CHAPTER

15 Formalizing Proof

What You'll Learn

Key Ideas

- Find the truth values of simple and compound statements. *(Lesson 15–1)*
- Use the Law of Detachment and the Law of Syllogism in deductive reasoning. *(Lesson 15–2)*
- Use paragraph proofs, two-column proofs, and coordinate proofs to prove theorems. *(Lessons 15–3, 15–5, and 15–6)*
- Use properties of equality in algebraic and geometric proofs. *(Lesson 15–4)*

Key Vocabulary

coordinate proof *(p. 660)*
deductive reasoning *(p. 639)*
paragraph proof *(p. 644)*
proof *(p. 644)*
two-column proof *(p. 649)*

Why It's Important

Engineering Designers and engineers use Computer-Aided Design, or CAD, to prepare drawings and make specifications for products in fields such as architecture, medical equipment, and automotive design. An engineer can also use CAD to simulate an operation to test a design, rather than building a prototype.

Geometric proof allows you to determine relationships among figures. You will investigate a CAD design in Lesson 15–6.

CONTENTS

Study these lessons to improve your skills.

✓Check Your Readiness

✓ **Lesson 1–4, pp. 24–28**

Identify the hypothesis and conclusion of each statement.

1. If it rains, then we will not have soccer practice.
2. If two segments are congruent, then they have the same measure.
3. I will advance to the semi-finals if I win this game.
4. All dogs are mammals.

✓ **Lesson 2–3, pp. 62–67**

Determine whether each statement is *true* or *false.* Explain your reasoning.

5. If $AB = DE$, then $\overline{AB} \cong \overline{DE}$.
6. If $\overline{MN} \cong \overline{OP}$ and $\overline{MN} \cong \overline{RT}$, then $\overline{OP} \cong \overline{RT}$.
7. If $\overline{JK} \cong \overline{KL}$, then K is the midpoint of $\overline{JL}$.
8. If $\overline{XY} \cong \overline{WZ}$ and $\overline{WZ} \cong \overline{WY}$, then $\overline{XZ} \cong \overline{WY}$.

✓ **Lesson 6–7, pp. 262–267**

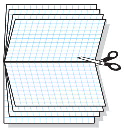
Find the distance between each pair of points. Round to the nearest tenth, if necessary.

9. $A(2, 1), B(5, 5)$
10. $C(-3, -2), D(2, 10)$
11. $E(0, 4), F(-3, 1)$
12. $G(9, 9), H(-1, 8)$
13. $I(-2, -4), J(-6, -5)$
14. $K(12, 10), (-5, -6)$

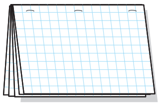
FOLDABLES™ Study Organizer

Make this Foldable to help you organize your Chapter 15 notes. Begin with four sheets of $8\frac{1}{2}$" by 11" grid paper.

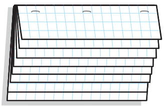
➊ **Fold** each sheet in half along the width. Then cut along the crease.



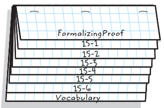
➋ **Staple** the eight half-sheets together to form a booklet.



➌ **Cut** seven lines from the bottom of the top sheet, six lines from the second sheet, and so on.



➍ **Label** each tab with a lesson title. The last tab is for vocabulary.



Reading and Writing As you read and study the chapter, use the pages to write main ideas, theorems, and examples for each lesson.

15–1 Logic and Truth Tables

What You'll Learn

You'll learn to find the truth values of simple and compound statements.

Why It's Important

Advertising Advertisers use conditional statements to sell products. *See Exercise 35.*

Every time you take a true-false test, you are using a building block of logic. Here's an example.

True or false:
Albany is the capital of New York.

A **statement** is any sentence that is either true or false, but not both. Therefore, every statement has a **truth value**, true (T) or false (F). The map shows that the statement above is true.

A convenient way to represent a statement is with a letter such as p or q.

p: Albany is the capital of New York.

Suppose you want to say that Albany is *not* the capital of New York. This statement is called *not* p.

not p: Albany is *not* the capital of New York.

The statement *not* p is the **negation** of p.

Definition of Negation

Words: If a statement is represented by p, then *not* p is the negation of the statement.

Symbols: $\sim p$

Examples

Let p represent "It is raining" and q represent "$15 - 8 = 5$." Write the statements for each negation.

① $\sim p$

p: It is raining.
$\sim p$: It is *not* raining.

② $\sim q$

q: $15 - 8 = 5$
$\sim q$: $15 - 8 \neq 5$

Your Turn

Let r represent "Today is Monday" and s represent "$4 + 3 = 7$."

a. $\sim r$

b. $\sim s$

CONTENTS

There is a relationship between the truth value of a statement and its negation. If a statement is true, its negation is false. If a statement is false, its negation is true. The truth values can be organized in a **truth table** like the one shown below.

Negation	
p	$\sim p$
T	F
F	T

If p is a true statement, then $\sim p$ is a false statement.
If p is a false statement, then $\sim p$ is a true statement.

Any two statements can be joined to form a **compound statement**. Consider the following two statements.

p: I am taking geometry. q: I am taking Spanish.

The two statements can be joined by the word *and*.

p *and* q: I am taking geometry, *and* I am taking Spanish.

Definition of Conjunction

Words: A **conjunction** is a compound statement formed by joining two statements with the word *and*.

Symbols: $p \wedge q$

The two statements can also be joined by the word *or*.

p *or* q: I am taking geometry, *or* I am taking Spanish.

Definition of Disjunction

Words: A **disjunction** is a compound statement formed by joining two statements with the word *or*.

Symbols: $p \vee q$

A conjunction is true only when *both* of the statements are true. In this case, the conjunction is true only if you are taking both geometry and Spanish. The disjunction is true if you are taking either geometry or Spanish, or both. In this case, the disjunction is false only if you are taking neither geometry nor Spanish. This information is summarized in the truth tables below.

Conjunction		
p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

A conjunction is true only when both statements are true.

Disjunction		
p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

A disjunction is false only when both statements are false.

Examples

Let p represent "10 + 3 = 13", q represent "June has 31 days," and r represent "A triangle has three sides." Write the statement for each conjunction or disjunction. Then find the truth value.

3 $p \wedge q$

10 + 3 = 13 and June has 31 days.

$p \wedge q$ is false because p is true and q is false.

4 $p \vee r$

10 + 3 = 13 or a triangle has three sides.

$p \vee r$ is true because both p and r are true.

5 $\sim q \wedge r$

June does not have 31 days and a triangle has three sides.

Because q is false, $\sim q$ is true. Therefore, $\sim q \wedge r$ is true because both $\sim q$ and r are true.

Your Turn

c. $q \wedge r$ **d.** $p \vee q$ **e.** $\sim p \vee q$

You can use truth values for conjunctions and disjunctions to construct truth tables for more complex compound statements.

Example

6 **Construct a truth table for the conjunction $p \wedge \sim q$.**

Step 1 Make columns with the headings p, q, $\sim q$, and $p \wedge \sim q$.

Step 2 List all of the possible combinations of truth values for p and q.

Step 3 Use the truth values for q to write the truth values for $\sim q$.

Step 4 Use the truth values for p and $\sim q$ to write the truth values for $p \wedge \sim q$.

Step 1 ⟶

p	q	$\sim q$	$p \wedge \sim q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

Step 2 Step 3 Step 4

Look Back

Conditional Statements, Lesson 1–4

Your Turn

f. Construct a truth table for the disjunction $\sim p \vee q$.

In this text, you have been using another compound statement that is formed by joining statements with *if. . . . then*. Recall that these statements are called *conditional statements*. Consider the following statements.

p: A figure is a rectangle. q: The diagonals are congruent.

If p, then q: *If* a figure is a rectangle, *then* the diagonals are congruent.

When is a conditional statement true? If a figure is a rectangle and its diagonals are congruent, the statement is true. If the figure is a rectangle, but its diagonals are *not* congruent, the statement is false.

Reading Geometry

Read $p \rightarrow q$ as *if p, then q*. The letter p represents the hypothesis, and the letter q represents the conclusion.

If the figure is *not* a rectangle, it is not possible to tell whether the diagonals are congruent. In this case, we will consider the conditional to be true.

A truth table for conditional statements is shown at the right.

Conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

A conditional is false only when p is true and q is false.

In Chapter 1, you learned about the *converse* of a conditional. The converse is formed by exchanging the hypothesis and the conclusion.

Conditional: *If* a figure is a rectangle, *then* the diagonals are congruent.

Converse: *If* the diagonals are congruent, *then* the figure is a rectangle.

Using symbols, if $p \rightarrow q$ is a conditional, $q \rightarrow p$ is its converse.

Example 7 **Construct a truth table for the converse $q \rightarrow p$.**

Converse

p	q	$q \rightarrow p$
T	T	T
T	F	T
F	T	F
F	F	T

The converse of a conditional is false when p is false and q is true.

Your Turn

g. The **inverse** of a conditional is formed by negating both p and q. So, if $p \rightarrow q$ is the conditional, $\sim p \rightarrow \sim q$ is its inverse. Construct a truth table for the inverse $\sim p \rightarrow \sim q$.

Check for Understanding

Communicating Mathematics

1. **Explain** the difference between a conjunction and a disjunction.

2. **Write** a compound sentence that meets each set of conditions.
   - a. a true conjunction
   - b. a false disjunction
   - c. a true conditional

Vocabulary

statement
truth value
negation
truth table
compound statement
conjunction
disjunction
inverse

Guided Practice

Getting Ready **Tell whether each statement is *true* or *false*.**

Sample: Abraham Lincoln was a president of the United States.
Solution: This statement is *true*.

3. $5 + 6 = 14$
4. France is a country in South America.
5. 0.5 is a rational number.

Examples 1 & 2

Let p represent "$5 + 8 = 13$" and q represent "Mark Twain is a famous author." Write the statements for each negation.

6. $\sim p$
7. $\sim q$

Examples 3–5

Let r represent "A square has congruent sides," s represent "A scalene triangle has congruent sides," and t represent "A parallelogram has parallel sides." Write a statement for each conjunction or disjunction. Then find the truth value.

8. $r \wedge s$
9. $r \vee t$
10. $\sim r \vee s$

Examples 6 & 7

Construct a truth table for each compound statement.

11. $p \vee \sim q$
12. $\sim p \rightarrow q$

Example 7

13. **Advertising** A cat food company's slogan is *If you love your cat, feed her Tasty Bits*. Let p represent "you love your cat" and q represent "feed her Tasty Bits." If p is false and q is true, find the truth value of $q \rightarrow p$.

Exercises

Practice

Use conditionals p, q, r, and s for Exercises 14–25.

p: Water freezes at 32°F. **q: Memorial Day is in July.**
r: $20 \times 5 = 90$ **s: A pentagon has five sides.**

Write the statements for each negation.

14. $\sim p$
15. $\sim q$
16. $\sim r$
17. $\sim s$

CONTENTS

Homework Help

For Exercises	See Examples
14–17	1, 2
18–25	3–5
26–29, 32–34	6
30, 31, 35	7
36, 37	6, 7

Extra Practice

See page 754.

Write a statement for each conjunction or disjunction. Then find the truth value.

18. $p \vee q$
19. $p \wedge q$
20. $q \vee r$
21. $p \wedge s$
22. $\sim p \vee r$
23. $\sim p \wedge \sim s$
24. $\sim q \wedge s$
25. $\sim q \vee \sim r$

Construct a truth table for each compound statement.

26. $\sim p \vee \sim q$
27. $\sim(p \vee q)$
28. $\sim p \wedge q$
29. $\sim p \wedge \sim q$
30. $p \rightarrow \sim q$
31. $\sim p \rightarrow q$
32. $\sim(p \vee \sim q)$
33. $\sim(\sim p \wedge q)$

Applications and Problem Solving

34. **Geography** Use the map on page 632 to determine whether each statement is true or false.
    - a. Albany is *not* located on the Hudson River.
    - b. Either Rochester or Syracuse is located on Lake Ontario.
    - c. It is false that Buffalo is located on Lake Erie.

35. **Advertising** *If you want clear skin, use Skin-So-Clear.*
    - a. Write the converse of the conditional.
    - b. What do you think the advertiser wants people to conclude about Skin-So-Clear?
    - c. Is the conclusion in Exercise 35b valid? Explain.

36. **Critical Thinking** The **contrapositive** of $p \rightarrow q$ is $\sim q \rightarrow \sim p$.
    - a. Construct a truth table for the contrapositive $\sim q \rightarrow \sim p$.
    - b. Two statements are **logically equivalent** if their truth tables are the same. Compare the truth tables for a conditional, converse, inverse, and contrapositive. Which of the statements is logically equivalent to a conditional?

