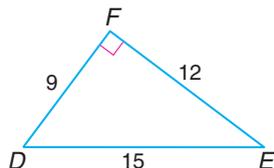
- 27. Find the missing measure. Round to the nearest tenth.



Objectives and Examples

- **Lesson 13–5** Use the sine and cosine ratios to solve problems.

Find $\sin D$ and $\cos D$.



$$\sin D = \frac{EF}{DE} \quad \begin{array}{l} \text{opposite} \\ \text{hypotenuse} \end{array}$$

$$= \frac{12}{15} \text{ or } 0.8 \quad \text{Substitute.}$$

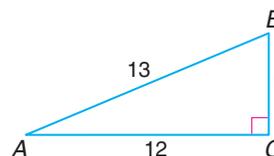
$$\cos D = \frac{DF}{DE} \quad \begin{array}{l} \text{adjacent} \\ \text{hypotenuse} \end{array}$$

$$= \frac{9}{15} \text{ or } 0.6 \quad \text{Substitute.}$$

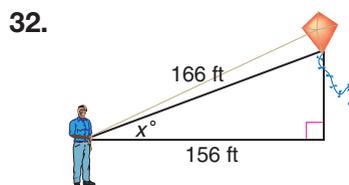
Review Exercises

Find each sine or cosine. Round to four decimal places, if necessary.

- 28. $\sin A$
- 29. $\cos B$
- 30. $\cos A$

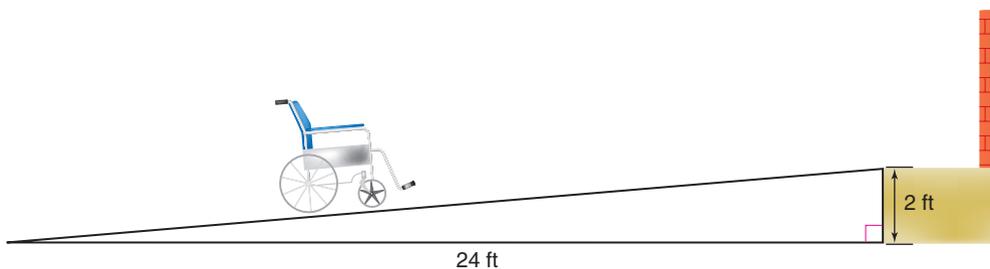


Find each missing measure. Round to the nearest tenth.



Applications and Problem Solving

- 33. **Arts and Crafts** Alyssa has a square piece of construction paper with a perimeter of 68 inches. Suppose she cuts the paper diagonally to form two congruent triangles. To the nearest inch, what is the sum of the perimeters of the two triangles? (*Lesson 13–2*)
- 34. **Surveying** A forest ranger sights a tree through a surveying instrument. The angle of elevation to the top of the tree is 27° . The instrument is 4 feet above the ground. The surveyor is 100 feet from the base of the tree. To the nearest foot, how tall is the tree? (*Lesson 13–4*)
- 35. **Construction** Mr. Boone is building a wooden ramp to allow people who use wheelchairs easier access to the public library. The ramp must be 2 feet tall. Find the angle of elevation if the ramp begins 24 feet away from the library. Round to the nearest tenth. (*Lesson 13–4*)



CHAPTER 13 Test

1. Define the term perfect square and list all perfect squares less than 100.
2. Compare and contrast angles of elevation and angles of depression.

Simplify each expression.

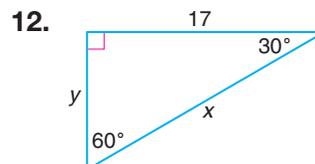
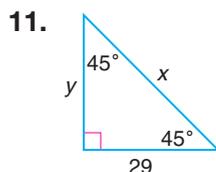
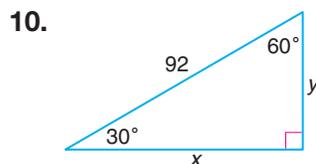
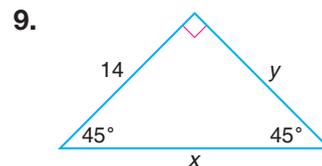
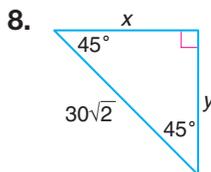
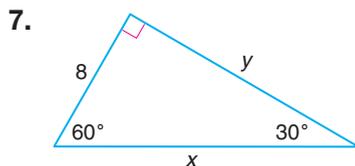
3. $\frac{\sqrt{3}}{\sqrt{7}}$