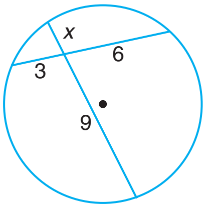
37. **Critical Thinking** The conjunction $(p \rightarrow q) \wedge (q \rightarrow p)$ is called a **biconditional**. For which values of p and q is a biconditional true?

Mixed Review

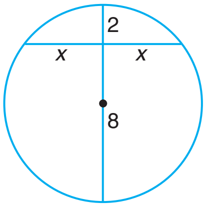
38. Write the equation of a circle with center $C(-2, 3)$ and a radius of 3 units. *(Lesson 14–6)*

Find each value of x. *(Lesson 14–5)*

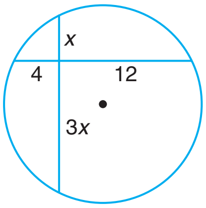
39.



40.

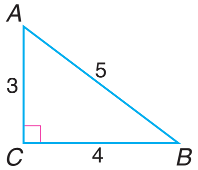


41.



Find each ratio in $\triangle ABC$. *(Lesson 13–5)*

42. sin A
43. sin B
44. cos A
45. cos B



Exercises 42–45

Standardized Test Practice

46. **Short Response** Draw a diagram in which the angle of depression to an object is 30°. *(Lesson 13–4)*

CONTENTS

15–2 Deductive Reasoning

What You'll Learn

You'll learn to use the Law of Detachment and the Law of Syllogism in deductive reasoning.

Why It's Important

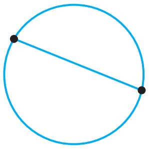
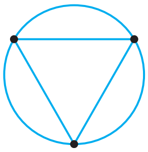
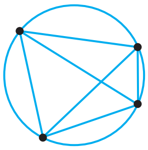
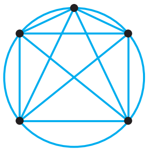
Literature Mystery writers use logical arguments. *See Exercise 25.*

When you make a prediction based on a pattern in data, you are using inductive reasoning. Inductive reasoning is very useful for developing conjectures in mathematics. In this text, you have developed the foundation of geometric definitions, postulates, and theorems using inductive reasoning with the help of models. Here's an example.

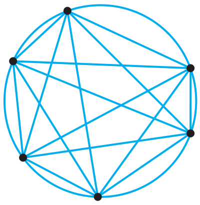
Suppose you place six points on a circle and draw each segment that connects a pair of points. What is the greatest number of regions within the circle that are formed by the segments? Look for a pattern.

Look Back

Inductive Reasoning: Lesson 1–1

Model				
Points	2	3	4	5
Regions	2	4	8	16

Make a conjecture about the number of regions formed by six points. It seems that the number of regions increases by a power of 2. Based on the pattern, you can reason that 6 points should determine 32 regions. Now, test your conjecture by counting the regions in the figure below.



Remember that only one counterexample is needed to disprove a conjecture.

The maximum number of regions is only 31. The counterexample shows that the conjecture based on inductive reasoning did not give the correct conclusion.

Even though patterns can help you make a conjecture, patterns alone do *not* guarantee that the conjecture will be true. In logic you can *prove* that a statement is true for all cases by using deductive reasoning. **Deductive reasoning** is the process of using facts, rules, definitions, or properties in a logical order.

CONTENTS

Here's an example of a conclusion that is arrived at by deductive reasoning using a conditional statement.

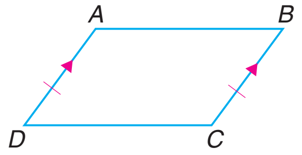
Words	Symbols	Meaning
If Marita obeys the speed limit, then she will not get a speeding ticket.	$p \rightarrow q$	If p is true, then q is true.
Marita obeyed the speed limit.	p	p is true.
Therefore, Marita did not get a speeding ticket.	q	Therefore, q is true.

In deductive reasoning, if you arrive at the conclusion using correct reasoning, then the conclusion is valid. The rule that allows us to reach a conclusion from conditional statements in the example is called the **Law of Detachment**.

Law of Detachment	If $p \rightarrow q$ is a true conditional and p is true, then q is true.

Examples

Use the Law of Detachment to determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, write *no valid conclusion*.



① **(1) If $\overline{AD} \parallel \overline{CB}$ and $\overline{AD} \cong \overline{CB}$, then $ABCD$ is a parallelogram.**

(2) $\overline{AD} \parallel \overline{CB}$ and $\overline{AD} \cong \overline{CB}$.

Let p and q represent the parts of the statements.

p: $\overline{AD} \parallel \overline{CB}$ and $\overline{AD} \cong \overline{CB}$

q: $ABCD$ is a parallelogram

Statement (1) indicates that $p \rightarrow q$ is true, and statement (2) indicates that p is true. So, q is true. Therefore, $ABCD$ is a parallelogram.

② **(1) If a figure is a square, it has four right angles.**

(2) A figure has four right angles.

p: a figure is a square

q: a figure has four right angles

Statement (1) is true, but statement (2) indicates that q is true. It does not provide information about p. Therefore, there is no valid conclusion.

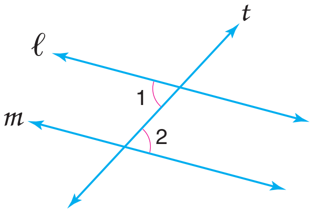
(continued on the next page)

Your Turn

a. Use the Law of Detachment to determine a conclusion that follows from statements (1) and (2).

(1) In a plane, if two lines are cut by a transversal so that a pair of alternate interior angles is congruent, then the lines are parallel.

(2) Lines ℓ and m are cut by transversal t and $\angle 1 \cong \angle 2$.



Look Back

Transitive Property: Lesson 2–2

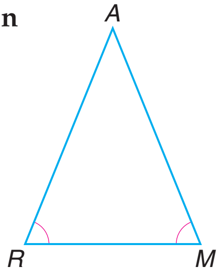
Another rule of logic is the **Law of Syllogism**. This rule is similar to the Transitive Property of Equality.

Law of Syllogism If $p \rightarrow q$ and $q \rightarrow r$ are true conditionals, then $p \rightarrow r$ is also true.

Example **3** **Use the Law of Syllogism to determine a conclusion that follows from statements (1) and (2).**

(1) If $\angle R \cong \angle M$, then $\triangle RAM$ is an isosceles triangle.

(2) If $\triangle RAM$ is an isosceles triangle, then $\overline{AR} \cong \overline{AM}$.



Let p, q, and r represent the parts of the statements.

p: $\angle R \cong \angle M$

q: $\triangle RAM$ is an isosceles triangle

r: $\overline{AR} \cong \overline{AM}$

Use the Law of Syllogism to conclude $p \rightarrow r$.

Therefore, if $\angle R \cong \angle M$, then $\overline{AR} \cong \overline{AM}$.

Your Turn

b. (1) If a triangle is a right triangle, the sum of the measures of the acute angles is 90.

(2) If the sum of the measures of two angles is 90, then the angles are complementary.

Check for Understanding

Communicating Mathematics

1. **Explain** the difference between inductive and deductive reasoning.
2. **Write** your own example to illustrate the correct use of the Law of Syllogism.

Vocabulary

deductive reasoning
Law of Detachment
Law of Syllogism

CONTENTS

3. **You Decide** Joel and Candace found the conclusion to this conditional using the Law of Detachment.

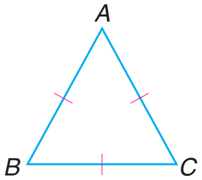
If crocuses are blooming, it must be spring.
Crocuses are not blooming.

Joel said the conclusion is *It must not be spring*. Candace said there is no valid conclusion. Who is correct? Explain your reasoning.

Guided Practice

Use the Law of Detachment to determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, write *no valid conclusion*.

Examples 1 & 2



Exercise 4

4. (1) If a triangle is equilateral, then the measure of each angle is 60.
   (2) $\triangle ABC$ is an equilateral triangle.
5. (1) If Amanda is taller than Teresa, then Amanda is at least 6 feet tall.
   (2) Amanda is older than Teresa.

Example 3

Use the Law of Syllogism to determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, write *no valid conclusion*.

6. (1) If I work part-time, I will save money.
   (2) If I save money, I can buy a computer.
7. (1) If two angles are vertical angles, then they are congruent.
   (2) If two angles are congruent, then their supplements are congruent.

Examples 1 & 2

8. **Media** The following statement is part of a message frequently played by radio stations across the country.

   If this had been an actual emergency, the attention signal you just heard would have been followed by official information, news, or instruction.

   Suppose there were an actual emergency. What would you expect to happen?

Exercises

Practice

Use the Law of Detachment to determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, write *no valid conclusion.*

9. (1) If I lose my textbook, I will fail my math test.
   (2) I did not lose my textbook.
10. (1) If x is an integer, then x is a real number.
    (2) x is an integer.

CONTENTS

Homework Help

For Exercises	See Examples
9–14, 25	1, 2
15–20	3

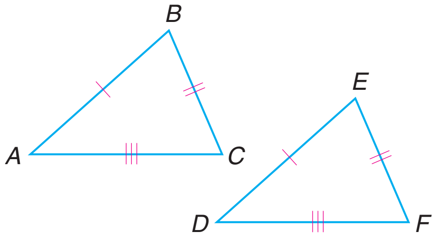
Extra Practice

See page 754.

Use the Law of Detachment to determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, write *no valid conclusion.*

11. (1) If two odd numbers are added, their sum is an even number.
(2) 5 and 3 are added.

12. (1) If three sides of one triangle are congruent to three corresponding sides of another triangle, then the triangles are congruent.
(2) In $\triangle ABC$ and $\triangle DEF$, $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{CA} \cong \overline{FD}$.



13. (1) If the measure of an angle is less than 90, it is an acute angle.
(2) $m\angle B = 45$.

14. (1) If a figure is a rectangle, then its opposite sides are congruent.
(2) $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$.

Use the Law of Syllogism to determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, write *no valid conclusion*.

15. (1) If a parallelogram has four congruent sides, it is a rhombus.
(2) If a figure is a rhombus, then the diagonals are perpendicular.

16. (1) If it is sunny tomorrow, I'll go swimming.
(2) If I go swimming, I'll miss the baseball game.

17. (1) If Morgan studies hard, she'll get a good grade on her test.
(2) If Morgan studies hard, she'll miss her favorite television show.

18. (1) If M is the midpoint of $\overline{AB}$, then $AM = MB$.
(2) If the measures of two segments are equal, then they are congruent.

19. (1) All integers are rational numbers.
(2) All integers are real numbers.

20. (1) All cheerleaders are athletes.
(2) All athletes can eat at the training table at lunch.

Determine whether each situation is an example of inductive or deductive reasoning.

21. Lessie's little sister found a nest of strange eggs near the beach. The first five eggs hatched into lizards. She concluded that all of the eggs were lizard eggs.

22. Carla has had a quiz in science every Friday for the last two months. She concludes that she will have a quiz this Friday.

23. Vincent's geometry teacher told his classes at the beginning of the year that there would be a quiz every Friday. Vincent concluded that he will have a quiz this Friday.

24. A number is divisible by 4 if its last two digits make a number that is divisible by 4. Dena concluded that 624 is divisible by 4.

CONTENTS

Applications and Problem Solving

25. **Literature** Sherlock Holmes was a master of deductive reasoning. Consider this argument from *The Hound of the Baskervilles*.

> If the initials C.C.H. mean Charing Cross Hospital, then the owner is a physician. The initials C.C.H. mean Charing Cross Hospital.

What conclusion can you draw from this argument?

26. **Logic** There are three women—Alicia, Brianne, and Charlita—each of whom has two occupations from the following: doctor, engineer, teacher, painter, writer, and lawyer. No two have the same occupation.

- The doctor had lunch with the teacher.
- The teacher and writer went to the movies with Alicia.
- The painter is related to the engineer.
- Brianne lives next door to the writer.
- The doctor hired the painter to do a job.
- Charlita beat Brianne and the painter at tennis.

Which two occupations does each woman have?

Sherlock Holmes with Dr. Watson in *The Hound of the Baskervilles*

27. **Critical Thinking** In addition to being the author of *Alice in Wonderland*, Lewis Carroll also wrote a book called *Symbolic Logic*. What conclusion can you draw from the following argument that is adapted from his book on logic?

> Babies are illogical.
> Nobody is despised who can manage a crocodile.
> Illogical people are despised.

Mixed Review

Let *p* represent "Dogs are mammals," *q* represent "Snakes are reptiles," and *r* represent "Birds are insects." Write a statement for each compound sentence. Then find the truth value. *(Lesson 15–1)*

28. $p \wedge \sim r$

29. $p \vee q$

30. $q \rightarrow r$

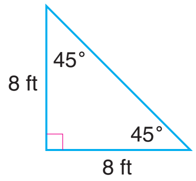
Find the coordinates of the center and measure of the radius of each circle whose equation is given. *(Lesson 14–6)*

31. $(x - 3)^2 + (y - 5)^2 = 1$

32. $(x + 5)^2 + y^2 = 49$

Standardized Test Practice

33. **Grid In** A small kitchen garden is shaped like a 45°-45°-90° triangle. If the legs of the triangle each measure 8 feet, find the length of the hypotenuse to the nearest tenth of a foot. *(Lesson 13–2)*



34. **Multiple Choice** Suppose a triangle has two sides measuring 12 units and 15 units. If the third side has a length of *x* units, which inequality must be true? *(Lesson 7–4)*

Ⓐ $4 < x < 26$ Ⓑ $4 < x < 29$ Ⓒ $3 < x < 27$ Ⓓ $2 < x < 27$

15–3 Paragraph Proofs

What You'll Learn

You'll learn to use paragraph proofs to prove theorems.

Why It's Important

Law When prosecuting attorneys present closing arguments in trials, they are using a form of paragraph proof. *See Exercise 15.*

In mathematics, proofs are used to show that a conjecture is valid. A **proof** is a logical argument in which each statement you make is backed up by a reason that is accepted as true. Throughout this text, some informal proofs have been presented, and you have been preparing to write proofs. In the next few lessons, you will learn to write them.

One type of proof is a **paragraph proof**. In this kind of proof, you write your statements and reasons in paragraph form. The following is a paragraph proof of a theorem you studied in Lesson 5–2.

Conjecture: If $\triangle PQR$ is an equiangular triangle, then the measure of each angle is 60.

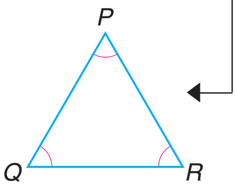
Paragraph Proof

The given information comes from the hypothesis of the conditional. It is the starting point of the proof.

The statement you want to prove comes from the conclusion of the conditional.

In geometry, a proof usually includes a figure. It may be provided, or you may need to draw it.

Given: $\triangle PQR$ is an equiangular triangle.
Prove: The measure of each angle is 60.



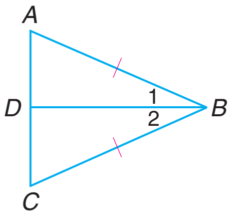
Definitions, postulates, and previously proven theorems can be used to justify each statement.

You know that $\triangle PQR$ is an equilangular triangle. All of the angles of an equiangular triangle are congruent. The Angle Sum Theorem states that the sum of the measures of the angles of a triangle is 180. Since all of the angles have equal measure, the measure of each angle is 180 ÷ 3 or 60. Therefore, the measure of each angle of an equiangular triangle is 60.

Before you begin to write a paragraph proof, you should make a plan. One problem-solving strategy that you might use is *work backward*. Start with what you want to prove, and work backward step-by-step until you can decide on a plan for completing the proof.

Examples

Write a paragraph proof for each conjecture.



1 **In $\triangle ABC$, if $\overline{AB} \cong \overline{CB}$ and D is the midpoint of $\overline{AC}$, then $\angle 1 \cong \angle 2$.**

Given: $\overline{AB} \cong \overline{CB}$
D is the midpoint of $\overline{AC}$.

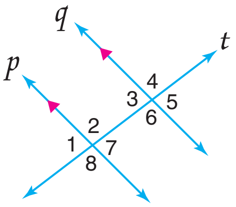
Prove: $\angle 1 \cong \angle 2$

Plan: $\angle 1 \cong \angle 2$ if they are corresponding parts of congruent triangles. Try to prove $\triangle ABD \cong \triangle CBD$ by SSS, SAS, ASA, or AAS.

Look Back

Isosceles Triangles: Lesson 6–4,
SAS: Lesson 5–5

You know that $\overline{AB} \cong \overline{CB}$. If two sides of a triangle are congruent, then the angles opposite those sides are congruent. So, $\angle BAD \cong \angle BCD$. Also, $\overline{DA} \cong \overline{DC}$ because D is the midpoint of $\overline{AC}$. Since $\overline{AB} \cong \overline{CB}$, $\angle BAD \cong \angle BCD$, and $\overline{DA} \cong \overline{DC}$, the triangles are congruent by SAS. Therefore, $\angle 1 \cong \angle 2$ because corresponding parts of congruent triangles are congruent (*CPCTC*).



2 **If $p \parallel q$, then $\angle 1$ is supplementary to $\angle 4$.**

Given: $p \parallel q$

Prove: $\angle 1$ is supplementary to $\angle 4$

Plan: $\angle 1$ is supplementary to $\angle 4$ if $m\angle 1 + m\angle 4 = 180$. Use corresponding angles and linear pairs to show that the angles are supplementary.

Look Back

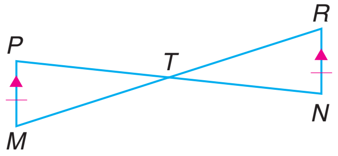
Corresponding Angles: Lesson 4–3,
Supplementary Angles: Lesson 3–5

You know that $p \parallel q$. If two parallel lines are cut by a transversal, their corresponding angles are congruent. So, $\angle 1 \cong \angle 3$. Also, $\angle 3$ and $\angle 4$ are supplementary because they are a linear pair. Since $m\angle 3 + m\angle 4 = 180$ and $m\angle 1 = m\angle 3$, $m\angle 1 + m\angle 4 = 180$ by substitution. Therefore, $\angle 1$ and $\angle 4$ are supplementary.

Your Turn

If $\overline{PM} \parallel \overline{RN}$ and $\overline{PM} \cong \overline{RN}$, then $\triangle MPT \cong \triangle RNT$.

Plan: Use alternate interior angles to show $\angle P \cong \angle N$ or $\angle M \cong \angle R$.



Check for Understanding

Vocabulary

proof
paragraph proof

Communicating Mathematics

1. **List** three things that can be used to justify a statement in a paragraph proof.

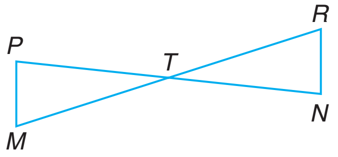
2. **Explain** how deductive reasoning is used in paragraph proofs.

Guided Practice

Write a paragraph proof for each conjecture.

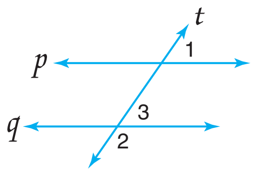
Example 1

3. If T bisects $\overline{PN}$ and $\overline{RM}$, then $\angle M \cong \angle R$.
Plan: Use a triangle congruence postulate.



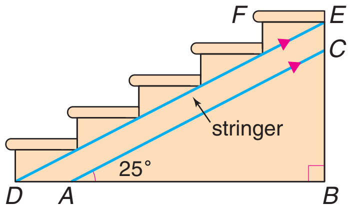
Example 2

4. If p and q are cut by transversal t, and $\angle 1$ is supplementary to $\angle 2$, then $p \parallel q$.



Example 2

5. **Carpentry** A carpenter is building a flight of stairs. The tops of the steps are parallel to the floor, and the bottom of the stringer makes a 25° angle with the floor. Prove that the top of the steps makes a 25° angle with the top of the stringer.



Exercises

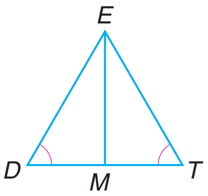
Practice

Write a paragraph proof for each conjecture.

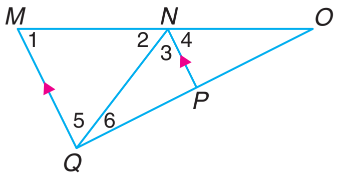
Homework Help	
For Exercises	See Examples
6–10, 12–14	1
7	2

Extra Practice
See page 754.

6. If $\angle D \cong \angle T$ and M is the midpoint of $\overline{DT}$, then $\triangle DEM \cong \triangle TEM$.
Plan: Use a triangle congruence postulate.

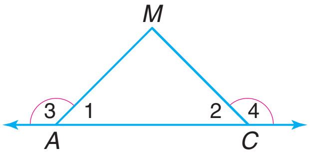


7. If $\overline{MQ} \parallel \overline{NP}$ and $m\angle 4 = m\angle 3$, then $m\angle 1 = m\angle 5$.
Plan: Use corresponding angles and alternate interior angles.

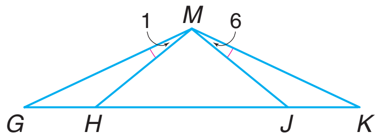


CONTENTS

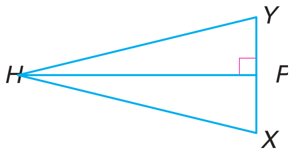
8. If $\angle 3 \cong \angle 4$, then $\overline{MA} \cong \overline{MC}$.



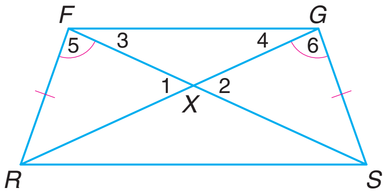
9. If $\triangle GMK$ is an isosceles triangle with vertex $\angle GMK$ and $\angle 1 \cong \angle 6$, then $\triangle GMH \cong \triangle KMJ$.



10. If $\overline{PH}$ bisects $\angle YHX$ and $\overline{HP} \perp \overline{YX}$, then $\triangle YHX$ is an isosceles triangle.



11. If $\angle 5 \cong \angle 6$ and $\overline{FR} \cong \overline{GS}$, then $\angle 4 \cong \angle 3$.



Draw and label a figure for each conjecture. Then write a paragraph proof.

12. In quadrilateral $EFGH$, if $\overline{EF} \cong \overline{GH}$ and $\overline{EH} \cong \overline{GF}$, then $\triangle EFH \cong \triangle GHF$.

13. If an angle bisector of a triangle is also an altitude, then the triangle is isosceles.

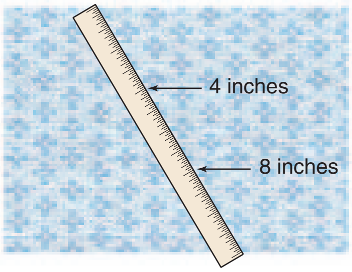
14. The medians drawn to the congruent sides of an isosceles triangle are congruent.

Applications and Problem Solving

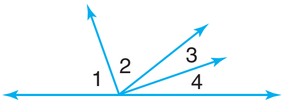
15. Law When a prosecuting attorney presents a closing argument in a trial, he or she gives a summary of the trial. Explain how the closing argument is like a paragraph proof.

An attorney presenting a summary to the jury

16. Sewing Abby needs to divide a rectangular piece of fabric into three strips, each having the same width. The width of the fabric is 10.5 inches. Instead of dividing 10.5 by 3, Abby angles her ruler as shown in the figure, divides 12 by 3, and makes marks at 4 inches and 8 inches. Explain why this method divides the fabric into three strips having the same width.



17. **Critical Thinking** What conclusion can you draw about the sum of $m\angle 1$ and $m\angle 4$ if $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$?



Mixed Review

18. If two lines are perpendicular, then they form four right angles. Lines ℓ and m are perpendicular. What conclusion can you derive from these statements? *(Lesson 15–2)*

Construct a truth table for each compound statement. *(Lesson 15–1)*

19. $\sim p \wedge \sim q$
20. $\sim q \rightarrow \sim p$
21. $p \vee \sim q$

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

22. **Short Response** Simplify $\sqrt{8} \cdot \sqrt{9}$. *(Lesson 13–1)*

23. **Multiple Choice** The median of a trapezoid is 8 meters. The height of the trapezoid is 4 meters. How is the area of this trapezoid changed when the median is doubled? *(Lesson 10–4)*

Ⓐ The area is halved.
Ⓑ The area is not changed.
Ⓒ The area is doubled.
Ⓓ The area is tripled.

Quiz 1 Lessons 15–1 through 15–3

Suppose *p* is a true statement and *q* is a false statement. Find the truth value of each compound statement. *(Lesson 15–1)*

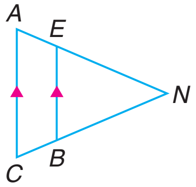
1. $p \rightarrow q$
2. $p \vee q$

Use the Law of Detachment or the Law of Syllogism to determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, write *no valid conclusion*. *(Lesson 15–2)*

3. (1) If school is in session, then it is not Saturday.
   (2) It is not Saturday.