4. $\sqrt{44}$

5. $\sqrt{6} \cdot \sqrt{3}$

6. $\sqrt{\frac{16}{3}}$

Find the missing measures. Write all radicals in simplest form.

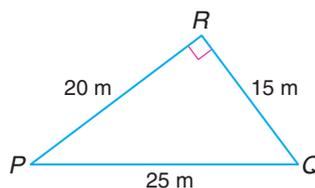


Find each trigonometric ratio. Round to four decimal places, if necessary.

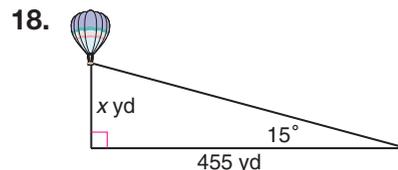
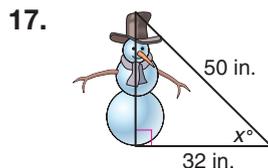
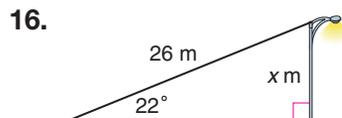
13. $\sin Q$

14. $\tan P$

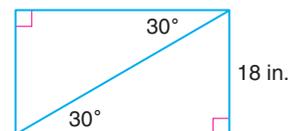
15. $\cos Q$



Find each missing measure. Round to the nearest tenth.



19. **Pets** Vincent's rectangular hamster cage is 18 inches wide. He would like to divide the cage into two triangular areas to separate his two hamsters. How long must the divider be in order to completely separate the two areas?



Exercise 19

20. **Transportation** A train travels 5000 meters along a track whose angle of elevation has a measurement of 3° . How much did the train rise during this distance? Round to the nearest tenth.



Perimeter, Circumference, and Area Problems

Standardized test problems often ask you to calculate the perimeter, circumference, or area of geometric shapes. You need to apply formulas for triangles, quadrilaterals, and circles.

Be sure you understand the following concepts.

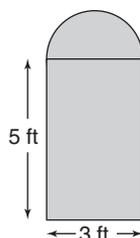
area	base	circumference
diameter	height	perimeter
radius		

Test-Taking Tip

Standardized tests usually provide formulas for perimeter, circumference, area, and volume, but you can save time by memorizing the formulas before the test.

Example 1

Maxine's family is replacing a window with one in the shape shown in the drawing. If the top is a semicircle, what is the area, to the nearest tenth, of the glass needed for the window?



Hint Break a complicated problem into smaller parts and solve each part.

Solution Find the area of the rectangle and the area of the semicircle. Then add them.

The formula for the area of a rectangle is $A = \ell w$. The area of the rectangle is $(5)(3)$ or 15 square feet.

The area of the semicircle is one-half the area of the circle. Find the area of the circle.

$$A = \pi r^2 \quad \text{The diameter is 3 feet, so}$$

$$A = (\pi)(1.5)^2 \quad \text{the radius is 1.5 feet.}$$

$$A \approx 7.1$$

The area of the circle is about 7.1 square feet.

Now find the area of the window.

$$A = (\text{area of rectangle}) + \frac{1}{2}(\text{area of circle})$$

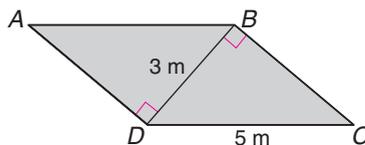
$$A \approx 15 + \frac{1}{2}(7.1)$$

$$A \approx 18.55$$

The answer is about 18.6 square feet.

Example 2

What is the area of parallelogram $ABCD$?