4. (1) If a parallelogram has four right angles, it is a rectangle.
   (2) If a figure is a rectangle, its diagonals are congruent.

5. If $\triangle CAN$ is an isosceles triangle with vertex $\angle N$ and $\overline{CA} \parallel \overline{BE}$, write a paragraph proof that shows $\triangle NEB$ is also an isosceles triangle. *(Lesson 15–3)*



CONTENTS

15–4 Preparing for Two-Column Proofs

What You'll Learn

You'll learn to use properties of equality in algebraic and geometric proofs.

Why It's Important

Science Scientists use properties of equality when they solve formulas for a specific variable. *See Example 2.*

When you solve an equation, you are using a deductive argument. Each step can be justified by an algebraic property.

$3(y + 2) = 12$	*Given*
$3y + 6 = 12$	*Distributive Property*
$3y = 6$	*Subtraction Property of Equality*
$y = 2$	*Division Property of Equality*

Notice that the column on the left is a step-by-step process that leads to a solution. The column on the right contains the reasons for each statement.

In geometry, you can use a similar format to prove theorems. A **two-column proof** is a deductive argument with statements and reasons organized in two columns. A two-column proof and a paragraph proof contain the same information. They are just organized differently. The following is an example of a two-column proof. You may want to compare it to the paragraph proof on page 644.

Conjecture: If $\triangle PQR$ is an equiangular triangle, the measure of each angle is 60.

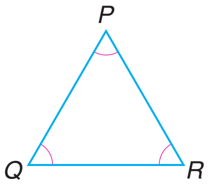
Two-Column Proof

Given: $\triangle PQR$ is an equiangular triangle.

Prove: The measure of each angle is 60.

There is a reason for each statement.

Proof:



The first statement(s) contains the given information.

Statements	**Reasons**
1. $\triangle PQR$ is an equiangular triangle.	**1.** Given
2. $\angle P \cong \angle Q \cong \angle R$	**2.** Definition of equiangular triangle
3. $m\angle P = m\angle Q = m\angle R$	**3.** Definition of congruent angles
4. $m\angle P + m\angle Q + m\angle R = 180$	**4.** Angle Sum Theorem
5. $m\angle P + m\angle P + m\angle P = 180$	**5.** Substitution Property of Equality
6. $3(m\angle P) = 180$	**6.** Combining like terms
7. $m\angle P = 60$	**7.** Division Property of Equality
8. The measure of each angle of $\triangle PQR$ is 60.	**8.** Substitution Property of Equality

The last statement is what you want to prove.

Notice that algebraic properties are used as reasons in the proof of a geometric theorem. Algebraic properties can be used because segment measures and angle measures are real numbers, so they obey the algebraic properties.

Example 1

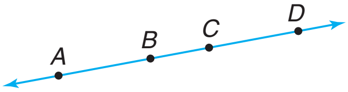
Justify the steps for the proof of the conditional.
If AC = BD, then AB = CD.

Reading Geometry

Remember that *AC*, *BD*, *AB*, and *CD* represent real numbers.

Given: $AC = BD$

Prove: $AB = CD$



Proof:

Statements	**Reasons**
1. $AC = BD$	1. ?
2. $AB + BC = AC$ $BC + CD = BD$	2. ?
3. $AB + BC = BC + CD$	3. ? *Hint: Use statements 1 and 2.*
4. $BC = BC$	4. ?
5. $AB = CD$	5. ?

Look Back

Properties of Equality: Lesson 2–2

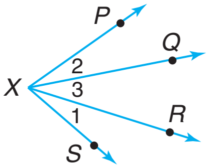
Reason 1: Given
Reason 2: Segment Addition Postulate
Reason 3: Substitution Property of Equality
Reason 4: Reflexive Property of Equality
Reason 5: Subtraction Property of Equality

Your Turn

a. Justify the steps for the proof of the conditional.
If $m\angle 1 = m\angle 2$, *then* $m\angle PXR = m\angle SXQ$.

Given: $m\angle 1 = m\angle 2$

Prove: $m\angle PXR = m\angle SXQ$



Proof:

Statements	**Reasons**
1. $m\angle 1 = m\angle 2$	1. ?
2. $m\angle 3 = m\angle 3$	2. ?
3. $m\angle 1 + m\angle 3 =$ $m\angle 2 + m\angle 3$	3. ?
4. $m\angle PXR = m\angle 2 + m\angle 3$ $m\angle SXQ = m\angle 1 + m\angle 3$	4. ?
5. $m\angle PXR = m\angle SXQ$	5. ?

Real World

Example **2**

Science Link

The formula $d = rt$ describes the relationship between distance, speed, and time. In the formula, d is the distance, r is the speed, and t is the time. Show that if $d = rt$, then $r = \frac{d}{t}$.

Given: $d = rt$

Prove: $r = \frac{d}{t}$

Proof:

Statements	Reasons
1. $d = rt$	1. Given
2. $\frac{d}{t} = \frac{rt}{t}$	2. Division Property of Equality
3. $\frac{d}{t} = r$	3. Substitution Property of Equality
4. $r = \frac{d}{t}$	4. Symmetric Property of Equality

Check for Understanding

Communicating Mathematics

1. **List** the parts of a two-column proof.
2. **Explain** why algebraic properties can be used in geometric proofs.
3. Writing Math Compare and contrast paragraph proofs and two-column proofs.

Vocabulary

two-column proof

Guided Practice

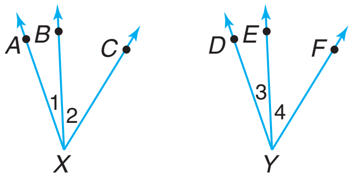
Example 1

4. Copy and complete the proof.
If $m\angle AXC = m\angle DYF$ and $m\angle 1 = m\angle 3$, then $m\angle 2 = m\angle 4$.

Given: $m\angle AXC = m\angle DYF$ and
$m\angle 1 = m\angle 3$

Prove: $m\angle 2 = m\angle 4$

Proof:



Statements	Reasons
a. $m\angle AXC = m\angle DYF$ $m\angle 1 = m\angle 3$	a. ___?___
b. $m\angle AXC = m\angle 1 + m\angle 2$ $m\angle DYF = m\angle 3 + m\angle 4$	b. ___?___
c. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	c. ___?___
d. $m\angle 3 + m\angle 2 = m\angle 3 + m\angle 4$	d. ___?___
e. $m\angle 2 = m\angle 4$	e. ___?___

Example 2

5. **Algebra** Solve the equation $-2x + 5 = -13$ by using a two-column proof.

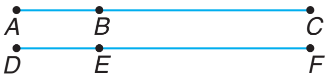
Exercises

Practice

Homework Help	
For Exercises	See Examples
6, 8	1
7, 9	2
Extra Practice	
See page 755.	

Copy and complete each proof.

6. If $AC = DF$ and $AB = DE$, then $BC = EF$.



Given: $AC = DF$ and $AB = DE$

Prove: $BC = EF$

Proof:

Statements	**Reasons**
a. $AC = DF$	a. ___?___
b. $AC = AB + BC$ $DF = DE + EF$	b. ___?___
c. $AB + BC = DE + EF$	c. ___?___
d. $AB = DE$	d. ___?___
e. $BC = EF$	e. ___?___

7. If $\frac{5x}{3} = 15$, then $x = 9$.

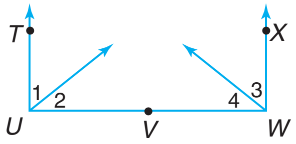
Given: $\frac{5x}{3} = 15$

Prove: $x = 9$

Proof:

Statements	**Reasons**
a. $\frac{5x}{3} = 15$	a. ___?___
b. $5x = 45$	b. ___?___
c. $x = 9$	c. ___?___

8. If $m\angle TUV = 90$, $m\angle XWV = 90$, and $m\angle 1 = m\angle 3$, then $m\angle 2 = m\angle 4$.



Given: $m\angle TUV = 90$, $m\angle XWV = 90$, and $m\angle 1 = m\angle 3$

Prove: $m\angle 2 = m\angle 4$

Proof:

Statements	**Reasons**
a. ___?___	a. Given
b. $m\angle TUV = m\angle XWV$	b. ___?___
c. $m\angle TUV = m\angle 1 + m\angle 2$ $m\angle XWV = m\angle 3 + m\angle 4$	c. ___?___
d. ___?___	d. Substitution Property, =
e. $m\angle 1 + m\angle 2 = m\angle 1 + m\angle 4$	e. ___?___
f. ___?___	f. Subtraction Property, =

CONTENTS

Applications and Problem Solving

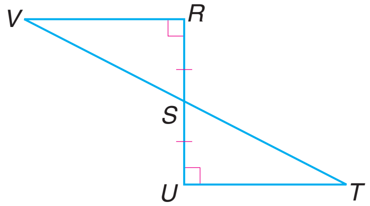
9. **Physics** The mass, force, and acceleration of a motorcycle and its rider are related by the formula $F = ma$, where F is the force, m is the mass, and a is the acceleration. Show that if $F = ma$, then $m = \frac{F}{a}$.

10. **Critical Thinking** There are ten boys lined up in gym class. They are arranged in order from the shortest to the tallest. Max is taller than Nate, Nate is taller than Rey, and Rey is taller than Ted. Brian is taller than Rey, but shorter than Nate. Mike is standing between Sal and Chet. Chet is shorter than Max but taller than Mike. Van is standing between Max and Omar. Omar is standing next to Chet. There are seven boys standing between Van and Ted. Name the ten boys in order from shortest to tallest.

Mixed Review

11. Write a paragraph proof for the conjecture. *(Lesson 15–3)*

If $\overline{VT}$ *and* $\overline{RU}$ *intersect at* S, $\overline{VR} \perp \overline{RS}$, $\overline{UT} \perp \overline{SU}$, *and* $\overline{RS} \cong \overline{US}$, *then* $\overline{VR} \cong \overline{TU}$.



12. **Biology** Use the Law of Syllogism to determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, write *no valid conclusion*. *(Lesson 15–2)*

(1) Sponges belong to the phylum porifera.

(2) Sponges are animals.

Solve each equation. *(Lesson 9–1)*

13. $\frac{120}{b} = \frac{24}{60}$

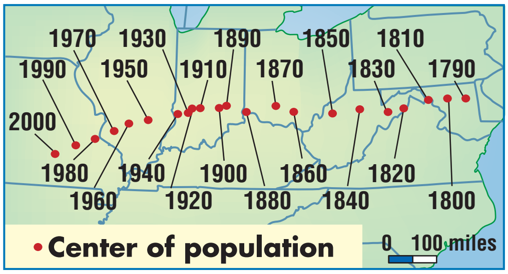
14. $\frac{n}{2} = \frac{0.7}{0.4}$

15. $\frac{18}{x + 1} = \frac{9}{4}$

interNET CONNECTION

Data Update For the latest information on population trends, visit: www.geomconcepts.com

16. **Geography** The graph shows how the center of population in the United States has changed since 1790. Half of the population lives north of this point and half lives south of it; half lives west of the point and half lives east of it. Use inductive reasoning to predict the position of the center of population in 2010. *(Lesson 1–1)*



Source: Statistical Abstract of the United States, 2003

Standardized Test Practice

17. **Multiple Choice** If x and y are positive integers and $x < y$, then $x - y$— *(Algebra Review)*

Ⓐ is positive.
Ⓑ is negative.
Ⓒ equals zero.
Ⓓ cannot be determined.

15–5 Two-Column Proofs

What You'll Learn

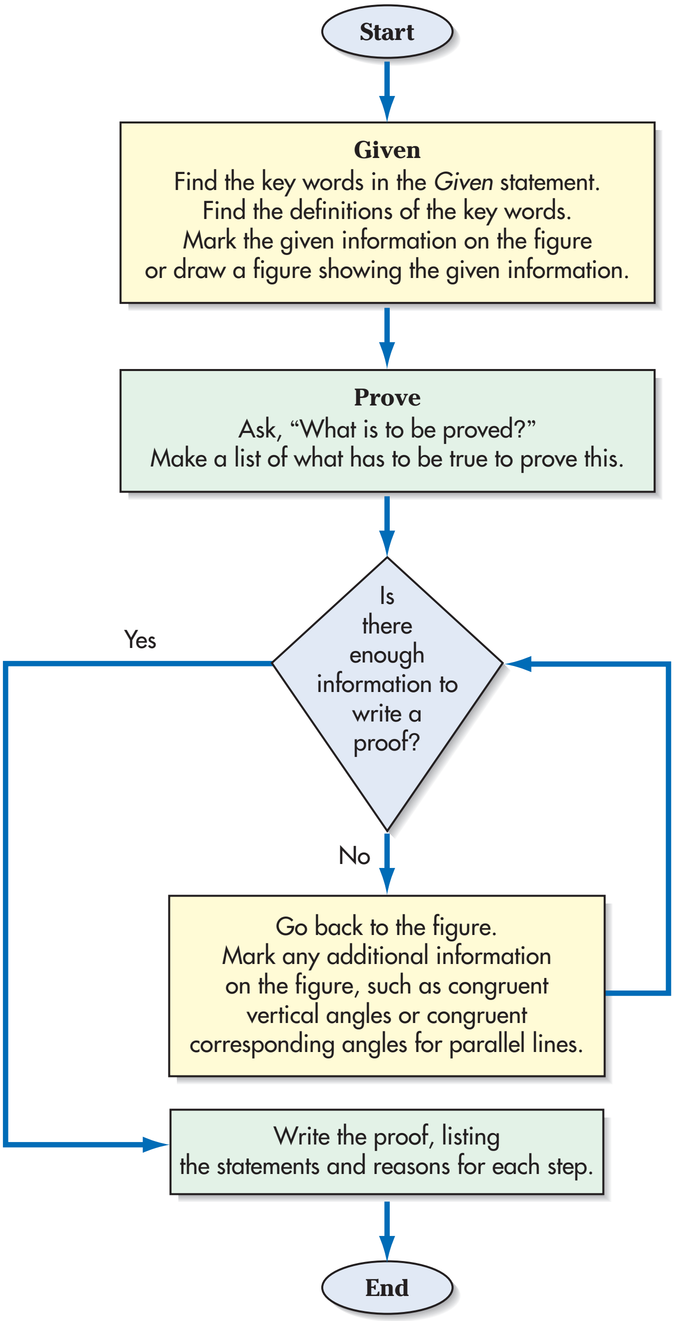
You'll learn to use two-column proofs to prove theorems.

Why It's Important

Programming
Computer programmers use a format similar to two-column proofs to plan their programs.

Before a computer programmer writes a program, a flowchart is developed to organize the main steps of the program. A flowchart helps identify what the computer program needs to do at each step.

You can use the steps in the flowchart below to organize your thoughts before you begin to write a two-column proof.



CONTENTS

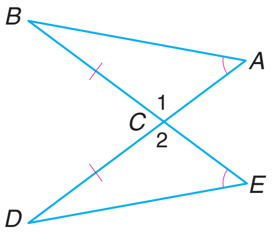
Example ①

Write a two-column proof for the conjecture.

If $\overline{BC} \cong \overline{DC}$ and $\angle A \cong \angle E$, then $\overline{AB} \cong \overline{ED}$.

Given: $\overline{BC} \cong \overline{DC}$
$\angle A \cong \angle E$

Prove: $\overline{AB} \cong \overline{ED}$



Explore You know that $\overline{BC} \cong \overline{DC}$ and $\angle A \cong \angle E$. Even though it is not mentioned in the given statement, $\angle 1 \cong \angle 2$ because they are vertical angles. Mark this information on the figure. You want to prove that $\overline{AB} \cong \overline{ED}$.

Plan $\overline{AB} \cong \overline{ED}$ if they are corresponding parts of congruent triangles. You can use the AAS Theorem to show that $\triangle BAC \cong \triangle DEC$.

Solve

Statements	Reasons
1. $\overline{BC} \cong \overline{DC}$	**1.** Given
2. $\angle A \cong \angle E$	**2.** Given
3. $\angle 1 \cong \angle 2$	**3.** Vertical angles are congruent.
4. $\triangle BAC \cong \triangle DEC$	**4.** AAS Theorem
5. $\overline{AB} \cong \overline{ED}$	**5.** CPCTC

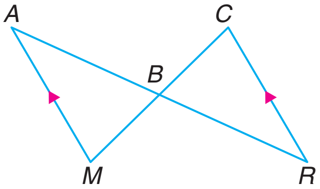
Examine Check your proof to be sure you haven't used any information that is not given or derived from definitions, postulates, or previously proved theorems. Never assume information that is not given.

Look Back

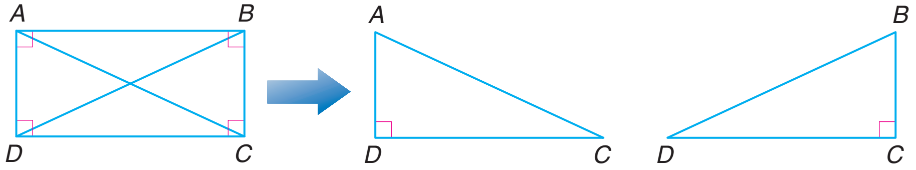
AAS Theorem:
Lesson 5–6

Your Turn

a. If $\overline{AM} \parallel \overline{CR}$ and B is the midpoint of $\overline{AR}$, then $\overline{AM} \cong \overline{RC}$.



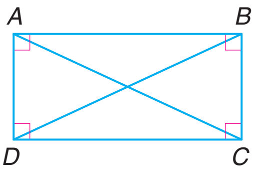
Sometimes the figure you are given contains triangles that overlap. If so, try to visualize them as two separate triangles. You may want to redraw them so they are separate.



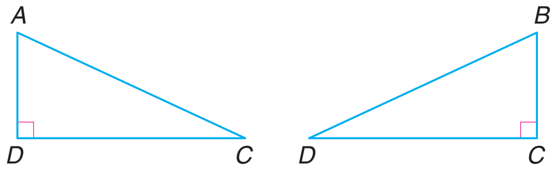
Example ② **Write a two-column proof.**

Given: $ABCD$ is a rectangle with diagonals $\overline{AC}$ and $\overline{BD}$.

Prove: $\overline{AC} \cong \overline{BD}$



Plan: Show that $\overline{AC}$ and $\overline{BD}$ are corresponding parts of congruent triangles. Redraw the figure as two separate triangles.



Statements	Reasons
1. $ABCD$ is a rectangle with diagonals $\overline{AC}$ and $\overline{BD}$.	1. Given
2. $\overline{DC} \cong \overline{DC}$	2. Congruence of segments is reflexive.
3. $\overline{AD} \cong \overline{BC}$	3. Definition of rectangle
4. $\angle ADC$ and $\angle BCD$ are right angles.	4. Definition of rectangle
5. $\triangle ADC$ and $\triangle BCD$ are right triangles.	5. Definition of right triangle
6. $\triangle ADC \cong \triangle BCD$	6. LL Theorem
7. $\overline{AC} \cong \overline{BD}$	7. CPCTC

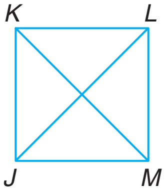
Look Back

LL Theorem: Lesson 6–5

Your Turn

b. **Given:** $JKLM$ is a square with diagonals $\overline{KM}$ and $\overline{LJ}$.

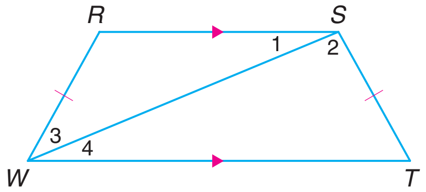
Prove: $\overline{KM} \cong \overline{LJ}$



Check for Understanding

Communicating Mathematics

1. Suppose you are given that $\overline{RS} \parallel \overline{WT}$ and $\overline{RW} \cong \overline{ST}$. Can you use $\angle 1 \cong \angle 4$ as a statement in a proof? Explain your reasoning.



2. Writing Math Write a paragraph in which you explain the process of writing a two-column proof.

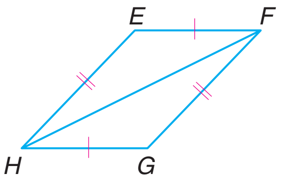
CONTENTS

Guided Practice

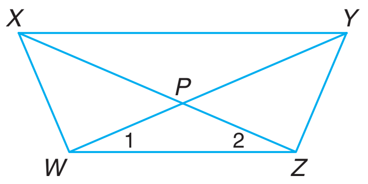
Write a two-column proof.

Examples 1 & 2

3. **Given:** $\overline{EF} \cong \overline{GH}$
$\overline{EH} \cong \overline{GF}$
Prove: $\triangle EFH \cong \triangle GHF$

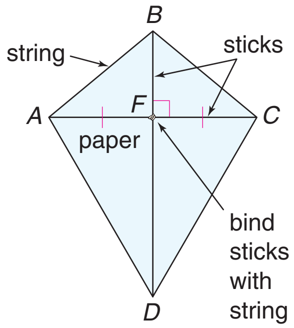


4. **Given:** $XYZW$ is an isosceles trapezoid with bases $\overline{XY}$ and $\overline{WZ}$.
Prove: $\angle 1 \cong \angle 2$



Example 1

5. **Kites** You can make a simple kite using paper, two sticks, and some string. The sticks meet so that $\overline{AC} \perp \overline{BD}$ and $\overline{AF} \cong \overline{CF}$. Prove that $\overline{AB} \cong \overline{CB}$ and $\overline{CD} \cong \overline{AD}$.



Exercises

Practice

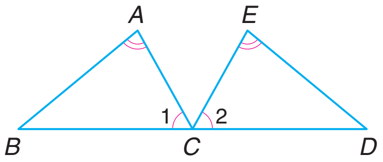
Write a two-column proof.

For Exercises	See Examples
6–15	1, 2

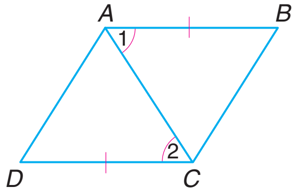
Homework Help

Extra Practice
See page 755.

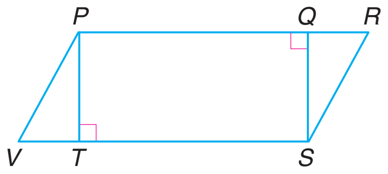
6. **Given:** $\angle A \cong \angle E$
$\angle 1 \cong \angle 2$
$\overline{AC}$ bisects $\overline{BD}$.
Prove: $\overline{AB} \cong \overline{ED}$



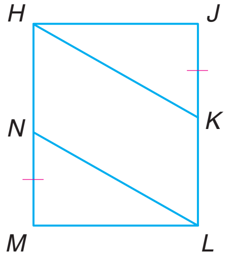
7. **Given:** $\overline{AB} \cong \overline{CD}$
$\angle 1 \cong \angle 2$
Prove: $\overline{AD} \cong \overline{CB}$



8. **Given:** $PRSV$ is a parallelogram.
$\overline{PT} \perp \overline{SV}$
$\overline{QS} \perp \overline{PR}$
Prove: $\triangle PTV \cong \triangle SQR$



9. **Given:** $HJLM$ is a rectangle.
$\overline{KJ} \cong \overline{NM}$
Prove: $\overline{HK} \cong \overline{LN}$

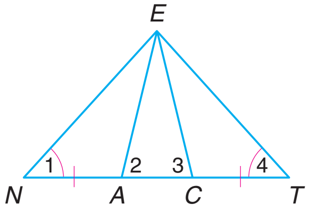


CONTENTS

Write a two-column proof.

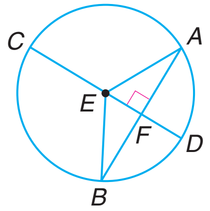
10. **Given:** $\angle 1 \cong \angle 4$
$\overline{NA} \cong \overline{TC}$

Prove: $\angle 3 \cong \angle 2$



11. **Given:** $\overline{CD}$ is a diameter of $\odot E$.
$\overline{CD} \perp \overline{AB}$

Prove: $\overline{AF} \cong \overline{BF}$



Draw and label a figure for each conjecture. Then write a two-column proof.

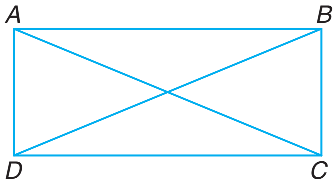
12. The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

13. The diagonals of an isosceles trapezoid are congruent.

14. The median from the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

Applications and Problem Solving

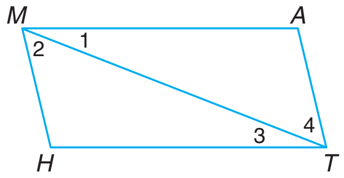
15. **Construction** Before laying the foundation of a rectangular house, the construction supervisor sets the corner points so that $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$. In order to guarantee that the corners are right angles, the supervisor measures both diagonals to be sure they are congruent.



a. Write the conjecture that the supervisor is using.

b. Write a two-column proof for the conjecture.