- (A) 12 m^2 (B) 15 m^2 (C) 18 m^2 (D) 20 m^2
 (E) It cannot be determined from the information given.

Hint Look for triangles inside quadrilaterals.

Solution Recall that the formula for the area of a parallelogram is $A = bh$. You know that the height is 3 meters, but its base is *not* 5 meters, because \overline{BD} is *not* perpendicular to \overline{DC} .

However, \overline{BD} is perpendicular to \overline{AD} . Therefore, $\triangle ABD$ is a right triangle. The hypotenuse of the triangle is 5 meters long, and one side is 3 meters long. Note that this is a 3-4-5 right triangle. So, the base is 4 meters long.

Use the formula for the area of a parallelogram.

$$A = bh \quad \text{Area formula}$$

$$A = (4)(3) \quad \text{Substitution}$$

$$A = 12 \quad \text{Simplify.}$$

The area of the parallelogram is 12 square meters.

The answer is A.

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

- If you double the length and the width of a rectangle, how does its perimeter change? (Lesson 9–2)

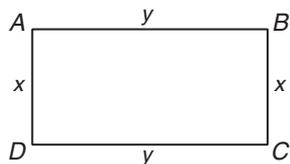
(A) It increases by $1\frac{1}{2}$.
 (B) It doubles.
 (C) It quadruples.
 (D) It does not change.
- Bacteria are growing at a rate of 1.3×10^5 per half hour. There are 5×10^5 bacteria. How many are there after 2 hours? (Algebra Review)

(A) 2.26×10^5 (B) 5.2×10^5
 (C) 6.3×10^5 (D) 1.02×10^6
- S is the set of all n such that $0 < n < 100$ and \sqrt{n} is an integer. What is the median of the members of set S ? (Statistics Review)

(A) 5 (B) 5.5 (C) 25
 (D) 50 (E) 99
- For all integers $n \neq 1$, let $\langle n \rangle = \frac{n+1}{n-1}$. Which is greatest? (Algebra Review)

(A) $\langle 0 \rangle$ (B) $\langle 2 \rangle$ (C) $\langle 3 \rangle$
 (D) $\langle 4 \rangle$ (E) $\langle 5 \rangle$
- What is the value of $\frac{2 \times 4}{36 \div 2 - 5 \times 2}$? (Algebra Review)

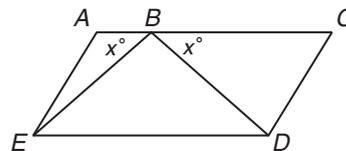
(A) -9.9 (B) 1 (C) $\frac{4}{11}$ (D) $-\frac{1}{3}$
- If the perimeter of rectangle $ABCD$ is p and $x = \frac{2}{3}y$, what is y ? (Lesson 9–2)



Note: Figure not drawn to scale.

- (A) $\frac{p}{10}$ (B) $\frac{3p}{10}$ (C) $\frac{p}{3}$ (D) $\frac{2p}{5}$ (E) $\frac{3p}{5}$

- In the figure, $\overline{AC} \parallel \overline{ED}$. If $BD = 3$, what is BE ? (Lesson 4–3)



- (A) 3 (B) 4
 (C) 5 (D) $3\sqrt{3}$
 (E) It cannot be determined from the information given.

- What is the difference between the mean of Set B and the median of Set A? (Statistics Review)

Set A: {2, -1, 7, -4, 11, 3}

Set B: {12, 5, -3, 4, 7, -7}

- (A) -0.5 (B) 0
 (C) 0.5 (D) 1

Grid In

- For a July 4th celebration, members of the school band wrap red, white, and blue ribbons around a circular bandstand. Its radius is 25 feet. If each colored ribbon is used once around the bandstand, about how many feet of ribbon are needed to encircle it? (Lesson 11–5)

Extended Response

- Mr. Huang has a rectangular garden that measures 15 meters by 20 meters. He wants to build a concrete walk of the same width around the garden. His budget for the project allows him to buy enough concrete to cover an area of 74 m^2 . (Lesson 1–6)

Part A Draw a diagram of the walk.

Part B How wide can he build the walk?