16. **Critical Thinking** Consider the following proof of the conjecture *A diagonal of a parallelogram bisects opposite angles.*



Given: ▱$MATH$ with diagonal $\overline{MT}$.

Prove: $\overline{MT}$ bisects $\angle AMH$ and $\angle ATH$.

Statements	Reasons
a. ▱$MATH$ is a parallelogram.	**a.** Given
b. $\overline{MH} \cong \overline{AT}$, $\overline{MA} \cong \overline{HT}$	**b.** Definition of parallelogram
c. $\overline{MT} \cong \overline{MT}$	**c.** Congruence of segments is reflexive.
d. $\triangle MHT \cong \triangle MAT$	**d.** SSS
e. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	**e.** CPCTC
f. $\overline{MT}$ bisects $\angle AMH$ and $\overline{MT}$ bisects $\angle ATH$.	**f.** Definition of angle bisector

Is this proof correct? Explain your reasoning.

CONTENTS

Mixed Review

17. **Algebra** Solve $-4x + 5 = -15$ using a two-column proof. *(Lesson 15–4)*

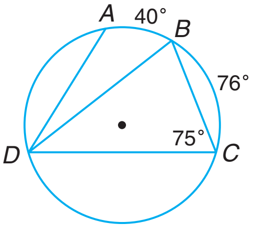
18. Use a paragraph proof to prove the following conjecture. *If two sides of a quadrilateral are parallel and congruent, the quadrilateral is a parallelogram.* *(Lesson 15–3)*

Find each measure. *(Lesson 14–1)*

19. $m\angle ADB$

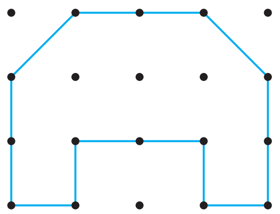
20. $m\angle BDC$

21. $m\overset{\frown}{AD}$



Standardized Test Practice

22. **Short Response** Draw a polygon with the same area as, but not congruent to, the figure at the right. *(Lesson 10–3)*

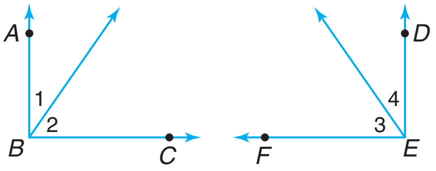


Quiz 2 Lessons 15–4 and 15–5

Copy and complete the proof of the conditional.
If m∠ABC = m∠DEF and m∠1 = m∠4, then m∠2 = m∠3. *(Lesson 15–4)*

Given: $m\angle ABC = m\angle DEF$
$m\angle 1 = m\angle 4$

Prove: $m\angle 2 = m\angle 3$

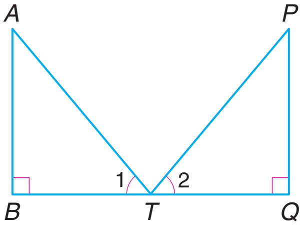


Statements	Reasons
1. $m\angle ABC = m\angle DEF$ $m\angle 1 = m\angle 4$	1. ___?___
2. $m\angle ABC = m\angle 1 + m\angle 2$ $m\angle DEF = m\angle 3 + m\angle 4$	2. ___?___
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. ___?___
4. $m\angle 2 = m\angle 3$	4. ___?___

5. Write a two-column proof. *(Lesson 15–5)*

Given: T is the midpoint of $\overline{BQ}$.
$\angle B$ and $\angle Q$ are right angles.
$\angle 1 \cong \angle 2$

Prove: $\overline{AT} \cong \overline{PT}$



15–6 Coordinate Proofs

What You'll Learn

You'll learn to use coordinate proofs to prove theorems.

Why It's Important

Computer-Aided Design A coordinate system is used to plot points on a CAD drawing. *See Exercise 23.*

Some recent movies have been made entirely with computer graphics. Animators first make wireframe drawings and assign numbers to control points in the drawings based on a coordinate system. The animator then makes objects move by using commands that move the control points.

Important relationships in geometry can also be demonstrated using a coordinate system.

Look Back

Distance Formula: Lesson 6–7

Hands-On Geometry

Materials: grid paper straightedge

Step 1: Draw and label the first quadrant of a rectangular coordinate system.

Step 2: Graph the vertices of $\triangle ABC$ and $\triangle RST$ at $A(2, 6)$, $B(5, 5)$, $C(3, 3)$, and $R(9, 5)$, $S(8, 2)$, $T(6, 4)$. Draw the triangles.

Try These

1. Use the Distance Formula to find the length of each side of each triangle.
2. Based on your calculations, what conclusion can you make about the two triangles?

In this activity, you used a coordinate plane and the Distance Formula to show that two triangles are congruent. You can also use a coordinate plane to prove many theorems in geometry. A proof that uses figures on a coordinate plane is called a **coordinate proof**.

One of the most important steps in planning a coordinate proof is deciding how to place the figure on a coordinate plane.

Guidelines for Placing Figures on a Coordinate Plane

1. Use the origin as a vertex or center.
2. Place at least one side of a polygon on an axis.
3. Keep the figure within the first quadrant, if possible.
4. Use coordinates that make computations as simple as possible.

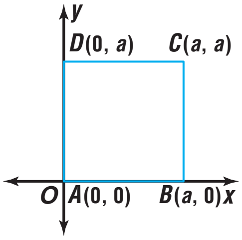
CONTENTS

Examples

Algebra Link

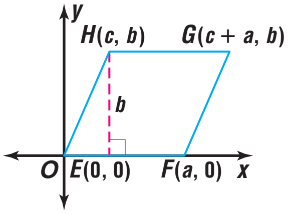
① **Position and label a square with sides a units long on a coordinate plane.**

- Use the origin as a vertex.
- Place one side on the x-axis and one side on the y-axis.
- Label the vertices A, B, C, and D.
- B is on the x-axis. So, its y-coordinate is 0, and its x-coordinate is a.
- D is on the y-axis. So, its x-coordinate is 0, and its y-coordinate is a.
- Since the sides of a square are congruent, the x-coordinate of C is a, and the y-coordinate of C is a.



② **Position and label a parallelogram with base a units long and a height of b units on a coordinate plane.**

- Use the origin as a vertex.
- Place the base along the x-axis.
- Label the vertices E, F, G, and H.
- Since F is on the x-axis, its y-coordinate is 0, and its x-coordinate is a.
- Since the height of the parallelogram is b units, the y-coordinate of H and G is b.
- Let the x-coordinate of H be c. Therefore, the x-coordinate of G is $c + a$.



Your Turn

Position and label a right triangle with legs a units long and b units long on a coordinate plane.

In coordinate proofs, you use algebraic tools like variables, equations, and formulas.

- First, draw the figure on a coordinate plane.
- Next, use all of the known properties of the figure to assign coordinates to the vertices. Use as few letters as possible.
- Finally, use variables, equations, and formulas to show the relationships among segments in the figure. The most common formulas are shown below. In the formulas, (x_1, y_1) and (x_2, y_2) are coordinates of the endpoints of a segment.

Midpoint Formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Examples

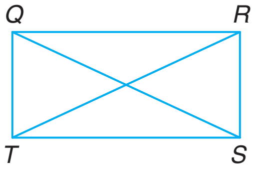
Algebra Link

Write a coordinate proof for each conjecture.

3 **The diagonals of a rectangle are congruent.**

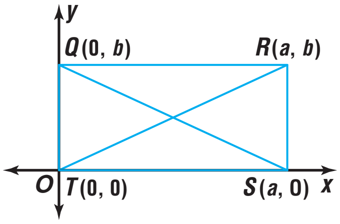
Given: rectangle $QRST$ with diagonals $\overline{QS}$ and $\overline{RT}$

Prove: $\overline{QS} \cong \overline{RT}$



Plan: Use the Distance Formula to find the length of each diagonal.

Place rectangle $QRST$ with width a and height b on a coordinate plane and label the coordinates as shown. Use the Distance Formula to find QS and RT.



First, find QS.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ *Distance Formula*

$QS = \sqrt{(a - 0)^2 + (0 - b)^2}$ $(x_1, y_1) = (0, b), (x_2, y_2) = (a, 0)$

$QS = \sqrt{a^2 + b^2}$ *Simplify.*

Find RT.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ *Distance Formula*

$RT = \sqrt{(0 - a)^2 + (0 - b)^2}$ $(x_1, y_1) = (a, b), (x_2, y_2) = (0, 0)$

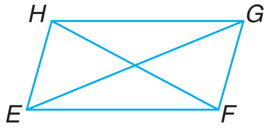
$RT = \sqrt{a^2 + b^2}$ *Simplify.*

The measures of the diagonals are equal. Therefore, $\overline{QS} \cong \overline{RT}$.

4 **The diagonals of a parallelogram bisect each other.**

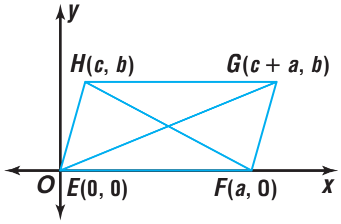
Given: parallelogram $HGFE$ with diagonals $\overline{HF}$ and $\overline{GE}$

Prove: $\overline{HF}$ and $\overline{GE}$ bisect each other.



Plan: Use the Midpoint Formula to find the coordinates of the midpoint of each diagonal. If the coordinates are the same, the midpoint of each diagonal is the same point, and the diagonals bisect each other.

Place parallelogram $HGFE$ on a coordinate plane and label the coordinates as shown. Use the Midpoint Formula to find the coordinates of the midpoint of each diagonal.



Look Back

Midpoint Formula: Lesson 2–5

CONTENTS

First, find the midpoint of $\overline{HF}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{c + a}{2}, \frac{b + 0}{2}\right) \quad \textit{Use the Midpoint Formula.}$$

$$= \left(\frac{c + a}{2}, \frac{b}{2}\right) \quad \textit{Simplify.}$$

Find the midpoint of $\overline{GE}$.

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{c + a + 0}{2}, \frac{b + 0}{2}\right) \quad \textit{Use the Midpoint Formula.}$$

$$= \left(\frac{c + a}{2}, \frac{b}{2}\right) \quad \textit{Simplify.}$$

The midpoints of the diagonals have the same coordinates. Therefore, they name the same point, and the diagonals bisect each other.

Check for Understanding

Communicating Mathematics

1. **Explain** how you should position a figure on the coordinate plane if it is to be used in a coordinate proof.

Vocabulary

coordinate proof

2. **You Decide?** Michael placed a right triangle on a coordinate plane with an acute angle at the origin. Latisha placed the right angle at the origin. Whose placement is best? Explain your reasoning.

Guided Practice

Getting Ready **If the coordinates of the endpoints of a segment are given, find the coordinates of the midpoint.**

Sample: $(a, 0), (0, a)$ **Solution:** $\left(\frac{a + 0}{2}, \frac{0 + a}{2}\right) = \left(\frac{a}{2}, \frac{a}{2}\right)$

3. (6, 3), (2, −5)
4. (a, b), (0, 0)
5. $(2e, 0)$, $(0, 2f)$

Examples 1 & 2

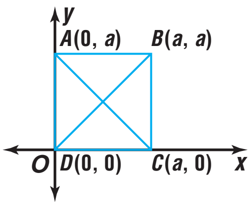
Position and label each figure on a coordinate plane.

6. a rectangle with length a units and width b units
7. a parallelogram with base m units and height n units

Examples 3 & 4

Write a coordinate proof for each conjecture.

8. The diagonals of a square are congruent.
9. The diagonals of a rectangle bisect each other.



Exercises

Practice

Homework Help

For Exercises	See Examples
10–15	1, 2
16–22	3, 4
23, 24	3

Extra Practice

See page 755.

Position and label each figure on a coordinate plane.

10. a square with sides r units long

11. a rectangle with base x units and width y units

12. an isosceles right triangle with legs b units long

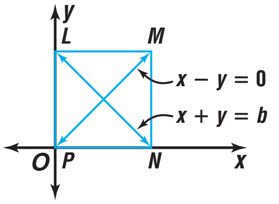
13. a right triangle with legs c and d units long

14. an isosceles triangle with base b units long and height h units long

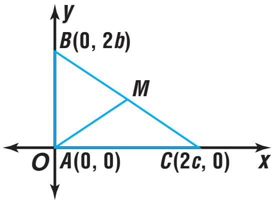
15. a parallelogram with base r units long and height t units long

Write a coordinate proof for each conjecture.

16. The diagonals of a square are perpendicular.



17. The midpoint of the hypotenuse of a right triangle is equidistant from each of the vertices.



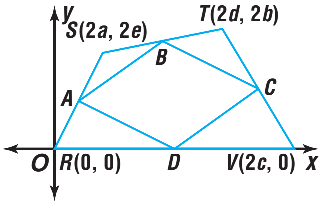
18. The line segments joining the midpoints of the sides of a rectangle form a rhombus.

19. The medians to the legs of an isosceles triangle are congruent.

20. The measure of the median to the hypotenuse of a right triangle is one-half the measure of the hypotenuse. (*Hint:* Use the figure from Exercise 17.)

21. The diagonals of an isosceles trapezoid are congruent.

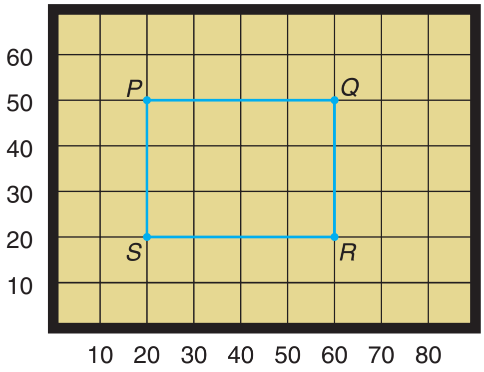
22. The line segments joining the midpoints of the sides of any quadrilateral form a parallelogram.



Applications and Problem Solving

A CAD Designer

23. Computer-Aided Design CAD systems produce accurate drawings because the user can input exact points, such as the ends of line segments. In CAD, the *digitizing tablet* and its *puck* act as a keyboard and mouse. The figure at the right shows an outline of the foundation of a house. Prove that $PQRS$ is a rectangle.

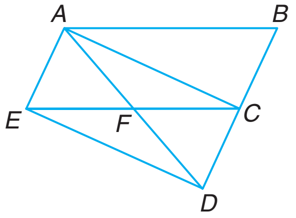


CONTENTS

24. **Critical Thinking** Point A has coordinates (0, 0), and B has coordinates (a, b). Find the coordinates of point C so $\triangle ABC$ is a right triangle.

Mixed Review

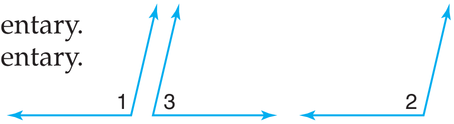
25. Write a two-column proof. *(Lesson 15–5)*



Given: $ACDE$ is a rectangle.
$ABCE$ is a parallelogram.

Prove: $\triangle ABD$ is isosceles.

26. Copy and complete the proof. *(Lesson 15–4)*



Given: $\angle 1$ and $\angle 3$ are supplementary.
$\angle 2$ and $\angle 3$ are supplementary.

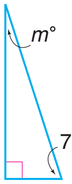
Prove: $\angle 1 \cong \angle 2$

Statements	Reasons
a. $\angle 1$ and $\angle 3$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	a. ___?___
b. $m\angle 1 + m\angle 3 = 180$ $m\angle 2 + m\angle 3 = 180$	b. ___?___
c. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	c. ___?___
d. $m\angle 1 = m\angle 2$	d. ___?___
e. $\angle 1 \cong \angle 2$	e. ___?___

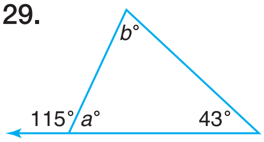
27. **Manufacturing** Many baking pans are given a special coating to make food stick less to the surface. A rectangular cake pan is 9 inches by 13 inches and 2 inches deep. What is the area of the surface to be coated? *(Lesson 12–2)*

Find the value of each variable. *(Lesson 5–2)*

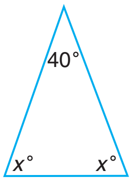
28.



29.



30.



Standardized Test Practice

31. **Grid In** Use the pattern in the table to find the unit digit for 7^{41}. *(Lesson 1–1)*

Power	7^1	7^2	7^3	7^4	7^5	7^6	7^7	7^8	7^9
Unit Digit	7	9	3	1	7	9	3	1	7

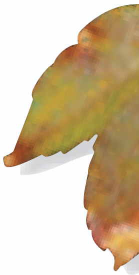
32. **Multiple Choice** Lawana's math test scores are 90, 85, 78, 92, and 99. What must she score on the next math test so that her average is exactly 90? *(Statistics Review)*

Ⓐ 90   Ⓑ 91   Ⓒ 95   Ⓓ 96

www.geomconcepts.com/self_check_quiz

CONTENTS

Chapter 15 Investigation



DON'T TOUCH THE POISON IVY

Indirect Proofs

If you've ever felt the itch of a rash from poison ivy, you are very careful about what plants you touch. You probably also know that poison ivy leaves are grouped in threes. What about the plant shown below at the left? How could you prove that the plant is *not* poison ivy?

You can use a technique called **indirect reasoning**. The following steps summarize the process of indirect reasoning. Use these steps when writing an **indirect proof**. *Indirect proofs are also called proofs by contradiction.*

Step 1: Assume that the *opposite* of what you want to prove is true.

Step 2: Show that this assumption leads to a contradiction of the hypothesis or some other fact. Therefore, the assumption must be false.

Step 3: Finally, state that what you want to prove is true.

Indirect Proof

Given: You have a picture of a plant.
Prove: The plant is *not* poison ivy.
Assume: The plant *is* poison ivy.

Proof: If the plant is poison ivy, its leaves would be in groups of three. However, the picture shows that the leaves are *not* in groups of three. This is a contradiction. Therefore the assumption *The plant is poison ivy* is false. So the statement *The plant is not poison ivy* is true.

Investigate

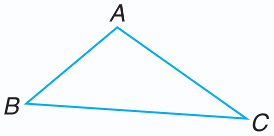
1. State the assumption you would make to start an indirect proof of each statement.
   a. Kiley ate the pizza.
   b. The defendant is guilty.
   c. Lines ℓ and m intersect at point X.

CONTENTS

- **d.** If two lines are cut by a transversal and the alternate interior angles are congruent, then the lines are parallel.
- **e.** Angle B is not a right angle.
- **f.** $\overline{XY} \cong \overline{AB}$
- **g.** If a number is odd, its square is odd.
- **h.** $m\angle 1 < m\angle 2$

2. Fill in the blanks with a word, symbol, or phrase to complete an indirect proof of this conjecture.

A triangle has no more than one right angle.



Given: $\triangle ABC$

Prove: $\triangle ABC$ has no more than one right angle.

Assume: ______

Assume that $\angle A$ and $\angle B$ are both right angles. So, $m\angle A =$ ______ and $m\angle B =$ ______. According to the Angle Sum Theorem, $m\angle A + m\angle B + m\angle C =$ ______. By substitution, ______ $+$ ______ $+ m\angle C = 180$. Therefore, $m\angle C =$ ______. This is a contradiction because ______. Therefore, the assumption $\triangle$ *ABC has more than one right angle* is ______. The statement ______ is true.

Extending the Investigation

In this extension, you will write an indirect proof and compare indirect proofs to other forms of proof. Here are some suggestions.

- Write an indirect proof of the following statement.

  A quadrilateral has no more than three acute angles.
- Work with a partner and choose a theorem from Chapters 4–8. Write a paragraph proof, a two-column proof, a coordinate proof, or an indirect proof of your theorem.

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Put your theorem on poster board and explain the method you used to prove it.
- Make a notebook that contains your proofs.
- Make a bulletin board that compares the different kinds of proof.

Investigation For more information on proofs, visit: www.geomconcepts.com

CHAPTER 15 Study Guide and Assessment

Understanding and Using the Vocabulary

Review Activities
For more review activities, visit:
www.geomconcepts.com

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

Geometry

coordinate proof *(p. 660)*
indirect proof *(p. 666)*
paragraph proof *(p. 644)*
proof *(p. 644)*
proof by contradiction *(p. 666)*
two-column proof *(p. 649)*

Logic

compound statement *(p. 633)*
conjunction *(p. 633)*
contrapositive *(p. 637)*
deductive reasoning *(p. 638)*
disjunction *(p. 633)*
indirect reasoning *(p. 666)*
inverse *(p. 635)*
Law of Detachment *(p. 639)*
Law of Syllogism *(p. 640)*
logically equivalent *(p. 637)*
negation *(p. 632)*
statement *(p. 632)*
truth table *(p. 633)*
truth value *(p. 632)*

Choose the letter of the term that best matches each phrase.

1. a rule similar to the Transitive Property of Equality
2. the process of using facts, rules, definitions, or properties in a logical order
3. a proof that uses figures on a coordinate plane
4. a proof containing statements and reasons that are organized with numbered steps and reasons
5. a logical argument in which each statement made is backed up by a reason that is accepted as true
6. $p \vee q$
7. if $p \rightarrow q$ is a true conditional and p is true, then q is true
8. $p \wedge q$
9. where a vertex or center of the figure should be placed in a coordinate proof
10. a table that lists all the truth values of a statement

a. proof
b. Law of Detachment
c. deductive reasoning
d. truth table
e. Law of Syllogism
f. disjunction
g. two-column proof
h. origin
i. coordinate proof
j. conjunction

Skills and Concepts

Objectives and Examples

- **Lesson 15–1** Find the truth values of simple and compound statements.

p	q	$p \vee q$	$p \wedge q$	$p \rightarrow q$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	T
F	F	F	F	T

Review Exercises

Let *p* represent a true statement and *q* represent a false statement. Find the truth value of each compound statement.

11. $p \vee q$
12. $p \wedge q$
13. $p \rightarrow q$
14. $p \wedge \sim q$
15. $\sim p \vee q$
16. $p \rightarrow \sim q$

17. *True* or *false*: $3 + 4 = 7$ and $2 + 5 = 8$.

www.geomconcepts.com/vocabulary_review

CONTENTS

Objectives and Examples

Review Exercises

- **Lesson 15–2** Use the Law of Detachment and the Law of Syllogism in deductive reasoning.

Law of Detachment: If $p \rightarrow q$ is a true conditional and p is true, then q is true.

Law of Syllogism: If $p \rightarrow q$ and $q \rightarrow r$ are true conditionals, then $p \rightarrow r$ is also true.

Determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, write *no valid conclusion*.

18. (1) If $x < 0$, then x is a negative number.
(2) $x < 0$

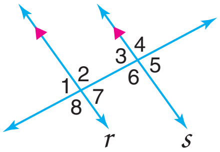
19. (1) Sean is on a field trip.
(2) All art students are on a field trip.

20. (1) If today is Tuesday, Katie has basketball practice.
(2) If Katie has basketball practice, she will eat dinner at 7:00.

- **Lesson 15–3** Use paragraph proofs to prove theorems.

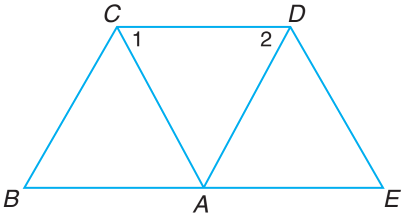
Given: $r \parallel s$

Prove: $\angle 4 \cong \angle 8$



You know that $r \parallel s$. If two parallel lines are cut by a transversal, their corresponding angles are congruent. So, $\angle 6 \cong \angle 8$. Also, $\angle 6 \cong \angle 4$ because vertical angles are congruent. Since $\angle 6 \cong \angle 8$ and $\angle 6 \cong \angle 4$, then $\angle 4 \cong \angle 8$ by substitution.

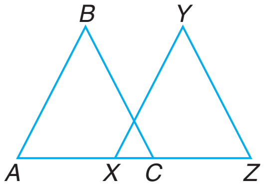
21. If $m\angle BCD = m\angle EDC$, $\overline{AC}$ bisects $\angle BCD$, and $\overline{AD}$ bisects $\angle EDC$, write a paragraph proof that shows $\triangle ACD$ is isosceles.



- **Lesson 15–4** Use properties of equality in algebraic and geometric proofs.

Given: $AX = ZC$

Prove: $AC = ZX$

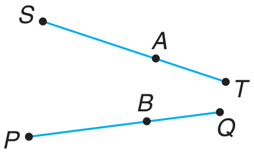


Statements	Reasons
1. $AX = ZC$	1. Given
2. $XC = XC$	2. Reflexive, =
3. $AX + XC = ZC + XC$	3. Addition, =
4. $AC = ZX$	4. Substitution, =

22. Copy and complete the proof.

Given: $SA = BP$
$AT = QB$

Prove: $ST = QP$



Statements	Reasons
a. $SA = BP$, $AT = QB$	a. ?
b. $SA + AT = BP + QB$	b. ?
c. $ST = SA + AT$ $QP = BP + QB$	c. ?
d. $ST = QP$	d. ?

CONTENTS

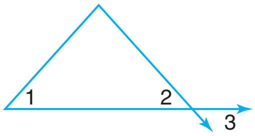
Mixed Problem Solving
See pages 758–765.

Objectives and Examples

- **Lesson 15–5** Use two-column proofs to prove theorems.

Given: $\angle 1 \cong \angle 2$

Prove: $\angle 1 \cong \angle 3$

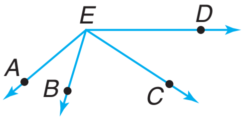


Statements	Reasons
1. $\angle 1 \cong \angle 2$	**1.** Given
2. $\angle 2 \cong \angle 3$	**2.** Vertical angles are $\cong$.
3. $\angle 1 \cong \angle 3$	**3.** Transitive Prop, $\cong$

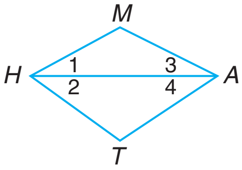
Review Exercises

Write a two-column proof.

23. Given:
$m\angle AEC = m\angle DEB$
Prove:
$m\angle AEB = m\angle DEC$



24. Given:
$m\angle 1 = m\angle 3$,
$m\angle 2 = m\angle 4$
Prove:
$m\angle MHT = m\angle MAT$



- **Lesson 15–6** Use coordinate proofs to prove theorems.

Guidelines for placing figures on a coordinate plane:

1. Use the origin as a vertex or center.
2. Place at least one side of a polygon on an axis.
3. Keep the figure within the first quadrant if possible.
4. Use coordinates that make computations as simple as possible.

Position and label each figure on a coordinate plane.

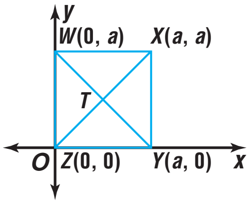
25. an isosceles right triangle with legs *a* units long

26. a parallelogram with base *b* units long and height *h* units

Write a coordinate proof for each conjecture.

27. The diagonals of a square bisect each other.

28. The opposite sides of a parallelogram are congruent.



Exercise 27

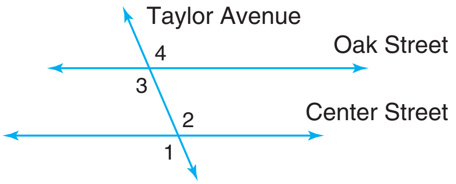
Applications and Problem Solving

29. School Use the Law of Detachment to determine a conclusion that follows from statements (1) and (2).

(1) If Julia scores at least 90 on the math final exam, she will earn an A for the semester.

(2) Julia scored 93 on the math final exam. *(Lesson 15–2)*

30. Maps If Oak Street is parallel to Center Street, write a paragraph proof to show that $\angle 1 \cong \angle 4$. *(Lesson 15–3)*



CHAPTER 15 Test

1. **List** three types of proofs and describe each type.
2. **Compare and contrast** a conjunction and a disjunction.

Construct a truth table for each compound statement.

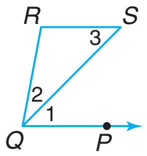
3. $\sim p \vee q$
4. $\sim(p \wedge \sim q)$

Use the Law of Detachment or the Law of Syllogism to determine a conclusion that follows from statements (1) and (2). If a valid conclusion does not follow, write *no valid conclusion*.

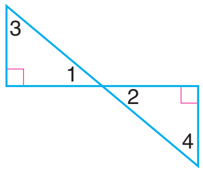
5. (1) Central Middle School's mascot is a polar bear.
   (2) Dan is on the baseball team at Central Middle School.
6. (1) If I watch television, I will waste time.
   (2) If I waste time, I will not be able to complete my homework.

Write a paragraph proof for each conjecture.

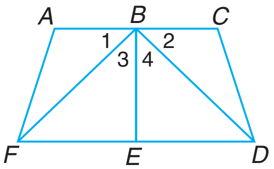
7. If $\overline{QS}$ bisects $\angle PQR$ and $\overline{RS} \parallel \overline{QP}$, then $\angle 2 \cong \angle 3$.



8. If $\angle 3$ is complementary to $\angle 1$ and $\angle 4$ is complementary to $\angle 2$, then $\angle 3 \cong \angle 4$.



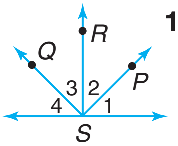
Copy and complete the proof of the conditional.
If $m\angle ABE = m\angle CBE$ and $m\angle 1 = m\angle 2$, then $m\angle 3 = m\angle 4$.



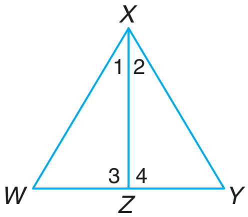
Statements	Reasons
9. $m\angle ABE = m\angle CBE$, $m\angle 1 = m\angle 2$	**9.** ?
10. $m\angle ABE - m\angle 1 = m\angle CBE - m\angle 2$	**10.** ?
11. $m\angle 3 = m\angle 4$	**11.** ?

Write a two-column proof.

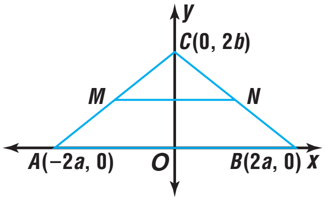
12. **Given:** $\overrightarrow{SR}$ bisects $\angle QSP$, $\angle 1$ is complementary to $\angle 2$, $\angle 4$ is complementary to $\angle 3$.
    Prove: $\angle 1 \cong \angle 4$



13. **Given:** $\overline{XZ}$ bisects $\angle WXY$, $\overline{XZ} \perp \overline{WY}$
    Prove: $\angle W \cong \angle Y$



14. Position and label a rectangle with base b units long and width w units long on a coordinate plane.
15. Write a coordinate proof for this conjecture.
    The segment joining the midpoints of two legs of an isosceles triangle is half the length of the base.
16. **Algebra** Write a two-column proof to show that if $3x + 5 = 23$, then $x = 6$.



Exercise 15

CHAPTER 15 Preparing for Standardized Tests

Solid Figure Problems

State proficiency tests often include geometry problems with 3-dimensional shapes. The ACT and SAT may contain just one or two problems with solid figures. Formulas for surface area and volume are usually provided in the test itself, but you'll save time if you understand and memorize these formulas.

Test-Taking Tip

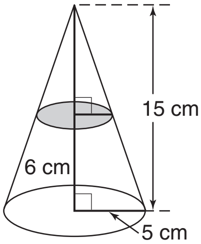
The volume of a right prism and a right cylinder is the area of the base times the height.

You'll need to know the following concepts.

cone cylinder prism pyramid surface area volume

Example 1

The right circular cone at the right has radius of 5 centimeters and height of 15 centimeters. A cross section of the cone is parallel to and 6 centimeters above the base of the cone. What is the area of this cross section, to the nearest square centimeter?



Ⓐ **13** Ⓑ **19** Ⓒ **28** Ⓓ **50**

Hint Look carefully for similar triangles and right triangles.

Solution There are two right triangles. These triangles are similar by AA Similarity, since the radii of the cross section and the base are parallel.

Find the length of the radius. Use a proportion of the sides of the two similar triangles. The height of the larger triangle is 15. The height of the smaller triangle is 15 − 6 or 9.

$$\frac{5}{15} = \frac{x}{9} \quad \rightarrow \quad 15x = 45 \quad \rightarrow \quad x = 3$$

The radius is 3 centimeters.

Now calculate the area.

$$A = \pi r^2 \quad \rightarrow \quad A = \pi(3)^2 \quad \rightarrow \quad A \approx 28$$

The answer is C.

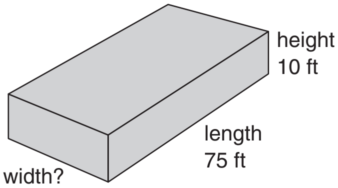
Example 2

A rectangular swimming pool has a volume of 16,500 cubic feet, a depth of 10 feet, and a length of 75 feet. What is the width of the pool, in feet?

Ⓐ **22** Ⓑ **26** Ⓒ **32**
Ⓓ **110** Ⓔ **1650**

Hint Draw a figure to help you understand the problem.

Solution



Use the formula for the volume of a rectangular prism. Use the given information for volume, length, and height. Solve for width.

$$V = Bh$$

$$V = \ell wh$$

$$16{,}500 = 75 \times w \times 10$$

$$16{,}500 = 750w$$

$$22 = w$$

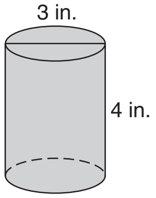
The answer is A.

CONTENTS

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 766–781.

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice



1. About how much paper is needed for the label on this can? *(Lesson 12–2)*

Ⓐ 6 in^2 Ⓑ 12 in^2
Ⓒ 18 in^2 Ⓓ 38 in^2

2. Which is equivalent to $\sqrt{72}$? *(Algebra Review)*

Ⓐ $2\sqrt{6}$ Ⓑ $6\sqrt{2}$ Ⓒ 12 Ⓓ 36

3. If $\tan \theta = \frac{4}{3}$, what is $\sin \theta$? *(Lesson 13–5)*

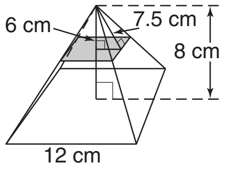
Ⓐ $\frac{3}{4}$ Ⓑ $\frac{4}{5}$ Ⓒ $\frac{5}{4}$ Ⓓ $\frac{5}{3}$ Ⓔ $\frac{7}{3}$

4. A rectangular garden is surrounded by a 60-foot fence. One side of the garden is 6 feet longer than the other side. Which equation could be used to find s, the shorter side of the garden? *(Lesson 1–6)*

Ⓐ $60 = 8s + s$ Ⓑ $4s = 60 + 12$
Ⓒ $60 = s(s + 6)$ Ⓓ $60 = 2(s - 6) + 2s$
Ⓔ $60 = 2(s + 6) + 2s$

5. A plane intersects the square pyramid so that the smaller pyramid formed has a height of 6 meters and a slant height of 7.5 meters. What is the area of the shaded cross section? *(Lesson 12–4)*

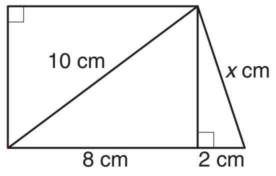
Ⓐ 20.25 m^2
Ⓑ 36 m^2
Ⓒ 56.25 m^2
Ⓓ 81 m^2



6. Ashley subscribes to four magazines that cost \$12.90, \$16.00, \$18.00, and \$21.90 per year. If she makes a down payment of one-half the total amount and pays the rest in four equal payments, how much is each payment? *(Algebra Review)*

Ⓐ \$8.60 Ⓑ \$9.20 Ⓒ \$9.45
Ⓓ \$17.20 Ⓔ \$34.40

7. Which equation could you use to find the length of the missing side? *(Lesssson 6–6)*



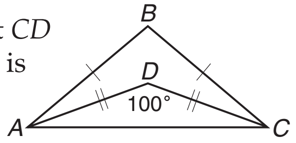
Ⓐ $6^2 + 2^2 = x^2$ Ⓑ $6^2 + 10^2 = x^2$
Ⓒ $8 + 2 = x$ Ⓓ $6 + 2 = x$

8. A rectangle with area 4 square units has sides of length r units and s units. Which expression shows the perimeter of the rectangle in terms of s? *(Lesson 1–6)*

Ⓐ $4s$ units
Ⓑ $2r + 2s$ units
Ⓒ $\frac{4}{r}$ units
Ⓓ $\frac{8}{s} + 2s$ units

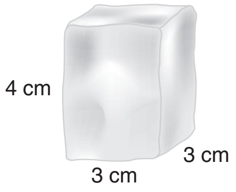
Grid In

9. Segment AD bisects $\angle BAC$, and segment CD bisects $\angle BCA$. What is the measure of $\angle B$? *(Lesson 3–3)*

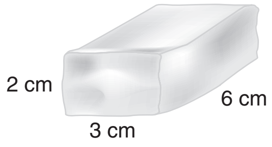


Extended Response

10. Adding heat to ice causes it to change from a solid to a liquid. The figures below represent two pieces of ice. *(Lesson 12–2)*



X



Y

Part A Make a conjecture as to which piece of ice will melt faster. Explain.

Part B Defend your reasoning in a short paragraph.